Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.3-d+e-x^2-^m-a+b-x^2+c-x^4-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [413]. This is test number [40].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.76 (412)	% 0.24 (1)
Mathematica	% 91.53 (378)	% 8.47 (35)
Maple	% 96.61 (399)	% 3.39 (14)
Maxima	% 27.36 (113)	% 72.64 (300)
Fricas	% 50.85 (210)	% 49.15 (203)
Sympy	% 45.52 (188)	% 54.48 (225)
Giac	% 42.86 (177)	% 57.14 (236)
Mupad	% 44.55 (184)	% 55.45 (229)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

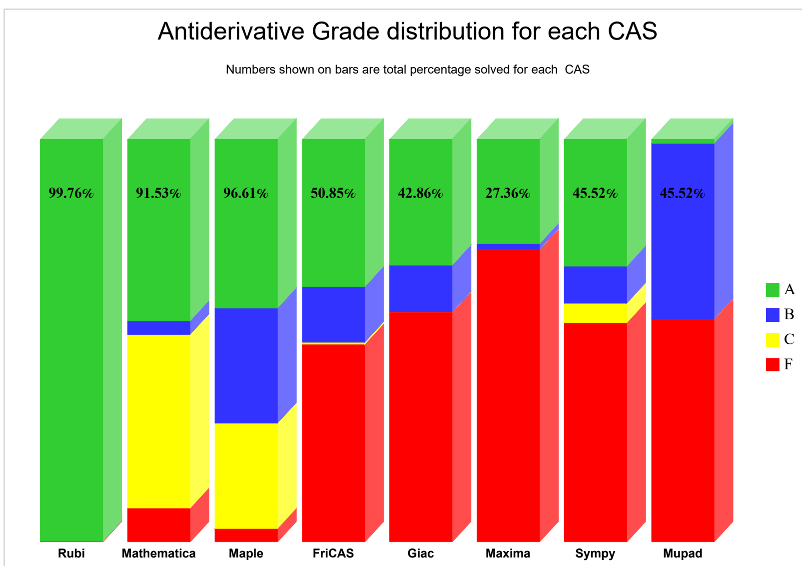
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

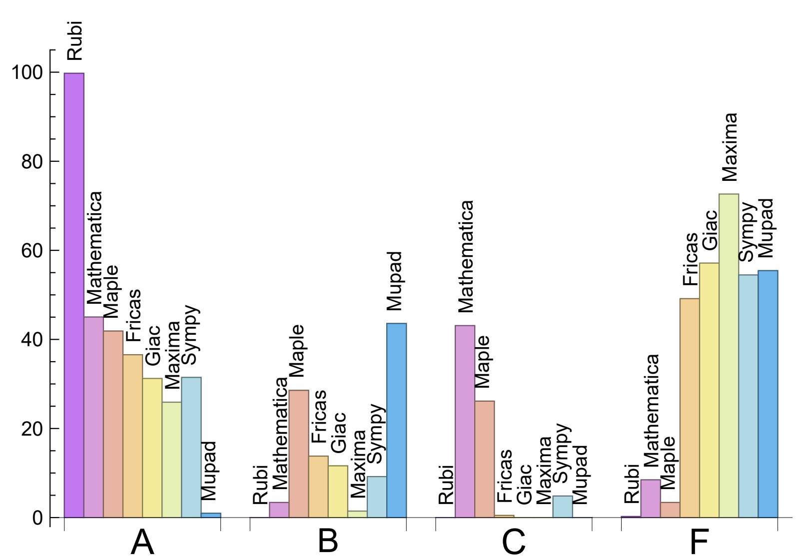
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.76	0.00	0.00	0.24
Mathematica	45.04	3.39	43.10	8.47
Maple	41.89	28.57	26.15	3.39
Maxima	25.91	1.45	0.00	72.64
Fricas	36.56	13.80	0.48	49.15
Sympy	31.48	9.20	4.84	54.48
Giac	31.23	11.62	0.00	57.14
Mupad	0.97	43.58	0.00	55.45

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	35	17.14 %	82.86 %	0.00 %
Maple	14	100.00 %	0.00 %	0.00 %
Maxima	300	97.67 %	0.00 %	2.33 %
Fricas	203	90.64 %	9.36 %	0.00 %
Sympy	225	83.56 %	13.78 %	2.67 %
Giac	236	91.53 %	1.69 %	6.78 %
Mupad	229	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

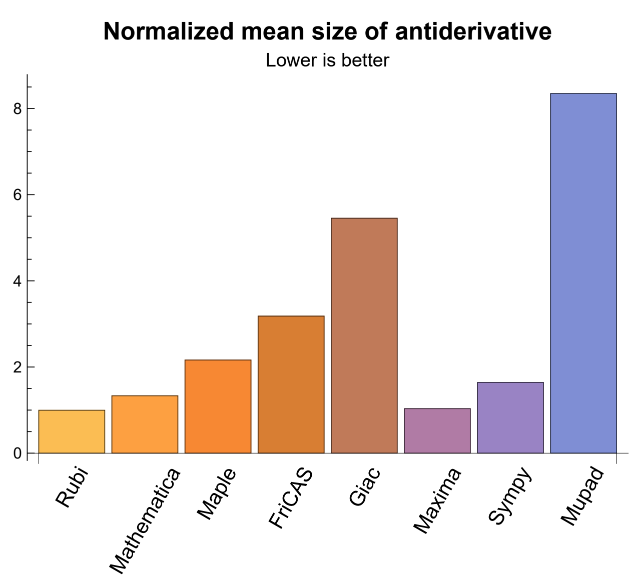
1.3 Performance

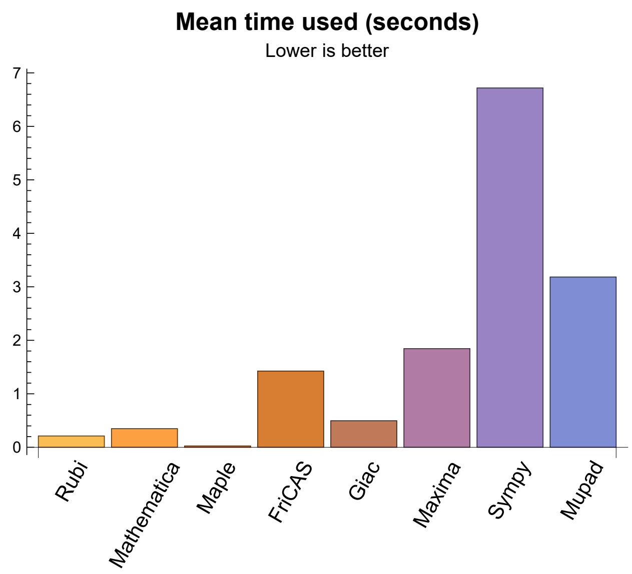
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.21	154.91	1.00	112.50	1.00
Mathematica	0.35	182.53	1.33	111.00	1.00
Maple	0.02	303.91	2.16	169.00	1.35
Maxima	1.84	117.20	1.03	74.00	0.97
Fricas	1.42	625.39	3.18	148.00	1.84
Sympy	6.72	151.93	1.64	82.50	1.09
Giac	0.50	762.54	5.45	79.00	0.99
Mupad	3.18	3780.42	8.35	67.00	0.93

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{175, 399, 404, 405}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {98, 99, 113, 114, 115, 116, 118, 198, 223, 224, 400, 401, 402, 403, 408, 409, 410, 411}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
```

```
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))
```

```
    except Exception as ee:
           leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

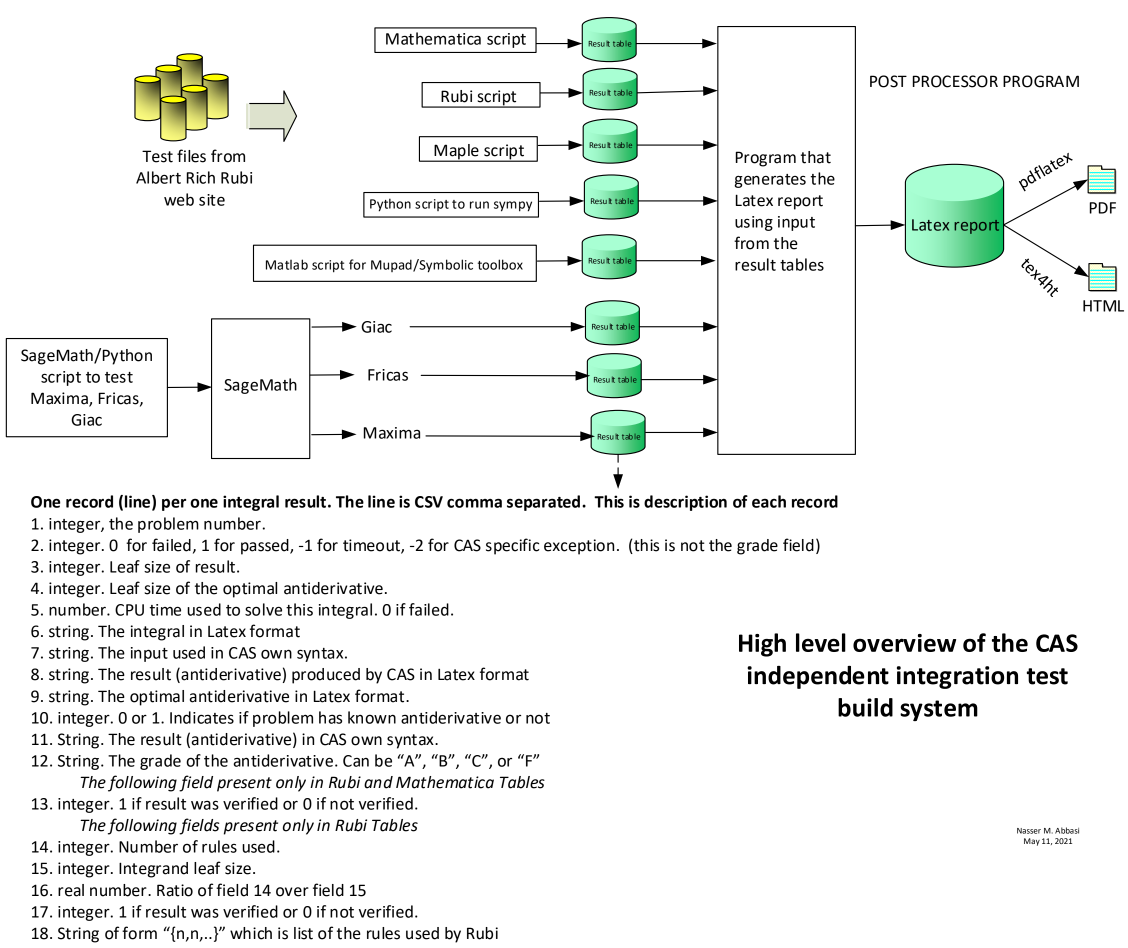
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

B grade: { }

C grade: { }

F grade: { 174 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 22, 24, 34, 35, 36, 37, 41, 42, 43, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 94, 95, 96, 97, 104, 105, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 170, 175, 176, 177, 178, 179, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 399, 400, 401, 402, 403, 404, 405 }

B grade: { 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 65, 80, 88 }

C grade: { 14, 15, 16, 17, 18, 19, 20, 21, 23, 25, 44, 45, 46, 48, 49, 73, 92, 93, 98, 99, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 198, 199, 200, 201, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 406, 407, 408, 409, 410, 411, 412, 413 }

F grade: { 174, 180, 181, 186, 187, 188, 238, 239, 240, 293, 294, 295, 307, 308, 309, 310, 311, 324, 325, 326, 327, 339, 340, 341, 342, 343, 344, 356, 357, 358, 359, 372, 373, 374, 375 }

2.1.3 Maple

A grade: { 1, 2, 7, 8, 13, 17, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 72, 73, 75, 76, 77, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 98, 99, 102, 103, 106, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 158, 159, 160, 164, 165, 175, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 199, 200, 201, 205, 206, 207, 211, 212, 213, 215, 216, 217, 218, 219, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 260, 261, 262, 267, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 316, 323, 324, 325, 331, 338, 347, 382, 383, 384, 386, 387, 388, 390, 391, 392, 393, 399, 404, 405, 407, 412, 413 }

B grade: { 3, 4, 5, 6, 9, 10, 11, 12, 14, 15, 16, 23, 25, 34, 35, 36, 39, 40, 41, 53, 54, 55, 67, 68, 69, 70, 71, 74, 78, 79, 80, 81, 82, 83, 91, 96, 97, 100, 101, 104, 105, 113, 114, 115, 116, 117, 118, 137, 144, 157, 161, 162, 163, 166, 167, 170, 172, 195, 196, 197, 198, 202, 203, 204, 208, 209, 210, 214, 220, 221, 222, 223, 224, 256, 257, 258, 259, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 317, 318, 319, 320, 321, 322, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 380, 381, 385, 389 }

C grade: { 18, 19, 20, 21, 22, 24, 150, 151, 152, 153, 154, 155, 156, 168, 169, 171, 173, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 394, 395, 396, 397, 398, 406, 408, 409, 410, 411 }

F grade: { 174, 176, 177, 178, 179, 180, 181, 186, 187, 188, 400, 401, 402, 403 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 37, 42, 43, 44, 47, 51, 52, 56, 57, 58, 61, 66, 72, 73, 74, 76, 77, 84, 85, 86, 89, 92, 93, 94, 96, 98, 99, 108, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 175, 189, 190, 191, 192, 193, 194, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 276, 277, 278, 279, 280, 281, 284, 285, 399, 404, 405 }

B grade: { 7, 65, 88, 95, 282, 283 }

C grade: { }

F grade: { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 45, 46, 48, 49, 50, 53, 54, 55, 59, 60, 62, 63, 64, 67, 68, 69, 70, 71, 75, 78, 79, 80, 81, 82, 83, 87, 90, 91, 97, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.5 FriCAS

A grade: { 5, 6, 8, 9, 10, 11, 12, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 98, 99, 108, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 147, 175, 189, 190, 191, 192, 193, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 215, 216, 217, 218, 220, 221, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 399, 404, 405 }

B grade: { 1, 2, 3, 4, 7, 65, 88, 94, 95, 96, 97, 100, 101, 104, 105, 106, 107, 109, 110, 111, 112, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 194, 197, 198, 211, 212, 213, 214, 219, 222, 223, 224, 258, 259, 260, 264, 265, 266, 267, 270, 271, 272, 273, 408, 409, 410, 411 }

C grade: { 102, 103 }

F grade: { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 113, 114, 115, 116, 117, 118, 119, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 268, 269, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 412, 413 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 100, 101, 106, 109, 120, 121, 122, 123, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 157, 158, 159, 164, 166, 189, 190, 191, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 258, 259, 266, 267, 273, 279, 280 }

B grade: { 7, 14, 15, 23, 25, 38, 42, 65, 88, 95, 96, 104, 105, 124, 125, 126, 132, 133, 167, 192, 193, 194, 214, 215, 216, 217, 248, 249, 256, 257, 261, 262, 276, 277, 278, 281, 282, 283 }

C grade: { 18, 19, 20, 21, 98, 99, 108, 150, 151, 152, 153, 168, 176, 177, 178, 179, 182, 183, 184, 185 }

F grade: { 22, 24, 102, 103, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 142, 143, 148, 149, 154, 155, 156, 160, 161, 162, 163, 165, 169, 170, 171, 172, 173, 174, 175, 180, 181, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 263, 264, 265, 268, 269, 270, 271, 272, 274, 275, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.7 Giac

A grade: { 1, 2, 5, 6, 8, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 98, 99, 102, 103, 108, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 175, 195, 197, 220, 221, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 399, 404, 405 }

B grade: { 3, 4, 7, 11, 12, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 47, 50, 53, 65, 88, 95, 96, 100, 101, 104, 105, 189, 190, 191, 192, 196, 214, 215, 216, 217, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273 }

C grade: { }

F grade: { 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 68, 80, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 218, 219, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.8 Mupad

A grade: { 175, 399, 404, 405 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 179, 185, 189, 190, 191, 192, 193, 194, 214, 215, 216, 217, 218, 219, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 282, 283, 284, 285 }

C grade: { }

F grade: { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 276, 277, 278, 279, 280, 281, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	183	260	221	767	109	241	599
normalized size	1	1.00	0.74	1.05	0.89	3.11	0.44	0.98	2.43
time (sec)	N/A	0.151	0.078	0.009	2.395	0.659	0.691	0.188	0.377

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	184	260	221	767	110	241	603
normalized size	1	1.00	0.74	1.05	0.89	3.11	0.45	0.98	2.44
time (sec)	N/A	0.138	0.045	0.003	2.342	0.940	0.680	0.172	0.257

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	95	122	109	755	110	230	579
normalized size	1	1.00	1.10	1.42	1.27	8.78	1.28	2.67	6.73
time (sec)	N/A	0.045	0.030	0.005	2.288	0.624	0.726	0.182	4.643

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	95	122	109	755	110	228	579
normalized size	1	1.00	1.10	1.42	1.27	8.78	1.28	2.65	6.73
time (sec)	N/A	0.040	0.023	0.003	2.339	0.768	0.943	0.325	4.578

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	33	122	39	33	41	52	29
normalized size	1	1.00	0.82	3.05	0.98	0.82	1.02	1.30	0.72
time (sec)	N/A	0.020	0.014	0.006	2.388	0.516	0.124	0.201	0.090

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	82	39	42	49	40	21
normalized size	1	1.00	0.86	1.61	0.76	0.82	0.96	0.78	0.41
time (sec)	N/A	0.021	0.014	0.003	2.417	0.693	0.125	0.170	4.433

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	32	13	25	29	32	29	12
normalized size	1	1.00	2.00	0.81	1.56	1.81	2.00	1.81	0.75
time (sec)	N/A	0.003	0.015	0.002	2.393	0.670	0.115	0.159	0.092

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	12	12
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.94	0.75	0.75
time (sec)	N/A	0.003	0.005	0.003	2.310	0.663	0.113	0.159	0.027

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	254	100	148	138	0	57
normalized size	1	1.00	0.80	3.39	1.33	1.97	1.84	0.00	0.76
time (sec)	N/A	0.037	0.021	0.005	2.305	0.639	0.394	0.000	4.793

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	91	254	70	151	131	0	43
normalized size	1	1.00	0.86	2.40	0.66	1.42	1.24	0.00	0.41
time (sec)	N/A	0.047	0.022	0.004	2.369	0.464	0.457	0.000	4.757

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	290	74	137	87	222	57
normalized size	1	1.00	0.80	3.87	0.99	1.83	1.16	2.96	0.76
time (sec)	N/A	0.050	0.035	0.010	2.476	0.409	0.223	0.171	4.406

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	75	290	62	140	80	222	41
normalized size	1	1.00	0.83	3.22	0.69	1.56	0.89	2.47	0.46
time (sec)	N/A	0.047	0.023	0.004	2.412	0.409	0.234	0.224	0.086

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	18	17	17	22	19	9
normalized size	1	1.00	1.92	1.38	1.31	1.31	1.69	1.46	0.69
time (sec)	N/A	0.006	0.006	0.006	2.350	0.405	0.197	0.159	0.040

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	45	100	0	0	70	0	-1
normalized size	1	1.00	2.81	6.25	0.00	0.00	4.38	0.00	-0.06
time (sec)	N/A	0.016	0.012	0.017	0.000	0.418	2.552	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	45	99	0	0	70	0	-1
normalized size	1	1.00	1.29	2.83	0.00	0.00	2.00	0.00	-0.03
time (sec)	N/A	0.033	0.012	0.007	0.000	0.438	2.927	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	74	107	0	0	61	0	-1
normalized size	1	1.00	1.72	2.49	0.00	0.00	1.42	0.00	-0.02
time (sec)	N/A	0.025	0.023	0.013	0.000	0.418	2.317	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	74	108	0	0	60	0	-1
normalized size	1	1.00	0.83	1.21	0.00	0.00	0.67	0.00	-0.01
time (sec)	N/A	0.046	0.020	0.006	0.000	0.419	2.190	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	47	120	0	0	66	0	-1
normalized size	1	1.00	0.53	1.35	0.00	0.00	0.74	0.00	-0.01
time (sec)	N/A	0.014	0.012	0.013	0.000	0.412	2.200	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	47	120	0	0	66	0	-1
normalized size	1	1.00	0.31	0.79	0.00	0.00	0.43	0.00	-0.01
time (sec)	N/A	0.031	0.010	0.004	0.000	0.453	2.603	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	122	0	0	70	0	-1
normalized size	1	1.00	0.84	1.36	0.00	0.00	0.78	0.00	-0.01
time (sec)	N/A	0.015	0.027	0.012	0.000	0.445	2.098	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	76	122	0	0	71	0	-1
normalized size	1	1.00	0.49	0.78	0.00	0.00	0.46	0.00	-0.01
time (sec)	N/A	0.032	0.018	0.004	0.000	0.418	2.057	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	15	0	0	0	0	-1
normalized size	1	1.00	1.00	1.50	0.00	0.00	0.00	0.00	-0.10
time (sec)	N/A	0.009	0.006	0.031	0.000	0.434	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	47	118	0	0	71	0	-1
normalized size	1	1.00	4.70	11.80	0.00	0.00	7.10	0.00	-0.10
time (sec)	N/A	0.016	0.012	0.014	0.000	0.424	2.048	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	0	0	0	-1
normalized size	1	1.00	1.04	1.22	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.008	0.015	0.000	0.418	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	47	117	0	0	71	0	-1
normalized size	1	1.00	2.04	5.09	0.00	0.00	3.09	0.00	-0.04
time (sec)	N/A	0.033	0.012	0.008	0.000	0.448	2.031	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	181	71	0	162	122	1642	94
normalized size	1	1.00	2.21	0.87	0.00	1.98	1.49	20.02	1.15
time (sec)	N/A	0.100	0.110	0.039	0.000	0.430	0.538	1.119	4.433

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	181	71	0	162	122	1642	98
normalized size	1	1.00	2.21	0.87	0.00	1.98	1.49	20.02	1.20
time (sec)	N/A	0.110	0.111	0.037	0.000	0.419	0.561	1.087	4.515

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	189	75	0	176	110	1676	30
normalized size	1	1.00	2.42	0.96	0.00	2.26	1.41	21.49	0.38
time (sec)	N/A	0.098	0.105	0.034	0.000	0.415	0.572	1.122	0.128

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	189	75	0	179	121	1676	88
normalized size	1	1.00	2.20	0.87	0.00	2.08	1.41	19.49	1.02
time (sec)	N/A	0.104	0.107	0.032	0.000	0.425	0.550	1.139	4.394

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	182	88	0	172	121	1642	99
normalized size	1	1.00	2.33	1.13	0.00	2.21	1.55	21.05	1.27
time (sec)	N/A	0.052	0.121	0.023	0.000	0.399	0.583	1.163	0.087

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	182	69	0	173	110	1642	57
normalized size	1	1.00	2.33	0.88	0.00	2.22	1.41	21.05	0.73
time (sec)	N/A	0.048	0.125	0.025	0.000	0.417	0.568	1.254	4.436

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	168	112	1676	29
normalized size	1	1.00	2.71	0.87	0.00	2.40	1.60	23.94	0.41
time (sec)	N/A	0.044	0.130	0.023	0.000	0.424	0.598	1.127	4.442

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	168	112	1676	29
normalized size	1	1.00	2.71	0.87	0.00	2.40	1.60	23.94	0.41
time (sec)	N/A	0.047	0.128	0.024	0.000	0.444	0.609	1.081	0.110

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	250	582	0	244	158	2202	129
normalized size	1	1.00	1.87	4.34	0.00	1.82	1.18	16.43	0.96
time (sec)	N/A	0.101	0.161	0.081	0.000	0.420	0.862	1.372	0.181

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	248	582	0	232	160	2202	232
normalized size	1	1.00	1.91	4.48	0.00	1.78	1.23	16.94	1.78
time (sec)	N/A	0.166	0.120	0.027	0.000	0.427	0.772	1.402	4.522

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	248	582	0	232	160	2202	232
normalized size	1	1.00	1.91	4.48	0.00	1.78	1.23	16.94	1.78
time (sec)	N/A	0.131	0.044	0.013	0.000	0.421	0.794	1.350	0.129

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	25	25	26	25	12
normalized size	1	1.00	1.00	0.90	0.86	0.86	0.90	0.86	0.41
time (sec)	N/A	0.026	0.019	0.012	1.043	0.399	0.470	0.245	4.410

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	138	52	0	164	117	51	55
normalized size	1	1.00	2.30	0.87	0.00	2.73	1.95	0.85	0.92
time (sec)	N/A	0.069	0.202	0.007	0.000	0.456	0.464	0.176	0.075

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	126	277	0	110	95	77	66
normalized size	1	1.00	2.03	4.47	0.00	1.77	1.53	1.24	1.06
time (sec)	N/A	0.058	0.060	0.045	0.000	0.419	0.378	0.308	4.385

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	134	277	0	120	83	80	24
normalized size	1	1.00	2.03	4.20	0.00	1.82	1.26	1.21	0.36
time (sec)	N/A	0.058	0.059	0.032	0.000	0.427	0.385	0.312	4.407

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	83	136	0	31	42	39	29
normalized size	1	1.00	1.84	3.02	0.00	0.69	0.93	0.87	0.64
time (sec)	N/A	0.059	0.077	0.052	0.000	0.399	0.147	0.170	0.087

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	12	11	19	22	11	19
normalized size	1	1.00	1.13	0.80	0.73	1.27	1.47	0.73	1.27
time (sec)	N/A	0.009	0.007	0.009	2.495	0.393	0.122	0.151	0.066

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	11	11	14	11	11
normalized size	1	1.00	1.00	0.86	0.79	0.79	1.00	0.79	0.79
time (sec)	N/A	0.007	0.005	0.002	2.299	0.402	0.118	0.162	0.026

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	97	34	33	33	44	33	29
normalized size	1	1.00	2.55	0.89	0.87	0.87	1.16	0.87	0.76
time (sec)	N/A	0.035	0.184	0.008	2.392	0.402	0.135	0.174	0.086

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	99	40	0	29	42	45	29
normalized size	1	1.00	2.06	0.83	0.00	0.60	0.88	0.94	0.60
time (sec)	N/A	0.040	0.104	0.033	0.000	0.389	0.129	0.190	4.391

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	97	40	0	33	44	52	29
normalized size	1	1.00	2.11	0.87	0.00	0.72	0.96	1.13	0.63
time (sec)	N/A	0.043	0.224	0.032	0.000	0.408	0.131	0.257	4.355

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	18	17	15	14	46	15
normalized size	1	1.00	0.81	0.86	0.81	0.71	0.67	2.19	0.71
time (sec)	N/A	0.013	0.006	0.006	2.240	0.421	0.113	0.158	4.288

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	101	40	0	31	42	52	29
normalized size	1	1.00	2.20	0.87	0.00	0.67	0.91	1.13	0.63
time (sec)	N/A	0.041	0.275	0.033	0.000	0.384	0.144	0.242	4.372

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	99	40	0	26	29	46	21
normalized size	1	1.00	2.25	0.91	0.00	0.59	0.66	1.05	0.48
time (sec)	N/A	0.034	0.101	0.036	0.000	0.400	0.135	0.175	0.057

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	14	20	0	15	12	42	15
normalized size	1	1.00	0.61	0.87	0.00	0.65	0.52	1.83	0.65
time (sec)	N/A	0.026	0.007	0.035	0.000	0.381	0.115	0.193	4.347

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	12	11	12	12	8	12	12
normalized size	1	1.00	1.09	1.00	1.09	1.09	0.73	1.09	1.09
time (sec)	N/A	0.005	0.005	0.006	0.929	0.405	0.091	0.158	4.298

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	30	29	25	26	33	14
normalized size	1	1.00	0.74	0.77	0.74	0.64	0.67	0.85	0.36
time (sec)	N/A	0.018	0.006	0.009	1.045	0.386	0.126	0.171	0.297

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	82	0	47	46	77	20
normalized size	1	1.00	0.95	1.86	0.00	1.07	1.05	1.75	0.45
time (sec)	N/A	0.035	0.015	0.043	0.000	0.385	0.123	0.340	0.224

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	127	279	0	109	94	73	63
normalized size	1	1.00	1.92	4.23	0.00	1.65	1.42	1.11	0.95
time (sec)	N/A	0.029	0.073	0.019	0.000	0.398	0.378	0.314	0.068

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	84	136	0	28	39	39	30
normalized size	1	1.00	1.83	2.96	0.00	0.61	0.85	0.85	0.65
time (sec)	N/A	0.031	0.073	0.017	0.000	0.387	0.132	0.176	4.380

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	12	10	9	17	14	9	17
normalized size	1	1.00	1.33	1.11	1.00	1.89	1.56	1.00	1.89
time (sec)	N/A	0.009	0.007	0.007	2.356	0.409	0.118	0.174	4.363

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	11	11	7	11	11
normalized size	1	1.00	1.00	1.00	1.00	1.00	0.64	1.00	1.00
time (sec)	N/A	0.005	0.004	0.008	1.079	0.381	0.090	0.157	4.299

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	25	25	26	25	12
normalized size	1	1.00	1.00	0.90	0.86	0.86	0.90	0.86	0.41
time (sec)	N/A	0.016	0.006	0.004	1.002	0.390	0.114	0.154	0.063

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	34	20
normalized size	1	1.00	0.84	0.78	0.00	0.90	0.92	0.68	0.40
time (sec)	N/A	0.023	0.014	0.010	0.000	0.395	0.114	0.172	4.369

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	43	46	41	20
normalized size	1	1.00	0.84	0.78	0.00	0.86	0.92	0.82	0.40
time (sec)	N/A	0.023	0.015	0.011	0.000	0.394	0.120	0.262	0.074

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	27	27	22	34	15
normalized size	1	1.00	1.00	0.90	0.87	0.87	0.71	1.10	0.48
time (sec)	N/A	0.015	0.005	0.004	1.061	0.396	0.112	0.157	0.068

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	41	20
normalized size	1	1.00	0.84	0.78	0.00	0.90	0.92	0.82	0.40
time (sec)	N/A	0.023	0.014	0.011	0.000	0.401	0.116	0.242	4.350

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	40	20
normalized size	1	1.00	0.84	0.78	0.00	0.90	0.92	0.80	0.40
time (sec)	N/A	0.024	0.020	0.010	0.000	0.394	0.119	0.178	0.068

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	41	20
normalized size	1	1.00	0.84	0.78	0.00	0.90	0.92	0.82	0.40
time (sec)	N/A	0.022	0.016	0.010	0.000	0.417	0.130	0.206	4.390

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	32	12	25	29	32	29	11
normalized size	1	1.00	2.29	0.86	1.79	2.07	2.29	2.07	0.79
time (sec)	N/A	0.006	0.009	0.002	2.355	0.403	0.107	0.158	4.328

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	30	29	27	29	33	15
normalized size	1	1.00	0.79	0.77	0.74	0.69	0.74	0.85	0.38
time (sec)	N/A	0.017	0.006	0.009	0.957	0.399	0.122	0.153	0.101

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	82	0	45	46	77	20
normalized size	1	1.00	0.88	1.71	0.00	0.94	0.96	1.60	0.42
time (sec)	N/A	0.039	0.019	0.017	0.000	0.420	0.118	0.322	0.127

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	124	277	0	101	88	0	73
normalized size	1	1.00	2.00	4.47	0.00	1.63	1.42	0.00	1.18
time (sec)	N/A	0.056	0.058	0.043	0.000	0.412	0.376	0.000	0.065

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	83	136	0	31	41	26	29
normalized size	1	1.00	1.69	2.78	0.00	0.63	0.84	0.53	0.59
time (sec)	N/A	0.088	0.139	0.050	0.000	0.419	0.124	0.177	0.083

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	81	110	0	31	41	26	29
normalized size	1	1.00	1.88	2.56	0.00	0.72	0.95	0.60	0.67
time (sec)	N/A	0.050	0.070	0.048	0.000	0.396	0.141	0.186	0.085

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	83	104	0	31	41	26	29
normalized size	1	1.00	1.69	2.12	0.00	0.63	0.84	0.53	0.59
time (sec)	N/A	0.062	0.104	0.043	0.000	0.398	0.127	0.163	4.391

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
normalized size	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.002	0.003	0.002	2.419	0.395	0.100	0.155	4.332

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	99	34	33	31	41	26	29
normalized size	1	1.00	2.61	0.89	0.87	0.82	1.08	0.68	0.76
time (sec)	N/A	0.027	0.195	0.006	2.403	0.417	0.123	0.162	0.077

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	88	39	29	39	39	29
normalized size	1	1.00	0.86	2.51	1.11	0.83	1.11	1.11	0.83
time (sec)	N/A	0.018	0.015	0.004	2.418	0.404	0.123	0.192	4.368

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	12	20	0	7	7	30	7
normalized size	1	1.00	0.52	0.87	0.00	0.30	0.30	1.30	0.30
time (sec)	N/A	0.020	0.007	0.019	0.000	0.451	0.110	0.168	4.315

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	16	10	10	7	11	10
normalized size	1	1.00	0.91	1.45	0.91	0.91	0.64	1.00	0.91
time (sec)	N/A	0.003	0.004	0.005	1.063	0.603	0.087	0.152	4.341

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	29	22	21	21	19	43	12
normalized size	1	1.00	0.45	0.34	0.32	0.32	0.29	0.66	0.18
time (sec)	N/A	0.032	0.006	0.006	0.992	0.713	0.113	0.165	0.256

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	70	0	36	39	39	18
normalized size	1	1.00	0.93	1.63	0.00	0.84	0.91	0.91	0.42
time (sec)	N/A	0.033	0.014	0.040	0.000	0.619	0.114	0.215	4.395

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	82	0	39	39	39	18
normalized size	1	1.00	0.87	1.78	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.036	0.014	0.036	0.000	0.595	0.119	0.244	4.474

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	125	279	0	100	87	0	76
normalized size	1	1.00	2.02	4.50	0.00	1.61	1.40	0.00	1.23
time (sec)	N/A	0.029	0.073	0.018	0.000	0.647	0.348	0.000	4.338

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	87	136	0	31	42	26	31
normalized size	1	1.00	1.74	2.72	0.00	0.62	0.84	0.52	0.62
time (sec)	N/A	0.040	0.135	0.017	0.000	0.646	0.130	0.167	0.079

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	82	111	0	31	42	26	31
normalized size	1	1.00	1.86	2.52	0.00	0.70	0.95	0.59	0.70
time (sec)	N/A	0.029	0.071	0.017	0.000	0.648	0.131	0.162	0.079

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	10	104	0	13	10	26	13
normalized size	1	1.00	0.26	2.67	0.00	0.33	0.26	0.67	0.33
time (sec)	N/A	0.033	0.007	0.018	0.000	0.437	0.119	0.183	4.308

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	5	7	9
normalized size	1	1.00	1.00	1.11	1.00	1.00	0.56	0.78	1.00
time (sec)	N/A	0.004	0.004	0.007	0.998	0.383	0.095	0.181	0.030

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	19	35	10
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.76	1.40	0.40
time (sec)	N/A	0.013	0.006	0.004	1.038	0.392	0.116	0.149	0.060

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	62	34	34	39	34	18
normalized size	1	1.00	0.87	1.35	0.74	0.74	0.85	0.74	0.39
time (sec)	N/A	0.019	0.013	0.003	2.263	0.406	0.113	0.150	0.060

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	0	39	39	39	18
normalized size	1	1.00	0.87	0.76	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.021	0.013	0.012	0.000	0.408	0.121	0.179	4.305

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	2
normalized size	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00
time (sec)	N/A	0.002	0.002	0.001	1.072	0.403	0.109	0.148	4.305

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	34	55	39	39	39	18
normalized size	1	1.00	1.05	0.89	1.45	1.03	1.03	1.03	0.47
time (sec)	N/A	0.029	0.014	0.004	2.458	0.389	0.116	0.182	0.112

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	70	0	39	39	39	18
normalized size	1	1.00	0.85	1.49	0.00	0.83	0.83	0.83	0.38
time (sec)	N/A	0.036	0.018	0.018	0.000	0.394	0.118	0.322	4.323

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	82	0	39	39	39	18
normalized size	1	1.00	0.87	1.78	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.035	0.016	0.018	0.000	0.395	0.142	0.225	4.388

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	99	34	33	33	46	33	29
normalized size	1	1.00	2.30	0.79	0.77	0.77	1.07	0.77	0.67
time (sec)	N/A	0.035	0.102	0.008	2.354	0.390	0.142	0.160	4.378

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	99	34	33	33	46	33	29
normalized size	1	1.00	2.30	0.79	0.77	0.77	1.07	0.77	0.67
time (sec)	N/A	0.032	0.033	0.004	2.487	0.402	0.150	0.177	0.002

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	23	34	22	25	17
normalized size	1	1.00	1.29	1.33	1.10	1.62	1.05	1.19	0.81
time (sec)	N/A	0.005	0.010	0.009	1.104	0.392	0.129	0.173	0.033

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	53	26	38	49	53	44	17
normalized size	1	1.00	1.89	0.93	1.36	1.75	1.89	1.57	0.61
time (sec)	N/A	0.013	0.019	0.006	2.356	0.405	0.606	0.174	4.385

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	72	56	51	55	474	60	290
normalized size	1	1.00	2.00	1.56	1.42	1.53	13.17	1.67	8.06
time (sec)	N/A	0.040	0.041	0.008	2.407	0.454	1.502	0.155	4.389

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	104	0	137	46	41	117
normalized size	1	1.00	0.99	1.41	0.00	1.85	0.62	0.55	1.58
time (sec)	N/A	0.045	0.098	0.025	0.000	0.439	0.209	0.158	0.108

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	97	114	69	69	740	69	827
normalized size	1	1.00	1.17	1.37	0.83	0.83	8.92	0.83	9.96
time (sec)	N/A	0.055	0.127	0.005	2.427	0.432	1.264	0.154	4.496

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	147	168	105	185	874	109	897
normalized size	1	1.00	1.24	1.41	0.88	1.55	7.34	0.92	7.54
time (sec)	N/A	0.091	0.249	0.014	2.349	0.413	1.895	0.157	4.495

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	111	710	0	3406	122	544	771
normalized size	1	1.00	0.47	3.03	0.00	14.56	0.52	2.32	3.29
time (sec)	N/A	0.230	0.116	0.099	0.000	0.554	1.322	0.878	4.490

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	165	756	0	4346	165	988	1491
normalized size	1	1.00	0.52	2.39	0.00	13.75	0.52	3.13	4.72
time (sec)	N/A	0.289	0.216	0.313	0.000	0.614	1.801	0.946	4.499

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	53	199	0	97	0	122	121
normalized size	1	1.00	0.33	1.24	0.00	0.61	0.00	0.76	0.76
time (sec)	N/A	0.146	0.045	0.089	0.000	0.423	0.000	0.378	4.956

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	53	199	0	97	0	126	121
normalized size	1	1.00	0.31	1.16	0.00	0.56	0.00	0.73	0.70
time (sec)	N/A	0.137	0.035	0.088	0.000	0.446	0.000	0.328	4.953

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	137	285	0	451	1469	1501	1227
normalized size	1	1.00	0.86	1.78	0.00	2.82	9.18	9.38	7.67
time (sec)	N/A	0.119	0.090	0.023	0.000	0.487	2.862	0.322	1.067

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	136	283	0	455	1467	1501	1227
normalized size	1	1.00	0.85	1.77	0.00	2.84	9.17	9.38	7.67
time (sec)	N/A	0.103	0.057	0.019	0.000	0.488	2.729	0.353	5.246

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	115	92	0	517	27	0	133
normalized size	1	1.00	1.01	0.81	0.00	4.54	0.24	0.00	1.17
time (sec)	N/A	0.076	0.172	0.037	0.000	0.431	0.253	0.000	4.484

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	115	96	0	251	0	0	159
normalized size	1	1.00	0.94	0.79	0.00	2.06	0.00	0.00	1.30
time (sec)	N/A	0.080	0.153	0.048	0.000	0.418	0.000	0.000	5.060

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	115	89	88	264	143	92	133
normalized size	1	1.00	0.93	0.72	0.71	2.13	1.15	0.74	1.07
time (sec)	N/A	0.077	0.131	0.026	2.287	0.457	0.313	0.177	0.237

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	130	190	0	4551	172	0	1007
normalized size	1	1.00	0.96	1.40	0.00	33.46	1.26	0.00	7.40
time (sec)	N/A	0.104	0.150	0.026	0.000	1.094	1.907	0.000	4.587

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	138	198	0	1141	0	0	1155
normalized size	1	1.00	0.86	1.24	0.00	7.13	0.00	0.00	7.22
time (sec)	N/A	0.116	0.135	0.041	0.000	0.539	0.000	0.000	4.986

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	247	404	0	1457	0	0	3285
normalized size	1	1.00	0.60	0.98	0.00	3.52	0.00	0.00	7.93
time (sec)	N/A	0.453	0.202	0.064	0.000	0.657	0.000	0.000	5.219

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	163	320	0	1469	0	0	1575
normalized size	1	1.00	0.70	1.37	0.00	6.28	0.00	0.00	6.73
time (sec)	N/A	0.172	0.186	0.071	0.000	1.169	0.000	0.000	5.290

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	103	200	0	0	0	0	-1
normalized size	1	1.00	1.07	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.135	0.110	0.000	0.422	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	19	113	0	0	0	0	-1
normalized size	1	1.00	0.76	4.52	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.032	0.055	0.016	0.000	0.446	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	103	204	0	0	0	0	-1
normalized size	1	1.00	1.07	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.168	0.099	0.000	0.420	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	107	204	0	0	0	0	-1
normalized size	1	1.00	1.16	2.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.134	0.100	0.000	0.411	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	35	95	0	0	0	0	-1
normalized size	1	1.00	1.30	3.52	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.063	0.010	0.000	0.441	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	107	204	0	0	0	0	-1
normalized size	1	1.00	1.16	2.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.167	0.094	0.000	0.420	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	187	515	0	0	0	0	-1
normalized size	1	1.00	0.63	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.119	0.301	0.050	0.000	0.443	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	97	94	98	110	94	95
normalized size	1	1.00	1.00	0.92	0.89	0.92	1.04	0.89	0.90
time (sec)	N/A	0.083	0.020	0.002	1.051	0.355	0.093	0.153	4.350

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	72	71	73	78	71	71
normalized size	1	1.00	1.00	0.91	0.90	0.92	0.99	0.90	0.90
time (sec)	N/A	0.056	0.016	0.002	1.037	0.351	0.088	0.149	0.030

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	49	48	50	56	50	49
normalized size	1	1.00	1.00	0.88	0.86	0.89	1.00	0.89	0.88
time (sec)	N/A	0.032	0.011	0.000	0.967	0.351	0.077	0.199	0.024

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	28	26
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.88	0.81
time (sec)	N/A	0.014	0.002	0.001	1.061	0.350	0.076	0.178	0.042

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	57	47	131	104	44	45
normalized size	1	1.00	1.00	1.04	0.85	2.38	1.89	0.80	0.82
time (sec)	N/A	0.035	0.036	0.008	2.546	0.396	0.324	0.169	0.069

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	82	74	222	138	62	68
normalized size	1	1.00	1.05	1.11	1.00	3.00	1.86	0.84	0.92
time (sec)	N/A	0.052	0.051	0.011	2.238	0.427	0.510	0.157	4.442

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	92	99	102	306	219	77	97
normalized size	1	1.00	0.99	1.06	1.10	3.29	2.35	0.83	1.04
time (sec)	N/A	0.067	0.064	0.009	2.559	0.431	0.755	0.160	4.481

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	113	122	137	424	204	100	129
normalized size	1	1.00	0.92	0.99	1.11	3.45	1.66	0.81	1.05
time (sec)	N/A	0.114	0.081	0.009	2.355	0.421	0.952	0.154	4.483

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	133	130	129	131	144	128	127
normalized size	1	1.00	1.00	0.98	0.97	0.98	1.08	0.96	0.95
time (sec)	N/A	0.107	0.022	0.002	1.066	0.353	0.095	0.159	0.058

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	90	89	91	104	91	89
normalized size	1	1.00	1.00	0.93	0.92	0.94	1.07	0.94	0.92
time (sec)	N/A	0.068	0.018	0.002	1.027	0.350	0.088	0.149	0.048

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	60	53	50
normalized size	1	1.00	1.00	0.85	0.83	0.83	1.00	0.88	0.83
time (sec)	N/A	0.030	0.003	0.002	1.037	0.348	0.076	0.152	0.026

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.008	0.001	0.000	1.004	0.344	0.065	0.166	0.028

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	136	113	268	236	105	141
normalized size	1	1.00	0.90	1.26	1.05	2.48	2.19	0.97	1.31
time (sec)	N/A	0.077	0.079	0.005	2.453	0.400	0.498	0.156	4.394

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	134	170	142	394	314	128	183
normalized size	1	1.00	1.02	1.30	1.08	3.01	2.40	0.98	1.40
time (sec)	N/A	0.188	0.108	0.011	2.283	0.404	0.932	0.171	4.399

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	154	211	167	516	257	145	164
normalized size	1	1.00	0.99	1.36	1.08	3.33	1.66	0.94	1.06
time (sec)	N/A	0.252	0.110	0.011	2.313	0.408	1.713	0.172	4.410

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	174	262	205	662	292	167	199
normalized size	1	1.00	0.95	1.42	1.11	3.60	1.59	0.91	1.08
time (sec)	N/A	0.297	0.138	0.014	2.392	0.448	2.612	0.164	4.486

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	200	231	244	806	335	198	240
normalized size	1	1.00	0.90	1.04	1.09	3.61	1.50	0.89	1.08
time (sec)	N/A	0.339	0.189	0.012	2.413	0.453	4.114	0.252	4.492

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	444	741	432	2878	500	498	4022
normalized size	1	1.00	1.02	1.70	0.99	6.59	1.14	1.14	9.20
time (sec)	N/A	0.453	0.339	0.011	2.453	4.262	3.752	0.185	5.081

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	360	572	342	2133	350	405	2712
normalized size	1	1.00	0.97	1.55	0.92	5.76	0.95	1.09	7.33
time (sec)	N/A	0.501	0.277	0.004	2.484	1.267	2.273	0.205	4.877

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	269	412	288	1480	238	318	1479
normalized size	1	1.00	0.91	1.39	0.97	4.98	0.80	1.07	4.98
time (sec)	N/A	0.293	0.257	0.004	2.361	0.609	1.478	0.179	4.794

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	183	260	221	767	109	245	599
normalized size	1	1.00	0.74	1.05	0.89	3.11	0.44	0.99	2.43
time (sec)	N/A	0.152	0.054	0.003	2.533	0.427	0.680	0.176	4.682

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	134	128	169	121	20	179	33
normalized size	1	1.00	0.72	0.69	0.91	0.65	0.11	0.97	0.18
time (sec)	N/A	0.111	0.018	0.003	2.439	0.407	0.174	0.181	4.409

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	234	363	268	4084	0	339	4802
normalized size	1	1.00	0.70	1.08	0.80	12.15	0.00	1.01	14.29
time (sec)	N/A	0.270	0.154	0.007	2.385	1.049	0.000	0.207	5.706

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	362	650	403	8409	0	517	16369
normalized size	1	1.00	0.80	1.43	0.89	18.56	0.00	1.14	36.13
time (sec)	N/A	0.384	0.468	0.012	2.445	16.306	0.000	0.253	6.548

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	371	624	292	2116	352	425	2560
normalized size	1	1.00	1.02	1.72	0.80	5.83	0.97	1.17	7.05
time (sec)	N/A	0.410	0.260	0.011	2.364	0.554	3.371	0.188	4.940

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	295	464	324	1596	275	350	1565
normalized size	1	1.00	0.85	1.33	0.93	4.57	0.79	1.00	4.48
time (sec)	N/A	0.313	0.167	0.009	2.589	0.627	2.069	0.190	4.786

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	267	303	253	873	136	273	637
normalized size	1	1.00	0.97	1.10	0.92	3.17	0.49	0.99	2.32
time (sec)	N/A	0.203	0.273	0.006	2.267	0.425	1.032	0.440	0.396

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	183	143	189	173	39	194	58
normalized size	1	1.00	0.91	0.71	0.94	0.86	0.19	0.96	0.29
time (sec)	N/A	0.133	0.115	0.005	2.430	0.409	0.348	0.182	0.084

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	429	873	506	9892	0	603	17945
normalized size	1	1.00	0.62	1.27	0.73	14.36	0.00	0.88	26.04
time (sec)	N/A	0.623	0.295	0.017	2.449	19.109	0.000	0.209	6.781

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	864	864	540	1169	732	0	0	855	28923
normalized size	1	1.00	0.62	1.35	0.85	0.00	0.00	0.99	33.48
time (sec)	N/A	0.906	0.584	0.020	2.607	0.000	0.000	0.247	8.330

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	386	203	506	0	0	214	0	-1
normalized size	1	0.99	0.52	1.30	0.00	0.00	0.55	0.00	-0.00
time (sec)	N/A	0.415	0.212	0.017	0.000	0.610	6.167	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	140	388	0	0	173	0	-1
normalized size	1	1.00	0.43	1.19	0.00	0.00	0.53	0.00	-0.00
time (sec)	N/A	0.287	0.138	0.010	0.000	0.917	4.730	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	120	266	0	0	124	0	-1
normalized size	1	1.00	0.45	1.01	0.00	0.00	0.47	0.00	-0.00
time (sec)	N/A	0.129	0.089	0.007	0.000	0.486	3.574	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	77	169	0	0	78	0	-1
normalized size	1	1.00	0.34	0.75	0.00	0.00	0.35	0.00	-0.00
time (sec)	N/A	0.069	0.032	0.004	0.000	0.653	2.061	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	95	107	0	0	0	0	-1
normalized size	1	1.00	0.28	0.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	0.150	0.038	0.000	11.319	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	522	556	0	0	0	0	-1
normalized size	1	1.00	0.90	0.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.762	0.764	0.031	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	729	729	332	1018	0	0	0	0	-1
normalized size	1	1.00	0.46	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.250	1.097	0.031	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	141	360	0	0	180	0	-1
normalized size	1	1.00	0.66	1.69	0.00	0.00	0.85	0.00	-0.00
time (sec)	N/A	0.282	0.160	0.031	0.000	0.916	4.890	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	121	246	0	0	129	0	-1
normalized size	1	1.00	0.75	1.52	0.00	0.00	0.80	0.00	-0.01
time (sec)	N/A	0.145	0.102	0.009	0.000	0.667	3.774	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	77	154	0	0	82	0	-1
normalized size	1	1.00	0.62	1.24	0.00	0.00	0.66	0.00	-0.01
time (sec)	N/A	0.089	0.031	0.005	0.000	0.851	2.236	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	91	97	0	0	0	0	-1
normalized size	1	1.00	1.26	1.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.149	0.029	0.000	10.029	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	508	523	0	0	0	0	-1
normalized size	1	1.00	1.70	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.356	0.958	0.030	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	321	961	0	0	0	0	-1
normalized size	1	1.00	0.76	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.751	1.236	0.032	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	458	1420	0	0	0	0	-1
normalized size	1	1.00	0.81	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.206	1.927	0.035	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	78	160	0	0	73	0	-1
normalized size	1	1.00	0.62	1.27	0.00	0.00	0.58	0.00	-0.01
time (sec)	N/A	0.083	0.035	0.010	0.000	1.054	2.182	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	92	99	0	0	0	0	-1
normalized size	1	1.00	1.26	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.146	0.022	0.000	13.112	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	86	158	0	0	70	0	-1
normalized size	1	1.00	1.59	2.93	0.00	0.00	1.30	0.00	-0.02
time (sec)	N/A	0.049	0.037	0.053	0.000	0.700	2.414	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	85	165	0	0	76	0	-1
normalized size	1	1.00	1.63	3.17	0.00	0.00	1.46	0.00	-0.02
time (sec)	N/A	0.046	0.030	0.044	0.000	0.669	2.296	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	80	175	0	0	83	0	-1
normalized size	1	1.00	0.34	0.74	0.00	0.00	0.35	0.00	-0.00
time (sec)	N/A	0.069	0.036	0.011	0.000	0.624	2.098	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	98	110	0	0	0	0	-1
normalized size	1	1.00	0.28	0.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	0.149	0.022	0.000	10.931	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	79	0	0	0	0	-1
normalized size	1	1.00	1.00	1.98	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.063	0.126	0.065	0.000	7.828	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	50	86	0	0	0	0	-1
normalized size	1	1.00	0.16	0.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	0.102	0.084	0.000	7.289	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	59	78	0	0	0	0	-1
normalized size	1	1.00	1.48	1.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.124	0.034	0.000	177.333	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	65	86	0	0	0	0	-1
normalized size	1	1.00	0.22	0.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.223	0.111	0.033	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.009	0.060	0.101	0.000	0.948	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.008	0.076	0.097	0.000	1.067	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	196	136	0	0	0	167	0	-1
normalized size	1	0.96	0.67	0.00	0.00	0.00	0.82	0.00	-0.00
time (sec)	N/A	0.230	0.074	0.087	0.000	0.678	139.104	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	142	106	0	0	0	119	0	-1
normalized size	1	0.95	0.71	0.00	0.00	0.00	0.79	0.00	-0.01
time (sec)	N/A	0.132	0.043	0.086	0.000	0.900	79.159	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	75	0	0	0	75	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.78	0.00	-0.01
time (sec)	N/A	0.051	0.023	0.082	0.000	0.947	42.838	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	41
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	0.93
time (sec)	N/A	0.010	0.003	0.082	0.000	0.765	9.182	0.000	4.364

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.127	0.091	0.000	0.559	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.243	0.090	0.000	0.537	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	103	86	75	0	0	129	0	-1
normalized size	1	0.95	0.80	0.69	0.00	0.00	1.19	0.00	-0.01
time (sec)	N/A	0.115	0.017	0.150	0.000	0.823	120.366	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	79	65	56	0	0	94	0	-1
normalized size	1	0.92	0.76	0.65	0.00	0.00	1.09	0.00	-0.01
time (sec)	N/A	0.068	0.011	0.093	0.000	0.665	69.116	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	0	0	61	0	-1
normalized size	1	1.00	1.00	0.88	0.00	0.00	1.45	0.00	-0.02
time (sec)	N/A	0.021	0.007	0.084	0.000	0.614	35.982	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	0	0	29	0	15
normalized size	1	1.00	1.00	0.94	0.00	0.00	1.61	0.00	0.83
time (sec)	N/A	0.004	0.002	0.084	0.000	0.680	7.707	0.000	0.070

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.081	0.113	0.000	0.636	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.143	0.095	0.000	0.725	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.257	0.093	0.000	0.640	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	42	56	116	75	144	42
normalized size	1	1.00	1.00	0.82	1.10	2.27	1.47	2.82	0.82
time (sec)	N/A	0.041	0.023	0.003	2.248	0.819	0.241	0.213	0.091

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	45	90	58	123	28
normalized size	1	1.00	1.00	0.82	1.18	2.37	1.53	3.24	0.74
time (sec)	N/A	0.034	0.018	0.003	2.453	0.647	0.201	0.232	0.055

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	36	73	34	118	21
normalized size	1	1.00	1.00	0.76	1.24	2.52	1.17	4.07	0.72
time (sec)	N/A	0.023	0.009	0.002	2.451	0.805	0.183	0.206	4.432

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	31	68	46	116	16
normalized size	1	1.00	1.00	0.67	1.29	2.83	1.92	4.83	0.67
time (sec)	N/A	0.012	0.005	0.002	2.351	0.539	0.155	0.290	0.058

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	65	55	71	189	226	0	74
normalized size	1	1.00	0.90	0.76	0.99	2.62	3.14	0.00	1.03
time (sec)	N/A	0.057	0.037	0.011	2.445	0.687	0.452	0.000	0.159

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	76	73	92	278	257	0	96
normalized size	1	1.00	0.85	0.82	1.03	3.12	2.89	0.00	1.08
time (sec)	N/A	0.083	0.059	0.011	2.493	0.857	0.715	0.000	0.163

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	1442	0	199	0	24	-1
normalized size	1	1.00	0.98	23.26	0.00	3.21	0.00	0.39	-0.02
time (sec)	N/A	0.044	0.029	0.059	0.000	0.737	0.000	0.246	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	986	0	138	0	131	-1
normalized size	1	1.00	1.00	25.95	0.00	3.63	0.00	3.45	-0.03
time (sec)	N/A	0.026	0.155	0.024	0.000	0.622	0.000	0.528	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	108	441	0	209	0	1	-1
normalized size	1	1.00	1.77	7.23	0.00	3.43	0.00	0.02	-0.02
time (sec)	N/A	0.039	0.126	0.022	0.000	0.609	0.000	0.328	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	345	911	0	279	0	0	-1
normalized size	1	1.00	4.31	11.39	0.00	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.069	3.343	0.028	0.000	0.772	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	98	132	0	251	0	0	-1
normalized size	1	1.00	0.64	0.86	0.00	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.165	0.073	0.000	0.728	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	107	0	223	0	0	-1
normalized size	1	1.00	0.78	0.97	0.00	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.072	0.022	0.000	0.716	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	69	0	121	0	0	-1
normalized size	1	1.00	0.77	1.06	0.00	1.86	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.043	0.024	0.000	0.851	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	249	0	152	0	0	-1
normalized size	1	1.00	1.00	3.19	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.052	0.055	0.000	0.990	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	111	488	0	297	0	0	-1
normalized size	1	1.00	0.89	3.90	0.00	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.085	0.056	0.000	0.907	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	123	711	0	365	0	0	-1
normalized size	1	1.00	0.73	4.23	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.107	0.059	0.000	0.931	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	123	105	0	265	0	0	-1
normalized size	1	1.00	0.81	0.69	0.00	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.200	0.020	0.000	0.954	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	110	85	0	236	0	0	-1
normalized size	1	1.00	1.01	0.78	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.110	0.014	0.000	1.009	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	67	54	0	125	0	0	-1
normalized size	1	1.00	1.05	0.84	0.00	1.95	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.040	0.014	0.000	0.938	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	267	0	155	0	0	-1
normalized size	1	1.00	1.00	3.47	0.00	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.050	0.063	0.000	0.735	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	110	510	0	302	0	0	-1
normalized size	1	1.00	0.89	4.11	0.00	2.44	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.082	0.042	0.000	0.648	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	122	739	0	376	0	0	-1
normalized size	1	1.00	0.73	4.43	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.106	0.053	0.000	0.960	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	38	25	0	73	0	0	-1
normalized size	1	1.00	1.27	0.83	0.00	2.43	0.00	0.00	-0.03
time (sec)	N/A	0.010	0.023	0.013	0.000	0.952	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	40	34	33	0	65	0	0	-1
normalized size	1	1.67	1.42	1.38	0.00	2.71	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.020	0.011	0.000	0.882	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	72	71	59	0	137	0	0	-1
normalized size	1	0.99	0.97	0.81	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.056	0.002	0.000	1.376	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	121	226	0	446	345	10312	182
normalized size	1	1.00	1.00	1.87	0.00	3.69	2.85	85.22	1.50
time (sec)	N/A	0.160	0.075	0.010	0.000	0.754	0.996	5.854	4.532

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	142	0	311	275	8680	113
normalized size	1	1.00	0.98	1.65	0.00	3.62	3.20	100.93	1.31
time (sec)	N/A	0.107	0.046	0.005	0.000	0.948	0.720	5.304	4.522

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	79	0	210	212	7051	52
normalized size	1	1.00	0.98	1.23	0.00	3.28	3.31	110.17	0.81
time (sec)	N/A	0.078	0.055	0.004	0.000	1.011	0.485	4.818	0.069

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	33	0	134	124	3276	38
normalized size	1	1.00	0.98	0.67	0.00	2.73	2.53	66.86	0.78
time (sec)	N/A	0.029	0.012	0.002	0.000	0.900	0.325	6.095	4.491

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	133	155	0	895	0	0	3901
normalized size	1	1.00	0.98	1.14	0.00	6.58	0.00	0.00	28.68
time (sec)	N/A	0.178	0.201	0.013	0.000	1.534	0.000	0.000	5.403

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	177	319	0	1765	0	0	6267
normalized size	1	1.00	0.95	1.71	0.00	9.44	0.00	0.00	33.51
time (sec)	N/A	0.278	0.411	0.013	0.000	2.909	0.000	0.000	6.453

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	134	7043	0	1079	0	54	-1
normalized size	1	1.00	0.96	50.67	0.00	7.76	0.00	0.39	-0.01
time (sec)	N/A	0.276	0.257	0.063	0.000	1.971	0.000	2.394	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	103	4308	0	940	0	27	-1
normalized size	1	1.00	0.95	39.89	0.00	8.70	0.00	0.25	-0.01
time (sec)	N/A	0.126	0.086	0.023	0.000	1.218	0.000	2.385	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	2252	0	432	0	0	-1
normalized size	1	1.00	1.00	29.63	0.00	5.68	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.065	0.023	0.000	0.796	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	418	771	0	701	0	0	-1
normalized size	1	1.00	3.94	7.27	0.00	6.61	0.00	0.00	-0.01
time (sec)	N/A	0.117	1.049	0.021	0.000	1.174	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	1058	1637	0	1063	0	0	-1
normalized size	1	1.00	7.10	10.99	0.00	7.13	0.00	0.00	-0.01
time (sec)	N/A	0.269	4.136	0.021	0.000	2.638	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	169	263	0	0	0	0	-1
normalized size	1	1.00	0.92	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.316	0.155	0.000	0.861	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	162	248	0	0	0	0	-1
normalized size	1	1.00	0.99	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.155	0.009	0.000	0.938	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	168	233	0	0	0	0	-1
normalized size	1	1.00	1.16	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.182	0.005	0.000	0.826	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	117	293	0	0	0	0	-1
normalized size	1	1.00	0.85	2.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.095	0.107	0.000	0.966	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	164	224	0	0	0	0	-1
normalized size	1	1.00	3.35	4.57	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.347	0.020	0.000	0.942	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	176	333	0	0	0	0	-1
normalized size	1	1.00	1.89	3.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.508	0.304	0.025	0.000	0.839	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	240	438	0	0	0	0	-1
normalized size	1	1.00	1.45	2.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.615	0.408	0.026	0.000	0.564	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	157	233	0	0	0	0	-1
normalized size	1	1.00	0.99	1.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.170	0.031	0.000	0.660	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	143	218	0	0	0	0	-1
normalized size	1	1.00	1.04	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.140	0.009	0.000	0.689	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	94	205	0	0	0	0	-1
normalized size	1	1.00	0.82	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.071	0.007	0.000	0.796	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	72	104	0	0	0	0	-1
normalized size	1	1.00	1.04	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.065	0.020	0.000	0.807	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	226	397	0	0	0	0	-1
normalized size	1	1.00	1.92	3.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.405	0.024	0.000	0.783	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	235	418	0	0	0	0	-1
normalized size	1	1.00	1.65	2.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.277	0.343	0.024	0.000	0.918	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	0	268	0	0	0	0	-1
normalized size	1	1.00	0.00	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.000	0.037	0.000	0.689	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	0	268	0	0	0	0	-1
normalized size	1	1.00	0.00	2.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.000	0.008	0.000	0.729	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	0	247	0	0	0	0	-1
normalized size	1	1.00	0.00	2.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.000	0.009	0.000	0.650	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	204	398	0	0	0	0	-1
normalized size	1	1.00	1.23	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.205	0.022	0.000	0.569	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	168	419	0	0	0	0	-1
normalized size	1	1.00	1.51	3.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.278	0.385	0.026	0.000	0.773	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	192	439	0	0	0	0	-1
normalized size	1	1.00	1.01	2.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.571	0.337	0.028	0.000	0.657	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	135	136	135	148	156	142	131
normalized size	1	1.00	1.00	1.01	1.00	1.10	1.16	1.05	0.97
time (sec)	N/A	0.126	0.037	0.001	0.962	0.683	0.113	0.154	0.063

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	103	102	111	112	108	101
normalized size	1	1.00	1.01	1.00	0.99	1.08	1.09	1.05	0.98
time (sec)	N/A	0.095	0.028	0.001	1.034	0.587	0.291	0.155	4.628

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	76	78	76	70
normalized size	1	1.00	1.00	0.96	0.95	1.04	1.07	1.04	0.96
time (sec)	N/A	0.060	0.020	0.001	1.074	0.492	0.108	0.149	4.587

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	40	39	43	38
normalized size	1	1.00	1.00	0.88	0.86	0.95	0.93	1.02	0.90
time (sec)	N/A	0.027	0.009	0.001	0.904	0.480	0.101	0.148	0.044

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	84	58	159	117	56	57
normalized size	1	1.00	0.98	1.27	0.88	2.41	1.77	0.85	0.86
time (sec)	N/A	0.045	0.052	0.004	2.409	0.609	0.729	0.151	0.085

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	84	268	153	75	77
normalized size	1	1.00	1.06	1.42	1.01	3.23	1.84	0.90	0.93
time (sec)	N/A	0.093	0.056	0.009	2.245	0.745	1.233	0.170	4.670

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	110	131	121	391	196	101	112
normalized size	1	1.00	0.96	1.14	1.05	3.40	1.70	0.88	0.97
time (sec)	N/A	0.107	0.097	0.008	2.253	0.595	2.266	0.232	4.847

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	142	158	162	530	241	134	144
normalized size	1	1.00	0.95	1.05	1.08	3.53	1.61	0.89	0.96
time (sec)	N/A	0.205	0.133	0.009	2.507	0.572	4.409	0.159	4.509

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	223	219	218	261	272	255	220
normalized size	1	1.00	1.00	0.98	0.98	1.17	1.22	1.14	0.99
time (sec)	N/A	0.199	0.087	0.001	1.044	0.437	0.220	0.157	4.484

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	156	155	147	181	192	181	148
normalized size	1	1.00	1.01	1.00	0.95	1.17	1.24	1.17	0.95
time (sec)	N/A	0.141	0.054	0.000	1.136	0.496	0.155	0.169	4.516

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	91	90	100	107	106	90
normalized size	1	1.00	1.00	0.95	0.94	1.04	1.11	1.10	0.94
time (sec)	N/A	0.068	0.024	0.000	1.009	0.527	0.247	0.170	0.038

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	45	43	48	43	42
normalized size	1	1.00	1.00	0.86	0.92	0.88	0.98	0.88	0.86
time (sec)	N/A	0.025	0.006	0.001	1.103	0.471	0.154	0.144	0.022

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	144	267	176	406	371	185	229
normalized size	1	1.00	1.01	1.87	1.23	2.84	2.59	1.29	1.60
time (sec)	N/A	0.140	0.066	0.005	2.377	0.691	1.530	0.162	4.469

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	183	320	205	600	484	207	293
normalized size	1	1.00	1.10	1.93	1.23	3.61	2.92	1.25	1.77
time (sec)	N/A	0.298	0.101	0.011	2.416	0.795	3.786	0.180	4.563

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	217	402	245	794	398	244	257
normalized size	1	1.00	1.08	2.00	1.22	3.95	1.98	1.21	1.28
time (sec)	N/A	0.419	0.113	0.013	2.363	0.649	17.717	0.182	0.118

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	267	506	300	1016	457	296	308
normalized size	1	1.00	1.07	2.02	1.20	4.06	1.83	1.18	1.23
time (sec)	N/A	0.543	0.148	0.014	2.390	0.585	94.000	0.182	4.599

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	345	412	366	1266	0	364	375
normalized size	1	1.00	1.09	1.30	1.15	3.99	0.00	1.15	1.18
time (sec)	N/A	0.650	0.225	0.013	2.519	0.738	0.000	0.195	4.574

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	84	268	153	75	77
normalized size	1	1.00	1.06	1.42	1.01	3.23	1.84	0.90	0.93
time (sec)	N/A	0.093	0.017	0.000	2.342	0.675	0.820	0.162	0.002

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	84	268	153	75	77
normalized size	1	1.00	1.06	1.42	1.01	3.23	1.84	0.90	0.93
time (sec)	N/A	0.084	0.017	0.009	2.369	0.613	0.862	0.154	0.115

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	570	1888	0	0	0	9285	29551
normalized size	1	1.00	1.24	4.11	0.00	0.00	0.00	20.23	64.38
time (sec)	N/A	1.537	0.687	0.049	0.000	0.000	0.000	1.628	9.313

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	402	1211	0	9584	0	6407	17954
normalized size	1	1.00	1.27	3.83	0.00	30.33	0.00	20.28	56.82
time (sec)	N/A	0.786	0.550	0.037	0.000	29.064	0.000	1.355	7.290

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	269	695	0	4690	0	4107	9600
normalized size	1	1.00	1.13	2.92	0.00	19.71	0.00	17.26	40.34
time (sec)	N/A	0.635	0.322	0.028	0.000	3.331	0.000	1.136	6.484

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	172	328	0	1525	314	1402	4109
normalized size	1	1.00	0.99	1.89	0.00	8.76	1.80	8.06	23.61
time (sec)	N/A	0.202	0.139	0.020	0.000	0.906	20.947	0.872	5.382

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	116	0	613	87	1024	763
normalized size	1	1.00	0.86	0.77	0.00	4.09	0.58	6.83	5.09
time (sec)	N/A	0.098	0.085	0.014	0.000	0.740	1.272	0.599	0.514

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	274	480	0	0	0	7650	23640
normalized size	1	1.00	1.08	1.89	0.00	0.00	0.00	30.12	93.07
time (sec)	N/A	0.586	0.272	0.022	0.000	0.000	0.000	2.535	9.446

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	354	1141	0	0	0	13225	91169
normalized size	1	1.00	0.83	2.66	0.00	0.00	0.00	30.83	212.52
time (sec)	N/A	1.415	0.753	0.029	0.000	0.000	0.000	2.510	10.280

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	540	1846	0	12117	0	8983	29030
normalized size	1	1.00	0.96	3.28	0.00	21.52	0.00	15.96	51.56
time (sec)	N/A	3.519	1.630	0.050	0.000	84.432	0.000	2.459	8.793

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	415	1223	0	7338	0	6390	18785
normalized size	1	1.00	1.08	3.17	0.00	19.01	0.00	16.55	48.67
time (sec)	N/A	2.079	1.111	0.042	0.000	11.077	0.000	1.846	9.845

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	310	1761	0	4573	0	4433	12350
normalized size	1	1.00	1.06	6.01	0.00	15.61	0.00	15.13	42.15
time (sec)	N/A	0.789	0.749	0.085	0.000	2.832	0.000	1.764	9.387

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	243	733	0	2309	394	2682	6404
normalized size	1	1.00	0.96	2.91	0.00	9.16	1.56	10.64	25.41
time (sec)	N/A	0.517	0.425	0.060	0.000	1.099	170.284	0.602	6.257

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	660	660	708	3841	0	0	0	0	237586
normalized size	1	1.00	1.07	5.82	0.00	0.00	0.00	0.00	359.98
time (sec)	N/A	2.873	2.789	0.064	0.000	0.000	0.000	0.000	16.455

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1077	1077	1020	5709	0	0	0	0	97073
normalized size	1	1.00	0.95	5.30	0.00	0.00	0.00	0.00	90.13
time (sec)	N/A	12.639	5.843	0.084	0.000	0.000	0.000	0.000	17.810

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	190	283	261	370	505	180	-1
normalized size	1	1.00	0.88	1.32	1.21	1.72	2.35	0.84	-0.00
time (sec)	N/A	0.161	0.388	0.010	1.123	1.131	63.830	0.226	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	157	229	207	304	413	145	-1
normalized size	1	1.00	0.90	1.31	1.18	1.74	2.36	0.83	-0.01
time (sec)	N/A	0.122	0.320	0.010	1.018	1.131	31.095	0.222	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	121	175	153	232	272	106	-1
normalized size	1	1.00	0.92	1.33	1.16	1.76	2.06	0.80	-0.01
time (sec)	N/A	0.109	0.234	0.010	0.984	0.989	12.265	0.219	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	82	122	100	174	230	79	-1
normalized size	1	1.00	0.85	1.26	1.03	1.79	2.37	0.81	-0.01
time (sec)	N/A	0.061	0.064	0.009	1.066	1.071	7.045	0.193	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	98	112	97	249	134	80	-1
normalized size	1	1.00	1.10	1.26	1.09	2.80	1.51	0.90	-0.01
time (sec)	N/A	0.073	0.105	0.009	1.130	0.883	9.984	0.205	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	112	124	135	289	450	88	-1
normalized size	1	1.00	1.11	1.23	1.34	2.86	4.46	0.87	-0.01
time (sec)	N/A	0.071	0.190	0.008	1.007	0.753	18.951	0.225	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	67	66	173	93	639	75	133
normalized size	1	1.00	0.78	0.77	2.01	1.08	7.43	0.87	1.55
time (sec)	N/A	0.107	0.052	0.005	1.156	0.667	45.985	0.211	4.704

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	101	100	227	136	1989	113	154
normalized size	1	1.00	0.80	0.79	1.80	1.08	15.79	0.90	1.22
time (sec)	N/A	0.146	0.093	0.004	1.199	0.895	119.187	0.270	4.667

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	164	132	136	281	177	0	148	189
normalized size	1	0.99	0.80	0.82	1.70	1.07	0.00	0.90	1.15
time (sec)	N/A	0.210	0.117	0.006	1.202	0.732	0.000	0.228	4.752

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	167	172	335	224	0	189	226
normalized size	1	1.00	0.80	0.82	1.60	1.07	0.00	0.90	1.08
time (sec)	N/A	0.222	0.142	0.008	1.111	1.096	0.000	0.235	4.760

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	119	172	0	0	0	0	-1
normalized size	1	1.00	0.62	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.102	0.025	0.000	0.725	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	114	155	0	0	0	0	-1
normalized size	1	1.00	0.68	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.079	0.007	0.000	0.508	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	109	137	0	0	0	0	-1
normalized size	1	1.00	0.73	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.063	0.008	0.000	0.841	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	102	121	0	0	0	0	-1
normalized size	1	1.00	0.72	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.036	0.004	0.000	0.782	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	232	90	138	0	0	0	0	-1
normalized size	1	1.30	0.51	0.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.145	0.035	0.000	0.877	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	208	162	0	0	0	0	-1
normalized size	1	1.00	1.00	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.123	0.275	0.021	0.000	0.738	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	174	186	0	0	0	0	-1
normalized size	1	1.00	0.73	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	0.355	0.023	0.000	0.484	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	0	206	0	0	0	0	-1
normalized size	1	1.00	0.00	0.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.000	0.020	0.000	0.468	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	0	189	0	0	0	0	-1
normalized size	1	1.00	0.00	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.000	0.009	0.000	0.423	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	0	172	0	0	0	0	-1
normalized size	1	1.00	0.00	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.000	0.005	0.000	0.409	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	114	155	0	0	0	0	-1
normalized size	1	1.00	0.66	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.043	0.005	0.000	0.423	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	148	170	0	0	0	0	-1
normalized size	1	1.00	0.71	0.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	0.177	0.016	0.000	0.476	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	333	213	177	0	0	0	0	-1
normalized size	1	1.50	0.96	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.444	0.293	0.022	0.000	0.478	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	288	174	186	0	0	0	0	-1
normalized size	1	1.25	0.75	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.668	0.379	0.022	0.000	0.510	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	106	138	0	0	0	0	-1
normalized size	1	1.00	0.68	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.103	0.018	0.000	0.414	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	104	121	0	0	0	0	-1
normalized size	1	1.00	0.73	0.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.088	0.007	0.000	0.417	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	69	106	0	0	0	0	-1
normalized size	1	1.00	0.57	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.060	0.004	0.000	0.429	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	0	0	0	0	-1
normalized size	1	1.00	1.04	0.96	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.015	0.004	0.000	0.426	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	55	47	0	0	0	0	-1
normalized size	1	1.00	0.52	0.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.096	0.014	0.000	0.477	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	208	162	0	0	0	0	-1
normalized size	1	1.00	1.00	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.188	0.257	0.020	0.000	0.469	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	186	186	0	0	0	0	-1
normalized size	1	1.00	0.78	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	0.343	0.021	0.000	0.494	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	0	274	0	0	0	0	-1
normalized size	1	1.00	0.00	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.000	0.036	0.000	0.418	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	0	234	0	0	0	0	-1
normalized size	1	1.00	0.00	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.000	0.009	0.000	0.417	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	0	196	0	0	0	0	-1
normalized size	1	1.00	0.00	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.000	0.008	0.000	0.411	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	0	173	0	0	0	0	-1
normalized size	1	1.00	0.00	1.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.000	0.008	0.000	0.423	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	150	0	0	0	0	-1
normalized size	1	1.00	0.00	1.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.000	0.006	0.000	0.430	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	99	129	0	0	0	0	-1
normalized size	1	1.00	0.66	0.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.041	0.005	0.000	0.440	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	207	138	161	0	0	0	0	-1
normalized size	1	1.20	0.80	0.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.169	0.019	0.000	0.470	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	208	185	0	0	0	0	-1
normalized size	1	1.00	0.89	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	0.281	0.024	0.000	0.475	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	159	209	0	0	0	0	-1
normalized size	1	1.00	0.60	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.760	0.501	0.025	0.000	0.465	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	193	0	0	0	0	-1
normalized size	1	1.00	0.97	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.132	0.025	0.000	0.420	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	107	176	0	0	0	0	-1
normalized size	1	1.00	1.13	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.104	0.009	0.000	0.422	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	159	0	0	0	0	-1
normalized size	1	1.00	1.38	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.089	0.008	0.000	0.420	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	94	141	0	0	0	0	-1
normalized size	1	1.00	2.04	3.07	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.081	0.006	0.000	0.418	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	90	125	0	0	0	0	-1
normalized size	1	1.00	2.05	2.84	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.045	0.004	0.000	0.416	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	51	141	0	0	0	0	-1
normalized size	1	1.00	1.11	3.07	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.079	0.131	0.019	0.000	0.491	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	196	165	0	0	0	0	-1
normalized size	1	1.00	2.65	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.283	0.022	0.000	0.478	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	244	189	0	0	0	0	-1
normalized size	1	1.00	2.39	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.414	0.357	0.020	0.000	0.471	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	0	227	0	0	0	0	-1
normalized size	1	1.00	0.00	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.000	0.023	0.000	0.408	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	210	0	0	0	0	-1
normalized size	1	1.00	0.00	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.000	0.007	0.000	0.435	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	0	193	0	0	0	0	-1
normalized size	1	1.00	0.00	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.000	0.009	0.000	0.429	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	176	0	0	0	0	-1
normalized size	1	1.00	0.00	2.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.000	0.007	0.000	0.416	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	159	0	0	0	0	-1
normalized size	1	1.00	1.38	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.055	0.004	0.000	0.415	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	130	173	0	0	0	0	-1
normalized size	1	1.00	1.81	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.196	0.017	0.000	0.484	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	201	180	0	0	0	0	-1
normalized size	1	1.00	2.16	1.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	0.314	0.020	0.000	0.478	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	244	189	0	0	0	0	-1
normalized size	1	1.00	2.39	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.497	0.407	0.022	0.000	0.477	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	97	142	0	0	0	0	-1
normalized size	1	1.00	1.49	2.18	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.114	0.018	0.000	0.402	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	92	125	0	0	0	0	-1
normalized size	1	1.00	2.00	2.72	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.097	0.009	0.000	0.436	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	34	110	0	0	0	0	-1
normalized size	1	1.00	1.36	4.40	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.059	0.005	0.000	0.439	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	47	0	0	0	0	-1
normalized size	1	1.00	1.90	4.70	0.00	0.00	0.00	0.00	-0.10
time (sec)	N/A	0.011	0.016	0.004	0.000	0.410	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	24	48	0	0	0	0	-1
normalized size	1	1.00	1.41	2.82	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.033	0.098	0.015	0.000	0.479	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	196	165	0	0	0	0	-1
normalized size	1	1.00	2.65	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.285	0.020	0.000	0.482	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	108	189	0	0	0	0	-1
normalized size	1	1.00	1.06	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.416	0.021	0.000	0.486	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	280	0	0	0	0	-1
normalized size	1	1.00	0.00	3.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.000	0.037	0.000	0.452	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	0	240	0	0	0	0	-1
normalized size	1	1.00	0.00	3.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.000	0.008	0.000	0.433	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	0	202	0	0	0	0	-1
normalized size	1	1.00	0.00	3.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.000	0.008	0.000	0.424	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	0	179	0	0	0	0	-1
normalized size	1	1.00	0.00	3.25	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.000	0.008	0.000	0.433	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	0	156	0	0	0	0	-1
normalized size	1	1.00	0.00	2.84	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.000	0.007	0.000	0.413	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	0	133	0	0	0	0	-1
normalized size	1	1.00	0.00	2.42	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.000	0.005	0.000	0.435	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	101	164	0	0	0	0	-1
normalized size	1	1.00	1.40	2.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.218	0.015	0.000	0.487	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	196	188	0	0	0	0	-1
normalized size	1	1.00	1.96	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.297	0.329	0.023	0.000	0.496	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	244	212	0	0	0	0	-1
normalized size	1	1.00	1.91	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.570	0.397	0.025	0.000	0.487	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	354	292	0	0	0	0	-1
normalized size	1	1.00	1.46	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.585	0.167	0.000	0.418	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	349	275	0	0	0	0	-1
normalized size	1	1.00	1.58	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.112	0.505	0.010	0.000	0.418	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	343	258	0	0	0	0	-1
normalized size	1	1.00	1.73	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.470	0.009	0.000	0.409	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	338	240	0	0	0	0	-1
normalized size	1	1.00	1.91	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.431	0.006	0.000	0.407	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	331	224	0	0	0	0	-1
normalized size	1	1.00	1.96	1.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.352	0.004	0.000	0.412	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	283	386	0	0	0	0	-1
normalized size	1	1.00	0.88	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.152	0.251	0.118	0.000	0.494	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	481	410	0	0	0	0	-1
normalized size	1	1.00	1.69	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.151	0.768	0.025	0.000	0.503	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	308	434	0	0	0	0	-1
normalized size	1	1.00	0.99	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.711	0.680	0.029	0.000	0.496	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	0	326	0	0	0	0	-1
normalized size	1	1.00	0.00	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.174	0.000	0.041	0.000	0.418	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	0	309	0	0	0	0	-1
normalized size	1	1.00	0.00	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.000	0.008	0.000	0.425	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	0	292	0	0	0	0	-1
normalized size	1	1.00	0.00	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.098	0.000	0.009	0.000	0.421	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	0	275	0	0	0	0	-1
normalized size	1	1.00	0.00	1.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.071	0.000	0.008	0.000	0.444	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	343	258	0	0	0	0	-1
normalized size	1	1.00	1.73	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.420	0.004	0.000	0.414	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	477	418	0	0	0	0	-1
normalized size	1	1.00	1.68	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.236	0.724	0.021	0.000	0.494	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	372	309	425	0	0	0	0	-1
normalized size	1	1.22	1.01	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.534	0.569	0.028	0.000	0.479	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	309	434	0	0	0	0	-1
normalized size	1	1.00	0.70	0.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.800	0.688	0.027	0.000	0.522	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	337	241	0	0	0	0	-1
normalized size	1	1.00	1.80	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.479	0.031	0.000	0.414	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	331	224	0	0	0	0	-1
normalized size	1	1.00	1.95	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.427	0.008	0.000	0.447	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	214	209	0	0	0	0	-1
normalized size	1	1.00	1.42	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.176	0.005	0.000	0.414	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	142	85	0	0	0	0	-1
normalized size	1	1.00	2.22	1.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.055	0.003	0.000	0.412	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	159	107	0	0	0	0	-1
normalized size	1	1.00	0.95	0.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.139	0.018	0.000	0.483	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	481	410	0	0	0	0	-1
normalized size	1	1.00	1.68	1.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.779	0.024	0.000	0.494	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	308	434	0	0	0	0	-1
normalized size	1	1.00	0.98	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.290	0.891	0.025	0.000	0.490	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	339	379	0	0	0	0	-1
normalized size	1	1.00	1.55	1.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.121	0.518	0.054	0.000	0.418	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	0	339	0	0	0	0	-1
normalized size	1	1.00	0.00	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.089	0.000	0.009	0.000	0.416	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	0	301	0	0	0	0	-1
normalized size	1	1.00	0.00	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.000	0.008	0.000	0.416	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	0	278	0	0	0	0	-1
normalized size	1	1.00	0.00	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.000	0.008	0.000	0.417	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	0	255	0	0	0	0	-1
normalized size	1	1.00	0.00	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.000	0.007	0.000	0.418	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	328	232	0	0	0	0	-1
normalized size	1	1.00	1.81	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.354	0.005	0.000	0.413	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	483	409	0	0	0	0	-1
normalized size	1	1.00	1.70	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.550	0.023	0.000	0.497	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	311	433	0	0	0	0	-1
normalized size	1	1.00	1.00	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.505	0.600	0.029	0.000	0.493	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	320	457	0	0	0	0	-1
normalized size	1	1.00	0.94	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.872	0.750	0.030	0.000	0.498	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	584	1186	0	0	0	0	-1
normalized size	1	1.00	1.25	2.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.425	2.871	0.022	0.000	0.449	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	488	756	0	0	0	0	-1
normalized size	1	1.00	1.37	2.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.191	1.615	0.008	0.000	0.414	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	0	0	0	-1
normalized size	1	1.00	1.07	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.083	0.257	0.005	0.000	0.409	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	214	200	0	0	0	0	-1
normalized size	1	1.00	0.53	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.220	0.036	0.000	45.979	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	1069	1279	0	0	0	0	-1
normalized size	1	1.00	1.49	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.081	1.863	0.036	0.000	125.492	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	596	1195	0	0	0	0	-1
normalized size	1	1.00	1.08	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.280	2.470	0.023	0.000	0.528	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	503	761	0	0	0	0	-1
normalized size	1	1.00	1.11	1.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.794	1.390	0.009	0.000	0.657	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	293	364	0	0	0	0	-1
normalized size	1	1.00	0.76	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.259	0.006	0.000	0.721	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	205	201	0	0	0	0	-1
normalized size	1	1.00	1.04	1.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.227	0.038	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	464	1293	0	0	0	0	-1
normalized size	1	1.00	0.65	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.015	5.530	0.036	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	304	355	0	0	0	0	-1
normalized size	1	1.00	0.63	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.475	0.302	0.033	0.000	1.105	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	216	198	0	0	0	0	-1
normalized size	1	1.00	1.06	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.223	0.033	0.000	76.165	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	295	357	0	0	0	0	-1
normalized size	1	1.00	1.01	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.090	0.306	0.035	0.000	0.634	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	207	199	0	0	0	0	-1
normalized size	1	1.00	0.50	0.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	0.221	0.033	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	154	380	0	0	0	0	-1
normalized size	1	1.00	0.67	1.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.230	0.011	0.000	0.550	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	127	235	0	0	0	0	-1
normalized size	1	1.00	0.76	1.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.157	0.010	0.000	0.549	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	73	108	0	0	0	0	-1
normalized size	1	1.00	0.60	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.074	0.005	0.000	0.831	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	59	55	0	0	0	0	-1
normalized size	1	1.00	0.48	0.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.110	0.025	0.000	1.120	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	399	175	443	0	0	0	0	-1
normalized size	1	1.26	0.55	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	0.599	0.031	0.000	1.926	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.098	0.096	0.000	1.215	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	373	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.814	0.509	0.088	0.000	0.629	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	358	345	303	0	0	0	0	0	-1
normalized size	1	0.96	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.356	0.370	0.086	0.000	0.593	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	232	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.222	0.255	0.027	0.000	0.611	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.156	0.015	0.000	0.427	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.129	0.093	0.000	0.454	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.260	0.115	0.000	0.457	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	446	446	258	251	0	0	0	0	-1
normalized size	1	1.00	0.58	0.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.497	0.684	0.021	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	719	247	0	0	0	0	-1
normalized size	1	1.00	3.30	1.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	1.260	0.020	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	685	327	0	323	0	0	-1
normalized size	1	1.00	10.54	5.03	0.00	4.97	0.00	0.00	-0.02
time (sec)	N/A	0.145	3.068	0.166	0.000	1.583	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	876	311	0	112	0	0	-1
normalized size	1	1.00	13.90	4.94	0.00	1.78	0.00	0.00	-0.02
time (sec)	N/A	0.141	7.827	0.161	0.000	1.267	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	623	336	0	328	0	0	-1
normalized size	1	1.00	8.65	4.67	0.00	4.56	0.00	0.00	-0.01
time (sec)	N/A	0.133	1.732	0.161	0.000	1.551	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	881	337	0	114	0	0	-1
normalized size	1	1.00	12.59	4.81	0.00	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.129	6.386	0.154	0.000	1.411	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	3652	437	0	0	0	0	-1
normalized size	1	1.00	6.52	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.617	7.869	0.025	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	3658	439	0	0	0	0	-1
normalized size	1	1.00	6.94	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.717	7.866	0.024	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [147] had the largest ratio of [.7778]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	6	1.00	17	0.353
2	A	9	6	1.00	18	0.333
3	A	3	3	1.00	18	0.167
4	A	3	3	1.00	19	0.158
5	A	5	3	1.00	17	0.176
6	A	3	2	1.00	17	0.118
7	A	2	2	1.00	17	0.118
8	A	2	2	1.00	17	0.118
9	A	5	3	1.00	27	0.111
10	A	3	2	1.00	28	0.071
11	A	5	3	1.00	21	0.143
12	A	3	2	1.00	22	0.091
13	A	3	3	1.00	15	0.200
14	A	2	2	1.00	22	0.091
15	A	5	5	1.00	23	0.217
16	A	3	3	1.00	21	0.143
17	A	6	6	1.00	22	0.273
18	A	1	1	1.00	22	0.045
19	A	3	3	1.00	21	0.143
20	A	1	1	1.00	23	0.043
21	A	3	3	1.00	22	0.136
22	A	1	1	1.00	28	0.036
23	A	2	2	1.00	24	0.083
24	A	4	4	1.00	28	0.143
25	A	5	5	1.00	25	0.200
26	A	5	3	1.00	26	0.115
27	A	5	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	5	3	1.00	27	0.111
29	A	5	3	1.00	27	0.111
30	A	3	2	1.00	27	0.074
31	A	3	2	1.00	27	0.074
32	A	3	2	1.00	28	0.071
33	A	3	2	1.00	28	0.071
34	A	3	2	1.00	30	0.067
35	A	5	3	1.00	29	0.103
36	A	6	4	1.00	29	0.138
37	A	3	2	1.00	32	0.062
38	A	5	3	1.00	31	0.097
39	A	5	3	1.00	22	0.136
40	A	5	3	1.00	23	0.130
41	A	3	2	1.00	22	0.091
42	A	3	2	1.00	22	0.091
43	A	3	3	1.00	22	0.136
44	A	5	3	1.00	22	0.136
45	A	5	3	1.00	22	0.136
46	A	5	3	1.00	20	0.150
47	A	5	3	1.00	17	0.176
48	A	5	3	1.00	22	0.136
49	A	5	3	1.00	22	0.136
50	A	5	3	1.00	22	0.136
51	A	2	2	1.00	22	0.091
52	A	7	3	1.00	22	0.136
53	A	5	3	1.00	22	0.136
54	A	3	2	1.00	22	0.091
55	A	3	2	1.00	22	0.091
56	A	3	2	1.00	22	0.091
57	A	2	2	1.00	22	0.091
58	A	3	2	1.00	22	0.091
59	A	3	2	1.00	22	0.091
60	A	3	2	1.00	20	0.100
61	A	3	2	1.00	17	0.118
62	A	3	2	1.00	22	0.091
63	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
64	A	3	2	1.00	22	0.091
65	A	3	3	1.00	22	0.136
66	A	7	3	1.00	22	0.136
67	A	5	3	1.00	22	0.136
68	A	5	3	1.00	18	0.167
69	A	3	2	1.00	18	0.111
70	A	3	2	1.00	18	0.111
71	A	3	2	1.00	18	0.111
72	A	2	2	1.00	18	0.111
73	A	5	3	1.00	16	0.188
74	A	5	3	1.00	13	0.231
75	A	5	3	1.00	18	0.167
76	A	2	2	1.00	18	0.111
77	A	7	3	1.00	18	0.167
78	A	5	3	1.00	18	0.167
79	A	5	3	1.00	18	0.167
80	A	3	2	1.00	20	0.100
81	A	3	2	1.00	20	0.100
82	A	3	2	1.00	20	0.100
83	A	3	2	1.00	20	0.100
84	A	2	2	1.00	20	0.100
85	A	3	2	1.00	18	0.111
86	A	3	2	1.00	15	0.133
87	A	3	2	1.00	20	0.100
88	A	3	3	1.00	20	0.150
89	A	5	3	1.00	20	0.150
90	A	5	3	1.00	20	0.150
91	A	5	3	1.00	20	0.150
92	A	5	3	1.00	23	0.130
93	A	5	3	1.00	22	0.136
94	A	3	3	1.00	20	0.150
95	A	3	2	1.00	22	0.091
96	A	3	2	1.00	22	0.091
97	A	3	2	1.00	18	0.111
98	A	9	5	1.00	18	0.278
99	A	10	6	1.00	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	9	5	1.00	18	0.278
101	A	10	6	1.00	18	0.333
102	A	9	5	1.00	29	0.172
103	A	9	5	1.00	26	0.192
104	A	9	5	1.00	24	0.208
105	A	9	5	1.00	22	0.227
106	A	9	5	1.00	25	0.200
107	A	9	5	1.00	31	0.161
108	A	9	5	1.00	32	0.156
109	A	9	5	1.00	23	0.217
110	A	9	5	1.00	25	0.200
111	A	9	5	1.00	29	0.172
112	A	9	5	1.00	32	0.156
113	A	4	4	1.00	22	0.182
114	A	5	5	1.00	24	0.208
115	A	4	4	1.00	24	0.167
116	A	4	4	1.00	24	0.167
117	A	4	4	1.00	24	0.167
118	A	4	4	1.00	24	0.167
119	A	3	3	1.00	39	0.077
120	A	2	1	1.00	17	0.059
121	A	2	1	1.00	17	0.059
122	A	2	1	1.00	17	0.059
123	A	2	1	1.00	15	0.067
124	A	3	2	1.00	17	0.118
125	A	3	3	1.00	17	0.176
126	A	3	3	1.00	17	0.176
127	A	4	4	1.00	17	0.235
128	A	2	1	1.00	19	0.053
129	A	2	1	1.00	19	0.053
130	A	2	1	1.00	17	0.059
131	A	2	1	1.00	9	0.111
132	A	3	2	1.00	19	0.105
133	A	4	3	1.00	19	0.158
134	A	5	4	1.00	19	0.210
135	A	5	5	1.00	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	5	5	1.00	19	0.263
137	A	11	7	1.00	19	0.368
138	A	11	7	1.00	19	0.368
139	A	11	7	1.00	19	0.368
140	A	9	6	1.00	17	0.353
141	A	9	6	1.00	9	0.667
142	A	12	8	1.00	19	0.421
143	A	14	9	1.00	19	0.474
144	A	11	8	1.00	19	0.421
145	A	11	8	1.00	19	0.421
146	A	10	7	1.00	17	0.412
147	A	10	7	1.00	9	0.778
148	A	22	9	1.00	19	0.474
149	A	24	10	1.00	19	0.526
150	A	6	5	0.99	21	0.238
151	A	5	5	1.00	21	0.238
152	A	4	4	1.00	21	0.190
153	A	3	3	1.00	19	0.158
154	A	3	3	1.00	21	0.143
155	A	6	6	1.00	21	0.286
156	A	7	7	1.00	21	0.333
157	A	8	8	1.00	22	0.364
158	A	7	7	1.00	22	0.318
159	A	6	6	1.00	20	0.300
160	A	2	2	1.00	22	0.091
161	A	10	10	1.00	22	0.454
162	A	11	11	1.00	22	0.500
163	A	12	11	1.00	22	0.500
164	A	6	6	1.00	21	0.286
165	A	2	2	1.00	23	0.087
166	A	3	3	1.00	29	0.103
167	A	3	3	1.00	29	0.103
168	A	3	3	1.00	22	0.136
169	A	3	3	1.00	24	0.125
170	A	2	2	1.00	21	0.095
171	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	1	1	1.00	22	0.045
173	A	3	3	1.00	21	0.143
174	F	0	0	N/A	0	N/A
175	A	0	0	0.00	0	0.000
176	A	9	6	0.96	19	0.316
177	A	7	6	0.95	19	0.316
178	A	6	5	1.00	17	0.294
179	A	2	2	1.00	9	0.222
180	A	6	5	1.00	19	0.263
181	A	8	5	1.00	19	0.263
182	A	6	4	0.95	19	0.210
183	A	5	4	0.92	19	0.210
184	A	4	3	1.00	17	0.176
185	A	1	1	1.00	9	0.111
186	A	4	3	1.00	19	0.158
187	A	5	3	1.00	19	0.158
188	A	6	3	1.00	19	0.158
189	A	4	3	1.00	24	0.125
190	A	4	3	1.00	24	0.125
191	A	3	3	1.00	24	0.125
192	A	2	2	1.00	22	0.091
193	A	5	5	1.00	24	0.208
194	A	6	6	1.00	24	0.250
195	A	6	6	1.00	26	0.231
196	A	3	3	1.00	26	0.115
197	A	4	4	1.00	26	0.154
198	A	6	6	1.00	26	0.231
199	A	5	5	1.00	28	0.179
200	A	4	4	1.00	28	0.143
201	A	3	3	1.00	28	0.107
202	A	3	3	1.00	28	0.107
203	A	4	4	1.00	28	0.143
204	A	6	6	1.00	28	0.214
205	A	5	5	1.00	29	0.172
206	A	4	4	1.00	29	0.138
207	A	3	3	1.00	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	3	3	1.00	29	0.103
209	A	4	4	1.00	29	0.138
210	A	6	6	1.00	29	0.207
211	A	2	2	1.00	19	0.105
212	A	3	3	1.67	19	0.158
213	A	7	5	0.99	31	0.161
214	A	4	3	1.00	39	0.077
215	A	4	3	1.00	39	0.077
216	A	3	3	1.00	39	0.077
217	A	2	2	1.00	37	0.054
218	A	5	5	1.00	39	0.128
219	A	6	6	1.00	39	0.154
220	A	7	7	1.00	41	0.171
221	A	6	6	1.00	41	0.146
222	A	3	3	1.00	41	0.073
223	A	4	4	1.00	41	0.098
224	A	6	6	1.00	41	0.146
225	A	6	6	1.00	20	0.300
226	A	5	5	1.00	20	0.250
227	A	4	4	1.00	18	0.222
228	A	8	7	1.00	20	0.350
229	A	1	1	1.00	20	0.050
230	A	23	13	1.00	20	0.650
231	A	26	14	1.00	20	0.700
232	A	5	5	1.00	20	0.250
233	A	4	4	1.00	20	0.200
234	A	3	3	1.00	18	0.167
235	A	4	4	1.00	20	0.200
236	A	8	8	1.00	20	0.400
237	A	9	9	1.00	20	0.450
238	A	4	4	1.00	20	0.200
239	A	2	2	1.00	20	0.100
240	A	2	2	1.00	18	0.111
241	A	9	8	1.00	20	0.400
242	A	16	11	1.00	20	0.550
243	A	23	14	1.00	20	0.700

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
244	A	2	1	1.00	22	0.045
245	A	2	1	1.00	22	0.045
246	A	2	1	1.00	22	0.045
247	A	2	1	1.00	20	0.050
248	A	3	2	1.00	22	0.091
249	A	3	3	1.00	22	0.136
250	A	3	3	1.00	22	0.136
251	A	4	4	1.00	22	0.182
252	A	2	1	1.00	24	0.042
253	A	2	1	1.00	24	0.042
254	A	2	1	1.00	22	0.045
255	A	2	1	1.00	14	0.071
256	A	3	2	1.00	24	0.083
257	A	4	3	1.00	24	0.125
258	A	5	4	1.00	24	0.167
259	A	5	4	1.00	24	0.167
260	A	5	4	1.00	24	0.167
261	A	3	3	1.00	22	0.136
262	A	3	3	1.00	23	0.130
263	A	5	3	1.00	24	0.125
264	A	5	3	1.00	24	0.125
265	A	5	3	1.00	24	0.125
266	A	3	2	1.00	22	0.091
267	A	3	2	1.00	14	0.143
268	A	6	3	1.00	24	0.125
269	A	8	4	1.00	24	0.167
270	A	4	3	1.00	24	0.125
271	A	4	3	1.00	24	0.125
272	A	4	3	1.00	22	0.136
273	A	4	3	1.00	14	0.214
274	A	10	4	1.00	24	0.167
275	A	12	5	1.00	24	0.208
276	A	7	5	1.00	24	0.208
277	A	6	5	1.00	24	0.208
278	A	5	5	1.00	24	0.208
279	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
280	A	4	4	1.00	24	0.167
281	A	4	4	1.00	24	0.167
282	A	4	4	1.00	24	0.167
283	A	5	5	1.00	24	0.208
284	A	6	5	0.99	24	0.208
285	A	7	5	1.00	24	0.208
286	A	6	6	1.00	24	0.250
287	A	5	5	1.00	24	0.208
288	A	4	4	1.00	22	0.182
289	A	4	4	1.00	14	0.286
290	A	8	7	1.30	24	0.292
291	A	8	7	1.00	24	0.292
292	A	25	10	1.00	24	0.417
293	A	7	6	1.00	24	0.250
294	A	6	5	1.00	24	0.208
295	A	5	4	1.00	22	0.182
296	A	5	5	1.00	14	0.357
297	A	13	8	1.00	24	0.333
298	A	21	10	1.50	24	0.417
299	A	27	10	1.25	24	0.417
300	A	5	5	1.00	24	0.208
301	A	4	4	1.00	24	0.167
302	A	3	3	1.00	22	0.136
303	A	1	1	1.00	14	0.071
304	A	4	4	1.00	24	0.167
305	A	9	8	1.00	24	0.333
306	A	10	9	1.00	24	0.375
307	A	6	5	1.00	24	0.208
308	A	5	5	1.00	24	0.208
309	A	4	4	1.00	24	0.167
310	A	4	4	1.00	24	0.167
311	A	4	4	1.00	22	0.182
312	A	4	4	1.00	14	0.286
313	A	9	8	1.20	24	0.333
314	A	19	10	1.00	24	0.417
315	A	29	11	1.00	24	0.458

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
316	A	8	7	1.00	24	0.292
317	A	7	7	1.00	24	0.292
318	A	6	6	1.00	24	0.250
319	A	5	5	1.00	22	0.227
320	A	5	5	1.00	14	0.357
321	A	7	7	1.00	24	0.292
322	A	7	7	1.00	24	0.292
323	A	21	10	1.00	24	0.417
324	A	9	7	1.00	24	0.292
325	A	8	7	1.00	24	0.292
326	A	7	6	1.00	24	0.250
327	A	6	5	1.00	22	0.227
328	A	6	6	1.00	14	0.429
329	A	13	8	1.00	24	0.333
330	A	21	13	1.00	24	0.542
331	A	27	13	1.00	24	0.542
332	A	6	6	1.00	24	0.250
333	A	5	5	1.00	24	0.208
334	A	4	4	1.00	22	0.182
335	A	2	2	1.00	14	0.143
336	A	2	2	1.00	24	0.083
337	A	8	8	1.00	24	0.333
338	A	9	9	1.00	24	0.375
339	A	7	6	1.00	24	0.250
340	A	6	6	1.00	24	0.250
341	A	5	5	1.00	24	0.208
342	A	5	5	1.00	24	0.208
343	A	5	5	1.00	22	0.227
344	A	5	5	1.00	14	0.357
345	A	8	8	1.00	24	0.333
346	A	17	10	1.00	24	0.417
347	A	26	11	1.00	24	0.458
348	A	7	6	1.00	24	0.250
349	A	6	6	1.00	24	0.250
350	A	5	5	1.00	24	0.208
351	A	4	4	1.00	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	4	4	1.00	14	0.286
353	A	7	6	1.00	24	0.250
354	A	7	6	1.00	24	0.250
355	A	18	9	1.00	24	0.375
356	A	8	6	1.00	24	0.250
357	A	7	6	1.00	24	0.250
358	A	6	5	1.00	24	0.208
359	A	5	4	1.00	22	0.182
360	A	5	5	1.00	14	0.357
361	A	12	7	1.00	24	0.292
362	A	19	11	1.22	24	0.458
363	A	22	10	1.00	24	0.417
364	A	5	5	1.00	24	0.208
365	A	4	4	1.00	24	0.167
366	A	3	3	1.00	22	0.136
367	A	1	1	1.00	14	0.071
368	A	3	3	1.00	24	0.125
369	A	6	6	1.00	24	0.250
370	A	7	7	1.00	24	0.292
371	A	6	5	1.00	24	0.208
372	A	5	5	1.00	24	0.208
373	A	4	4	1.00	24	0.167
374	A	4	4	1.00	24	0.167
375	A	4	4	1.00	22	0.182
376	A	4	4	1.00	14	0.286
377	A	8	7	1.00	24	0.292
378	A	15	10	1.00	24	0.417
379	A	22	11	1.00	24	0.458
380	A	5	5	1.00	26	0.192
381	A	4	4	1.00	26	0.154
382	A	3	3	1.00	24	0.125
383	A	3	3	1.00	26	0.115
384	A	6	6	1.00	26	0.231
385	A	6	6	1.00	27	0.222
386	A	5	5	1.00	27	0.185
387	A	4	4	1.00	25	0.160

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
388	A	2	2	1.00	27	0.074
389	A	8	8	1.00	27	0.296
390	A	5	5	1.00	26	0.192
391	A	2	2	1.00	28	0.071
392	A	3	3	1.00	27	0.111
393	A	3	3	1.00	29	0.103
394	A	5	5	1.00	24	0.208
395	A	4	4	1.00	24	0.167
396	A	3	3	1.00	22	0.136
397	A	4	4	1.00	24	0.167
398	A	9	8	1.26	24	0.333
399	A	0	0	0.00	0	0.000
400	A	8	7	1.00	24	0.292
401	A	7	6	0.96	24	0.250
402	A	6	5	1.00	22	0.227
403	A	2	2	1.00	14	0.143
404	A	0	0	0.00	0	0.000
405	A	0	0	0.00	0	0.000
406	A	8	8	1.00	24	0.333
407	A	10	10	1.00	26	0.385
408	A	2	2	1.00	40	0.050
409	A	2	2	1.00	40	0.050
410	A	2	2	1.00	46	0.043
411	A	2	2	1.00	46	0.043
412	A	8	8	1.00	29	0.276
413	A	10	10	1.00	31	0.323

Chapter 3

Listing of integrals

3.1 $\int \frac{c+dx^2}{a+bx^4}\,dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}\,d+\right.}{}$$

```
[Out] -1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-d*a^(1/2)+c*b^(1/
2))/a^(3/4)/b^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^
(1/2))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)
*x*2^(1/2)/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arcta
n(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2
)
```

Rubi [A] time = 0.15, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}\,d+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^4),x]
```

```
[Out] -((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)
) + ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rubi steps

$$\int \frac{c+dx^2}{a+bx^4}\,dx = \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b} + \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}$$

$$= \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} + \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}}{4\sqrt{2}a^{3/4}b^{3/4}}$$

$$= -\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$

$$= -\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{\right.}{4\sqrt{2}}$$

Mathematica [A] time = 0.08, size = 183, normalized size = 0.74

$$\frac{-\left(\sqrt{b}c-\sqrt{a}d\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\right)-2\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c -
 Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^(3
/4))
```

fricas [B] time = 0.66, size = 767, normalized size = 3.11

$$-\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*
d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^
2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^
2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a
*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log
(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d
^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b
*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt(-(b^2*
c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a
^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - a*b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^
3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 +
a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - 2*c*d)/(a*b)))
```

giac [A] time = 0.19, size = 241, normalized size = 0.98

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b
^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*log(
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^2*c - (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b^3)
```

maple [A] time = 0.01, size = 260, normalized size = 1.05

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}+\frac{\sqrt{2}\,d\arctan\left(\cdots\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2
-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*c*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x+1)+1/4*c*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x-1)+1/8*d/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2
))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*d/b/(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*d/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x-1)
```

maxima [A] time = 2.40, size = 221, normalized size = 0.89

$$\frac{\sqrt{2}\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\,\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\,\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{b}\,c-\sqrt{a}\,d\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 1/4*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)
*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*log(sqrt(b)*x^2 + s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(sqrt(b
)*c - sqrt(a)*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^
(3/4)*b^(3/4))
```

mupad [B] time = 0.38, size = 599, normalized size = 2.43

$$-2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}-\frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{-\frac{b}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^4),x)
```

```
[Out] - 2*atanh((8*b^3*c^2*x*((d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3) - (c^2*(-a^3*b^
3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (
2*b*c^3*(-a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^
2*x*((d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b
^2) - (c*d)/(8*a*b))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*b^3)^
(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a))*(-(b*c^2*(-a^3*b^3)^(1/2) - a*d
^2*(-a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) - 2*atanh((8*b^3*c
^2*x*((c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) - (d^2*(-a^3*b^3)
^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^3)
^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*x*((c^2*(-a^3*b^
3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) - (d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3
))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^3)^(1/2))/a^2 + (2*c*
d^2*(-a^3*b^3)^(1/2))/a))*(-(a*d^2*(-a^3*b^3)^(1/2) - b*c^2*(-a^3*b^3)^(1/2
) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

sympy [A] time = 0.69, size = 109, normalized size = 0.44

$$\mathrm{RootSum}\left(256t^4a^3b^3+64t^2a^2b^2cd+a^2d^4+2abc^2d^2+b^2c^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^3b^2d+12ta^2bcd^2-4tab^2c^3}{a^2d^4-b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c*
*2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d + 12*_t*a*
*2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

3.2 $\int \frac{c-dx^2}{a+bx^4} \, dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}\,c-\sqrt{a}\right.}{2}$$

```
[Out] 1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(
3/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))
/a^(3/4)/b^(3/4)*2^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1
/2))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}\,c-\sqrt{a}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)/(a + b*x^4),x]
```

```
[Out] -((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)
) + ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rubi steps

$$\int \frac{c-dx^2}{a+bx^4}\,dx = \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b} + \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}$$

$$= \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} + \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} - \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-}}{4\sqrt{2}a^{3/4}b^{3/4}}$$

$$= -\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$

$$= -\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log}{4}$$

Mathematica [A] time = 0.05, size = 184, normalized size = 0.74

$$\frac{-\left(\sqrt{a}d+\sqrt{b}c\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\right)+\left(2\sqrt{a}d-2\sqrt{b}c\right)\tan}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)/(a + b*x^4),x]
```

```
[Out] ((-2*Sqrt[b]*c + 2*Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c
 + Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^
(3/4))
```

fricas [B] time = 0.94, size = 767, normalized size = 3.11

$$-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bc\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d
)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*
c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*
sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(
b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)
/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4
 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*
d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) -
a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^
4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*
d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3
*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 + a
^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - 2*c*d)/(a*b)))
```

giac [**A**] time = 0.17, size = 241, normalized size = 0.98

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b
^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*log(
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^2*c + (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b^3)
```

maple [**A**] time = 0.00, size = 260, normalized size = 1.05

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}-\frac{\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)/(b*x^4+a),x)
```

```
[Out] 1/8*(a/b)^(1/4)*2^(1/2)/a*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2
-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4*(a/b)^(1/4)*2^(1/2)/a*c*arctan(2^(
1/2)/(a/b)^(1/4)*x+1)+1/4*(a/b)^(1/4)*2^(1/2)/a*c*arctan(2^(1/2)/(a/b)^(1/4
)*x-1)-1/8/(a/b)^(1/4)*2^(1/2)/b*d*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2
))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))-1/4/(a/b)^(1/4)*2^(1/2)/b*d*arc
tan(2^(1/2)/(a/b)^(1/4)*x+1)-1/4/(a/b)^(1/4)*2^(1/2)/b*d*arctan(2^(1/2)/(a/
b)^(1/4)*x-1)
```

maxima [A] time = 2.34, size = 221, normalized size = 0.89

$$\frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 1/4*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)
*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*log(sqrt(b)*x^2 + s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(sqrt(b
)*c + sqrt(a)*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^
(3/4)*b^(3/4))
```

mupad [B] time = 0.26, size = 603, normalized size = 2.44

$$2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\sqrt{-a^3\,b^3}}{a}}-\frac{8\,a\,b^2\,d^2\,x\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{\frac{a}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)/(a + b*x^4),x)
```

```
[Out] 2*atanh((8*b^3*c^2*x*((c*d)/(8*a*b) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) +
 (d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*
b*c^3*(-a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*
x*((c*d)/(8*a*b) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(-a^3*b^3)^(1
/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^3)^(1
/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a))*((a*d^2*(-a^3*b^3)^(1/2) - b*c^2*
(-a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) + 2*atanh((8*b^3*c^2*
x*((c*d)/(8*a*b) + (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(-a^3*b^3)^(1
/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*b^3)^(1
/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*x*((c*d)/(8*a*b) +
(c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3))^
(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*b^3)^(1/2))/a^2 - (2*c*d^2
*(-a^3*b^3)^(1/2))/a))*((b*c^2*(-a^3*b^3)^(1/2) - a*d^2*(-a^3*b^3)^(1/2) +
2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

sympy [A] time = 0.68, size = 110, normalized size = 0.45

$$-\operatorname{RootSum}\left(256t^4a^3b^3-64t^2a^2b^2cd+a^2d^4+2abc^2d^2+b^2c^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^3b^2d-12ta^2bcd^2+4tab^2}{a^2d^4-b^2c^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)/(b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c
**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d - 12*_t*a
**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

3.3 $\int \frac{c+dx^2}{a-bx^4} dx$

Optimal. Leaf size=86

$$\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

```
[Out] 1/2*arctan(b^(1/4)*x/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*ar
ctanh(b^(1/4)*x/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1167, 205, 208}

$$\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{a-bx^4}\,dx &= \frac{1}{2}\left(-\frac{\sqrt{b}\,c}{\sqrt{a}}+d\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+d\right)\int \frac{1}{\sqrt{a}\,\sqrt{b}-bx^2}\,dx\\ &= \frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 95, normalized size = 1.10

$$\frac{2\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + Sqrt[
a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b^(
3/4))
```

fricas [B] time = 0.62, size = 755, normalized size = 8.78

$$\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\vphantom{x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/
(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^
2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4
+ 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt
((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c
^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*
b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 +
a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b
*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x +
 (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3
 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^
3)) - 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d
^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt
((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)*
sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*
b)))
```

giac [B] time = 0.18, size = 230, normalized size = 2.67

$$-\frac{\sqrt{2}\left(b^2c+\sqrt{-ab}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c-\sqrt{\vphantom{x}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a
/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c - sqrt(-a*b)*b
*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^
(3/4) - 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*log(x^2 + sqrt(2)*x*(-a/b)^(1/
4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*log(
x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

maple [B] time = 0.00, size = 122, normalized size = 1.42

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a}-\frac{d\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*c*(a/b)^(1/4)/a
*arctan(x/(a/b)^(1/4))-1/2*d/b/(a/b)^(1/4)*arctan(x/(a/b)^(1/4))+1/4*d/b/(a
/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 2.29, size = 109, normalized size = 1.27

$$\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(b)*c - sqrt(a)*d)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(b)*c + sqrt(a)*d)*log((sqrt(b)
*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b))
```

mupad [B] time = 4.64, size = 579, normalized size = 6.73

$$2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}+\frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{-\frac{a\,d^2\,\sqrt{a}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a - b*x^4),x)
```

```
[Out] 2*atanh((8*b^3*c^2*x*((c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) -
(d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*
c^3*(a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((
c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(a^3*b^3)^(1/2))/(
16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2))/a^
2 - (2*c*d^2*(a^3*b^3)^(1/2))/a))*(-(a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3
)^(1/2) - 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) + 2*atanh((8*b^3*c^2*x*((c*d)/
(8*a*b) + (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(a^3*b^3)^(1/2))/(16*a^
2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (
2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((c*d)/(8*a*b) + (c^2*(a^3*b^3
)^(1/2))/(16*a^3*b^2) + (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c
^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2)
)/a))*((a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*
a^3*b^3))^(1/2)
```

sympy [A] time = 0.73, size = 110, normalized size = 1.28

$$-\operatorname{RootSum}\left(256t^4a^3b^3-64t^2a^2b^2cd-a^2d^4+2abc^2d^2-b^2c^4,\left(t\mapsto t\log\left(x+\frac{-64t^3a^3b^2d+12ta^2bcd^2+4tab^2c^3}{a^2d^4-b^2c^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c
**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d + 12*_t*
a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

3.4 $\int \frac{c-dx^2}{a-bx^4}\,dx$

Optimal. Leaf size=86

$$\frac{\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

```
[Out] 1/2*arctanh(b^(1/4)*x/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*a
rctan(b^(1/4)*x/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1167, 205, 208}

$$\frac{\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}\int \frac{c-dx^2}{a-bx^4}\,dx&=\frac{1}{2}\left(-\frac{\sqrt{b}\,c}{\sqrt{a}}-d\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(\frac{\sqrt{b}\,c}{\sqrt{a}}-d\right)\int \frac{1}{\sqrt{a}\,\sqrt{b}-bx^2}\,dx\\&=\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 95, normalized size = 1.10

$$\frac{2\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c - Sqrt[
a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b^(
3/4))
```

fricas [B] time = 0.77, size = 755, normalized size = 8.78

$$\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)
/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d
^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^
4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*s
qrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^
2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a
^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^
2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*
a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*
x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*
c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*
b^3)) - 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqr
t((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)
*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*
b)))
```

giac [B] time = 0.32, size = 228, normalized size = 2.65

$$-\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c+\sqrt{-ab}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c+\sqrt{-al\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a
/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c + sqrt(-a*b)*b
*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^
(3/4) - 1/8*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*log(x^2 + sqrt(2)*x*(-a/b)^(1/
4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*log(
x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

maple [B] time = 0.00, size = 122, normalized size = 1.42

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a}+\frac{d\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}-\frac{d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*(a/b)^(1/4)/a*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*(a/b)^(1/4)/a*c
*arctan(1/(a/b)^(1/4)*x)+1/2/(a/b)^(1/4)/b*d*arctan(1/(a/b)^(1/4)*x)-1/4/(a
/b)^(1/4)/b*d*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 2.34, size = 109, normalized size = 1.27

$$\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(b)*c + sqrt(a)*d)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(b)*c - sqrt(a)*d)*log((sqrt(b)
*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b))
```

mupad [B] time = 4.58, size = 579, normalized size = 6.73

$$-2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{-\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}+\frac{8\,a\,b^2\,d^2\,x\,\sqrt{-\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{-\frac{a}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)/(a - b*x^4),x)
```

```
[Out] - 2*atanh((8*b^3*c^2*x*(- (c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2
) - (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (
2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*
x*(- (c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(a^3*b^3)^(1
/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/
2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a))*(-(a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a
^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) - 2*atanh((8*b^3*c^2*x*(
(c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)^(1/2))/
(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2))/a
^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((c^2*(a^3*b^3)^(1/2))/(
16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2
*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(a^3*b^3)
^(1/2))/a))*((a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3)^(1/2) - 2*a^2*b^2*c*d
)/(16*a^3*b^3))^(1/2)
```

sympy [A] time = 0.94, size = 110, normalized size = 1.28

$$\mathrm{RootSum}\left(256t^4a^3b^3+64t^2a^2b^2cd-a^2d^4+2abc^2d^2-b^2c^4,\left(t\mapsto t\log\left(x+\frac{-64t^3a^3b^2d-12ta^2bcd^2-4tab^2c}{a^2d^4-b^2c^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c*
*2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d - 12*_t*a
**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

3.5 $\int \frac{2+3x^2}{4+9x^4} \, dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\sqrt{3}\,x+1\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(1-\sqrt{3}\,x\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctan(-1+x*3^(1/2))*3^(1/2)+1/6*arctan(1+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(\sqrt{3}\,x+1\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(1-\sqrt{3}\,x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] -ArcTan[1 - Sqrt[3]*x]/(2*Sqrt[3]) + ArcTan[1 + Sqrt[3]*x]/(2*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{4+9x^4}\,dx &= \frac{1}{6}\int \frac{1}{\frac{2}{3}-\frac{2x}{\sqrt{3}}+x^2}\,dx+\frac{1}{6}\int \frac{1}{\frac{2}{3}+\frac{2x}{\sqrt{3}}+x^2}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{3}\,x\right)}{2\sqrt{3}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{3}\,x\right)}{2\sqrt{3}}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{3}\,x\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(1+\sqrt{3}\,x\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.82

$$\frac{\tan^{-1}\left(\sqrt{3}\,x+1\right)-\tan^{-1}\left(1-\sqrt{3}\,x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] (-ArcTan[1 - Sqrt[3]*x] + ArcTan[1 + Sqrt[3]*x])/(2*Sqrt[3])
```

fricas [A] time = 0.52, size = 33, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{4}\sqrt{3}\left(3x^3+2x\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/4*sqrt(3)*(3*x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/2*sqrt
(3)*x)
```

giac [A] time = 0.20, size = 52, normalized size = 1.30

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{9}{8}\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{9}{8}\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x + sqrt(2)*(4/9)^(1/4))) + 1
/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x - sqrt(2)*(4/9)^(1/4)))
```

maple [B] time = 0.01, size = 122, normalized size = 3.05

$$\frac{\sqrt{6}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{6}\,\sqrt{2}\,x}{2}-1\right)}{12}+\frac{\sqrt{6}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{6}\,\sqrt{2}\,x}{2}+1\right)}{12}+\frac{\sqrt{6}\,\sqrt{2}\,\ln\left(\frac{x^2-\frac{\sqrt{6}\,\sqrt{2}\,x}{3}+\frac{2}{3}}{x^2+\frac{\sqrt{6}\,\sqrt{2}\,x}{3}+\frac{2}{3}}\right)}{48}+\frac{\sqrt{6}\,\sqrt{2}\,\ln\left(\frac{x^2+\frac{\sqrt{6}\,\sqrt{2}\,x}{3}+\frac{2}{3}}{x^2-\frac{\sqrt{6}\,\sqrt{2}\,x}{3}+\frac{2}{3}}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(9*x^4+4),x)
```

```
[Out] 1/12*6^(1/2)*2^(1/2)*arctan(1/2*6^(1/2)*x*2^(1/2)-1)+1/48*6^(1/2)*2^(1/2)*l
n((x^2+1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2-1/3*6^(1/2)*x*2^(1/2)+2/3))+1/12*6^(
1/2)*2^(1/2)*arctan(1/2*6^(1/2)*x*2^(1/2)+1)+1/48*6^(1/2)*2^(1/2)*ln((x^2-1
/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2+1/3*6^(1/2)*x*2^(1/2)+2/3))
```

maxima [A] time = 2.39, size = 39, normalized size = 0.98

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(3x+\sqrt{3}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(3x-\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(3*x + sqrt(3))) + 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(3*x - sqrt(3)))
```

mupad [B] time = 0.09, size = 29, normalized size = 0.72

$$\frac{\sqrt{3}\,\left(\mathrm{atan}\left(\frac{3\,\sqrt{3}\,x^3}{4}+\frac{\sqrt{3}\,x}{2}\right)+\mathrm{atan}\left(\frac{\sqrt{3}\,x}{2}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(9*x^4 + 4),x)
[Out] (3^(1/2)*(atan((3^(1/2)*x)/2 + (3*3^(1/2)*x^3)/4) + atan((3^(1/2)*x)/2)))/6
```

sympy [A] time = 0.12, size = 41, normalized size = 1.02

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}\,x}{2}\right)+2\operatorname{atan}\left(\frac{3\sqrt{3}\,x^{3}}{4}+\frac{\sqrt{3}\,x}{2}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(9*x**4+4),x)
[Out] sqrt(3)*(2*atan(sqrt(3)*x/2) + 2*atan(3*sqrt(3)*x**3/4 + sqrt(3)*x/2))/12
```

3.6 $\int \frac{2-3x^2}{4+9x^4}\,dx$

Optimal. Leaf size=51

$$\frac{\log\left(3x^2+2\sqrt{3}\,x+2\right)}{4\sqrt{3}}-\frac{\log\left(3x^2-2\sqrt{3}\,x+2\right)}{4\sqrt{3}}$$

```
[Out] -1/12*ln(2+3*x^2-2*x*3^(1/2))*3^(1/2)+1/12*ln(2+3*x^2+2*x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1165, 628}

$$\frac{\log\left(3x^2+2\sqrt{3}\,x+2\right)}{4\sqrt{3}}-\frac{\log\left(3x^2-2\sqrt{3}\,x+2\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] -Log[2 - 2*Sqrt[3]*x + 3*x^2]/(4*Sqrt[3]) + Log[2 + 2*Sqrt[3]*x + 3*x^2]/(4
*Sqrt[3])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{2-3x^2}{4+9x^4}\,dx &= -\frac{\int \frac{\frac{2}{\sqrt{3}}+2x}{-\frac{2}{3}-\frac{2x}{\sqrt{3}}-x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\frac{2}{\sqrt{3}}-2x}{-\frac{2}{3}+\frac{2x}{\sqrt{3}}-x^2}\,dx}{4\sqrt{3}}\\ &= -\frac{\log\left(2-2\sqrt{3}\,x+3x^2\right)}{4\sqrt{3}}+\frac{\log\left(2+2\sqrt{3}\,x+3x^2\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.86

$$\frac{\log\left(3x^2+2\sqrt{3}\,x+2\right)-\log\left(-3x^2+2\sqrt{3}\,x-2\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] (-Log[-2 + 2*Sqrt[3]*x - 3*x^2] + Log[2 + 2*Sqrt[3]*x + 3*x^2])/(4*Sqrt[3])
```

fricas [A] time = 0.69, size = 42, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\log\left(\frac{9x^4+24x^2+4\sqrt{3}\left(3x^3+2x\right)+4}{9x^4+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((9*x^4 + 24*x^2 + 4*sqrt(3)*(3*x^3 + 2*x) + 4)/(9*x^4 + 4)
)
```

giac [A] time = 0.17, size = 40, normalized size = 0.78

$$\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x+\frac{2}{3}\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(2)*(4/9)^(1/4)*x + 2/3) - 1/12*sqrt(3)*log(x^2
- sqrt(2)*(4/9)^(1/4)*x + 2/3)
```

maple [B] time = 0.00, size = 82, normalized size = 1.61

$$-\frac{\sqrt{6}\,\sqrt{2}\,\ln\left(\frac{x^2-\frac{\sqrt{6}\,\sqrt{2}\,x}{3}+\frac{2}{3}}{x^2+\frac{\sqrt{6}\,\sqrt{2}\,x}{3}+\frac{2}{3}}\right)}{48}+\frac{\sqrt{6}\,\sqrt{2}\,\ln\left(\frac{x^2+\frac{\sqrt{6}\,\sqrt{2}\,x}{3}+\frac{2}{3}}{x^2-\frac{\sqrt{6}\,\sqrt{2}\,x}{3}+\frac{2}{3}}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+2)/(9*x^4+4),x)
```

```
[Out] 1/48*6^(1/2)*2^(1/2)*ln((x^2+1/3*6^(1/2)*2^(1/2)*x+2/3)/(x^2-1/3*6^(1/2)*2^
(1/2)*x+2/3))-1/48*6^(1/2)*2^(1/2)*ln((x^2-1/3*6^(1/2)*2^(1/2)*x+2/3)/(x^2+
1/3*6^(1/2)*2^(1/2)*x+2/3))
```

maxima [A] time = 2.42, size = 39, normalized size = 0.76

$$\frac{1}{12}\sqrt{3}\log\left(3x^2+2\sqrt{3}x+2\right)-\frac{1}{12}\sqrt{3}\log\left(3x^2-2\sqrt{3}x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*log(3*x^2 + 2*sqrt(3)*x + 2) - 1/12*sqrt(3)*log(3*x^2 - 2*sqrt
(3)*x + 2)
```

mupad [B] time = 4.43, size = 21, normalized size = 0.41

$$\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3x^2+2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 - 2)/(9*x^4 + 4),x)
```

```
[Out] (3^(1/2)*atanh((2*3^(1/2)*x)/(3*x^2 + 2)))/6
```

sympy [A] time = 0.12, size = 49, normalized size = 0.96

$$-\frac{\sqrt{3}\log\left(x^2-\frac{2\sqrt{3}\,x}{3}+\frac{2}{3}\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\frac{2\sqrt{3}\,x}{3}+\frac{2}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+2)/(9*x**4+4),x)
```

```
[Out] -sqrt(3)*log(x**2 - 2*sqrt(3)*x/3 + 2/3)/12 + sqrt(3)*log(x**2 + 2*sqrt(3)*
x/3 + 2/3)/12
```

3.7 $\int \frac{2+3x^2}{4-9x^4}\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {26, 206}

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] ArcTanh[Sqrt[3/2]*x]/Sqrt[6]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{4-9x^4}\,dx &= \int \frac{1}{2-3x^2}\,dx\\ &= \frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 2.00

$$\frac{\log\left(3x+\sqrt{6}\right)-\log\left(\sqrt{6}-3x\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] (-Log[Sqrt[6] - 3*x] + Log[Sqrt[6] + 3*x])/(2*Sqrt[6])
```

fricas [B] time = 0.67, size = 29, normalized size = 1.81

$$\frac{1}{12}\sqrt{6}\log\left(\frac{3\,x^2+2\,\sqrt{6}\,x+2}{3\,x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2))
```

giac [B] time = 0.16, size = 29, normalized size = 1.81

$$\frac{1}{12}\,\sqrt{6}\,\log\left(\left|x+\frac{1}{3}\,\sqrt{6}\right|\right)-\frac{1}{12}\,\sqrt{6}\,\log\left(\left|x-\frac{1}{3}\,\sqrt{6}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*log(abs(x + 1/3*sqrt(6))) - 1/12*sqrt(6)*log(abs(x - 1/3*sqrt(
6)))
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\sqrt{6}\,\operatorname{arctanh}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(-9*x^4+4),x)
```

```
[Out] 1/6*arctanh(1/2*6^(1/2)*x)*6^(1/2)
```

maxima [B] time = 2.39, size = 25, normalized size = 1.56

$$-\frac{1}{12}\,\sqrt{6}\,\log\left(\frac{3\,x-\sqrt{6}}{3\,x+\sqrt{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6)))
```

mupad [B] time = 0.09, size = 12, normalized size = 0.75

$$\frac{\sqrt{6}\,\operatorname{atanh}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 + 2)/(9*x^4 - 4),x)
```

```
[Out] (6^(1/2)*atanh((6^(1/2)*x)/2))/6
```

sympy [B] time = 0.11, size = 32, normalized size = 2.00

$$-\frac{\sqrt{6}\,\log\left(x-\frac{\sqrt{6}}{3}\right)}{12}+\frac{\sqrt{6}\,\log\left(x+\frac{\sqrt{6}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(-9*x**4+4),x)
```

```
[Out] -sqrt(6)*log(x - sqrt(6)/3)/12 + sqrt(6)*log(x + sqrt(6)/3)/12
```

3.8 $\int \frac{2-3x^2}{4-9x^4} \, dx$

Optimal. Leaf size=16

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {26, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2-3x^2}{4-9x^4} \, dx &= \int \frac{1}{2+3x^2} \, dx\\ &= \frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

fricas [A] time = 0.66, size = 12, normalized size = 0.75

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

giac [A] time = 0.16, size = 12, normalized size = 0.75

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\sqrt{6}\arctan\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+2)/(-9*x^4+4),x)
[Out] 1/6*arctan(1/2*6^(1/2)*x)*6^(1/2)
```

maxima [A] time = 2.31, size = 12, normalized size = 0.75

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.75

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 2)/(9*x^4 - 4),x)
[Out] (6^(1/2)*atan((6^(1/2)*x)/2))/6
```

sympy [A] time = 0.11, size = 15, normalized size = 0.94

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+2)/(-9*x**4+4),x)
[Out] sqrt(6)*atan(sqrt(6)*x/2)/6
```

3.9 $\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

```
[Out] 1/2*b^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)*2^(1/2)+1/2*b^(1/4
)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1162, 617, 204}

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))) + (b
^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx &= \frac{1}{2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx+\frac{1}{2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx\\
&= \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}\\
&= -\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.80

$$\frac{\sqrt[4]{b}\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)-\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right)}{\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

```
[Out] (b^(1/4)*(-ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)]))/(Sqrt[2]*a^(1/4))
```

fricas [A] time = 0.64, size = 148, normalized size = 1.97

$$\left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4-4\sqrt{a}\sqrt{b}x^2+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3-ax\right)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right),\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*log((b*x^4 - 4*sqrt(a)*sqrt(b)*x^2 +
4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a)) + a)/(b*x^4
+ a)), sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(sqrt(b)/sqrt
(a))) + sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(b)*x
^3 + a*x)*sqrt(sqrt(b)/sqrt(a))/a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.00, size = 254, normalized size = 3.39

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\sqrt{a}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\sqrt{a}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{b}\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{8\sqrt{a}}+\frac{\sqrt{2}\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x)
```

```
[Out] 1/8/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)
^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4/a^(1/2)*b^(1/2)*(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/a^(1/2)*b^(1/2)*(a/b)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/8/(a/b)^(1/4)*2^(1/2)*ln((x^2-
(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))
+1/4/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/(a/b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 2.31, size = 100, normalized size = 1.33

$$\frac{\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 1/2*sqrt(2)*sqrt(b)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/sqrt(sqrt(a)*sqrt(b))
```

mupad [B] time = 4.79, size = 57, normalized size = 0.76

$$\frac{\sqrt{2}\,b^{1/4}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{2\,a^{1/4}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,b^{3/4}\,x^3}{2\,a^{3/4}}+\frac{\sqrt{2}\,b^{1/4}\,x}{2\,a^{1/4}}\right)\right)}{4\,a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + a^(1/2)*b^(1/2))/(a + b*x^4),x)
```

```
[Out] (2^(1/2)*b^(1/4)*(2*atan((2^(1/2)*b^(1/4)*x)/(2*a^(1/4))) + 2*atan((2^(1/2)
*b^(3/4)*x^3)/(2*a^(3/4)) + (2^(1/2)*b^(1/4)*x)/(2*a^(1/4)))))/(4*a^(1/4))
```

sympy [A] time = 0.39, size = 138, normalized size = 1.84

$$-\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a
))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log
(sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)
/4
```

3.10 $\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx$

Optimal. Leaf size=106

$$\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

```
[Out] -1/4*b^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/4)*2^(
1/2)+1/4*b^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/4)*
2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1165, 628}

$$\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

```
[Out] -(b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(2*Sqrt[2
]*a^(1/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(2*Sqrt[2]*a^(1/4))
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx &= -\frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}}\\ &= -\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 91, normalized size = 0.86

$$\frac{\sqrt[4]{b}\left(\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x-\sqrt{a}-\sqrt{b}x^2\right)\right)}{2\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

```
[Out] (b^(1/4)*(-Log[-Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x - Sqrt[b]*x^2] + Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(2*Sqrt[2]*a^(1/4))
```

fricas [A] time = 0.46, size = 151, normalized size = 1.42

$$\left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4 + 4\sqrt{a}\sqrt{b}x^2 + 4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3 + ax\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} + a}{bx^4 + a}\right), -\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*log((b*x^4 + 4*sqrt(a)*sqrt(b)*x^2 + 4
*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 + a*x)*sqrt(sqrt(b)/sqrt(a)) + a)/(b*x^4 +
a)), -sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(-sqrt(b)/sqr
t(a))) + sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(b)
*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a))/a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.00, size = 254, normalized size = 2.40

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\sqrt{a}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\sqrt{a}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{b}\,\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\sqrt{a}}-\frac{\sqrt{2}\,\arctan}{4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x)
```

```
[Out] 1/8*(a/b)^(1/4)*2^(1/2)/a^(1/2)*b^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)
^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4*(a/b)^(1/4)*2^(1/2)/a^
(1/2)*b^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*(a/b)^(1/4)*2^(1/2)/a^(1/
2)*b^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/8/(a/b)^(1/4)*2^(1/2)*ln((x^2-
(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))
-1/4/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/4/(a/b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 2.37, size = 70, normalized size = 0.66

$$\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{b}\,x^2 + \sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{4\,a^{\frac{1}{4}}} - \frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{b}\,x^2 - \sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{4\,a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*b^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/
a^(1/4) - 1/4*sqrt(2)*b^(1/4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x +
 sqrt(a))/a^(1/4)
```

mupad [B] time = 4.76, size = 43, normalized size = 0.41

$$\frac{\sqrt{2}\,b^{1/4}\,\mathrm{atanh}\left(\frac{2\,\sqrt{2}\,a^{1/4}\,b^{11/4}\,x}{2\,\sqrt{a}\,b^{5/2}+2\,b^3\,x^2}\right)}{2\,a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^2 - a^(1/2)*b^(1/2))/(a + b*x^4),x)
```

```
[Out] (2^(1/2)*b^(1/4)*atanh((2*2^(1/2)*a^(1/4)*b^(11/4)*x)/(2*a^(1/2)*b^(5/2) +
2*b^3*x^2)))/(2*a^(1/4))
```

sympy [A] time = 0.46, size = 131, normalized size = 1.24

$$-\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}\,x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\,\sqrt{a}\,x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))
/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(sq
rt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4
```

3.11 $\int \frac{d+ex^2}{d^2+e^2x^4}\,dx$

Optimal. Leaf size=75

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[Out] 1/2*arctan(-1+x*2^(1/2)*e^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)+1/2*arctan
(1+x*2^(1/2)*e^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])) + ArcT
an[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{d^2+e^2x^4}\,dx &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{e}}+x^2}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{e}}+x^2}\,dx}{2e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}\\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.80

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)-\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] (-ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]] + ArcTan[1 + (Sqrt[2]*Sqrt[e]*x)/
Sqrt[d]])/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

fricas [A] time = 0.41, size = 137, normalized size = 1.83

$$\left[-\frac{\sqrt{2}\sqrt{-de}\log\left(\frac{e^2x^4-4\,dex^2-2\,\sqrt{2}\left(ex^3-dx\right)\sqrt{-de}+d^2}{e^2x^4+d^2}\right)}{4\,de}, \frac{\sqrt{2}\sqrt{de}\arctan\left(\frac{\sqrt{2}\sqrt{de}\,x}{2\,d}\right)+\sqrt{2}\sqrt{de}\arctan\left(\frac{\sqrt{2}\left(ex^3+dx\right)\sqrt{de}}{2\,d^2}\right)}{2\,de}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(2)*sqrt(-d*e)*log((e^2*x^4 - 4*d*e*x^2 - 2*sqrt(2)*(e*x^3 - d*x)
*sqrt(-d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), 1/2*(sqrt(2)*sqrt(d*e)*arctan(1/
2*sqrt(2)*sqrt(d*e)*x/d) + sqrt(2)*sqrt(d*e)*arctan(1/2*sqrt(2)*(e*x^3 + d*
x)*sqrt(d*e)/d^2))/(d*e)]
```

giac [B] time = 0.17, size = 222, normalized size = 2.96

$$\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+2\,x\right)e^{\frac{1}{2}}}{2\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-6)}}{4\,d^2}+\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{\ldots}\right.}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*arctan(1/2*sqrt
(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) + 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 +
1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*arctan(-1/2*sqr
t(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) - 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 +
 1/8*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*e^(-6)*log(sqr
t(2)*(d^2)^(1/4)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2 - 1/8*sqrt(2)*((d
^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*e^(-6)*log(-sqrt(2)*(d^2)^(1/4
)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2
```

maple [B] time = 0.01, size = 290, normalized size = 3.87

$$\frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}-1\right)}{4d}+\frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}+1\right)}{4d}+\frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{d^2}{e^2}}}{x^2-\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{d^2}{e^2}}}\right)}{8d}+\frac{\sqrt{2}\,\arctan \ldots}{4\left(\frac{d^2}{e^2}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4+d^2),x)
```

```
[Out] 1/8/d*(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(
1/2))/(x^2-(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2)))+1/4/d*(d^2/e^2)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+1)+1/4/d*(d^2/e^2)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)+1/8/e/(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2-
(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2))/(x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(
d^2/e^2)^(1/2)))+1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/
4)*x+1)+1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)
```

maxima [A] time = 2.48, size = 74, normalized size = 0.99

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ex+\sqrt{2}\,\sqrt{d}\,\sqrt{e}\right)}{2\,\sqrt{de}}\right)}{2\,\sqrt{de}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ex-\sqrt{2}\,\sqrt{d}\,\sqrt{e}\right)}{2\,\sqrt{de}}\right)}{2\,\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*e*x + sqrt(2)*sqrt(d)*sqrt(e))/sqrt(d*e))
/sqrt(d*e) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*e*x - sqrt(2)*sqrt(d)*sqrt(e
))/sqrt(d*e))/sqrt(d*e)
```

mupad [B] time = 4.41, size = 57, normalized size = 0.76

$$\frac{\sqrt{2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{3/2}\,x^3}{2\,d^{3/2}}+\frac{\sqrt{2}\,\sqrt{e}\,x}{2\,\sqrt{d}}\right)\right)}{4\,\sqrt{d}\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(d^2 + e^2*x^4),x)
```

```
[Out] (2^(1/2)*(2*atan((2^(1/2)*e^(1/2)*x)/(2*d^(1/2))) + 2*atan((2^(1/2)*e^(3/2)
*x^3)/(2*d^(3/2)) + (2^(1/2)*e^(1/2)*x)/(2*d^(1/2)))))/(4*d^(1/2)*e^(1/2))
```

sympy [A] time = 0.22, size = 87, normalized size = 1.16

$$-\frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(-\sqrt{2}\,dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(\sqrt{2}\,dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(-1/(d*e))*log(-sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4 + s
qrt(2)*sqrt(-1/(d*e))*log(sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4
```

3.12 $\int \frac{d-ex^2}{d^2+e^2x^4}\,dx$

Optimal. Leaf size=90

$$\frac{\log\left(\sqrt{2}\sqrt{d}\sqrt{e}\,x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\log\left(-\sqrt{2}\sqrt{d}\sqrt{e}\,x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[Out] -1/4*ln(d+e*x^2-x*2^(1/2)*d^(1/2)*e^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)+1/4*ln(d
+e*x^2+x*2^(1/2)*d^(1/2)*e^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1165, 628}

$$\frac{\log\left(\sqrt{2}\sqrt{d}\sqrt{e}\,x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\log\left(-\sqrt{2}\sqrt{d}\sqrt{e}\,x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e]) + L
og[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{d-ex^2}{d^2+e^2x^4}\,dx &= -\frac{\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{e}}+2x}{-\frac{d}{e}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}-x^2}\,dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{e}}-2x}{-\frac{d}{e}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}-x^2}\,dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}\\ &= -\frac{\log\left(d-\sqrt{2}\sqrt{d}\sqrt{e}\,x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\log\left(d+\sqrt{2}\sqrt{d}\sqrt{e}\,x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 0.83

$$\frac{\log\left(\sqrt{2}\sqrt{d}\sqrt{e}\,x+d+ex^2\right)-\log\left(\sqrt{2}\sqrt{d}\sqrt{e}\,x-d-ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] (-Log[-d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x - e*x^2] + Log[d + Sqrt[2]*Sqrt[d]*Sqr
t[e]*x + e*x^2])/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

fricas [A] time = 0.41, size = 140, normalized size = 1.56

$$\left[\frac{\sqrt{2}\sqrt{de}\log\left(\frac{e^2x^4+4\,dex^2+2\,\sqrt{2}\left(ex^3+dx\right)\sqrt{de}+d^2}{e^2x^4+d^2}\right)}{4\,de}, -\frac{\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\sqrt{-de}\,x}{2\,d}\right)-\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\left(ex^3-dx\right)\sqrt{-de}}{2\,d^2}\right)}{2\,de}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(d*e)*log((e^2*x^4 + 4*d*e*x^2 + 2*sqrt(2)*(e*x^3 + d*x)*s
qrt(d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), -1/2*(sqrt(2)*sqrt(-d*e)*arctan(1/2
*sqrt(2)*sqrt(-d*e)*x/d) - sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(e*x^3 - d
*x)*sqrt(-d*e)/d^2))/(d*e)]
```

giac [B] time = 0.22, size = 222, normalized size = 2.47

$$\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}-\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+2\,x\right)e^{\frac{1}{2}}}{2\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-6)}}{4\,d^2}+\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}-\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}}\right.}{2\left(\right.}\right.}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*arctan(1/2*sqrt
(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) + 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 +
1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*arctan(-1/2*sqr
t(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) - 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 +
 1/8*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-6)*log(sqr
t(2)*(d^2)^(1/4)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2 - 1/8*sqrt(2)*((d
^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-6)*log(-sqrt(2)*(d^2)^(1/4
)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2
```

maple [B] time = 0.00, size = 290, normalized size = 3.22

$$\frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}-1\right)}{4d}+\frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}+1\right)}{4d}+\frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{d^2}{e^2}}}{x^2-\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{d^2}{e^2}}}\right)}{8d}-\frac{\sqrt{2}\,\arctan\left(\right.}{4\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4+d^2),x)
```

```
[Out] 1/8*(d^2/e^2)^(1/4)*2^(1/2)/d*ln((x^2+(d^2/e^2)^(1/4)*2^(1/2)*x+(d^2/e^2)^(
1/2))/(x^2-(d^2/e^2)^(1/4)*2^(1/2)*x+(d^2/e^2)^(1/2)))+1/4*(d^2/e^2)^(1/4)*
2^(1/2)/d*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+1)+1/4*(d^2/e^2)^(1/4)*2^(1/2)/d
*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)-1/8/(d^2/e^2)^(1/4)*2^(1/2)/e*ln((x^2-
(d^2/e^2)^(1/4)*2^(1/2)*x+(d^2/e^2)^(1/2))/(x^2+(d^2/e^2)^(1/4)*2^(1/2)*x+(
d^2/e^2)^(1/2)))-1/4/(d^2/e^2)^(1/4)*2^(1/2)/e*arctan(2^(1/2)/(d^2/e^2)^(1/
4)*x+1)-1/4/(d^2/e^2)^(1/4)*2^(1/2)/e*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)
```

maxima [A] time = 2.41, size = 62, normalized size = 0.69

$$\frac{\sqrt{2}\log\left(ex^2+\sqrt{2}\sqrt{d}\sqrt{e}\,x+d\right)}{4\sqrt{d}\sqrt{e}}-\frac{\sqrt{2}\log\left(ex^2-\sqrt{2}\sqrt{d}\sqrt{e}\,x+d\right)}{4\sqrt{d}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(e*x^2 + sqrt(2)*sqrt(d)*sqrt(e)*x + d)/(sqrt(d)*sqrt(e)) -
1/4*sqrt(2)*log(e*x^2 - sqrt(2)*sqrt(d)*sqrt(e)*x + d)/(sqrt(d)*sqrt(e))
```

mupad [B] time = 0.09, size = 41, normalized size = 0.46

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{2\sqrt{2}\sqrt{d}\,e^{7/2}\,x}{2\,e^4\,x^2+2\,d\,e^3}\right)}{2\sqrt{d}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(d^2 + e^2*x^4),x)
```

```
[Out] (2^(1/2)*atanh((2*2^(1/2)*d^(1/2)*e^(7/2)*x)/(2*d*e^3 + 2*e^4*x^2)))/(2*d^(
1/2)*e^(1/2))
```

sympy [A] time = 0.23, size = 80, normalized size = 0.89

$$-\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(-\sqrt{2}\,dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(\sqrt{2}\,dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(1/(d*e))*log(-sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4 + sqr
t(2)*sqrt(1/(d*e))*log(sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4
```

3.13 $\int \frac{5+2x^2}{-1+x^4} \, dx$

Optimal. Leaf size=13

$$-\frac{3}{2}\tan^{-1}(x)-\frac{7}{2}\tanh^{-1}(x)$$

```
[Out] -3/2*arctan(x)-7/2*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1167, 207, 203}

$$-\frac{3}{2}\tan^{-1}(x)-\frac{7}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x^2)/(-1 + x^4),x]
[Out] (-3*ArcTan[x])/2 - (7*ArcTanh[x])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}\int \frac{5+2x^2}{-1+x^4}\,dx &= -\left(\frac{3}{2}\int \frac{1}{1+x^2}\,dx\right)+\frac{7}{2}\int \frac{1}{-1+x^2}\,dx\\ &= -\frac{3}{2}\tan^{-1}(x)-\frac{7}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.92

$$\frac{7}{4}\log(1-x)-\frac{7}{4}\log(x+1)-\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x^2)/(-1 + x^4),x]
[Out] (-3*ArcTan[x])/2 + (7*Log[1 - x])/4 - (7*Log[1 + x])/4
```

fricas [A] time = 0.40, size = 17, normalized size = 1.31

$$-\frac{3}{2}\arctan(x)-\frac{7}{4}\log(x+1)+\frac{7}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="fricas")
[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

giac [B] time = 0.16, size = 19, normalized size = 1.46

$$-\frac{3}{2}\arctan(x)-\frac{7}{4}\log(|x+1|)+\frac{7}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="giac")
[Out] -3/2*arctan(x) - 7/4*log(abs(x + 1)) + 7/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 18, normalized size = 1.38

$$-\frac{3\arctan(x)}{2}-\frac{7\ln(x+1)}{4}+\frac{7\ln(x-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+5)/(x^4-1),x)
[Out] 7/4*ln(x-1)-7/4*ln(x+1)-3/2*arctan(x)
```

maxima [A] time = 2.35, size = 17, normalized size = 1.31

$$-\frac{3}{2}\arctan(x)-\frac{7}{4}\log(x+1)+\frac{7}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="maxima")
[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

mupad [B] time = 0.04, size = 9, normalized size = 0.69

$$-\frac{3\,\mathrm{atan}(x)}{2}-\frac{7\,\mathrm{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 5)/(x^4 - 1),x)
[Out] - (3*atan(x))/2 - (7*atanh(x))/2
```

sympy [A] time = 0.20, size = 22, normalized size = 1.69

$$\frac{7\log(x-1)}{4}-\frac{7\log(x+1)}{4}-\frac{3\,\mathrm{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+5)/(x**4-1),x)
[Out] 7*log(x - 1)/4 - 7*log(x + 1)/4 - 3*atan(x)/2
```

3.14 $\int \frac{1+bx^2}{\sqrt{1-b^2x^4}} dx$

Optimal. Leaf size=16

$$\frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}$$

```
[Out] EllipticE(x*b^(1/2),I)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1199, 424}

$$\frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx &= \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx\\ &= \frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 2.81

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)+\frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + (b*x^3*Hypergeometric2F1[1/2,
 3/4, 7/4, b^2*x^4])/3
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-b^2x^4+1}}{bx^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b^2*x^4 + 1)/(b*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

maple [B] time = 0.02, size = 100, normalized size = 6.25

$$\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{\sqrt{-b^2x^4+1}\,\sqrt{b}}-\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(-\operatorname{EllipticE}\left(\sqrt{b}\,x,i\right)+\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{-b^2x^4+1}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(-b^2*x^4+1)^(1/2),x)
```

```
[Out] -1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/2)/(-b^2*x^4+1)^(1/2)*(EllipticF(b
^(1/2)*x,I)-EllipticE(b^(1/2)*x,I))+1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1
/2)/(-b^2*x^4+1)^(1/2)*EllipticF(b^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{b\,x^2+1}{\sqrt{1-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + 1)/(1 - b^2*x^4)^(1/2),x)
```

```
[Out] int((b*x^2 + 1)/(1 - b^2*x^4)^(1/2), x)
```

sympy [B] time = 2.55, size = 70, normalized size = 4.38

$$\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

```
[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(4
*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(2
*I*pi))/(4*gamma(5/4))
```

3.15 $\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx$

Optimal. Leaf size=35

$$\frac{2F\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}-\frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}$$

```
[Out] -EllipticE(x*b^(1/2),I)/b^(1/2)+2*EllipticF(x*b^(1/2),I)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1199, 423, 424, 248, 221}

$$\frac{2F\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}-\frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] -(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*EllipticF[ArcSin[Sqrt[b]*x
], -1])/Sqrt[b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = \int \frac{\sqrt{1-bx^2}}{\sqrt{1+bx^2}}\,dx$$
$$= 2\int \frac{1}{\sqrt{1-bx^2}\,\sqrt{1+bx^2}}\,dx - \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx$$
$$= -\frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}} + 2\int \frac{1}{\sqrt{1-b^2x^4}}\,dx$$
$$= -\frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}} + \frac{2F\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 1.29

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right) - \frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] - (b*x^3*Hypergeometric2F1[1/2,
 3/4, 7/4, b^2*x^4])/3
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b^2x^4+1}}{bx^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b^2*x^4 + 1)/(b*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^2-1}{\sqrt{-b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

maple [B] time = 0.01, size = 99, normalized size = 2.83

$$\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\text{EllipticF}\left(\sqrt{b}\,x, i\right)}{\sqrt{-b^2x^4+1}\,\sqrt{b}} + \frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(-\text{EllipticE}\left(\sqrt{b}\,x, i\right)+\text{EllipticF}\left(\sqrt{b}\,x, i\right)\right)}{\sqrt{-b^2x^4+1}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x)
```

```
[Out] 1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/2)/(-b^2*x^4+1)^(1/2)*(EllipticF(b^
(1/2)*x,I)-EllipticE(b^(1/2)*x,I))+1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/
2)/(-b^2*x^4+1)^(1/2)*EllipticF(b^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx^2 - 1}{\sqrt{-b^2x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{b\,x^2 - 1}{\sqrt{1 - b^2\,x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^2 - 1)/(1 - b^2*x^4)^(1/2),x)
```

```
[Out] -int((b*x^2 - 1)/(1 - b^2*x^4)^(1/2), x)
```

sympy [B] time = 2.93, size = 70, normalized size = 2.00

$$-\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\,b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

```
[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(
4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(
2*I*pi))/(4*gamma(5/4))
```

3.16 $\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{b^2x^4-1}}$$

```
[Out] EllipticE(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1200, 1199, 424}

$$\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] (Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2
*x^4])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx &= \frac{\sqrt{1-b^2x^4}\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}}\\ &= \frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx}{\sqrt{-1+b^2x^4}}\\ &= \frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{-1+b^2x^4}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 74, normalized size = 1.72

$$\frac{\sqrt{1-b^2x^4}\left(3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)+bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)\right)}{3\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] (Sqrt[1 - b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b*x^3*H
ypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[-1 + b^2*x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^4-1}}{bx^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^4 - 1)/(b*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2+1}{\sqrt{b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

maple [B] time = 0.01, size = 107, normalized size = 2.49

$$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\operatorname{EllipticF}\left(\sqrt{-b}\,x, i\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(-\operatorname{EllipticE}\left(\sqrt{-b}\,x, i\right)+\operatorname{EllipticF}\left(\sqrt{-b}\right.\right.}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(b^2*x^4-1)^(1/2),x)
```

```
[Out] 1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*(EllipticF(
x*(-b)^(1/2),I)-EllipticE(x*(-b)^(1/2),I))+1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b
*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*EllipticF(x*(-b)^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2+1}{\sqrt{b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{b\,x^2+1}{\sqrt{b^2\,x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + 1)/(b^2*x^4 - 1)^(1/2),x)
```

```
[Out] int((b*x^2 + 1)/(b^2*x^4 - 1)^(1/2), x)
```

sympy [**A**] time = 2.32, size = 61, normalized size = 1.42

$$-\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) -
I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

3.17 $\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{1-b^2x^4}\,F\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{b^2x^4-1}}-\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{b^2x^4-1}}$$

```
[Out] -EllipticE(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)+2*Elli
pticF(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1200, 1199, 423, 424, 248, 221}

$$\frac{2\sqrt{1-b^2x^4}\,F\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{b^2x^4-1}}-\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] -((Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b
^2*x^4])) + (2*Sqrt[1 - b^2*x^4]*EllipticF[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]
*Sqrt[-1 + b^2*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx &= \frac{\sqrt{1-b^2x^4}\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= \frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1-bx^2}}{\sqrt{1+bx^2}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= -\frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx}{\sqrt{-1+b^2x^4}}+\frac{\left(2\sqrt{1-b^2x^4}\right)\int \frac{1}{\sqrt{1-bx^2}\sqrt{1+bx^2}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= -\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}+\frac{\left(2\sqrt{1-b^2x^4}\right)\int \frac{1}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= -\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}+\frac{2\sqrt{1-b^2x^4}\,F\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 74, normalized size = 0.83

$$
-\frac{\sqrt{1-b^2x^4}\left(bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)-3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)\right)}{3\sqrt{b^2x^4-1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] -1/3*(Sqrt[1 - b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/Sqrt[-1 + b^2*x^4]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$
\text{integral}\left(-\frac{\sqrt{b^2x^4-1}}{bx^2+1},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b^2*x^4 - 1)/(b*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int -\frac{bx^2-1}{\sqrt{b^2x^4-1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

maple [A] time = 0.01, size = 108, normalized size = 1.21

$$\frac{\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{-b}\,x,i\right)}{\sqrt{-b}\,\sqrt{b^{2}x^{4}-1}}-\frac{\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\,\left(-\operatorname{EllipticE}\left(\sqrt{-b}\,x,i\right)+\operatorname{EllipticF}\left(\sqrt{-b}\right.\right.}{\sqrt{-b}\,\sqrt{b^{2}x^{4}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(b^2*x^4-1)^(1/2),x)
```

```
[Out] -1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*(EllipticF
((-b)^(1/2)*x,I)-EllipticE((-b)^(1/2)*x,I))+1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-
b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*EllipticF((-b)^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx^{2}-1}{\sqrt{b^{2}x^{4}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{b\,x^{2}-1}{\sqrt{b^{2}\,x^{4}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^2 - 1)/(b^2*x^4 - 1)^(1/2),x)
```

```
[Out] -int((b*x^2 - 1)/(b^2*x^4 - 1)^(1/2), x)
```

sympy [A] time = 2.19, size = 60, normalized size = 0.67

$$\frac{ibx^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|b^{2}x^{4}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|b^{2}x^{4}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) - I
*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

3.18 $\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

```
[Out] -x*(b^2*x^4+1)^(1/2)/(b*x^2+1)+(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2)
/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*(
(b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1196}

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] -((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 +
 b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx=-\frac{x\sqrt{1+b^2x^4}}{1+bx^2}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.53

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)-\frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] - (b*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, -(b^2*x^4)])/3
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{bx^2-1}{\sqrt{b^2x^4+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^2-1}{\sqrt{b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

maple [C] time = 0.01, size = 120, normalized size = 1.35

$$\frac{\sqrt{-ib\,x^2+1}\;\sqrt{ib\,x^2+1}\;\operatorname{EllipticF}\left(\sqrt{ib}\;x,i\right)}{\sqrt{ib}\;\sqrt{b^2x^4+1}}-\frac{i\sqrt{-ib\,x^2+1}\;\sqrt{ib\,x^2+1}\;\left(-\operatorname{EllipticE}\left(\sqrt{ib}\;x,i\right)+\operatorname{EllipticF}\left(\sqrt{ib}\right.\right.}{\sqrt{ib}\;\sqrt{b^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(b^2*x^4+1)^(1/2),x)
[Out] -I/(I*b)^(1/2)*(1-I*b*x^2)^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*(Ellip
ticF(x*(I*b)^(1/2),I)-EllipticE(x*(I*b)^(1/2),I))+1/(I*b)^(1/2)*(1-I*b*x^2)
^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*EllipticF(x*(I*b)^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx^2-1}{\sqrt{b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{b\,x^2-1}{\sqrt{b^2\,x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^2 - 1)/(b^2*x^4 + 1)^(1/2),x)
[Out] -int((b*x^2 - 1)/(b^2*x^4 + 1)^(1/2), x)
```

sympy [C] time = 2.20, size = 66, normalized size = 0.74

$$-\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I*
pi))/(4*gamma(5/4))
```

3.19 $\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx$

Optimal. Leaf size=152

$$\frac{x\sqrt{b^2x^4+1}}{bx^2+1}+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}$$

```
[Out] x*(b^2*x^4+1)^(1/2)/(b*x^2+1)-(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2)/
cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*((
b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x^4+1)^(1/2)+(b*x^2+1)*(cos(2*ar
ctan(x*b^(1/2)))^2)^(1/2)/cos(2*arctan(x*b^(1/2)))*EllipticF(sin(2*arctan(x
*b^(1/2))),1/2*2^(1/2))*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x^4+1)
^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1198, 220, 1196}

$$\frac{x\sqrt{b^2x^4+1}}{bx^2+1}+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] (x*Sqrt[1 + b^2*x^4])/(1 + b*x^2) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*
x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]) +
((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x
], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = 2\int \frac{1}{\sqrt{1+b^2x^4}}\,dx - \int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx$$

$$= \frac{x\sqrt{1+b^2x^4}}{1+bx^2} - \frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{1+b^2x^4}} + \frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt{\right.\right.}{\sqrt{b}\,\sqrt{1+b^2x^4}}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.31

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right) + \frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + (b*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, -(b^2*x^4)])/3
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2+1}{\sqrt{b^2x^4+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2+1}{\sqrt{b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

maple [C] time = 0.00, size = 120, normalized size = 0.79

$$\frac{\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\operatorname{EllipticF}\left(\sqrt{ib}\,x,i\right)}{\sqrt{ib}\,\sqrt{b^2x^4+1}} + \frac{i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\left(-\operatorname{EllipticE}\left(\sqrt{ib}\,x,i\right)+\operatorname{EllipticF}\left(\sqrt{ib}\,x,\right.\right.}{\sqrt{ib}\,\sqrt{b^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(b^2*x^4+1)^(1/2),x)
```

```
[Out] I/(I*b)^(1/2)*(-I*b*x^2+1)^(1/2)*(I*b*x^2+1)^(1/2)/(b^2*x^4+1)^(1/2)*(Ellip
ticF((I*b)^(1/2)*x,I)-EllipticE((I*b)^(1/2)*x,I))+1/(I*b)^(1/2)*(-I*b*x^2+1
)^(1/2)*(I*b*x^2+1)^(1/2)/(b^2*x^4+1)^(1/2)*EllipticF((I*b)^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2+1}{\sqrt{b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2+1}{\sqrt{b^2\,x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + 1)/(b^2*x^4 + 1)^(1/2),x)
```

```
[Out] int((b*x^2 + 1)/(b^2*x^4 + 1)^(1/2), x)
```

sympy [C] time = 2.60, size = 66, normalized size = 0.43

$$\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*g
amma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I*p
i))/(4*gamma(5/4))
```

3.20 $\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-b^2x^4-1}}$$

```
[Out] x*(-b^2*x^4-1)^(1/2)/(b*x^2+1)+(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2)
/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*(
(b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1196}

$$\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] (x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b
*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx=\frac{x\sqrt{-1-b^2x^4}}{1+bx^2}+\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-1-b^2x^4}}$$

Mathematica [C] time = 0.03, size = 76, normalized size = 0.84

$$-\frac{\sqrt{b^2x^4+1}\left(bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)-3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)\right)}{3\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] -1/3*(Sqrt[1 + b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)]
+ b*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/Sqrt[-1 - b^2*x^4]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\frac{bx\mathrm{integral}\left(-\frac{\sqrt{-b^2x^4-1}\left(bx^2-1\right)}{b^3x^6+bx^2},x\right)+\sqrt{-b^2x^4-1}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*x*integral(-sqrt(-b^2*x^4 - 1)*(b*x^2 - 1)/(b^3*x^6 + b*x^2), x) + sqrt(
-b^2*x^4 - 1))/(b*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

maple [C] time = 0.01, size = 122, normalized size = 1.36

$$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\mathrm{EllipticF}\left(\sqrt{-ib}\,x,i\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{-ib}\,x,i\right)+\mathrm{EllipticF}\left(\right.\right.}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x)
```

```
[Out] I/(-I*b)^(1/2)*(I*b*x^2+1)^(1/2)*(-I*b*x^2+1)^(1/2)/(-b^2*x^4-1)^(1/2)*(Ell
ipticF(x*(-I*b)^(1/2),I)-EllipticE(x*(-I*b)^(1/2),I))+1/(-I*b)^(1/2)*(I*b*x
^2+1)^(1/2)*(-I*b*x^2+1)^(1/2)/(-b^2*x^4-1)^(1/2)*EllipticF(x*(-I*b)^(1/2),
I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{b\,x^2-1}{\sqrt{-b^2\,x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^2 - 1)/(- b^2*x^4 - 1)^(1/2),x)
```

```
[Out] -int((b*x^2 - 1)/(- b^2*x^4 - 1)^(1/2), x)
```

sympy [C] time = 2.10, size = 70, normalized size = 0.78

$$\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4
*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar
(I*pi))/(4*gamma(5/4))
```

3.21 $\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal. Leaf size=156

$$-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-b^2x^4-1}}-\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-b^2x^4-1}}$$

```
[Out] -x*(-b^2*x^4-1)^(1/2)/(b*x^2+1)-(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2
)/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*
((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b^2*x^4-1)^(1/2)+(b*x^2+1)*(cos(2
*arctan(x*b^(1/2)))^2)^(1/2)/cos(2*arctan(x*b^(1/2)))*EllipticF(sin(2*arcta
n(x*b^(1/2))),1/2*2^(1/2))*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b^2*x^
4-1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1198, 220, 1196}

$$-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-b^2x^4-1}}-\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] -((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2)) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1
+ b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4
]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt
[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = 2\int \frac{1}{\sqrt{-1-b^2x^4}}\,dx - \int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx$$
$$= -\frac{x\sqrt{-1-b^2x^4}}{1+bx^2} - \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,F\left(2\tan^{-1}\right.}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

Mathematica [C] time = 0.02, size = 76, normalized size = 0.49

$$\frac{\sqrt{b^2x^4+1}\left(3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)+bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)\right)}{3\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] (Sqrt[1 + b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + b*x^
3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/(3*Sqrt[-1 - b^2*x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\frac{bx\text{integral}\left(-\frac{\sqrt{-b^2x^4-1}\,(bx^2+1)}{b^3x^6+bx^2},x\right)-\sqrt{-b^2x^4-1}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*x*integral(-sqrt(-b^2*x^4 - 1)*(b*x^2 + 1)/(b^3*x^6 + b*x^2), x) - sqrt(
-b^2*x^4 - 1))/(b*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

maple [C] time = 0.00, size = 122, normalized size = 0.78

$$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\;\mathrm{EllipticF}\left(\sqrt{-ib}\;x,i\right)}{\sqrt{-ib}\;\sqrt{-b^2x^4-1}} - \frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\;\left(-\mathrm{EllipticE}\left(\sqrt{-ib}\;x,i\right)+\mathrm{EllipticF}\left(\sqrt{-\right.\right.}{\sqrt{-ib}\;\sqrt{-b^2x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(-b^2*x^4-1)^(1/2),x)
```

```
[Out] -I/(-I*b)^(1/2)*(I*b*x^2+1)^(1/2)*(-I*b*x^2+1)^(1/2)/(-b^2*x^4-1)^(1/2)*(El
lipticF((-I*b)^(1/2)*x,I)-EllipticE((-I*b)^(1/2)*x,I))+1/(-I*b)^(1/2)*(I*b*
x^2+1)^(1/2)*(-I*b*x^2+1)^(1/2)/(-b^2*x^4-1)^(1/2)*EllipticF((-I*b)^(1/2)*x
,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2+1}{\sqrt{-b^2\,x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + 1)/(- b^2*x^4 - 1)^(1/2),x)
```

```
[Out] int((b*x^2 + 1)/(- b^2*x^4 - 1)^(1/2), x)
```

sympy [C] time = 2.06, size = 71, normalized size = 0.46

$$-\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(
4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_pola
r(I*pi))/(4*gamma(5/4))
```

3.22 $\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] EllipticE(c*x,I)/c
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {424}

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

maple [C] time = 0.03, size = 15, normalized size = 1.50

$$\frac{\mathrm{EllipticE}\left(cx\,\mathrm{csgn}(c), i\right)\mathrm{csgn}(c)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] EllipticE(x*csgn(c)*c,I)*csgn(c)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{\sqrt{c^2\,x^2+1}}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(1 - c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.23 $\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=10

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] EllipticE(c*x,I)/c
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1199, 424}

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx &= \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx\\ &= \frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 4.70

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^4x^4\right)+\frac{1}{3}c^2x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] + (c^2*x^3*Hypergeometric2F1[1/
2, 3/4, 7/4, c^4*x^4])/3
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^4x^4+1}}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^4*x^4 + 1)/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c^2x^2+1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

maple [B] time = 0.01, size = 118, normalized size = 11.80

$$\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\,\mathrm{EllipticF}\left(\sqrt{c^2}\,x,i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}-\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{c^2}\,x,i\right)+\mathrm{EllipticF}\left(\sqrt{c^2}\right.\right.}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] 1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*Ellip
ticF(x*(c^2)^(1/2),I)-1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-
c^4*x^4+1)^(1/2)*(EllipticF(x*(c^2)^(1/2),I)-EllipticE(x*(c^2)^(1/2),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c^2x^2+1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{c^2\,x^2+1}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((c^2*x^2 + 1)/(1 - c^4*x^4)^(1/2), x)
```

sympy [B] time = 2.05, size = 71, normalized size = 7.10

$$\frac{c^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi))
/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_pola
r(2*I*pi))/(4*gamma(5/4))
```

3.24 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] -EllipticE(c*x,I)/c+2*EllipticF(c*x,I)/c
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {423, 424, 248, 221}

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx &= 2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}\,dx-\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx\\ &= -\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}+2\int \frac{1}{\sqrt{1-c^4x^4}}\,dx\\ &= -\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}+\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{E\left(\sin^{-1}\left(\sqrt{-c^2}\,x\right)\middle|-1\right)}{\sqrt{-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
[Out] EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

maple [C] time = 0.02, size = 28, normalized size = 1.22

$$\frac{\left(-\mathrm{EllipticE}\left(cx\,\mathrm{csgn}(c), i\right)+2\,\mathrm{EllipticF}\left(cx\,\mathrm{csgn}(c), i\right)\right)\mathrm{csgn}(c)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x)
[Out] (2*EllipticF(c*x*csgn(c),I)-EllipticE(c*x*csgn(c),I))*csgn(c)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-c^2\,x^2}}{\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)
```

3.25 $\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] -EllipticE(c*x,I)/c+2*EllipticF(c*x,I)/c
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1199, 423, 424, 248, 221}

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$$
$$= 2\int \frac{1}{\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}\,dx - \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx$$
$$= -\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c} + 2\int \frac{1}{\sqrt{1-c^4x^4}}\,dx$$
$$= -\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c} + \frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 2.04

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^4x^4\right) - \frac{1}{3}c^2x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] - (c^2*x^3*Hypergeometric2F1[1/
2, 3/4, 7/4, c^4*x^4])/3
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^4x^4+1}}{c^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^4*x^4 + 1)/(c^2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{c^2x^2-1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

maple [B] time = 0.01, size = 117, normalized size = 5.09

$$\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\,\text{EllipticF}\left(\sqrt{c^2}\,x,i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}+\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(-\text{EllipticE}\left(\sqrt{c^2}\,x,i\right)+\text{EllipticF}\left(\sqrt{c^2}\right.\right.}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] 1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*Ellip
ticF((c^2)^(1/2)*x,I)+1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-
c^4*x^4+1)^(1/2)*(EllipticF((c^2)^(1/2)*x,I)-EllipticE((c^2)^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c^2x^2-1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{c^2\,x^2-1}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(c^2*x^2 - 1)/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] -int((c^2*x^2 - 1)/(1 - c^4*x^4)^(1/2), x)
```

sympy [B] time = 2.03, size = 71, normalized size = 3.09

$$-\frac{c^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] -c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi)
)/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_pol
ar(2*I*pi))/(4*gamma(5/4))
```

3.26 $\int \frac{d+ex^2}{d^2+bx^2+e^2x^4} dx$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

```
[Out] -arctan((-2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+arctan((2
*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[-b + 2*d*e] - 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]) + Arc
Tan[(Sqrt[-b + 2*d*e] + 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{d^2+bx^2+e^2x^4} dx &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{-b+2de}\,x}{e}+x^2} dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{-b+2de}\,x}{e}+x^2} dx}{2e}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{b+2de}{e^2}-x^2} dx, x, -\frac{\sqrt{-b+2de}}{e}+2x\right)}{e}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{b+2de}{e^2}-x^2} dx, x, \frac{\sqrt{-b+2de}}{e}+2x\right)}{e}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}+\frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}
\end{aligned}$$

Mathematica [B] time = 0.11, size = 181, normalized size = 2.21

$$\frac{\frac{\left(\sqrt{b^2-4d^2e^2}-b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}+\frac{\left(\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] (((-b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b
^2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] + ((b - 2*d*e + Sqrt[b^2
 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[
b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

fricas [A] time = 0.43, size = 162, normalized size = 1.98

$$\left[-\frac{\sqrt{-2\,de-b}\,\log\left(\frac{e^2x^4-(4\,de+b)x^2+d^2-2\left(ex^3-dx\right)\sqrt{-2\,de-b}}{e^2x^4+bx^2+d^2}\right)}{2\,(2\,de+b)},\ \frac{\sqrt{2\,de+b}\,\arctan\left(\frac{ex}{\sqrt{2\,de+b}}\right)+\sqrt{2\,de+b}\,\arctan\left(\frac{\left(e^2x^3+(de+\cdots}{2\,d^2\cdots}\right)}{2\,de+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e - b)*log((e^2*x^4 - (4*d*e + b)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e - b))/(e^2*x^4 + b*x^2 + d^2))/(2*d*e + b), (sqrt(2*d*e + b)
*arctan(e*x/sqrt(2*d*e + b)) + sqrt(2*d*e + b)*arctan((e^2*x^3 + (d*e + b)*
x)*sqrt(2*d*e + b)/(2*d^2*e + b*d)))/(2*d*e + b)]
```

giac [B] time = 1.12, size = 1642, normalized size = 20.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/4*(16*sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d^4*e^4 - 8*sqrt(2
)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*d^2*e^2 + 4*sqrt(2)*sqrt(-4*
d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^2 + sqrt(2)
*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^4 - 32*d^4*e^6 + 8*sqrt(2)*sqrt
(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^4 + 16*b^2*d^2*e^4 - 2*sqrt(2)
*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3*e^2 - 2*b^4*e^2 - sqrt(2)*sqr
t(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3 + 2*sqrt(2
)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^2 -
 4*sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d^2*e^6 - 8*b*d^2*e^6 +
 sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^4 + 2*b^3*e^4 + 8*(
4*d^2*e^2 - b^2)*d^2*e^4 - 2*(4*d^2*e^2 - b^2)*b^2*e^2 - sqrt(2)*sqrt(-4*d^
2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*e^4 + 2*(4*d^2*e^2
- b^2)*b*e^4 - 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2
*e^2 + b^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt
(-4*d^2*e^2 + b^2)*e^2)*b^2*d + 2*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2
 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d*e^2 - 8*d^3*e^6 + 2*b^2*d*e^4 - sqrt(2)*
sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d*e^4 + 2*(
4*d^2*e^2 - b^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x*e/sqrt(b + sqrt(-4*d^2*e^2
+ b^2)))/(16*d^5*e^6 - 8*b^2*d^3*e^4 + b^4*d*e^2 + 8*b*d^3*e^6 - 2*b^3*d*e^
4 - 4*d^3*e^8 + b^2*d*e^6) + 1/4*(16*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 +
 b^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2
```

```
*d^2*e^2 - 4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 +
b^2)*e^2)*b*d^2*e^2 + sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^4
+ 32*d^4*e^6 + 8*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^4
 - 16*b^2*d^2*e^4 - 2*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3*
e^2 + 2*b^4*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e
^2 + b^2)*e^2)*b^3 - 2*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*
d^2*e^2 + b^2)*e^2)*b^2*e^2 - 4*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)
*e^2)*d^2*e^6 + 8*b*d^2*e^6 + sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e
^2)*b^2*e^4 - 2*b^3*e^4 - 8*(4*d^2*e^2 - b^2)*d^2*e^4 + 2*(4*d^2*e^2 - b^2)
*b^2*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^
2)*e^2)*b*e^4 - 2*(4*d^2*e^2 - b^2)*b*e^4 + 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 +
b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2
*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*d + 2*sqrt(2)*sqrt
(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d*e^2 - 8*d^3
*e^6 + 2*b^2*d*e^4 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^
2*e^2 + b^2)*e^2)*d*e^4 + 2*(4*d^2*e^2 - b^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*
x*e/sqrt(b - sqrt(-4*d^2*e^2 + b^2)))/(16*d^5*e^6 - 8*b^2*d^3*e^4 + b^4*d*e
^2 + 8*b*d^3*e^6 - 2*b^3*d*e^4 - 4*d^3*e^8 + b^2*d*e^6)
```

maple [A] time = 0.04, size = 71, normalized size = 0.87

$$-\frac{\arctan\left(\frac{-2ex+\sqrt{2de-b}}{\sqrt{2de+b}}\right)}{\sqrt{2de+b}}+\frac{\arctan\left(\frac{2ex+\sqrt{2de-b}}{\sqrt{2de+b}}\right)}{\sqrt{2de+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x)
```

```
[Out] -arctan((-2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+arctan((2
*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{e^2x^4+bx^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 + b*x^2 + d^2), x)
```

mupad [B] time = 4.43, size = 94, normalized size = 1.15

$$\frac{\mathrm{atan}\left(\frac{e\,x}{\sqrt{b+2\,d\,e}}\right)+\mathrm{atan}\left(\frac{b^2\,x-\frac{x\,{\left(b+2\,d\,e\right)}^2}{2}+\frac{b\,x\,\left(b+2\,d\,e\right)}{2}+2\,b\,e^2\,x^3-e^2\,x^3\,\left(b+2\,d\,e\right)}{\left(b\,d-2\,d^2\,e\right)\,\sqrt{b+2\,d\,e}}\right)}{\sqrt{b+2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(b*x^2 + d^2 + e^2*x^4),x)
```

```
[Out] (atan((e*x)/(b + 2*d*e)^(1/2)) + atan((b^2*x - (x*(b + 2*d*e)^2)/2 + (b*x*(
b + 2*d*e))/2 + 2*b*e^2*x^3 - e^2*x^3*(b + 2*d*e))/((b*d - 2*d^2*e)*(b + 2*
d*e)^(1/2))))/(b + 2*d*e)^(1/2)
```

sympy [A] time = 0.54, size = 122, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b+2de}}-2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b+2de}}+2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(-1/(b + 2*d*e)) - 2*d*e*
sqrt(-1/(b + 2*d*e)))/e)/2 + sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(b*sq
rt(-1/(b + 2*d*e)) + 2*d*e*sqrt(-1/(b + 2*d*e)))/e)/2
```

3.27 $\int \frac{d+ex^2}{d^2+fx^2+e^2x^4} dx$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

```
[Out] -arctan((-2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+arctan((2
*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2*d*e - f] - 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]) + ArcT
an[(Sqrt[2*d*e - f] + 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de-f}\,x}{e}+x^2}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de-f}\,x}{e}+x^2}\,dx}{2e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2}-x^2}\,dx, x, -\frac{\sqrt{2de-f}}{e}+2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2}-x^2}\,dx, x, \frac{\sqrt{2de-f}}{e}+2x\right)}{e}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} + \frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Mathematica [B] time = 0.11, size = 181, normalized size = 2.21

$$\frac{\frac{\left(\sqrt{f^2-4d^2e^2}+2de-f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}\right)}{\sqrt{f-\sqrt{f^2-4d^2e^2}}} + \frac{\left(\sqrt{f^2-4d^2e^2}-2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e - f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[-
4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e + f + Sqrt[
-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]])/
Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

fricas [A] time = 0.42, size = 162, normalized size = 1.98

$$\left[-\frac{\sqrt{-2\,de-f}\,\log\left(\frac{e^2x^4-\left(4\,de+f\right)x^2+d^2-2\left(ex^3-dx\right)\sqrt{-2\,de-f}}{e^2x^4+fx^2+d^2}\right)}{2\left(2\,de+f\right)}, \frac{\sqrt{2\,de+f}\,\arctan\left(\frac{ex}{\sqrt{2\,de+f}}\right)+\sqrt{2\,de+f}\,\arctan\left(\frac{\left(e^2x^3+\left(de\right.\right.}{2\ldots}\right.}{2\,de+f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e - f)*log((e^2*x^4 - (4*d*e + f)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e - f))/(e^2*x^4 + f*x^2 + d^2))/(2*d*e + f), (sqrt(2*d*e + f)
*arctan(e*x/sqrt(2*d*e + f)) + sqrt(2*d*e + f)*arctan((e^2*x^3 + (d*e + f)*
x)*sqrt(2*d*e + f)/(2*d^2*e + d*f)))/(2*d*e + f)]
```

giac [B] time = 1.09, size = 1642, normalized size = 20.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/4*(16*sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^4*e^4 - 8*sqrt(2
)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f^2*e^2 + 4*sqrt(2)*sqrt(-4*
d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^2 + sqrt(2)
*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^4 - 32*d^4*e^6 + 8*sqrt(2)*sqrt
(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^4 + 16*d^2*f^2*e^4 - 2*sqrt(2)
*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3*e^2 - 2*f^4*e^2 - sqrt(2)*sqr
```

```
t(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3 + 2*sqrt(2
)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^2*e^2 -
 4*sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*e^6 - 8*d^2*f*e^6 +
 8*(4*d^2*e^2 - f^2)*d^2*e^4 + sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*
e^2)*f^2*e^4 + 2*f^3*e^4 - 2*(4*d^2*e^2 - f^2)*f^2*e^2 - sqrt(2)*sqrt(-4*d^
2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f*e^4 + 2*(4*d^2*e^2
- f^2)*f*e^4 - 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2
*e^2 + f^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt
(-4*d^2*e^2 + f^2)*e^2)*d*f^2 + 2*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2
 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f*e^2 - 8*d^3*e^6 + 2*d*f^2*e^4 - sqrt(2)*
sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d*e^4 + 2*(
4*d^2*e^2 - f^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x*e/sqrt(f + sqrt(-4*d^2*e^2
+ f^2)))/(16*d^5*e^6 - 8*d^3*f^2*e^4 + d*f^4*e^2 + 8*d^3*f*e^6 - 2*d*f^3*e^
4 - 4*d^3*e^8 + d*f^2*e^6) + 1/4*(16*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 +
 f^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2
*f^2*e^2 - 4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 +
f^2)*e^2)*d^2*f*e^2 + sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^4
+ 32*d^4*e^6 + 8*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^4
 - 16*d^2*f^2*e^4 - 2*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3*
e^2 + 2*f^4*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e
^2 + f^2)*e^2)*f^3 - 2*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*
d^2*e^2 + f^2)*e^2)*f^2*e^2 - 4*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)
*e^2)*d^2*e^6 + 8*d^2*f*e^6 - 8*(4*d^2*e^2 - f^2)*d^2*e^4 + sqrt(2)*sqrt(f*
e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^2*e^4 - 2*f^3*e^4 + 2*(4*d^2*e^2 - f^2)
*f^2*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^
2)*e^2)*f*e^4 - 2*(4*d^2*e^2 - f^2)*f*e^4 + 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 +
f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2
*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f^2 + 2*sqrt(2)*sqrt
(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f*e^2 - 8*d^3
*e^6 + 2*d*f^2*e^4 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^
2*e^2 + f^2)*e^2)*d*e^4 + 2*(4*d^2*e^2 - f^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*
x*e/sqrt(f - sqrt(-4*d^2*e^2 + f^2)))/(16*d^5*e^6 - 8*d^3*f^2*e^4 + d*f^4*e
^2 + 8*d^3*f*e^6 - 2*d*f^3*e^4 - 4*d^3*e^8 + d*f^2*e^6)
```

maple [A] time = 0.04, size = 71, normalized size = 0.87

$$-\frac{\arctan\left(\frac{-2ex+\sqrt{2de-f}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}+\frac{\arctan\left(\frac{2ex+\sqrt{2de-f}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x)
```

```
[Out] -arctan((-2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+arctan((2
*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{e^2x^4+fx^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 + f*x^2 + d^2), x)
```

mupad [B] time = 4.52, size = 98, normalized size = 1.20

$$\frac{\operatorname{atan}\left(\frac{f^2\,x-\frac{x\,\left(f+2\,d\,e\right)^2}{2}+\frac{f\,x\,\left(f+2\,d\,e\right)}{2}+2\,e^2\,f\,x^3-e^2\,x^3\,\left(f+2\,d\,e\right)}{\left(2\,d\,f-d\,\left(f+2\,d\,e\right)\right)\,\sqrt{f+2\,d\,e}}\right)+\operatorname{atan}\left(\frac{e\,x}{\sqrt{f+2\,d\,e}}\right)}{\sqrt{f+2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(f*x^2 + d^2 + e^2*x^4),x)
```

```
[Out] (atan((f^2*x - (x*(f + 2*d*e)^2)/2 + (f*x*(f + 2*d*e))/2 + 2*e^2*f*x^3 - e^
2*x^3*(f + 2*d*e))/((2*d*f - d*(f + 2*d*e))*(f + 2*d*e)^(1/2))) + atan((e*x
)/(f + 2*d*e)^(1/2)))/(f + 2*d*e)^(1/2)
```

sympy [A] time = 0.56, size = 122, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{-\frac{1}{2de+f}}-f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{-\frac{1}{2de+f}}+f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e + f)) - f*
sqrt(-1/(2*d*e + f)))/e)/2 + sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(2*d*
e*sqrt(-1/(2*d*e + f)) + f*sqrt(-1/(2*d*e + f)))/e)/2
```

3.28 $\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

```
[Out] arctanh((-2*e*x+(2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2)-arctanh
((2*e*x+(2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[b + 2*d*e] - 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e] - ArcTan
h[(Sqrt[b + 2*d*e] + 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{b+2de}\,x}{e}+x^2}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{b+2de}\,x}{e}+x^2}\,dx}{2e}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2}-x^2}\,dx,x,-\frac{\sqrt{b+2de}}{e}+2x\right)}{e}-\frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2}-x^2}\,dx,x,\frac{\sqrt{b+2de}}{e}+2x\right)}{e}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}
\end{aligned}$$

Mathematica [B] time = 0.11, size = 189, normalized size = 2.42

$$\frac{\frac{\left(\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}+\frac{\left(\sqrt{b^2-4d^2e^2}-b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] (((b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b - Sqrt[b
^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]] + ((-b - 2*d*e + Sqrt[b
^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]]])/Sq
rt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

fricas [A] time = 0.42, size = 176, normalized size = 2.26

$$\left[-\frac{\sqrt{-2\,de+b}\log\left(\frac{e^2x^4-(4\,de-b)x^2+d^2-2\left(ex^3-dx\right)\sqrt{-2\,de+b}}{e^2x^4-bx^2+d^2}\right)}{2\,(2\,de-b)},\ \frac{\sqrt{2\,de-b}\arctan\left(\frac{ex}{\sqrt{2\,de-b}}\right)+\sqrt{2\,de-b}\arctan\left(\frac{(e^2x^3+(de-b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e + b)*log((e^2*x^4 - (4*d*e - b)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e + b))/(e^2*x^4 - b*x^2 + d^2))/(2*d*e - b), (sqrt(2*d*e - b)
*arctan(e*x/sqrt(2*d*e - b)) + sqrt(2*d*e - b)*arctan((e^2*x^3 + (d*e - b)*
x)*sqrt(2*d*e - b)/(2*d^2*e - b*d)))/(2*d*e - b)]
```

giac [B] time = 1.12, size = 1676, normalized size = 21.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/4*(16*sqrt(2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*d^4*e^4 - 8*sqrt(
2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*d^2*e^2 + 4*sqrt(2)*sqrt(-
4*d^2*e^2 + b^2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^2 + sqrt
(2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^4 + 32*d^4*e^6 - 8*sqrt(2)*
sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^4 - 16*b^2*d^2*e^4 + 2*sq
rt(2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3*e^2 + 2*b^4*e^2 - sqrt(
2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3 - 2
*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b
^2*e^2 - 4*sqrt(2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*d^2*e^6 - 8*b*
d^2*e^6 + sqrt(2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^4 + 2*b^3
*e^4 - 8*(4*d^2*e^2 - b^2)*d^2*e^4 + 2*(4*d^2*e^2 - b^2)*b^2*e^2 - sqrt(2)*
sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*e^4 + 2*
(4*d^2*e^2 - b^2)*b*e^4 + 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 -
 sqrt(-4*d^2*e^2 + b^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(
-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*d - 2*sqrt(2)*sqrt(-4*d^2*e^2 + b^
2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d*e^2 - 8*d^3*e^6 + 2*b^2*d*
e^4 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e
^2)*d*e^4 + 2*(4*d^2*e^2 - b^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x/sqrt(-(b + s
qrt(-4*d^2*e^2 + b^2))*e^(-2)))/(16*d^5*e^6 - 8*b^2*d^3*e^4 + b^4*d*e^2 - 8
*b*d^3*e^6 + 2*b^3*d*e^4 - 4*d^3*e^8 + b^2*d*e^6) + 1/4*(16*sqrt(2)*sqrt(-b
*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(-b*e^2 + sqrt(-
```

```
4*d^2*e^2 + b^2)*e^2)*b^2*d^2*e^2 - 4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-
b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^2 + sqrt(2)*sqrt(-b*e^2 + sqrt(
-4*d^2*e^2 + b^2)*e^2)*b^4 - 32*d^4*e^6 - 8*sqrt(2)*sqrt(-b*e^2 + sqrt(-4*d
^2*e^2 + b^2)*e^2)*b*d^2*e^4 + 16*b^2*d^2*e^4 + 2*sqrt(2)*sqrt(-b*e^2 + sqr
t(-4*d^2*e^2 + b^2)*e^2)*b^3*e^2 - 2*b^4*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + b^
2)*sqrt(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3 + 2*sqrt(2)*sqrt(-4*d^2*e^
2 + b^2)*sqrt(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^2 - 4*sqrt(2)*sqrt
(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d^2*e^6 + 8*b*d^2*e^6 + sqrt(2)*sqrt(
-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^4 - 2*b^3*e^4 + 8*(4*d^2*e^2 - b
^2)*d^2*e^4 - 2*(4*d^2*e^2 - b^2)*b^2*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*
sqrt(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*e^4 - 2*(4*d^2*e^2 - b^2)*b*e^4
 - 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)
*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 + sqrt(-4*d^2*e^
2 + b^2)*e^2)*b^2*d - 2*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 + sqrt(-
4*d^2*e^2 + b^2)*e^2)*b*d*e^2 - 8*d^3*e^6 + 2*b^2*d*e^4 - sqrt(2)*sqrt(-4*d
^2*e^2 + b^2)*sqrt(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d*e^4 + 2*(4*d^2*e^
2 - b^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x/sqrt(-(b - sqrt(-4*d^2*e^2 + b^2))*
e^(-2)))/(16*d^5*e^6 - 8*b^2*d^3*e^4 + b^4*d*e^2 - 8*b*d^3*e^6 + 2*b^3*d*e^
4 - 4*d^3*e^8 + b^2*d*e^6)
```

maple [A] time = 0.03, size = 75, normalized size = 0.96

$$-\frac{\arctan\left(\frac{-2ex+\sqrt{2de+b}}{\sqrt{2de-b}}\right)}{\sqrt{2de-b}}+\frac{\arctan\left(\frac{2ex+\sqrt{2de+b}}{\sqrt{2de-b}}\right)}{\sqrt{2de-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x)
```

```
[Out] -1/(2*d*e-b)^(1/2)*arctan((-2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))+1/(2*d*
e-b)^(1/2)*arctan((2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{e^2x^4-bx^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 - b*x^2 + d^2), x)
```

mupad [B] time = 0.13, size = 30, normalized size = 0.38

$$\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{b-2\,d\,e}}{d-e\,x^2}\right)}{\sqrt{b-2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x)
```

```
[Out] atanh((x*(b - 2*d*e)^(1/2))/(d - e*x^2))/(b - 2*d*e)^(1/2)
```

sympy [A] time = 0.57, size = 110, normalized size = 1.41

$$\frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{\frac{1}{b-2de}}+2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}-\frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{\frac{1}{b-2de}}-2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(1/(b - 2*d*e)) + 2*d*e*sqr
t(1/(b - 2*d*e)))/e)/2 - sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(b*sqrt(1/
(b - 2*d*e)) - 2*d*e*sqrt(1/(b - 2*d*e)))/e)/2
```

3.29 $\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx$

Optimal. Leaf size=86

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

```
[Out] -arctan((-2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+arctan((2
*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2*d*e + f] - 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]) + ArcT
an[(Sqrt[2*d*e + f] + 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de+f}\,x}{e}+x^2}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de+f}\,x}{e}+x^2}\,dx}{2e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2}-x^2}\,dx, x, -\frac{\sqrt{2de+f}}{e}+2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2}-x^2}\,dx, x, \frac{\sqrt{2de+f}}{e}+2x\right)}{e}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}} + \frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Mathematica [B] time = 0.11, size = 189, normalized size = 2.20

$$\frac{\frac{\left(\sqrt{f^2-4d^2e^2}+2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}} + \frac{\left(\sqrt{f^2-4d^2e^2}-2de-f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f - Sqrt[
-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e - f + Sqr
t[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]]
])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

fricas [A] time = 0.42, size = 179, normalized size = 2.08

$$\left[-\frac{\sqrt{-2\,de+f}\,\log\left(\frac{e^2x^4-\left(4\,de-f\right)x^2+d^2-2\left(ex^3-dx\right)\sqrt{-2\,de+f}}{e^2x^4-fx^2+d^2}\right)}{2\left(2\,de-f\right)}, -\frac{\sqrt{2\,de-f}\,\arctan\left(-\frac{ex}{\sqrt{2\,de-f}}\right)+\sqrt{2\,de-f}\,\arctan\left(-\frac{(e^2x\right.}{}}{2\,de-f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e + f)*log((e^2*x^4 - (4*d*e - f)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e + f))/(e^2*x^4 - f*x^2 + d^2))/(2*d*e - f), -(sqrt(2*d*e - f
)*arctan(-e*x/sqrt(2*d*e - f)) + sqrt(2*d*e - f)*arctan(-(e^2*x^3 + (d*e -
f)*x)*sqrt(2*d*e - f)/(2*d^2*e - d*f)))/(2*d*e - f)]
```

giac [B] time = 1.14, size = 1676, normalized size = 19.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/4*(16*sqrt(2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^4*e^4 - 8*sqrt(
2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f^2*e^2 + 4*sqrt(2)*sqrt(-
4*d^2*e^2 + f^2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^2 + sqrt
(2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^4 + 32*d^4*e^6 - 8*sqrt(2)*
sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^4 - 16*d^2*f^2*e^4 + 2*sq
rt(2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3*e^2 + 2*f^4*e^2 - sqrt(
```

```
2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3 - 2
*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f
^2*e^2 - 4*sqrt(2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*e^6 - 8*d^
2*f*e^6 - 8*(4*d^2*e^2 - f^2)*d^2*e^4 + sqrt(2)*sqrt(-f*e^2 - sqrt(-4*d^2*e
^2 + f^2)*e^2)*f^2*e^4 + 2*f^3*e^4 + 2*(4*d^2*e^2 - f^2)*f^2*e^2 - sqrt(2)*
sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f*e^4 + 2*
(4*d^2*e^2 - f^2)*f*e^4 + 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 -
 sqrt(-4*d^2*e^2 + f^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(
-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f^2 - 2*sqrt(2)*sqrt(-4*d^2*e^2 + f^
2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f*e^2 - 8*d^3*e^6 + 2*d*f^2*
e^4 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e
^2)*d*e^4 + 2*(4*d^2*e^2 - f^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x/sqrt(-(f + s
qrt(-4*d^2*e^2 + f^2))*e^(-2)))/(16*d^5*e^6 - 8*d^3*f^2*e^4 + d*f^4*e^2 - 8
*d^3*f*e^6 + 2*d*f^3*e^4 - 4*d^3*e^8 + d*f^2*e^6) + 1/4*(16*sqrt(2)*sqrt(-f
*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(-f*e^2 + sqrt(-
4*d^2*e^2 + f^2)*e^2)*d^2*f^2*e^2 - 4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-
f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^2 + sqrt(2)*sqrt(-f*e^2 + sqrt(
-4*d^2*e^2 + f^2)*e^2)*f^4 - 32*d^4*e^6 - 8*sqrt(2)*sqrt(-f*e^2 + sqrt(-4*d
^2*e^2 + f^2)*e^2)*d^2*f*e^4 + 16*d^2*f^2*e^4 + 2*sqrt(2)*sqrt(-f*e^2 + sqr
t(-4*d^2*e^2 + f^2)*e^2)*f^3*e^2 - 2*f^4*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + f^
2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3 + 2*sqrt(2)*sqrt(-4*d^2*e^
2 + f^2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^2*e^2 - 4*sqrt(2)*sqrt
(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*e^6 + 8*d^2*f*e^6 + 8*(4*d^2*e^2
- f^2)*d^2*e^4 + sqrt(2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^2*e^4
- 2*f^3*e^4 - 2*(4*d^2*e^2 - f^2)*f^2*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*
sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f*e^4 - 2*(4*d^2*e^2 - f^2)*f*e^4
 - 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)
*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^
2 + f^2)*e^2)*d*f^2 - 2*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 + sqrt(-
4*d^2*e^2 + f^2)*e^2)*d*f*e^2 - 8*d^3*e^6 + 2*d*f^2*e^4 - sqrt(2)*sqrt(-4*d
^2*e^2 + f^2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d*e^4 + 2*(4*d^2*e^
2 - f^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x/sqrt(-(f - sqrt(-4*d^2*e^2 + f^2))*
e^(-2)))/(16*d^5*e^6 - 8*d^3*f^2*e^4 + d*f^4*e^2 - 8*d^3*f*e^6 + 2*d*f^3*e^
4 - 4*d^3*e^8 + d*f^2*e^6)
```

maple [A] time = 0.03, size = 75, normalized size = 0.87

$$-\frac{\arctan\left(\frac{-2ex+\sqrt{2de+f}}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}+\frac{\arctan\left(\frac{2ex+\sqrt{2de+f}}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x)
```

```
[Out] -arctan((-2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+arctan((2
*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{e^2x^4-fx^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 - f*x^2 + d^2), x)
```

mupad [B] time = 4.39, size = 88, normalized size = 1.02

$$-\frac{\operatorname{atan}\left(\frac{e^2\,x^3\,\sqrt{2\,d\,e-f}-f\,x\,\sqrt{2\,d\,e-f}+d\,e\,x\,\sqrt{2\,d\,e-f}}{d\,\left(f-2\,d\,e\right)}\right)-\operatorname{atan}\left(\frac{e\,x}{\sqrt{2\,d\,e-f}}\right)}{\sqrt{2\,d\,e-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x)
```

```
[Out] -(atan((e^2*x^3*(2*d*e - f)^(1/2) - f*x*(2*d*e - f)^(1/2) + d*e*x*(2*d*e -
f)^(1/2))/(d*(f - 2*d*e))) - atan((e*x)/(2*d*e - f)^(1/2)))/(2*d*e - f)^(1/
2)
```

sympy [A] time = 0.55, size = 121, normalized size = 1.41

$$-\frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{-\frac{1}{2de-f}}+f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{-\frac{1}{2de-f}}-f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e - f)) + f*
sqrt(-1/(2*d*e - f)))/e)/2 + sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(2*d*
e*sqrt(-1/(2*d*e - f)) - f*sqrt(-1/(2*d*e - f)))/e)/2
```

3.30 $\int \frac{d-ex^2}{d^2+bx^2+e^2x^4} \, dx$

Optimal. Leaf size=78

$$\frac{\log\left(x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}-\frac{\log\left(-x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e-b)^(1/2))/(2*d*e-b)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e-
b)^(1/2))/(2*d*e-b)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1164, 628}

$$\frac{\log\left(x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}-\frac{\log\left(-x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[-b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e]) + Log[d + Sqrt[-b
 + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{d-ex^2}{d^2+bx^2+e^2x^4} \, dx &= -\frac{\int \frac{\frac{\sqrt{-b+2de}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{-b+2de}\,x}{e}-x^2} \, dx}{2\sqrt{-b+2de}}-\frac{\int \frac{\frac{\sqrt{-b+2de}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{-b+2de}\,x}{e}-x^2} \, dx}{2\sqrt{-b+2de}}\\ &= -\frac{\log\left(d-\sqrt{-b+2de}\,x+ex^2\right)}{2\sqrt{-b+2de}}+\frac{\log\left(d+\sqrt{-b+2de}\,x+ex^2\right)}{2\sqrt{-b+2de}}\end{aligned}$$

Mathematica [B] time = 0.12, size = 182, normalized size = 2.33

$$\frac{\frac{\left(-\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}-\frac{\left(\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] (((b + 2*d*e - Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b^
2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] - ((b + 2*d*e + Sqrt[b^2
- 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b
 + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

fricas [A] time = 0.40, size = 172, normalized size = 2.21

$$\left[\frac{\log\left(\frac{e^2x^4+(4\,de-b)x^2+d^2+2\left(ex^3+dx\right)\sqrt{2\,de-b}}{e^2x^4+bx^2+d^2}\right)}{2\,\sqrt{2\,de-b}},-\frac{\sqrt{-2\,de+b}\,\arctan\left(\frac{\sqrt{-2\,de+b}\,ex}{2\,de-b}\right)-\sqrt{-2\,de+b}\,\arctan\left(\frac{\left(e^2x^3-(de-b)x\right)\sqrt{-2\,d}}{2\,d^2e-bd}\right.}{2\,de-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e - b)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - b)
)/(e^2*x^4 + b*x^2 + d^2))/sqrt(2*d*e - b), -(sqrt(-2*d*e + b)*arctan(sqrt(
-2*d*e + b)*e*x/(2*d*e - b)) - sqrt(-2*d*e + b)*arctan((e^2*x^3 - (d*e - b)
*x)*sqrt(-2*d*e + b)/(2*d^2*e - b*d)))/(2*d*e - b)]
```

giac [B] time = 1.16, size = 1642, normalized size = 21.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/4*(16*sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d^4*e^4 - 8*sqrt(2
)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*d^2*e^2 + 4*sqrt(2)*sqrt(-4*
d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^2 + sqrt(2)
*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^4 - 32*d^4*e^6 + 8*sqrt(2)*sqrt
(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^4 + 16*b^2*d^2*e^4 - 2*sqrt(2)
*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3*e^2 - 2*b^4*e^2 - sqrt(2)*sqr
t(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3 + 2*sqrt(2
)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^2 -
 4*sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d^2*e^6 - 8*b*d^2*e^6 +
 sqrt(2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^4 + 2*b^3*e^4 + 8*(
4*d^2*e^2 - b^2)*d^2*e^4 - 2*(4*d^2*e^2 - b^2)*b^2*e^2 - sqrt(2)*sqrt(-4*d^
2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*e^4 + 2*(4*d^2*e^2
- b^2)*b*e^4 + 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2
*e^2 + b^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt
(-4*d^2*e^2 + b^2)*e^2)*b^2*d + 2*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2
 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d*e^2 - 8*d^3*e^6 + 2*b^2*d*e^4 - sqrt(2)*
sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d*e^4 + 2*(
4*d^2*e^2 - b^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x*e/sqrt(b + sqrt(-4*d^2*e^2
+ b^2)))/(16*d^5*e^6 - 8*b^2*d^3*e^4 + b^4*d*e^2 + 8*b*d^3*e^6 - 2*b^3*d*e^
4 - 4*d^3*e^8 + b^2*d*e^6) + 1/4*(16*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 +
 b^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2
*d^2*e^2 - 4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 +
b^2)*e^2)*b*d^2*e^2 + sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^4
+ 32*d^4*e^6 + 8*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^4
 - 16*b^2*d^2*e^4 - 2*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3*
e^2 + 2*b^4*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e
^2 + b^2)*e^2)*b^3 - 2*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*
d^2*e^2 + b^2)*e^2)*b^2*e^2 - 4*sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)
*e^2)*d^2*e^6 + 8*b*d^2*e^6 + sqrt(2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e
^2)*b^2*e^4 - 2*b^3*e^4 - 8*(4*d^2*e^2 - b^2)*d^2*e^4 + 2*(4*d^2*e^2 - b^2)
*b^2*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^
```

```
2)*e^2)*b*e^4 - 2*(4*d^2*e^2 - b^2)*b*e^4 - 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 +
b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2
*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*d + 2*sqrt(2)*sqrt
(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d*e^2 - 8*d^3
*e^6 + 2*b^2*d*e^4 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(b*e^2 - sqrt(-4*d^
2*e^2 + b^2)*e^2)*d*e^4 + 2*(4*d^2*e^2 - b^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*
x*e/sqrt(b - sqrt(-4*d^2*e^2 + b^2)))/(16*d^5*e^6 - 8*b^2*d^3*e^4 + b^4*d*e
^2 + 8*b*d^3*e^6 - 2*b^3*d*e^4 - 4*d^3*e^8 + b^2*d*e^6)
```

maple [A] time = 0.02, size = 88, normalized size = 1.13

$$-\frac{\sqrt{2de-b}\,\ln\left(e\,x^2+d+\sqrt{2de-b}\,x\right)}{-4de+2b}+\frac{\sqrt{2de-b}\,\ln\left(-e\,x^2-d+\sqrt{2de-b}\,x\right)}{-4de+2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x)
```

```
[Out] 1/(-4*d*e+2*b)*(2*d*e-b)^(1/2)*ln(-e*x^2+x*(2*d*e-b)^(1/2)-d)-1/(-4*d*e+2*b
)*(2*d*e-b)^(1/2)*ln(d+e*x^2+x*(2*d*e-b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^2-d}{e^2x^4+bx^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 + b*x^2 + d^2), x)
```

mupad [B] time = 0.09, size = 99, normalized size = 1.27

$$\frac{\mathrm{atan}\left(\frac{b\,x\,(b-2\,d\,e)+2\,b\,e^2\,x^3+4\,d^2\,e^2\,x-e^2\,x^3\,(b-2\,d\,e)+3\,d\,e\,x\,(b-2\,d\,e)}{\left(2\,e\,d^2+b\,d\right)\,\sqrt{b-2\,d\,e}}\right)-\mathrm{atan}\left(\frac{e\,x}{\sqrt{b-2\,d\,e}}\right)}{\sqrt{b-2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(b*x^2 + d^2 + e^2*x^4),x)
```

```
[Out] (atan((b*x*(b - 2*d*e) + 2*b*e^2*x^3 + 4*d^2*e^2*x - e^2*x^3*(b - 2*d*e) +
3*d*e*x*(b - 2*d*e))/((b*d + 2*d^2*e)*(b - 2*d*e)^(1/2))) - atan((e*x)/(b -
 2*d*e)^(1/2)))/(b - 2*d*e)^(1/2)
```

sympy [A] time = 0.58, size = 121, normalized size = 1.55

$$\frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b-2de}}+2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}-\frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b-2de}}-2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(-1/(b - 2*d*e)) + 2*d*e*sq
rt(-1/(b - 2*d*e)))/e)/2 - sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(b*sqrt(
-1/(b - 2*d*e)) - 2*d*e*sqrt(-1/(b - 2*d*e)))/e)/2
```

3.31 $\int \frac{d-ex^2}{d^2+fx^2+e^2x^4} \, dx$

Optimal. Leaf size=78

$$\frac{\log\left(x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}-\frac{\log\left(-x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e-
f)^(1/2))/(2*d*e-f)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1164, 628}

$$\frac{\log\left(x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}-\frac{\log\left(-x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[2*d*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f]) + Log[d + Sqrt[2*d*
e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx &= -\frac{\int \frac{\frac{\sqrt{2de-f}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{2de-f}\,x}{e}-x^2}\,dx}{2\sqrt{2de-f}}-\frac{\int \frac{\frac{\sqrt{2de-f}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{2de-f}\,x}{e}-x^2}\,dx}{2\sqrt{2de-f}}\\ &= -\frac{\log\left(d-\sqrt{2de-f}\,x+ex^2\right)}{2\sqrt{2de-f}}+\frac{\log\left(d+\sqrt{2de-f}\,x+ex^2\right)}{2\sqrt{2de-f}}\end{aligned}$$

Mathematica [B] time = 0.12, size = 182, normalized size = 2.33

$$\frac{\frac{\left(-\sqrt{f^2-4d^2e^2}+2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}\right)}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}-\frac{\left(\sqrt{f^2-4d^2e^2}+2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e + f - Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[-
4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] - ((2*d*e + f + Sqrt[-
4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]])/S
qrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

fricas [A] time = 0.42, size = 173, normalized size = 2.22

$$\left[\frac{\log\left(\frac{e^2x^4+(4\,de-f)x^2+d^2+2\left(ex^3+dx\right)\sqrt{2\,de-f}}{e^2x^4+fx^2+d^2}\right)}{2\sqrt{2\,de-f}}, \frac{\sqrt{-2\,de+f}\,\arctan\left(-\frac{\sqrt{-2\,de+f}\,ex}{2\,de-f}\right)-\sqrt{-2\,de+f}\,\arctan\left(-\frac{\left(e^2x^3-(de-f)\ldots}{2\,d^2e\ldots}\right.}{2\,de-f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e - f)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - f)
)/(e^2*x^4 + f*x^2 + d^2))/sqrt(2*d*e - f), (sqrt(-2*d*e + f)*arctan(-sqrt(
-2*d*e + f)*e*x/(2*d*e - f)) - sqrt(-2*d*e + f)*arctan(-(e^2*x^3 - (d*e - f
)*x)*sqrt(-2*d*e + f)/(2*d^2*e - d*f)))/(2*d*e - f)]
```

giac [B] time = 1.25, size = 1642, normalized size = 21.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/4*(16*sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^4*e^4 - 8*sqrt(2
)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f^2*e^2 + 4*sqrt(2)*sqrt(-4*
d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^2 + sqrt(2)
*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^4 - 32*d^4*e^6 + 8*sqrt(2)*sqrt
(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^4 + 16*d^2*f^2*e^4 - 2*sqrt(2)
*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3*e^2 - 2*f^4*e^2 - sqrt(2)*sqr
t(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3 + 2*sqrt(2
)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^2*e^2 -
 4*sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*e^6 - 8*d^2*f*e^6 +
 8*(4*d^2*e^2 - f^2)*d^2*e^4 + sqrt(2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*
e^2)*f^2*e^4 + 2*f^3*e^4 - 2*(4*d^2*e^2 - f^2)*f^2*e^2 - sqrt(2)*sqrt(-4*d^
2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f*e^4 + 2*(4*d^2*e^2
- f^2)*f*e^4 + 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2
*e^2 + f^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt
(-4*d^2*e^2 + f^2)*e^2)*d*f^2 + 2*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2
 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f*e^2 - 8*d^3*e^6 + 2*d*f^2*e^4 - sqrt(2)*
sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d*e^4 + 2*(
4*d^2*e^2 - f^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x*e/sqrt(f + sqrt(-4*d^2*e^2
+ f^2)))/(16*d^5*e^6 - 8*d^3*f^2*e^4 + d*f^4*e^2 + 8*d^3*f*e^6 - 2*d*f^3*e^
4 - 4*d^3*e^8 + d*f^2*e^6) + 1/4*(16*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 +
 f^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2
*f^2*e^2 - 4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 +
f^2)*e^2)*d^2*f*e^2 + sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^4
+ 32*d^4*e^6 + 8*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^4
 - 16*d^2*f^2*e^4 - 2*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3*
e^2 + 2*f^4*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e
^2 + f^2)*e^2)*f^3 - 2*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*
d^2*e^2 + f^2)*e^2)*f^2*e^2 - 4*sqrt(2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)
*e^2)*d^2*e^6 + 8*d^2*f*e^6 - 8*(4*d^2*e^2 - f^2)*d^2*e^4 + sqrt(2)*sqrt(f*
```

```
e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^2*e^4 - 2*f^3*e^4 + 2*(4*d^2*e^2 - f^2)
*f^2*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^
2)*e^2)*f*e^4 - 2*(4*d^2*e^2 - f^2)*f*e^4 - 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 +
f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2
*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f^2 + 2*sqrt(2)*sqrt
(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f*e^2 - 8*d^3
*e^6 + 2*d*f^2*e^4 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(f*e^2 - sqrt(-4*d^
2*e^2 + f^2)*e^2)*d*e^4 + 2*(4*d^2*e^2 - f^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*
x*e/sqrt(f - sqrt(-4*d^2*e^2 + f^2)))/(16*d^5*e^6 - 8*d^3*f^2*e^4 + d*f^4*e
^2 + 8*d^3*f*e^6 - 2*d*f^3*e^4 - 4*d^3*e^8 + d*f^2*e^6)
```

maple [A] time = 0.02, size = 69, normalized size = 0.88

$$\frac{\ln\left(e\,x^{2}+d+\sqrt{2de-f}\;x\right)}{2\sqrt{2de-f}}-\frac{\ln\left(-e\,x^{2}-d+\sqrt{2de-f}\;x\right)}{2\sqrt{2de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x)
```

```
[Out] 1/2*ln(d+e*x^2+x*(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)-1/2/(2*d*e-f)^(1/2)*ln(-e
*x^2+x*(2*d*e-f)^(1/2)-d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{ex^{2}-d}{e^{2}x^{4}+fx^{2}+d^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 + f*x^2 + d^2), x)
```

mupad [B] time = 4.44, size = 57, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\frac{f\,x\,1\mathrm{i}-d\,e\,x\,2\mathrm{i}}{d\,\sqrt{2\,d\,e-f}+e\,x^2\,\sqrt{2\,d\,e-f}}\right)\,1\mathrm{i}}{\sqrt{2\,d\,e-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(f*x^2 + d^2 + e^2*x^4),x)
```

```
[Out] (atan((f*x*1i - d*e*x*2i)/(d*(2*d*e - f)^(1/2) + e*x^2*(2*d*e - f)^(1/2)))*
1i)/(2*d*e - f)^(1/2)
```

sympy [A] time = 0.57, size = 110, normalized size = 1.41

$$-\frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^{2}+\frac{x\left(-2de\sqrt{\frac{1}{2de-f}}+f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^{2}+\frac{x\left(2de\sqrt{\frac{1}{2de-f}}-f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] -sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e - f)) + f*sqr
t(1/(2*d*e - f)))/e)/2 + sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(2*d*e*sqrt
(1/(2*d*e - f)) - f*sqrt(1/(2*d*e - f)))/e)/2
```

3.32 $\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx$

Optimal. Leaf size=70

$$\frac{\log\left(x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}-\frac{\log\left(-x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e+
b)^(1/2))/(2*d*e+b)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1164, 628}

$$\frac{\log\left(x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}-\frac{\log\left(-x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[b + 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e]) + Log[d + Sqrt[b +
2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx &= -\frac{\int \frac{\frac{\sqrt{b+2de}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{b+2de}\,x}{e}-x^2}\,dx}{2\sqrt{b+2de}}-\frac{\int \frac{\frac{\sqrt{b+2de}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{b+2de}\,x}{e}-x^2}\,dx}{2\sqrt{b+2de}}\\
&= -\frac{\log\left(d-\sqrt{b+2de}\,x+ex^2\right)}{2\sqrt{b+2de}}+\frac{\log\left(d+\sqrt{b+2de}\,x+ex^2\right)}{2\sqrt{b+2de}}
\end{aligned}$$

Mathematica [B] time = 0.13, size = 190, normalized size = 2.71

$$\frac{\frac{\left(-\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}-\frac{\left(\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] (-(((b - 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b - Sqrt
[b^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]]) + ((b - 2*d*e - Sqrt
[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]]])/
Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

fricas [A] time = 0.42, size = 168, normalized size = 2.40

$$\left[\frac{\log\left(\frac{e^2x^4+(4\,de+b)x^2+d^2+2\left(ex^3+dx\right)\sqrt{2\,de+b}}{e^2x^4-bx^2+d^2}\right)}{2\sqrt{2\,de+b}}, -\frac{\sqrt{-2\,de-b}\,\arctan\left(\frac{\sqrt{-2\,de-b}\,ex}{2\,de+b}\right)-\sqrt{-2\,de-b}\,\arctan\left(\frac{\left(e^2x^3-(de+b)x\right)\sqrt{-2\,d}}{2\,d^2e+bd}\right.}{2\,de+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e + b)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e + b)
)/(e^2*x^4 - b*x^2 + d^2))/sqrt(2*d*e + b), -(sqrt(-2*d*e - b)*arctan(sqrt(
-2*d*e - b)*e*x/(2*d*e + b)) - sqrt(-2*d*e - b)*arctan((e^2*x^3 - (d*e + b)
*x)*sqrt(-2*d*e - b)/(2*d^2*e + b*d)))/(2*d*e + b)]
```

giac [B] time = 1.13, size = 1676, normalized size = 23.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/4*(16*sqrt(2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*d^4*e^4 - 8*sqrt(
2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*d^2*e^2 + 4*sqrt(2)*sqrt(-
4*d^2*e^2 + b^2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^2 + sqrt
(2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^4 + 32*d^4*e^6 - 8*sqrt(2)*
sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^4 - 16*b^2*d^2*e^4 + 2*sq
rt(2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3*e^2 + 2*b^4*e^2 - sqrt(
2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3 - 2
*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b
^2*e^2 - 4*sqrt(2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*d^2*e^6 - 8*b*
d^2*e^6 + sqrt(2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^4 + 2*b^3
*e^4 - 8*(4*d^2*e^2 - b^2)*d^2*e^4 + 2*(4*d^2*e^2 - b^2)*b^2*e^2 - sqrt(2)*
sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*e^4 + 2*
(4*d^2*e^2 - b^2)*b*e^4 - 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 -
 sqrt(-4*d^2*e^2 + b^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(
-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*d - 2*sqrt(2)*sqrt(-4*d^2*e^2 + b^
2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d*e^2 - 8*d^3*e^6 + 2*b^2*d*
e^4 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 - sqrt(-4*d^2*e^2 + b^2)*e
^2)*d*e^4 + 2*(4*d^2*e^2 - b^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x/sqrt(-(b + s
qrt(-4*d^2*e^2 + b^2))*e^(-2)))/(16*d^5*e^6 - 8*b^2*d^3*e^4 + b^4*d*e^2 - 8
*b*d^3*e^6 + 2*b^3*d*e^4 - 4*d^3*e^8 + b^2*d*e^6) + 1/4*(16*sqrt(2)*sqrt(-b
*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(-b*e^2 + sqrt(-
4*d^2*e^2 + b^2)*e^2)*b^2*d^2*e^2 - 4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-
b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*d^2*e^2 + sqrt(2)*sqrt(-b*e^2 + sqrt(
-4*d^2*e^2 + b^2)*e^2)*b^4 - 32*d^4*e^6 - 8*sqrt(2)*sqrt(-b*e^2 + sqrt(-4*d
^2*e^2 + b^2)*e^2)*b*d^2*e^4 + 16*b^2*d^2*e^4 + 2*sqrt(2)*sqrt(-b*e^2 + sqr
t(-4*d^2*e^2 + b^2)*e^2)*b^3*e^2 - 2*b^4*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + b^
2)*sqrt(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^3 + 2*sqrt(2)*sqrt(-4*d^2*e^
2 + b^2)*sqrt(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^2 - 4*sqrt(2)*sqrt
(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d^2*e^6 + 8*b*d^2*e^6 + sqrt(2)*sqrt(
-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b^2*e^4 - 2*b^3*e^4 + 8*(4*d^2*e^2 - b
^2)*d^2*e^4 - 2*(4*d^2*e^2 - b^2)*b^2*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*
```

```
sqrt(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*b*e^4 - 2*(4*d^2*e^2 - b^2)*b*e^4
 + 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)
*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 + sqrt(-4*d^2*e^
2 + b^2)*e^2)*b^2*d - 2*sqrt(2)*sqrt(-4*d^2*e^2 + b^2)*sqrt(-b*e^2 + sqrt(-
4*d^2*e^2 + b^2)*e^2)*b*d*e^2 - 8*d^3*e^6 + 2*b^2*d*e^4 - sqrt(2)*sqrt(-4*d
^2*e^2 + b^2)*sqrt(-b*e^2 + sqrt(-4*d^2*e^2 + b^2)*e^2)*d*e^4 + 2*(4*d^2*e^
2 - b^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x/sqrt(-(b - sqrt(-4*d^2*e^2 + b^2))*
e^(-2)))/(16*d^5*e^6 - 8*b^2*d^3*e^4 + b^4*d*e^2 - 8*b*d^3*e^6 + 2*b^3*d*e^
4 - 4*d^3*e^8 + b^2*d*e^6)
```

maple [A] time = 0.02, size = 61, normalized size = 0.87

$$\frac{\ln\left(e\,x^{2}+d+\sqrt{2de+b}\;x\right)}{2\sqrt{2de+b}}-\frac{\ln\left(-e\,x^{2}-d+\sqrt{2de+b}\;x\right)}{2\sqrt{2de+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x)
```

```
[Out] -1/2/(2*d*e+b)^(1/2)*ln(-e*x^2+x*(2*d*e+b)^(1/2)-d)+1/2*ln(d+e*x^2+x*(2*d*e
+b)^(1/2))/(2*d*e+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^{2}-d}{e^{2}x^{4}-bx^{2}+d^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 - b*x^2 + d^2), x)
```

mupad [B] time = 4.44, size = 29, normalized size = 0.41

$$\frac{\mathrm{atanh}\left(\frac{x\,\sqrt{b+2\,d\,e}}{e\,x^2+d}\right)}{\sqrt{b+2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x)
```

```
[Out] atanh((x*(b + 2*d*e)^(1/2))/(d + e*x^2))/(b + 2*d*e)^(1/2)
```

sympy [A] time = 0.60, size = 112, normalized size = 1.60

$$-\frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^{2}+\frac{x\left(-b\sqrt{\frac{1}{b+2de}}-2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^{2}+\frac{x\left(b\sqrt{\frac{1}{b+2de}}+2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] -sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(1/(b + 2*d*e)) - 2*d*e*sqr
t(1/(b + 2*d*e)))/e)/2 + sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(b*sqrt(1/(
b + 2*d*e)) + 2*d*e*sqrt(1/(b + 2*d*e)))/e)/2
```

3.33 $\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx$

Optimal. Leaf size=70

$$\frac{\log\left(x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}-\frac{\log\left(-x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e+
f)^(1/2))/(2*d*e+f)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1164, 628}

$$\frac{\log\left(x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}-\frac{\log\left(-x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[2*d*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f]) + Log[d + Sqrt[2*d*
e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx &= -\frac{\int \frac{\frac{\sqrt{2de+f}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{2de+f}\,x}{e}-x^2}\,dx}{2\sqrt{2de+f}}-\frac{\int \frac{\frac{\sqrt{2de+f}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{2de+f}\,x}{e}-x^2}\,dx}{2\sqrt{2de+f}}\\ &= -\frac{\log\left(d-\sqrt{2de+f}\,x+ex^2\right)}{2\sqrt{2de+f}}+\frac{\log\left(d+\sqrt{2de+f}\,x+ex^2\right)}{2\sqrt{2de+f}}\end{aligned}$$

Mathematica [B] time = 0.13, size = 190, normalized size = 2.71

$$\frac{\frac{\left(-\sqrt{f^2-4d^2e^2}-2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}-\frac{\left(\sqrt{f^2-4d^2e^2}-2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] (-(((-2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f - Sq
rt[-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]]) + ((-2*d*e + f -
 Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f + Sqrt[-4*d^2*e^2 + f
^2]]])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

fricas [A] time = 0.44, size = 168, normalized size = 2.40

$$\left[\frac{\log\left(\frac{e^2x^4+\left(4\,de+f\right)x^2+d^2+2\left(ex^3+dx\right)\sqrt{2\,de+f}}{e^2x^4-fx^2+d^2}\right)}{2\sqrt{2\,de+f}}, -\frac{\sqrt{-2\,de-f}\,\arctan\left(\frac{\sqrt{-2\,de-f}\,ex}{2\,de+f}\right)-\sqrt{-2\,de-f}\,\arctan\left(\frac{\left(e^2x^3-\left(de+f\right)x\right)}{2\,d^2e+d}\right.}{2\,de+f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e + f)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e + f)
)/(e^2*x^4 - f*x^2 + d^2))/sqrt(2*d*e + f), -(sqrt(-2*d*e - f)*arctan(sqrt(
-2*d*e - f)*e*x/(2*d*e + f)) - sqrt(-2*d*e - f)*arctan((e^2*x^3 - (d*e + f)
*x)*sqrt(-2*d*e - f)/(2*d^2*e + d*f)))/(2*d*e + f)]
```

giac [B] time = 1.08, size = 1676, normalized size = 23.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/4*(16*sqrt(2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^4*e^4 - 8*sqrt(
2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f^2*e^2 + 4*sqrt(2)*sqrt(-
4*d^2*e^2 + f^2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^2 + sqrt
(2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^4 + 32*d^4*e^6 - 8*sqrt(2)*
sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^4 - 16*d^2*f^2*e^4 + 2*sq
rt(2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3*e^2 + 2*f^4*e^2 - sqrt(
2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3 - 2
*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f
^2*e^2 - 4*sqrt(2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*e^6 - 8*d^
2*f*e^6 - 8*(4*d^2*e^2 - f^2)*d^2*e^4 + sqrt(2)*sqrt(-f*e^2 - sqrt(-4*d^2*e
^2 + f^2)*e^2)*f^2*e^4 + 2*f^3*e^4 + 2*(4*d^2*e^2 - f^2)*f^2*e^2 - sqrt(2)*
sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*f*e^4 + 2*
(4*d^2*e^2 - f^2)*f*e^4 - 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 -
 sqrt(-4*d^2*e^2 + f^2)*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(
-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f^2 - 2*sqrt(2)*sqrt(-4*d^2*e^2 + f^
2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e^2)*d*f*e^2 - 8*d^3*e^6 + 2*d*f^2*
e^4 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 - sqrt(-4*d^2*e^2 + f^2)*e
^2)*d*e^4 + 2*(4*d^2*e^2 - f^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x/sqrt(-(f + s
qrt(-4*d^2*e^2 + f^2))*e^(-2)))/(16*d^5*e^6 - 8*d^3*f^2*e^4 + d*f^4*e^2 - 8
*d^3*f*e^6 + 2*d*f^3*e^4 - 4*d^3*e^8 + d*f^2*e^6) + 1/4*(16*sqrt(2)*sqrt(-f
*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^4*e^4 - 8*sqrt(2)*sqrt(-f*e^2 + sqrt(-
4*d^2*e^2 + f^2)*e^2)*d^2*f^2*e^2 - 4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-
f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*f*e^2 + sqrt(2)*sqrt(-f*e^2 + sqrt(
-4*d^2*e^2 + f^2)*e^2)*f^4 - 32*d^4*e^6 - 8*sqrt(2)*sqrt(-f*e^2 + sqrt(-4*d
^2*e^2 + f^2)*e^2)*d^2*f*e^4 + 16*d^2*f^2*e^4 + 2*sqrt(2)*sqrt(-f*e^2 + sqr
t(-4*d^2*e^2 + f^2)*e^2)*f^3*e^2 - 2*f^4*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + f^
2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^3 + 2*sqrt(2)*sqrt(-4*d^2*e^
2 + f^2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^2*e^2 - 4*sqrt(2)*sqrt
(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d^2*e^6 + 8*d^2*f*e^6 + 8*(4*d^2*e^2
```

```
- f^2)*d^2*e^4 + sqrt(2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f^2*e^4
- 2*f^3*e^4 - 2*(4*d^2*e^2 - f^2)*f^2*e^2 + sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*
sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*f*e^4 - 2*(4*d^2*e^2 - f^2)*f*e^4
 + 2*(4*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)
*e^2)*d^3*e^2 - sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^
2 + f^2)*e^2)*d*f^2 - 2*sqrt(2)*sqrt(-4*d^2*e^2 + f^2)*sqrt(-f*e^2 + sqrt(-
4*d^2*e^2 + f^2)*e^2)*d*f*e^2 - 8*d^3*e^6 + 2*d*f^2*e^4 - sqrt(2)*sqrt(-4*d
^2*e^2 + f^2)*sqrt(-f*e^2 + sqrt(-4*d^2*e^2 + f^2)*e^2)*d*e^4 + 2*(4*d^2*e^
2 - f^2)*d*e^4)*e)*arctan(2*sqrt(1/2)*x/sqrt(-(f - sqrt(-4*d^2*e^2 + f^2))*
e^(-2)))/(16*d^5*e^6 - 8*d^3*f^2*e^4 + d*f^4*e^2 - 8*d^3*f*e^6 + 2*d*f^3*e^
4 - 4*d^3*e^8 + d*f^2*e^6)
```

maple [A] time = 0.02, size = 61, normalized size = 0.87

$$\frac{\ln\left(e x^2 + d + \sqrt{2de+f}\, x\right)}{2\sqrt{2de+f}} - \frac{\ln\left(-e x^2 - d + \sqrt{2de+f}\, x\right)}{2\sqrt{2de+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x)
```

```
[Out] 1/2*ln(d+e*x^2+x*(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)-1/2/(2*d*e+f)^(1/2)*ln(-e
*x^2+x*(2*d*e+f)^(1/2)-d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^2 - d}{e^2x^4 - fx^2 + d^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 - f*x^2 + d^2), x)
```

mupad [B] time = 0.11, size = 29, normalized size = 0.41

$$\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{f+2\,d\,e}}{e\,x^2+d}\right)}{\sqrt{f+2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x)
```

```
[Out] atanh((x*(f + 2*d*e)^(1/2))/(d + e*x^2))/(f + 2*d*e)^(1/2)
```

sympy [A] time = 0.61, size = 112, normalized size = 1.60

$$-\frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e} + x^2 + \frac{x\left(-2de\sqrt{\frac{1}{2de+f}} - f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2} + \frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e} + x^2 + \frac{x\left(2de\sqrt{\frac{1}{2de+f}} + f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

```
[Out] -sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e + f)) - f*sqr
t(1/(2*d*e + f)))/e)/2 + sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(2*d*e*sqrt
(1/(2*d*e + f)) + f*sqrt(1/(2*d*e + f)))/e)/2
```

3.34 $\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$

Optimal. Leaf size=134

$$\frac{e^{3/2}\log\left(\sqrt{e}\,x\sqrt{2cd-be}+\sqrt{c}\,d+\sqrt{c}\,ex^2\right)}{2\sqrt{c}\,\sqrt{2cd-be}}-\frac{e^{3/2}\log\left(-\sqrt{e}\,x\sqrt{2cd-be}+\sqrt{c}\,d+\sqrt{c}\,ex^2\right)}{2\sqrt{c}\,\sqrt{2cd-be}}$$

```
[Out] -1/2*e^(3/2)*ln(d*c^(1/2)+e*x^2*c^(1/2)-x*e^(1/2)*(-b*e+2*c*d)^(1/2))/c^(1/
2)/(-b*e+2*c*d)^(1/2)+1/2*e^(3/2)*ln(d*c^(1/2)+e*x^2*c^(1/2)+x*e^(1/2)*(-b*
e+2*c*d)^(1/2))/c^(1/2)/(-b*e+2*c*d)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1164, 628}

$$\frac{e^{3/2}\log\left(\sqrt{e}\,x\sqrt{2cd-be}+\sqrt{c}\,d+\sqrt{c}\,ex^2\right)}{2\sqrt{c}\,\sqrt{2cd-be}}-\frac{e^{3/2}\log\left(-\sqrt{e}\,x\sqrt{2cd-be}+\sqrt{c}\,d+\sqrt{c}\,ex^2\right)}{2\sqrt{c}\,\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] -(e^(3/2)*Log[Sqrt[c]*d - Sqrt[e]*Sqrt[2*c*d - b*e]*x + Sqrt[c]*e*x^2])/(2*
Sqrt[c]*Sqrt[2*c*d - b*e]) + (e^(3/2)*Log[Sqrt[c]*d + Sqrt[e]*Sqrt[2*c*d -
b*e]*x + Sqrt[c]*e*x^2])/(2*Sqrt[c]*Sqrt[2*c*d - b*e])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx &= -\frac{e^{3/2}\int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\,\sqrt{e}}+2x}{-\frac{d}{e}-\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\,\sqrt{e}}-x^2}\,dx}{2\sqrt{c}\,\sqrt{2cd-be}}-\frac{e^{3/2}\int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\,\sqrt{e}}-2x}{-\frac{d}{e}+\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\,\sqrt{e}}-x^2}\,dx}{2\sqrt{c}\,\sqrt{2cd-be}}\\ &= -\frac{e^{3/2}\log\left(\sqrt{c}\,d-\sqrt{e}\,\sqrt{2cd-be}\,x+\sqrt{c}\,ex^2\right)}{2\sqrt{c}\,\sqrt{2cd-be}}+\frac{e^{3/2}\log\left(\sqrt{c}\,d+\sqrt{e}\,\sqrt{2cd-be}\,x+\sqrt{c}\,ex^2\right)}{2\sqrt{c}\,\sqrt{2cd-be}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 250, normalized size = 1.87

$$\frac{e^{3/2}\left(-\frac{\left(\sqrt{b^2e^2-4c^2d^2}-be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}\right)}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}-\frac{\left(\sqrt{b^2e^2-4c^2d^2}+be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2e^2-4c^2d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*(-(((-2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sq
rt[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-
4*c^2*d^2 + b^2*e^2]]) - ((2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[
b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*
e^2])
```

fricas [A] time = 0.42, size = 244, normalized size = 1.82

$$\left[\frac{1}{2}e\sqrt{\frac{e}{2c^2d-bce}}\log\left(\frac{ce^2x^4+cd^2+\left(4cde-be^2\right)x^2+2\left(\left(2c^2de-bce^2\right)x^3+\left(2c^2d^2-bcde\right)x\right)\sqrt{\frac{e}{2c^2d-bce}}}{ce^2x^4+be^2x^2+cd^2}\right),-e\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(e/(2*c^2*d - b*c*e))*log((c*e^2*x^4 + c*d^2 + (4*c*d*e - b*e^2)
*x^2 + 2*((2*c^2*d*e - b*c*e^2)*x^3 + (2*c^2*d^2 - b*c*d*e)*x)*sqrt(e/(2*c^
2*d - b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), -e*sqrt(-e/(2*c^2*d - b*c*
e))*arctan(c*x*sqrt(-e/(2*c^2*d - b*c*e))) + e*sqrt(-e/(2*c^2*d - b*c*e))*a
rctan((c*e*x^3 - (c*d - b*e)*x)*sqrt(-e/(2*c^2*d - b*c*e))/d)]
```

giac [B] time = 1.37, size = 2202, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")
```

```
[Out] -1/4*(32*c^5*d^4*e^4 - 16*sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*
e^4)*c*e^2)*c^4*d^4*e^2 - 16*b^2*c^3*d^2*e^6 + 8*b*c^4*d^2*e^6 + 8*sqrt(2)*
sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c^2*d^2*e^4 - 8*sq
rt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b*c^3*d^2*e^4 +
4*sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^4*d^2*e^4
- 4*sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e
^2 + b^2*e^4)*c*e^2)*b*c^2*d^2*e^2 - 8*(4*c^2*d^2*e^2 - b^2*e^4)*c^3*d^2*e^
2 + 2*b^4*c*e^8 - 2*b^3*c^2*e^8 - sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^
2 + b^2*e^4)*c*e^2)*b^4*e^6 + 2*sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2
+ b^2*e^4)*c*e^2)*b^3*c*e^6 - sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 +
b^2*e^4)*c*e^2)*b^2*c^2*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b
*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^3*e^4 - 2*sqrt(2)*sqrt(-4*
c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)
*b^2*c*e^4 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*
c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b*c^2*e^4 + 2*(4*c^2*d^2*e^2 - b^2*e^4)*b^2*c
*e^4 - 2*(4*c^2*d^2*e^2 - b^2*e^4)*b*c^2*e^4 + 2*(8*c^5*d^3*e^4 - 4*sqrt(2)
*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^
4)*c*e^2)*c^3*d^3 - 2*b^2*c^3*d*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4
```

```
)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c*d*e^2 - 2*sqrt
(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2
*e^4)*c*e^2)*b*c^2*d*e^2 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*
e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^3*d*e^2 - 2*(4*c^2*d^2*e^2 -
b^2*e^4)*c^3*d*e^2)*e)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(-4*c^2*d^2*e^(-2
) + b^2))/c))/((16*c^5*d^5*e^2 - 8*b^2*c^3*d^3*e^4 + 8*b*c^4*d^3*e^4 - 4*c^
5*d^3*e^4 + b^4*c*d*e^6 - 2*b^3*c^2*d*e^6 + b^2*c^3*d*e^6)*abs(c)) + 1/4*(3
2*c^5*d^4*e^4 + 16*sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*
e^2)*c^4*d^4*e^2 - 16*b^2*c^3*d^2*e^6 + 8*b*c^4*d^2*e^6 - 8*sqrt(2)*sqrt(b*
c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c^2*d^2*e^4 + 8*sqrt(2)*s
qrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b*c^3*d^2*e^4 - 4*sqrt(
2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^4*d^2*e^4 - 4*sqr
t(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^
2*e^4)*c*e^2)*b*c^2*d^2*e^2 - 8*(4*c^2*d^2*e^2 - b^2*e^4)*c^3*d^2*e^2 + 2*b
^4*c*e^8 - 2*b^3*c^2*e^8 + sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2
*e^4)*c*e^2)*b^4*e^6 - 2*sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e
^4)*c*e^2)*b^3*c*e^6 + sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4
)*c*e^2)*b^2*c^2*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4
- sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^3*e^4 - 2*sqrt(2)*sqrt(-4*c^2*d^2
*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c*
e^4 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2
*e^2 + b^2*e^4)*c*e^2)*b*c^2*e^4 + 2*(4*c^2*d^2*e^2 - b^2*e^4)*b^2*c*e^4 -
2*(4*c^2*d^2*e^2 - b^2*e^4)*b*c^2*e^4 + 2*(8*c^5*d^3*e^4 - 4*sqrt(2)*sqrt(-
4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^
2)*c^3*d^3 - 2*b^2*c^3*d*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(
b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c*d*e^2 - 2*sqrt(2)*sqr
t(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c
*e^2)*b*c^2*d*e^2 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - s
qrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^3*d*e^2 - 2*(4*c^2*d^2*e^2 - b^2*e^4
)*c^3*d*e^2)*e)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(-4*c^2*d^2*e^(-2) + b^2
))/c))/((16*c^5*d^5*e^2 - 8*b^2*c^3*d^3*e^4 + 8*b*c^4*d^3*e^4 - 4*c^5*d^3*e
^4 + b^4*c*d*e^6 - 2*b^3*c^2*d*e^6 + b^2*c^3*d*e^6)*abs(c))
```

maple [B] time = 0.08, size = 582, normalized size = 4.34

$$-\frac{\sqrt{2}\, b e^{4} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c e x}{\sqrt{\left(-b e^{2}+\sqrt{(b e-2 c d)(b e+2 c d) e^{2}}\right) c}}\right)}{2 \sqrt{(b e-2 c d)(b e+2 c d) e^{2}}\, \sqrt{\left(-b e^{2}+\sqrt{(b e-2 c d)(b e+2 c d) e^{2}}\right) c}}-\frac{\sqrt{2}\, b e^{4} \arctan\left(\frac{\sqrt{2}\, c \cdots}{\sqrt{\left(b e^{2}+\sqrt{(b e-2 c \cdots}\right.}}\right.}{2 \sqrt{(b e-2 c d)(b e+2 c d) e^{2}}\, \sqrt{\left(b e^{2}+\sqrt{(b \cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x)
```

```
[Out] -1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c
*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x*2^(1/2)/((-e^2*b+(e^2*(b*e-2
*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(
1/2)*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh
(c*e*x*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d+1/
2*e^2*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan
h(c*e*x*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))-1/2
*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((e^2*b+(e^2*(b*e-2*c*d)*(
b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/2)/((e^2*b+(e^2*(b*e-2*c*d)*(
b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^
(1/2)/((e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^
(1/2)/((e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d-1/2*e^2*2^(1
/2)/((e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1
/2)/((e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^2 - d}{cx^4 + bx^2 + \frac{cd^2}{e^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)
```

mupad [B] time = 0.18, size = 129, normalized size = 0.96

$$-\frac{e^{3/2}\left(\operatorname{atan}\left(\frac{\sqrt{e}\,x\,\sqrt{b\,c\,e-2\,c^2\,d}}{b\,e-2\,c\,d}\right)+\operatorname{atan}\left(\frac{c\,e^{3/2}\,x^3\,\sqrt{b\,c\,e-2\,c^2\,d}+b\,e^{3/2}\,x\,\sqrt{b\,c\,e-2\,c^2\,d}-c\,d\,\sqrt{e}\,x\,\sqrt{b\,c\,e-2\,c^2\,d}}{d\left(2\,c^2\,d-b\,c\,e\right)}\right)\right)}{\sqrt{b\,c\,e-2\,c^2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(b*x^2 + c*x^4 + (c*d^2)/e^2),x)
```

```
[Out] -(e^(3/2)*(atan((e^(1/2)*x*(b*c*e - 2*c^2*d)^(1/2))/(b*e - 2*c*d)) + atan((
c*e^(3/2)*x^3*(b*c*e - 2*c^2*d)^(1/2) + b*e^(3/2)*x*(b*c*e - 2*c^2*d)^(1/2)
 - c*d*e^(1/2)*x*(b*c*e - 2*c^2*d)^(1/2))/(d*(2*c^2*d - b*c*e)))))/(b*c*e -
 2*c^2*d)^(1/2)
```

sympy [A] time = 0.86, size = 158, normalized size = 1.18

$$\frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be-2cd)}}+2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2}-\frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be-2cd)}}-2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] sqrt(-e**3/(c*(b*e - 2*c*d)))*log(d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e -
 2*c*d))) + 2*c*d*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2 - sqrt(-e**3/(c*(b
*e - 2*c*d)))*log(d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e - 2*c*d))) - 2*c*d
*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2
```

3.35 $\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$

Optimal. Leaf size=130

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

```
[Out] -e^(3/2)*arctan((-2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d)^(1/2))/(b*e+2*c*d)^(1/2)
)/c^(1/2)/(b*e+2*c*d)^(1/2)+e^(3/2)*arctan((2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d
)^(1/2))/(b*e+2*c*d)^(1/2))/c^(1/2)/(b*e+2*c*d)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1161, 618, 204}

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*
e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] + 2*
Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \frac{e\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c} + \frac{e\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c}$$

$$= -\frac{e\,\mathrm{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, -\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{c} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, \frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{c}$$

$$= -\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}$$

Mathematica [A] time = 0.12, size = 248, normalized size = 1.91

$$\frac{e^{3/2}\left(\frac{\left(\sqrt{b^2e^2-4c^2d^2}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}\,x}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}\right)}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}} + \frac{\left(\sqrt{b^2e^2-4c^2d^2}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}\,x}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2e^2-4c^2d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4*c
^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e
 + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2
])
```

fricas [A] time = 0.43, size = 232, normalized size = 1.78

$$\left[\frac{1}{2}e\sqrt{-\frac{e}{2c^2d+bce}}\log\left(\frac{ce^2x^4+cd^2-\left(4cde+be^2\right)x^2+2\left(\left(2c^2de+bce^2\right)x^3-\left(2c^2d^2+bcde\right)x\right)\sqrt{-\frac{e}{2c^2d+bce}}}{ce^2x^4+be^2x^2+cd^2}\right), e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d*e + b*e^2
)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c*d*e)*x)*sqrt(-e/(2*
c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), e*sqrt(e/(2*c^2*d + b*c*
e))*arctan(c*x*sqrt(e/(2*c^2*d + b*c*e))) + e*sqrt(e/(2*c^2*d + b*c*e))*arc
tan((c*e*x^3 + (c*d + b*e)*x)*sqrt(e/(2*c^2*d + b*c*e))/d)]
```

giac [B] time = 1.40, size = 2202, normalized size = 16.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")
```

```
[Out] -1/4*(32*c^5*d^4*e^4 - 16*sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*
e^4)*c*e^2)*c^4*d^4*e^2 - 16*b^2*c^3*d^2*e^6 + 8*b*c^4*d^2*e^6 + 8*sqrt(2)*
sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c^2*d^2*e^4 - 8*sq
```

```
rt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b*c^3*d^2*e^4 +
4*sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^4*d^2*e^4
- 4*sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e
^2 + b^2*e^4)*c*e^2)*b*c^2*d^2*e^2 - 8*(4*c^2*d^2*e^2 - b^2*e^4)*c^3*d^2*e^
2 + 2*b^4*c*e^8 - 2*b^3*c^2*e^8 - sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^
2 + b^2*e^4)*c*e^2)*b^4*e^6 + 2*sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2
+ b^2*e^4)*c*e^2)*b^3*c*e^6 - sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 +
b^2*e^4)*c*e^2)*b^2*c^2*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b
*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^3*e^4 - 2*sqrt(2)*sqrt(-4*
c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)
*b^2*c*e^4 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*
c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b*c^2*e^4 + 2*(4*c^2*d^2*e^2 - b^2*e^4)*b^2*c
*e^4 - 2*(4*c^2*d^2*e^2 - b^2*e^4)*b*c^2*e^4 - 2*(8*c^5*d^3*e^4 - 4*sqrt(2)
*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^
4)*c*e^2)*c^3*d^3 - 2*b^2*c^3*d*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4
)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c*d*e^2 - 2*sqrt
(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2
*e^4)*c*e^2)*b*c^2*d*e^2 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*
e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^3*d*e^2 - 2*(4*c^2*d^2*e^2 -
b^2*e^4)*c^3*d*e^2)*e)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(-4*c^2*d^2*e^(-2
) + b^2))/c))/((16*c^5*d^5*e^2 - 8*b^2*c^3*d^3*e^4 + 8*b*c^4*d^3*e^4 - 4*c^
5*d^3*e^4 + b^4*c*d*e^6 - 2*b^3*c^2*d*e^6 + b^2*c^3*d*e^6)*abs(c)) + 1/4*(3
2*c^5*d^4*e^4 + 16*sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*
e^2)*c^4*d^4*e^2 - 16*b^2*c^3*d^2*e^6 + 8*b*c^4*d^2*e^6 - 8*sqrt(2)*sqrt(b*
c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c^2*d^2*e^4 + 8*sqrt(2)*s
qrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b*c^3*d^2*e^4 - 4*sqrt(
2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^4*d^2*e^4 - 4*sqr
t(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^
2*e^4)*c*e^2)*b*c^2*d^2*e^2 - 8*(4*c^2*d^2*e^2 - b^2*e^4)*c^3*d^2*e^2 + 2*b
^4*c*e^8 - 2*b^3*c^2*e^8 + sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2
*e^4)*c*e^2)*b^4*e^6 - 2*sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e
^4)*c*e^2)*b^3*c*e^6 + sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4
)*c*e^2)*b^2*c^2*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4
- sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^3*e^4 - 2*sqrt(2)*sqrt(-4*c^2*d^2
*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c*
e^4 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2
*e^2 + b^2*e^4)*c*e^2)*b*c^2*e^4 + 2*(4*c^2*d^2*e^2 - b^2*e^4)*b^2*c*e^4 -
2*(4*c^2*d^2*e^2 - b^2*e^4)*b*c^2*e^4 - 2*(8*c^5*d^3*e^4 - 4*sqrt(2)*sqrt(-
4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^
2)*c^3*d^3 - 2*b^2*c^3*d*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(
b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c*d*e^2 - 2*sqrt(2)*sqr
t(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c
*e^2)*b*c^2*d*e^2 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - s
qrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^3*d*e^2 - 2*(4*c^2*d^2*e^2 - b^2*e^4
)*c^3*d*e^2)*e)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(-4*c^2*d^2*e^(-2) + b^2
))/c))/((16*c^5*d^5*e^2 - 8*b^2*c^3*d^3*e^4 + 8*b*c^4*d^3*e^4 - 4*c^5*d^3*e
^4 + b^4*c*d*e^6 - 2*b^3*c^2*d*e^6 + b^2*c^3*d*e^6)*abs(c))
```

maple [B] time = 0.03, size = 582, normalized size = 4.48

$$\frac{\sqrt{2}\, b e^{4} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c e x}{\sqrt{\left(-b e^{2}+\sqrt{(b e-2 c d)(b e+2 c d) e^{2}}\right) c}}\right)}{2 \sqrt{(b e-2 c d)(b e+2 c d) e^{2}}\, \sqrt{\left(-b e^{2}+\sqrt{(b e-2 c d)(b e+2 c d) e^{2}}\right) c}}+\frac{\sqrt{2}\, b e^{4} \arctan\left(\frac{\sqrt{2}\, c e \cdots}{\sqrt{\left(b e^{2}+\sqrt{(b e-2 c d) \cdots}\right.}}\right)}{2 \sqrt{(b e-2 c d)(b e+2 c d) e^{2}}\, \sqrt{\left(b e^{2}+\sqrt{(b e \cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x)
```

```
[Out] 1/2*e^4/((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2)*2^(1/2)/((-b*e^2+((b*e-2*c*d)*(
b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b*e^2+((b*e-2*c*d)*(b*e+
2*c*d)*e^2)^(1/2))*c)^(1/2)*c*e*x)*b-e^3*c/((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1
/2)*2^(1/2)/((-b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*c*e*x)*d-1/2
*e^2*2^(1/2)/((-b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*c*e*x)+1/2/
((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2)*2^(1/2)/((b*e^2+((b*e-2*c*d)*(b*e+2*c*d
)*e^2)^(1/2))*c)^(1/2)*b*e^4*arctan(2^(1/2)/((b*e^2+((b*e-2*c*d)*(b*e+2*c*d
)*e^2)^(1/2))*c)^(1/2)*c*e*x)-1/((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2)*2^(1/2)
/((b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*c*d*e^3*arctan(2^(1/
2)/((b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*c*e*x)+1/2*2^(1/2)
/((b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*e^2*arctan(2^(1/2)/(
(b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*c*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{cx^4 + bx^2 + \frac{cd^2}{e^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)
```

mupad [B] time = 4.52, size = 232, normalized size = 1.78

$$\frac{e^{3/2}\left(\mathrm{atan}\left(\frac{c\sqrt{e}\,x}{\sqrt{c\,(b\,e+2\,c\,d)}}\right)-\mathrm{atan}\left(\frac{\left(2\,d\,c^2+b\,e\,c\right)\left(x\left(\frac{\sqrt{e}\left(c\,d\,e^7-\frac{4\,c^3\,d^2\,e^7}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}+\frac{e^{3/2}\left(2\,c^2\,d\,e^6-b\,c\,e^7\right)}{c\,d\,\sqrt{2\,d\,c^2+b\,e\,c}\,(b\,e-2\,c\,d)}\right)+\frac{\sqrt{e}\,x^3\left(c\,e^8-\frac{2\,b\,c^2\,e^9}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}\right)}{c\,e^7}\right)\right)}{\sqrt{2\,d\,c^2+b\,e\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(b*x^2 + c*x^4 + (c*d^2)/e^2),x)
```

```
[Out] (e^(3/2)*(atan((c*e^(1/2)*x)/(c*(b*e + 2*c*d))^(1/2)) - atan(((2*c^2*d + b*
c*e)*(x*((e^(1/2)*(c*d*e^7 - (4*c^3*d^2*e^7)/(2*c^2*d + b*c*e)))/(d*(c*(b*e
 + 2*c*d))^(1/2)*(b*e - 2*c*d)) + (e^(3/2)*(2*c^2*d*e^6 - b*c*e^7))/(c*d*(2
*c^2*d + b*c*e)^(1/2)*(b*e - 2*c*d))) + (e^(1/2)*x^3*(c*e^8 - (2*b*c^2*e^9)
/(2*c^2*d + b*c*e)))/(d*(c*(b*e + 2*c*d))^(1/2)*(b*e - 2*c*d))))/(c*e^7))))
/(2*c^2*d + b*c*e)^(1/2)
```

sympy [A] time = 0.77, size = 160, normalized size = 1.23

$$-\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be+2cd)}}-2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}+\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be+2cd)}}+2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e
 + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(c*
(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) + 2*
c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2
```

3.36 $\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx$

Optimal. Leaf size=130

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\,\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\,\sqrt{be+2cd}}$$

```
[Out] -e^(3/2)*arctan((-2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d)^(1/2))/(b*e+2*c*d)^(1/2)
)/c^(1/2)/(b*e+2*c*d)^(1/2)+e^(3/2)*arctan((2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d
)^(1/2))/(b*e+2*c*d)^(1/2))/c^(1/2)/(b*e+2*c*d)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1990, 1161, 618, 204}

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\,\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\,\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
```

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*
e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] + 2*
Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rubi steps

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx = \int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$$

$$= \frac{e\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\,\sqrt{e}}+x^2}\,dx}{2c} + \frac{e\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\,\sqrt{e}}+x^2}\,dx}{2c}$$

$$= -\frac{e\,\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, -\frac{\sqrt{2cd-be}}{\sqrt{c}\,\sqrt{e}}+2x\right)}{c} - \frac{e\,\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, \frac{\sqrt{2cd-be}}{\sqrt{c}\,\sqrt{e}}+2x\right)}{c}$$

$$= -\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\,\sqrt{2cd+be}} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\,\sqrt{2cd+be}}$$

Mathematica [A] time = 0.04, size = 248, normalized size = 1.91

$$\frac{e^{3/2}\left(\frac{\left(\sqrt{b^2e^2-4c^2d^2}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{e}\,x}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}\right)}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}+\frac{\left(\sqrt{b^2e^2-4c^2d^2}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{e}\,x}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{2}\,\sqrt{c}\,\sqrt{b^2e^2-4c^2d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
```

```
[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4*c
^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e
 + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2
])
```

fricas [A] time = 0.42, size = 232, normalized size = 1.78

$$\left[\frac{1}{2}\,e\sqrt{-\frac{e}{2\,c^2d+bce}}\,\log\left(\frac{ce^2x^4+cd^2-\left(4\,cde+be^2\right)x^2+2\left(\left(2\,c^2de+bce^2\right)x^3-\left(2\,c^2d^2+bcde\right)x\right)\sqrt{-\frac{e}{2\,c^2d+bce}}}{ce^2x^4+be^2x^2+cd^2}\right), e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d*e + b*e^2
)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c*d*e)*x)*sqrt(-e/(2*
c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), e*sqrt(e/(2*c^2*d + b*c*
e))*arctan(c*x*sqrt(e/(2*c^2*d + b*c*e))) + e*sqrt(e/(2*c^2*d + b*c*e))*arc
tan((c*e*x^3 + (c*d + b*e)*x)*sqrt(e/(2*c^2*d + b*c*e))/d)]
```

giac [B] time = 1.35, size = 2202, normalized size = 16.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="giac")
```

```
[Out] -1/4*(32*c^5*d^4*e^4 - 16*sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*
e^4)*c*e^2)*c^4*d^4*e^2 - 16*b^2*c^3*d^2*e^6 + 8*b*c^4*d^2*e^6 + 8*sqrt(2)*
sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c^2*d^2*e^4 - 8*sq
rt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b*c^3*d^2*e^4 +
4*sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^4*d^2*e^4
- 4*sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e
^2 + b^2*e^4)*c*e^2)*b*c^2*d^2*e^2 - 8*(4*c^2*d^2*e^2 - b^2*e^4)*c^3*d^2*e^
2 + 2*b^4*c*e^8 - 2*b^3*c^2*e^8 - sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^
2 + b^2*e^4)*c*e^2)*b^4*e^6 + 2*sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2
+ b^2*e^4)*c*e^2)*b^3*c*e^6 - sqrt(2)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 +
b^2*e^4)*c*e^2)*b^2*c^2*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b
*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^3*e^4 - 2*sqrt(2)*sqrt(-4*
c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)
*b^2*c*e^4 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*
c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b*c^2*e^4 + 2*(4*c^2*d^2*e^2 - b^2*e^4)*b^2*c
*e^4 - 2*(4*c^2*d^2*e^2 - b^2*e^4)*b*c^2*e^4 - 2*(8*c^5*d^3*e^4 - 4*sqrt(2)
*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^
4)*c*e^2)*c^3*d^3 - 2*b^2*c^3*d*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4
)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c*d*e^2 - 2*sqrt
(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(-4*c^2*d^2*e^2 + b^2
*e^4)*c*e^2)*b*c^2*d*e^2 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*
e^4 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^3*d*e^2 - 2*(4*c^2*d^2*e^2 -
b^2*e^4)*c^3*d*e^2)*e)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(-4*c^2*d^2*e^(-2
) + b^2))/c))/((16*c^5*d^5*e^2 - 8*b^2*c^3*d^3*e^4 + 8*b*c^4*d^3*e^4 - 4*c^
5*d^3*e^4 + b^4*c*d*e^6 - 2*b^3*c^2*d*e^6 + b^2*c^3*d*e^6)*abs(c)) + 1/4*(3
2*c^5*d^4*e^4 + 16*sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*
e^2)*c^4*d^4*e^2 - 16*b^2*c^3*d^2*e^6 + 8*b*c^4*d^2*e^6 - 8*sqrt(2)*sqrt(b*
c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c^2*d^2*e^4 + 8*sqrt(2)*s
qrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b*c^3*d^2*e^4 - 4*sqrt(
2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^4*d^2*e^4 - 4*sqr
t(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^
2*e^4)*c*e^2)*b*c^2*d^2*e^2 - 8*(4*c^2*d^2*e^2 - b^2*e^4)*c^3*d^2*e^2 + 2*b
^4*c*e^8 - 2*b^3*c^2*e^8 + sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2
*e^4)*c*e^2)*b^4*e^6 - 2*sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e
^4)*c*e^2)*b^3*c*e^6 + sqrt(2)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4
)*c*e^2)*b^2*c^2*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4
- sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^3*e^4 - 2*sqrt(2)*sqrt(-4*c^2*d^2
*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c*
e^4 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2
*e^2 + b^2*e^4)*c*e^2)*b*c^2*e^4 + 2*(4*c^2*d^2*e^2 - b^2*e^4)*b^2*c*e^4 -
2*(4*c^2*d^2*e^2 - b^2*e^4)*b*c^2*e^4 - 2*(8*c^5*d^3*e^4 - 4*sqrt(2)*sqrt(-
4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^
2)*c^3*d^3 - 2*b^2*c^3*d*e^6 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(
b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*b^2*c*d*e^2 - 2*sqrt(2)*sqr
t(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*c
*e^2)*b*c^2*d*e^2 + sqrt(2)*sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*sqrt(b*c*e^4 - s
qrt(-4*c^2*d^2*e^2 + b^2*e^4)*c*e^2)*c^3*d*e^2 - 2*(4*c^2*d^2*e^2 - b^2*e^4
)*c^3*d*e^2)*e)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(-4*c^2*d^2*e^(-2) + b^2
))/c))/((16*c^5*d^5*e^2 - 8*b^2*c^3*d^3*e^4 + 8*b*c^4*d^3*e^4 - 4*c^5*d^3*e
^4 + b^4*c*d*e^6 - 2*b^3*c^2*d*e^6 + b^2*c^3*d*e^6)*abs(c))
```

maple [B] time = 0.01, size = 582, normalized size = 4.48

$$\frac{\sqrt{2}\, b\, e^{4} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c e x}{\sqrt{\left(-b\, e^{2}+\sqrt{(be-2cd)\,(be+2cd)\, e^{2}}\right) c}}\right)}{2\sqrt{(be-2cd)\,(be+2cd)\, e^{2}}\; \sqrt{\left(-b\, e^{2}+\sqrt{(be-2cd)\,(be+2cd)\, e^{2}}\right) c}}+\frac{\sqrt{2}\, b\, e^{4} \arctan\left(\frac{\sqrt{2}\, ce\cdots}{\sqrt{\left(b\, e^{2}+\sqrt{(be-2cd)\cdots}\right.}}\right.}{2\sqrt{(be-2cd)\,(be+2cd)\, e^{2}}\; \sqrt{\left(b\, e^{2}+\sqrt{(be\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x)
```

```
[Out] 1/2*e^4/((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2)*2^(1/2)/((-b*e^2+((b*e-2*c*d)*(
b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b*e^2+((b*e-2*c*d)*(b*e+
2*c*d)*e^2)^(1/2))*c)^(1/2)*c*e*x)*b-e^3*c/((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1
/2)*2^(1/2)/((-b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*c*e*x)*d-1/2
*e^2*2^(1/2)/((-b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*c*e*x)+1/2/
((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2)*2^(1/2)/((b*e^2+((b*e-2*c*d)*(b*e+2*c*d
)*e^2)^(1/2))*c)^(1/2)*b*e^4*arctan(2^(1/2)/((b*e^2+((b*e-2*c*d)*(b*e+2*c*d
)*e^2)^(1/2))*c)^(1/2)*c*e*x)-1/((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2)*2^(1/2)
/((b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*c*d*e^3*arctan(2^(1/
2)/((b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*c*e*x)+1/2*2^(1/2)
/((b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*e^2*arctan(2^(1/2)/(
(b*e^2+((b*e-2*c*d)*(b*e+2*c*d)*e^2)^(1/2))*c)^(1/2)*c*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{bx^2+\left(x^4+\frac{d^2}{e^2}\right)c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(b*x^2 + (x^4 + d^2/e^2)*c), x)
```

mupad [B] time = 0.13, size = 232, normalized size = 1.78

$$\frac{e^{3/2}\left(\operatorname{atan}\left(\frac{c\,\sqrt{e}\,x}{\sqrt{c\,(b\,e+2\,c\,d)}}\right)-\operatorname{atan}\left(\frac{\left(2\,d\,c^2+b\,e\,c\right)\left(x\left(\frac{\sqrt{e}\left(c\,d\,e^7-\frac{4\,c^3\,d^2\,e^7}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}+\frac{e^{3/2}\left(2\,c^2\,d\,e^6-b\,c\,e^7\right)}{c\,d\,\sqrt{2\,d\,c^2+b\,e\,c}\,(b\,e-2\,c\,d)}\right)+\frac{\sqrt{e}\,x^3\left(c\,e^8-\frac{2\,b\,c^2\,e^9}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}\right)}{c\,e^7}\right)\right)}{\sqrt{2\,d\,c^2+b\,e\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(b*x^2 + c*(x^4 + d^2/e^2)),x)
```

```
[Out] (e^(3/2)*(atan((c*e^(1/2)*x)/(c*(b*e + 2*c*d))^(1/2)) - atan(((2*c^2*d + b*
c*e)*(x*((e^(1/2)*(c*d*e^7 - (4*c^3*d^2*e^7)/(2*c^2*d + b*c*e)))/(d*(c*(b*e
 + 2*c*d))^(1/2)*(b*e - 2*c*d)) + (e^(3/2)*(2*c^2*d*e^6 - b*c*e^7))/(c*d*(2
*c^2*d + b*c*e)^(1/2)*(b*e - 2*c*d))) + (e^(1/2)*x^3*(c*e^8 - (2*b*c^2*e^9)
/(2*c^2*d + b*c*e)))/(d*(c*(b*e + 2*c*d))^(1/2)*(b*e - 2*c*d))))/(c*e^7))))
/(2*c^2*d + b*c*e)^(1/2)
```

sympy [A] time = 0.79, size = 160, normalized size = 1.23

$$-\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be+2cd)}}-2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}+\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be+2cd)}}+2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(b*x**2+c*(d**2/e**2+x**4)),x)
```

```
[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e
 + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(c*
(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) + 2*
c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2
```

3.37 $\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

```
[Out] -1/2*ln(b*x^2+a-x)+1/2*ln(b*x^2+a+x)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1164, 628}

$$\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] -Log[a - x + b*x^2]/2 + Log[a + x + b*x^2]/2
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx &= -\left(\frac{1}{2}\int \frac{\frac{1}{b}+2x}{-\frac{a}{b}-\frac{x}{b}-x^2}\,dx\right)-\frac{1}{2}\int \frac{\frac{1}{b}-2x}{-\frac{a}{b}+\frac{x}{b}-x^2}\,dx\\ &= -\frac{1}{2}\log\left(a-x+bx^2\right)+\frac{1}{2}\log\left(a+x+bx^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*Log[a - x + b*x^2] + Log[a + x + b*x^2]/2
```

fricas [A] time = 0.40, size = 25, normalized size = 0.86

$$\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

giac [A] time = 0.24, size = 25, normalized size = 0.86

$$\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")
```

```
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

maple [A] time = 0.01, size = 26, normalized size = 0.90

$$-\frac{\ln\left(b\,x^2+a-x\right)}{2}+\frac{\ln\left(b\,x^2+a+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x)
```

```
[Out] -1/2*ln(b*x^2+a-x)+1/2*ln(b*x^2+a+x)
```

maxima [A] time = 1.04, size = 25, normalized size = 0.86

$$\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

mupad [B] time = 4.41, size = 12, normalized size = 0.41

$$\mathrm{atanh}\left(\frac{x}{b\,x^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)/(x^2*(2*a*b - 1) + a^2 + b^2*x^4),x)
```

```
[Out] atanh(x/(a + b*x^2))
```

sympy [A] time = 0.47, size = 26, normalized size = 0.90

$$-\frac{\log\left(\frac{a}{b}+x^2-\frac{x}{b}\right)}{2}+\frac{\log\left(\frac{a}{b}+x^2+\frac{x}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
```

```
[Out] -log(a/b + x**2 - x/b)/2 + log(a/b + x**2 + x/b)/2
```

3.38 $\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

```
[Out] arctanh((-2*b*x+1)/(-4*a*b+1)^(1/2))/(-4*a*b+1)^(1/2)-arctanh((2*b*x+1)/(-4
*a*b+1)^(1/2))/(-4*a*b+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] ArcTanh[(1 - 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b] - ArcTanh[(1 + 2*b*x)/
Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx &= \frac{\int \frac{1}{\frac{a}{b}-\frac{x}{b}+x^2} dx}{2b}+\frac{\int \frac{1}{\frac{a}{b}+\frac{x}{b}+x^2} dx}{2b}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2}-x^2} dx, x, -\frac{1}{b}+2x\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2}-x^2} dx, x, \frac{1}{b}+2x\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{1+2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}
\end{aligned}$$

Mathematica [B] time = 0.20, size = 138, normalized size = 2.30

$$\frac{\frac{\left(\sqrt{1-4ab}+1\right)\tan^{-1}\left(\frac{bx}{\sqrt{ab-\frac{1}{2}\sqrt{1-4ab}-\frac{1}{2}}}\right)}{\sqrt{2ab-\sqrt{1-4ab}-1}}+\frac{\left(\sqrt{1-4ab}-1\right)\tan^{-1}\left(\frac{\sqrt{2}bx}{\sqrt{2ab+\sqrt{1-4ab}-1}}\right)}{\sqrt{2ab+\sqrt{1-4ab}-1}}}{\sqrt{2-8ab}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] (((1 + Sqrt[1 - 4*a*b])*ArcTan[(b*x)/Sqrt[-1/2 + a*b - Sqrt[1 - 4*a*b]/2]])
/Sqrt[-1 + 2*a*b - Sqrt[1 - 4*a*b]] + ((-1 + Sqrt[1 - 4*a*b])*ArcTan[(Sqrt[
2]*b*x)/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a*b]]])/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a
*b]])/Sqrt[2 - 8*a*b]
```

fricas [A] time = 0.46, size = 164, normalized size = 2.73

$$\left[-\frac{\sqrt{-4\,ab+1}\,\log\left(\frac{b^2x^4-(6\,ab-1)x^2+a^2-2\left(bx^3-ax\right)\sqrt{-4\,ab+1}}{b^2x^4+(2\,ab-1)x^2+a^2}\right)}{2\,(4\,ab-1)},\ \frac{\sqrt{4\,ab-1}\,\arctan\left(\frac{bx}{\sqrt{4\,ab-1}}\right)+\sqrt{4\,ab-1}\,\arctan\left(\frac{\left(b^2x^3+(3\,a\right.}{4}\right.}{4\,ab-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-4*a*b + 1)*log((b^2*x^4 - (6*a*b - 1)*x^2 + a^2 - 2*(b*x^3 - a*
x)*sqrt(-4*a*b + 1))/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2))/(4*a*b - 1), (sqrt(
4*a*b - 1)*arctan(b*x/sqrt(4*a*b - 1)) + sqrt(4*a*b - 1)*arctan((b^2*x^3 +
(3*a*b - 1)*x)*sqrt(4*a*b - 1)/(4*a^2*b - a)))/(4*a*b - 1)]
```

giac [A] time = 0.18, size = 51, normalized size = 0.85

$$\frac{\arctan\left(\frac{2\,bx+1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}}+\frac{\arctan\left(\frac{2\,bx-1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")
```

```
[Out] arctan((2*b*x + 1)/sqrt(4*a*b - 1))/sqrt(4*a*b - 1) + arctan((2*b*x - 1)/sq
rt(4*a*b - 1))/sqrt(4*a*b - 1)
```

maple [A] time = 0.01, size = 52, normalized size = 0.87

$$\frac{\arctan\left(\frac{2bx-1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}+\frac{\arctan\left(\frac{2bx+1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x)
```

```
[Out] 1/(4*a*b-1)^(1/2)*arctan((2*b*x-1)/(4*a*b-1)^(1/2))+1/(4*a*b-1)^(1/2)*arcta
n((2*b*x+1)/(4*a*b-1)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*b-0.25>0)', see `assume?` for mor
e details)Is a*b-0.25 positive or negative?
```

mupad [B] time = 0.07, size = 55, normalized size = 0.92

$$\frac{\operatorname{atan}\left(\frac{b\,x}{\sqrt{4\,a\,b-1}}\right)+\operatorname{atan}\left(\frac{\frac{3\,x\,(4\,a\,b-1)}{4}-\frac{x}{4}+b^2\,x^3}{a\,\sqrt{4\,a\,b-1}}\right)}{\sqrt{4\,a\,b-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2*(2*a*b - 1) + a^2 + b^2*x^4),x)
```

```
[Out] (atan((b*x)/(4*a*b - 1)^(1/2)) + atan(((3*x*(4*a*b - 1))/4 - x/4 + b^2*x^3)
/(a*(4*a*b - 1)^(1/2))))/(4*a*b - 1)^(1/2)
```

sympy [B] time = 0.46, size = 117, normalized size = 1.95

$$-\frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^2+\frac{x\left(-4ab\sqrt{-\frac{1}{4ab-1}}+\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^2+\frac{x\left(4ab\sqrt{-\frac{1}{4ab-1}}-\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
```

```
[Out] -sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(-4*a*b*sqrt(-1/(4*a*b - 1)) + sq
rt(-1/(4*a*b - 1)))/b)/2 + sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(4*a*b*
sqrt(-1/(4*a*b - 1)) - sqrt(-1/(4*a*b - 1)))/b)/2
```

3.39 $\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx$

Optimal. Leaf size=62

$$\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}-\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

```
[Out] -arctan((-4*x+(4-b)^(1/2))/(4+b)^(1/2))/(4+b)^(1/2)+arctan((4*x+(4-b)^(1/2)
)/(4+b)^(1/2))/(4+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}-\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]

[Out] -(ArcTan[(Sqrt[4 - b] - 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]) + ArcTan[(Sqrt[4 - b
] + 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{1}{2}\sqrt{4-b}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{1}{2}\sqrt{4-b}\,x+x^2}\,dx\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b)-x^2}\,dx, x, -\frac{\sqrt{4-b}}{2}+2x\right)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b)-x^2}\,dx, x,\right.\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}
\end{aligned}$$

Mathematica [B] time = 0.06, size = 126, normalized size = 2.03

$$\frac{\frac{\left(\sqrt{b^2-16}-b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{\sqrt{b-\sqrt{b^2-16}}}+\frac{\left(\sqrt{b^2-16}+b-4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}+b}}\right)}{\sqrt{\sqrt{b^2-16}+b}}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] (((4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]]
)/Sqrt[b - Sqrt[-16 + b^2]] + ((-4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]
*x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-1
6 + b^2])
```

fricas [A] time = 0.42, size = 110, normalized size = 1.77

$$\left[-\frac{\sqrt{-b-4}\,\log\left(\frac{4x^4-(b+8)x^2-2\left(2x^3-x\right)\sqrt{-b-4}+1}{4x^4+bx^2+1}\right)}{2(b+4)}, \frac{\sqrt{b+4}\,\arctan\left(\frac{4x^3+(b+2)x}{\sqrt{b+4}}\right)+\sqrt{b+4}\,\arctan\left(\frac{2x}{\sqrt{b+4}}\right)}{b+4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b - 4)*log((4*x^4 - (b + 8)*x^2 - 2*(2*x^3 - x)*sqrt(-b - 4) +
1)/(4*x^4 + b*x^2 + 1))/(b + 4), (sqrt(b + 4)*arctan((4*x^3 + (b + 2)*x)/sq
rt(b + 4)) + sqrt(b + 4)*arctan(2*x/sqrt(b + 4)))/(b + 4)]
```

giac [A] time = 0.31, size = 77, normalized size = 1.24

$$\frac{\sqrt{b+4}\,(b-8)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{b+\sqrt{b^2-16}}}\right)}{b^2-4b-32}+\frac{\sqrt{b+4}\,(b-8)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{b^2-4b-32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] sqrt(b + 4)*(b - 8)*arctan(4*sqrt(1/2)*x/sqrt(b + sqrt(b^2 - 16)))/(b^2 - 4
*b - 32) + sqrt(b + 4)*(b - 8)*arctan(4*sqrt(1/2)*x/sqrt(b - sqrt(b^2 - 16)
))/(b^2 - 4*b - 32)
```

maple [B] time = 0.04, size = 277, normalized size = 4.47

$$-\frac{b\arctan\left(\frac{4x}{\sqrt{2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\,\sqrt{2b-2\sqrt{(b-4)(b+4)}}}+\frac{b\arctan\left(\frac{4x}{\sqrt{2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\,\sqrt{2b+2\sqrt{(b-4)(b+4)}}}+\frac{4\arctan\left(\frac{}{\sqrt{2}}\right.}{\sqrt{(b-4)(b+4)}\,\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+b*x^2+1),x)
```

```
[Out] 4/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b
-4)*(4+b))^(1/2)+2*b)^(1/2))+1/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*
x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))-1/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b
))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b-4/((b-
```

```
4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b
))^(1/2)+2*b)^(1/2))+1/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-
4)*(4+b))^(1/2)+2*b)^(1/2))+1/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*
b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2+1}{4\,x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + b*x^2 + 1), x)
```

mupad [B] time = 4.39, size = 66, normalized size = 1.06

$$-\frac{\operatorname{atan}\left(\frac{-b^3\,x-4\,b^2\,x^3-2\,b^2\,x+16\,b\,x+64\,x^3+32\,x}{\left(b^2-16\right)\,\sqrt{b+4}}\right)-\operatorname{atan}\left(\frac{2\,x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(b*x^2 + 4*x^4 + 1),x)
```

```
[Out] -(atan((32*x + 16*b*x - 2*b^2*x - b^3*x + 64*x^3 - 4*b^2*x^3)/((b^2 - 16)*(
b + 4)^(1/2))) - atan((2*x)/(b + 4)^(1/2)))/(b + 4)^(1/2)
```

sympy [A] time = 0.38, size = 95, normalized size = 1.53

$$-\frac{\sqrt{-\frac{1}{b+4}}\log\left(x^2+x\left(-\frac{b\sqrt{-\frac{1}{b+4}}}{2}-2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2}+\frac{\sqrt{-\frac{1}{b+4}}\log\left(x^2+x\left(\frac{b\sqrt{-\frac{1}{b+4}}}{2}+2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] -sqrt(-1/(b + 4))*log(x**2 + x*(-b*sqrt(-1/(b + 4))/2 - 2*sqrt(-1/(b + 4)))
 - 1/2)/2 + sqrt(-1/(b + 4))*log(x**2 + x*(b*sqrt(-1/(b + 4))/2 + 2*sqrt(-1
/(b + 4))) - 1/2)/2
```

3.40 $\int \frac{1+2x^2}{1-bx^2+4x^4} \, dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

```
[Out] -arctan((-4*x+(4+b)^(1/2))/(4-b)^(1/2))/(4-b)^(1/2)+arctan((4*x+(4+b)^(1/2)
)/(4-b)^(1/2))/(4-b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[4 + b] - 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]) + ArcTan[(Sqrt[4 + b
] + 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1-bx^2+4x^4} \, dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{1}{2}\sqrt{4+b}\,x+x^2} \, dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{1}{2}\sqrt{4+b}\,x+x^2} \, dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b)-x^2} \, dx,x,-\frac{\sqrt{4+b}}{2}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b)-x^2} \, dx\right.\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{4+b}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}+\frac{\tan^{-1}\left(\frac{\sqrt{4+b}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}
\end{aligned}$$

Mathematica [B] time = 0.06, size = 134, normalized size = 2.03

$$\frac{\frac{\left(\sqrt{b^2-16}+b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{-\sqrt{b^2-16}-b}}\right)}{\sqrt{-\sqrt{b^2-16}-b}}+\frac{\left(\sqrt{b^2-16}-b-4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}-b}}\right)}{\sqrt{\sqrt{b^2-16}-b}}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

```
[Out] (((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[-b - Sqrt[-16 + b^2]]
])/Sqrt[-b - Sqrt[-16 + b^2]] + ((-4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[
2]*x)/Sqrt[-b + Sqrt[-16 + b^2]]])/Sqrt[-b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqr
t[-16 + b^2])
```

fricas [A] time = 0.43, size = 120, normalized size = 1.82

$$\left[\frac{\log\left(\frac{4x^4+(b-8)x^2-2\left(2x^3-x\right)\sqrt{b-4}+1}{4x^4-bx^2+1}\right)}{2\sqrt{b-4}}, \frac{\sqrt{-b+4}\arctan\left(\frac{\left(4x^3-(b-2)x\right)\sqrt{-b+4}}{b-4}\right)+\sqrt{-b+4}\arctan\left(\frac{2\sqrt{-b+4}x}{b-4}\right)}{b-4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/2*log((4*x^4 + (b - 8)*x^2 - 2*(2*x^3 - x)*sqrt(b - 4) + 1)/(4*x^4 - b*x
^2 + 1))/sqrt(b - 4), (sqrt(-b + 4)*arctan((4*x^3 - (b - 2)*x)*sqrt(-b + 4)
/(b - 4)) + sqrt(-b + 4)*arctan(2*sqrt(-b + 4)*x/(b - 4)))/(b - 4)]
```

giac [A] time = 0.31, size = 80, normalized size = 1.21

$$\frac{(b+8)\sqrt{-b+4}\arctan\left(\frac{x}{\sqrt{-\frac{1}{8}b+\frac{1}{8}\sqrt{b^2-16}}}\right)}{b^2+4b-32}-\frac{(b+8)\sqrt{-b+4}\arctan\left(\frac{x}{\sqrt{-\frac{1}{8}b-\frac{1}{8}\sqrt{b^2-16}}}\right)}{b^2+4b-32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="giac")
```

```
[Out] (b + 8)*sqrt(-b + 4)*arctan(x/sqrt(-1/8*b + 1/8*sqrt(b^2 - 16)))/(b^2 + 4*b
 - 32) - (b + 8)*sqrt(-b + 4)*arctan(x/sqrt(-1/8*b - 1/8*sqrt(b^2 - 16)))/(
b^2 + 4*b - 32)
```

maple [B] time = 0.03, size = 277, normalized size = 4.20

$$\frac{b\arctan\left(\frac{4x}{\sqrt{-2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{-2b-2\sqrt{(b-4)(b+4)}}}-\frac{b\arctan\left(\frac{4x}{\sqrt{-2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)(b+4)}\sqrt{-2b+2\sqrt{(b-4)(b+4)}}}+\frac{4\arctan\left(\frac{}{\sqrt{-2}}\right.}{\sqrt{(b-4)(b+4)}\sqrt{-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-b*x^2+1),x)
```

```
[Out] -4/((b-4)*(b+4))^(1/2)/(2*((b-4)*(b+4))^(1/2)-2*b)^(1/2)*arctan(4*x/(2*((b-
4)*(b+4))^(1/2)-2*b)^(1/2))+1/(2*((b-4)*(b+4))^(1/2)-2*b)^(1/2)*arctan(4*x/
(2*((b-4)*(b+4))^(1/2)-2*b)^(1/2))-1/((b-4)*(b+4))^(1/2)/(2*((b-4)*(b+4))^(
1/2)-2*b)^(1/2)*arctan(4*x/(2*((b-4)*(b+4))^(1/2)-2*b)^(1/2))*b+4/((b-4)*(b
```

```
+4))^(1/2)/(-2*((b-4)*(b+4))^(1/2)-2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(b+4))^
(1/2)-2*b)^(1/2))+1/(-2*((b-4)*(b+4))^(1/2)-2*b)^(1/2)*arctan(4*x/(-2*((b-4
)*(b+4))^(1/2)-2*b)^(1/2))+1/((b-4)*(b+4))^(1/2)/(-2*((b-4)*(b+4))^(1/2)-2*
b)^(1/2)*arctan(4*x/(-2*((b-4)*(b+4))^(1/2)-2*b)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2+1}{4\,x^4-bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - b*x^2 + 1), x)
```

mupad [B] time = 4.41, size = 24, normalized size = 0.36

$$-\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{b-4}}{2\,x^2-1}\right)}{\sqrt{b-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - b*x^2 + 1),x)
```

```
[Out] -atanh((x*(b - 4)^(1/2))/(2*x^2 - 1))/(b - 4)^(1/2)
```

sympy [A] time = 0.39, size = 83, normalized size = 1.26

$$\frac{\sqrt{\frac{1}{b-4}}\log\left(x^2+x\left(-\frac{b\sqrt{\frac{1}{b-4}}}{2}+2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2}-\frac{\sqrt{\frac{1}{b-4}}\log\left(x^2+x\left(\frac{b\sqrt{\frac{1}{b-4}}}{2}-2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-b*x**2+1),x)
```

```
[Out] sqrt(1/(b - 4))*log(x**2 + x*(-b*sqrt(1/(b - 4))/2 + 2*sqrt(1/(b - 4))) - 1
/2)/2 - sqrt(1/(b - 4))*log(x**2 + x*(b*sqrt(1/(b - 4))/2 - 2*sqrt(1/(b - 4
))) - 1/2)/2
```

3.41 $\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

```
[Out] 1/10*arctan(2*x/(1/2*10^(1/2)-1/2*2^(1/2)))*10^(1/2)+1/10*arctan(2*x/(1/2*1
0^(1/2)+1/2*2^(1/2)))*10^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[10] + ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/
Sqrt[10]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx &= \frac{1}{5}\left(5-\sqrt{5}\right)\int \frac{1}{3-\sqrt{5}+4x^2}\,dx+\frac{1}{5}\left(5+\sqrt{5}\right)\int \frac{1}{3+\sqrt{5}+4x^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 83, normalized size = 1.84

$$\frac{\left(\sqrt{5}-1\right)\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{2\sqrt{5\left(3-\sqrt{5}\right)}}+\frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{2\sqrt{5\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ((-1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]])/(2*Sqrt[5*(3 - Sqrt[5])])
+ ((1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(2*Sqrt[5*(3 + Sqrt[5])])
```

fricas [A] time = 0.40, size = 31, normalized size = 0.69

$$\frac{1}{10}\sqrt{10}\arctan\left(\frac{2}{5}\sqrt{10}\left(x^3+2x\right)\right)+\frac{1}{10}\sqrt{10}\arctan\left(\frac{1}{5}\sqrt{10}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*arctan(2/5*sqrt(10)*(x^3 + 2*x)) + 1/10*sqrt(10)*arctan(1/5*s
qrt(10)*x)
```

giac [A] time = 0.17, size = 39, normalized size = 0.87

$$\frac{1}{10}\sqrt{10}\arctan\left(\frac{4x}{\sqrt{10}+\sqrt{2}}\right)+\frac{1}{10}\sqrt{10}\arctan\left(\frac{4x}{\sqrt{10}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(10)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/10*sqrt(10)*arctan(4*x/(
sqrt(10) - sqrt(2)))
```

maple [B] time = 0.05, size = 136, normalized size = 3.02

$$-\frac{2\sqrt{5}\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}+\frac{2\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{2\sqrt{10}-2\sqrt{2}}+\frac{2\sqrt{5}\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}+\frac{2\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{2\sqrt{10}+2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+6*x^2+1),x)
```

```
[Out] 2/5*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctan(8*x/(2*10^(1/2)+2*2^(1/2)))+2/(2*
10^(1/2)+2*2^(1/2))*arctan(8*x/(2*10^(1/2)+2*2^(1/2)))-2/5*5^(1/2)/(2*10^(1
/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(1/2)))+2/(2*10^(1/2)-2*2^(1/2))*
arctan(8*x/(2*10^(1/2)-2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2+1}{4x^4+6x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + 6*x^2 + 1), x)
```

mupad [B] time = 0.09, size = 29, normalized size = 0.64

$$\frac{\sqrt{10}\left(\mathrm{atan}\left(\frac{2\sqrt{10}x^3}{5}+\frac{4\sqrt{10}x}{5}\right)+\mathrm{atan}\left(\frac{\sqrt{10}x}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(6*x^2 + 4*x^4 + 1),x)
```

```
[Out] (10^(1/2)*(atan((4*10^(1/2)*x)/5 + (2*10^(1/2)*x^3)/5) + atan((10^(1/2)*x)/
5)))/10
```

sympy [A] time = 0.15, size = 42, normalized size = 0.93

$$\frac{\sqrt{10}\left(2\operatorname{atan}\left(\frac{\sqrt{10}\,x}{5}\right)+2\operatorname{atan}\left(\frac{2\sqrt{10}\,x^{3}}{5}+\frac{4\sqrt{10}\,x}{5}\right)\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+6*x**2+1),x)
```

```
[Out] sqrt(10)*(2*atan(sqrt(10)*x/5) + 2*atan(2*sqrt(10)*x**3/5 + 4*sqrt(10)*x/5)
)/20
```

3.42 $\int \frac{1+2x^2}{1+5x^2+4x^4} \, dx$

Optimal. Leaf size=15

$$\frac{1}{3}\tan^{-1}(x) + \frac{1}{3}\tan^{-1}(2x)$$

```
[Out] 1/3*arctan(x)+1/3*arctan(2*x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1163, 203}

$$\frac{1}{3}\tan^{-1}(x) + \frac{1}{3}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[x]/3 + ArcTan[2*x]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{1+5x^2+4x^4} \, dx &= \frac{2}{3}\int \frac{1}{1+4x^2} \, dx + \frac{4}{3}\int \frac{1}{4+4x^2} \, dx \\ &= \frac{1}{3}\tan^{-1}(x) + \frac{1}{3}\tan^{-1}(2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.13

$$-\frac{1}{3}\tan^{-1}\left(\frac{3x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] -1/3*ArcTan[(3*x)/(-1 + 2*x^2)]
```

fricas [A] time = 0.39, size = 19, normalized size = 1.27

$$\frac{1}{3}\arctan\left(\frac{4}{3}x^3 + \frac{7}{3}x\right) + \frac{1}{3}\arctan\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")
[Out] 1/3*arctan(4/3*x^3 + 7/3*x) + 1/3*arctan(2/3*x)
```

giac [A]   time = 0.15, size = 11, normalized size = 0.73

$$\frac{1}{3}\arctan\left(2\,x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")
[Out] 1/3*arctan(2*x) + 1/3*arctan(x)
```

maple [A]   time = 0.01, size = 12, normalized size = 0.80

$$\frac{\arctan(x)}{3}+\frac{\arctan\left(2x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+5*x^2+1),x)
[Out] 1/3*arctan(x)+1/3*arctan(2*x)
```

maxima [A]   time = 2.49, size = 11, normalized size = 0.73

$$\frac{1}{3}\arctan\left(2\,x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")
[Out] 1/3*arctan(2*x) + 1/3*arctan(x)
```

mupad [B]   time = 0.07, size = 19, normalized size = 1.27

$$\frac{\mathrm{atan}\left(\frac{2\,x}{3}\right)}{3}+\frac{\mathrm{atan}\left(\frac{4\,x^3}{3}+\frac{7\,x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(5*x^2 + 4*x^4 + 1),x)
[Out] atan((2*x)/3)/3 + atan((7*x)/3 + (4*x^3)/3)/3
```

sympy [B]   time = 0.12, size = 22, normalized size = 1.47

$$\frac{\mathrm{atan}\left(\frac{2x}{3}\right)}{3}+\frac{\mathrm{atan}\left(\frac{4x^3}{3}+\frac{7x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+5*x**2+1),x)
[Out] atan(2*x/3)/3 + atan(4*x**3/3 + 7*x/3)/3
```

3.43 $\int \frac{1+2x^2}{1+4x^2+4x^4} \, dx$

Optimal. Leaf size=14

$$\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {28, 21, 203}

$$\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1+4x^2+4x^4} \, dx &= 4 \int \frac{1+2x^2}{\left(2+4x^2\right)^2} \, dx \\
&= \int \frac{1}{1+2x^2} \, dx \\
&= \frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

fricas [A] time = 0.40, size = 11, normalized size = 0.79

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

giac [A] time = 0.16, size = 11, normalized size = 0.79

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{\sqrt{2}\arctan\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+4*x^2+1),x)
[Out] 1/2*arctan(2^(1/2)*x)*2^(1/2)
```

maxima [A] time = 2.30, size = 11, normalized size = 0.79

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.79

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^2 + 4*x^4 + 1),x)
[Out] (2^(1/2)*atan(2^(1/2)*x))/2
```

sympy [A] time = 0.12, size = 14, normalized size = 1.00

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+4*x**2+1),x)
[Out] sqrt(2)*atan(sqrt(2)*x)/2
```

3.44 $\int \frac{1+2x^2}{1+3x^2+4x^4} \, dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -1/7*arctan(1/7*(1-4*x)*7^(1/2))*7^(1/2)+1/7*arctan(1/7*(1+4*x)*7^(1/2))*7^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(1 - 4*x)/Sqrt[7]]/Sqrt[7]) + ArcTan[(1 + 4*x)/Sqrt[7]]/Sqrt[7]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1+3x^2+4x^4} \, dx &= \frac{1}{4} \int \frac{1}{\frac{1}{2}-\frac{x}{2}+x^2} \, dx + \frac{1}{4} \int \frac{1}{\frac{1}{2}+\frac{x}{2}+x^2} \, dx \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{7}{4}-x^2} \, dx, x, -\frac{1}{2}+2x\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-\frac{7}{4}-x^2} \, dx, x, \frac{1}{2}+2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\frac{1+4x}{\sqrt{7}}\right)}{\sqrt{7}}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 97, normalized size = 2.55

$$\frac{\left(\sqrt{7}-i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(3-i\sqrt{7}\right)}}\right)}{\sqrt{42-14i\sqrt{7}}}+\frac{\left(\sqrt{7}+i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(3+i\sqrt{7}\right)}}\right)}{\sqrt{42+14i\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] ((-I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 - I*Sqrt[7])/2]])/Sqrt[42 - (14*I)*Sqr
t[7]] + ((I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 + I*Sqrt[7])/2]])/Sqrt[42 + (14
*I)*Sqrt[7]]
```

fricas [A] time = 0.40, size = 33, normalized size = 0.87

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(4x^3+5x\right)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{2}{7}\sqrt{7}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x^3 + 5*x)) + 1/7*sqrt(7)*arctan(2/7*sqrt
(7)*x)
```

giac [A] time = 0.17, size = 33, normalized size = 0.87

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+1)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)*
(4*x - 1))
```

maple [A] time = 0.01, size = 34, normalized size = 0.89

$$\frac{\sqrt{7}\arctan\left(\frac{(4x+1)\sqrt{7}}{7}\right)}{7}+\frac{\sqrt{7}\arctan\left(\frac{(4x-1)\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+3*x^2+1),x)
```

```
[Out] 1/7*7^(1/2)*arctan(1/7*(4*x-1)*7^(1/2))+1/7*arctan(1/7*(1+4*x)*7^(1/2))*7^(
1/2)
```

maxima [A] time = 2.39, size = 33, normalized size = 0.87

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+1)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)*
(4*x - 1))
```

mupad [B] time = 0.09, size = 29, normalized size = 0.76

$$\frac{\sqrt{7}\left(\operatorname{atan}\left(\frac{4\sqrt{7}x^3}{7}+\frac{5\sqrt{7}x}{7}\right)+\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(3*x^2 + 4*x^4 + 1),x)
```

```
[Out] (7^(1/2)*(atan((5*7^(1/2)*x)/7 + (4*7^(1/2)*x^3)/7) + atan((2*7^(1/2)*x)/7)
))/7
```

sympy [A] time = 0.14, size = 44, normalized size = 1.16

$$\frac{\sqrt{7}\left(2\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}\right)+2\operatorname{atan}\left(\frac{4\sqrt{7}x^3}{7}+\frac{5\sqrt{7}x}{7}\right)\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+3*x**2+1),x)
```

```
[Out] sqrt(7)*(2*atan(2*sqrt(7)*x/7) + 2*atan(4*sqrt(7)*x**3/7 + 5*sqrt(7)*x/7))/
14
```

3.45 $\int \frac{1+2x^2}{1+2x^2+4x^4} \, dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[Out] -1/6*arctan(1/3*(1-2*x*2^(1/2))*3^(1/2))*6^(1/2)+1/6*arctan(1/3*(1+2*x*2^(1
/2))*3^(1/2))*6^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(1 - 2*Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTan[(1 + 2*Sqrt[2]*x)/Sqr
t[3]]/Sqrt[6]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1+2x^2+4x^4} \, dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{x}{\sqrt{2}}+x^2} \, dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{x}{\sqrt{2}}+x^2} \, dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-x^2} \, dx,x,-\frac{1}{\sqrt{2}}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-x^2} \, dx,x,\frac{1}{\sqrt{2}}+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}
\end{aligned}$$

Mathematica [C] time = 0.10, size = 99, normalized size = 2.06

$$\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{2x}{\sqrt{1-i\sqrt{3}}}\right)}{2\sqrt{3\left(1-i\sqrt{3}\right)}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{2x}{\sqrt{1+i\sqrt{3}}}\right)}{2\sqrt{3\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] ((-I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 - I*Sqrt[3]]])/(2*Sqrt[3*(1 - I*Sqrt[3]
)]) + ((I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 + I*Sqrt[3]]])/(2*Sqrt[3*(1 + I*Sq
rt[3])])
```

fricas [A] time = 0.39, size = 29, normalized size = 0.60

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{2}{3}\sqrt{6}\left(x^3+x\right)\right)+\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{3}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*arctan(2/3*sqrt(6)*(x^3 + x)) + 1/6*sqrt(6)*arctan(1/3*sqrt(6)*
x)
```

giac [A] time = 0.19, size = 45, normalized size = 0.94

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{4}{3}\sqrt{3}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x+\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{6}\sqrt{6}\arctan\left(\frac{4}{3}\sqrt{3}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x-\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*arctan(4/3*sqrt(3)*(1/4)^(3/4)*(2*x + (1/4)^(1/4))) + 1/6*sqrt(
6)*arctan(4/3*sqrt(3)*(1/4)^(3/4)*(2*x - (1/4)^(1/4)))
```

maple [A] time = 0.03, size = 40, normalized size = 0.83

$$\frac{\sqrt{6}\arctan\left(\frac{\left(4x-\sqrt{2}\right)\sqrt{6}}{6}\right)}{6}+\frac{\sqrt{6}\arctan\left(\frac{\left(4x+\sqrt{2}\right)\sqrt{6}}{6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+2*x^2+1),x)
```

```
[Out] 1/6*6^(1/2)*arctan(1/6*(4*x+2^(1/2))*6^(1/2))+1/6*6^(1/2)*arctan(1/6*(4*x-2
^(1/2))*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2x^2+1}{4x^4+2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1), x)
```

mupad [B] time = 4.39, size = 29, normalized size = 0.60

$$\frac{\sqrt{6}\left(\mathrm{atan}\left(\frac{2\sqrt{6}\,x^3}{3}+\frac{2\sqrt{6}\,x}{3}\right)+\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(2*x^2 + 4*x^4 + 1),x)
```

```
[Out] (6^(1/2)*(atan((2*6^(1/2)*x)/3 + (2*6^(1/2)*x^3)/3) + atan((6^(1/2)*x)/3)))
/6
```

sympy [A] time = 0.13, size = 42, normalized size = 0.88

$$\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}\right)+2\,\mathrm{atan}\left(\frac{2\sqrt{6}\,x^3}{3}+\frac{2\sqrt{6}\,x}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+2*x**2+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3) + 2*atan(2*sqrt(6)*x**3/3 + 2*sqrt(6)*x/3))/12
```

3.46 $\int \frac{1+2x^2}{1+x^2+4x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -1/5*arctan(1/5*(-4*x+3^(1/2))*5^(1/2))*5^(1/2)+1/5*arctan(1/5*(4*x+3^(1/2)
)*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[3] - 4*x)/Sqrt[5]]/Sqrt[5]) + ArcTan[(Sqrt[3] + 4*x)/Sqrt[5]
]/Sqrt[5]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1+x^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{3}x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{3}x}{2}+x^2}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{5}{4}-x^2}\,dx,x,-\frac{\sqrt{3}}{2}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{5}{4}-x^2}\,dx,x,\frac{\sqrt{3}}{2}+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}+4x}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}$$

Mathematica [C] time = 0.22, size = 97, normalized size = 2.11

$$\frac{\left(\sqrt{15}-3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{15}\right)}}\right)}{\sqrt{30-30i\sqrt{15}}}+\frac{\left(\sqrt{15}+3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{15}\right)}}\right)}{\sqrt{30+30i\sqrt{15}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] ((-3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 - I*Sqrt[15])/2]])/Sqrt[30 - (30*I)
*Sqrt[15]] + ((3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 + I*Sqrt[15])/2]])/Sqrt
[30 + (30*I)*Sqrt[15]]
```

fricas [A] time = 0.41, size = 33, normalized size = 0.72

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\left(4x^3+3x\right)\right)+\frac{1}{5}\sqrt{5}\arctan\left(\frac{2}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(4*x^3 + 3*x)) + 1/5*sqrt(5)*arctan(2/5*sqrt
(5)*x)
```

giac [A] time = 0.26, size = 52, normalized size = 1.13

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{2}{5}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x+\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{5}\sqrt{5}\arctan\left(\frac{2}{5}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x-\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*arctan(2/5*sqrt(10)*(1/4)^(3/4)*(4*x + sqrt(6)*(1/4)^(1/4))) +
1/5*sqrt(5)*arctan(2/5*sqrt(10)*(1/4)^(3/4)*(4*x - sqrt(6)*(1/4)^(1/4)))
```

maple [A] time = 0.03, size = 40, normalized size = 0.87

$$\frac{\sqrt{5}\arctan\left(\frac{\left(4x-\sqrt{3}\right)\sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5}\arctan\left(\frac{\left(4x+\sqrt{3}\right)\sqrt{5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+x^2+1),x)
```

```
[Out] 1/5*arctan(1/5*(4*x+3^(1/2))*5^(1/2))*5^(1/2)+1/5*5^(1/2)*arctan(1/5*(4*x-3
^(1/2))*5^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2x^2+1}{4x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1), x)
```

mupad [B] time = 4.36, size = 29, normalized size = 0.63

$$\frac{\sqrt{5}\,\left(\operatorname{atan}\left(\frac{4\,\sqrt{5}\,x^3}{5}+\frac{3\,\sqrt{5}\,x}{5}\right)+\operatorname{atan}\left(\frac{2\,\sqrt{5}\,x}{5}\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(x^2 + 4*x^4 + 1),x)
```

```
[Out] (5^(1/2)*(atan((3*5^(1/2)*x)/5 + (4*5^(1/2)*x^3)/5) + atan((2*5^(1/2)*x)/5)
))/5
```

sympy [A] time = 0.13, size = 44, normalized size = 0.96

$$\frac{\sqrt{5}\left(2\operatorname{atan}\left(\frac{2\sqrt{5}x}{5}\right)+2\operatorname{atan}\left(\frac{4\sqrt{5}x^3}{5}+\frac{3\sqrt{5}x}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+x**2+1),x)
```

```
[Out] sqrt(5)*(2*atan(2*sqrt(5)*x/5) + 2*atan(4*sqrt(5)*x**3/5 + 3*sqrt(5)*x/5))/
10
```

3.47 $\int \frac{1+2x^2}{1+4x^4} \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\tan^{-1}(2x+1) - \frac{1}{2}\tan^{-1}(1-2x)$$

```
[Out] 1/2*arctan(-1+2*x)+1/2*arctan(1+2*x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1162, 617, 204}

$$\frac{1}{2}\tan^{-1}(2x+1) - \frac{1}{2}\tan^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -ArcTan[1 - 2*x]/2 + ArcTan[1 + 2*x]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1+4x^4} \, dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-x+x^2} \, dx + \frac{1}{4}\int \frac{1}{\frac{1}{2}+x+x^2} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1-2x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1+2x\right) \\
&= -\frac{1}{2}\tan^{-1}(1-2x) + \frac{1}{2}\tan^{-1}(1+2x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.81

$$-\frac{1}{2}\tan^{-1}\left(\frac{2x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -1/2*ArcTan[(2*x)/(-1 + 2*x^2)]
```

fricas [A] time = 0.42, size = 15, normalized size = 0.71

$$\frac{1}{2}\arctan\left(2x^3+x\right)+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(2*x^3 + x) + 1/2*arctan(x)
```

giac [B] time = 0.16, size = 46, normalized size = 2.19

$$\frac{1}{2}\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{2}\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(2*sqrt(2)*(1/4)^(3/4)*(2*x + sqrt(2)*(1/4)^(1/4))) + 1/2*arctan(
2*sqrt(2)*(1/4)^(3/4)*(2*x - sqrt(2)*(1/4)^(1/4)))
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$\frac{\arctan\left(2x+1\right)}{2}+\frac{\arctan\left(2x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+1),x)
```

```
[Out] 1/2*arctan(2*x-1)+1/2*arctan(2*x+1)
```

maxima [A] time = 2.24, size = 17, normalized size = 0.81

$$\frac{1}{2}\arctan\left(2x+1\right)+\frac{1}{2}\arctan\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(2*x + 1) + 1/2*arctan(2*x - 1)
```

mupad [B] time = 4.29, size = 15, normalized size = 0.71

$$\frac{\mathrm{atan}\left(2x^3+x\right)}{2}+\frac{\mathrm{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 + 1),x)
```

```
[Out] atan(x + 2*x^3)/2 + atan(x)/2
```

sympy [A] time = 0.11, size = 14, normalized size = 0.67

$$\frac{\mathrm{atan}\left(x\right)}{2}+\frac{\mathrm{atan}\left(2x^3+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+1),x)
```

```
[Out] atan(x)/2 + atan(2*x**3 + x)/2
```

3.48 $\int \frac{1+2x^2}{1-x^2+4x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctan(1/3*(-4*x+5^(1/2))*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(4*x+5^(1/2)
)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[5] - 4*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(Sqrt[5] + 4*x)/Sqrt[3]
]/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1-x^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}x}{2}+x^2}\,dx\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx,x,-\frac{\sqrt{5}}{2}+2x\right)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx,x,\frac{\sqrt{5}}{2}+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{5}+4x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.27, size = 101, normalized size = 2.20

$$\frac{\left(\sqrt{15}-5i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1-i\sqrt{15}\right)}}+\frac{\left(\sqrt{15}+5i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1+i\sqrt{15}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] ((-5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 - I*Sqrt[15])/2]])/Sqrt[30*(-1 - I
*Sqrt[15])] + ((5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 + I*Sqrt[15])/2]])/Sq
rt[30*(-1 + I*Sqrt[15])]
```

fricas [A] time = 0.38, size = 31, normalized size = 0.67

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(4x^3+x\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(4*x^3 + x)) + 1/3*sqrt(3)*arctan(2/3*sqrt(3
)*x)
```

giac [A] time = 0.24, size = 52, normalized size = 1.13

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{6}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x+\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{6}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x-\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(6)*(1/4)^(3/4)*(4*x + sqrt(10)*(1/4)^(1/4))) +
1/3*sqrt(3)*arctan(2/3*sqrt(6)*(1/4)^(3/4)*(4*x - sqrt(10)*(1/4)^(1/4)))
```

maple [A] time = 0.03, size = 40, normalized size = 0.87

$$\frac{\sqrt{3}\arctan\left(\frac{\left(4x-\sqrt{5}\right)\sqrt{3}}{3}\right)}{3}+\frac{\sqrt{3}\arctan\left(\frac{\left(4x+\sqrt{5}\right)\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-x^2+1),x)
```

```
[Out] 1/3*arctan(1/3*(4*x+5^(1/2))*3^(1/2))*3^(1/2)+1/3*3^(1/2)*arctan(1/3*(4*x-5
^(1/2))*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2x^2+1}{4x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1), x)
```

mupad [B] time = 4.37, size = 29, normalized size = 0.63

$$\frac{\sqrt{3}\,\left(\mathrm{atan}\left(\frac{4\,\sqrt{3}\,x^3}{3}+\frac{\sqrt{3}\,x}{3}\right)+\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - x^2 + 1),x)
```

```
[Out] (3^(1/2)*(atan((3^(1/2)*x)/3 + (4*3^(1/2)*x^3)/3) + atan((2*3^(1/2)*x)/3)))
/3
```

sympy [A] time = 0.14, size = 42, normalized size = 0.91

$$\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}\right)+2\,\mathrm{atan}\left(\frac{4\sqrt{3}\,x^3}{3}+\frac{\sqrt{3}\,x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-x**2+1),x)
```

```
[Out] sqrt(3)*(2*atan(2*sqrt(3)*x/3) + 2*atan(4*sqrt(3)*x**3/3 + sqrt(3)*x/3))/6
```

3.49 $\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx$

Optimal. Leaf size=44

$$\frac{\tan^{-1}\left(2\sqrt{2}\,x+\sqrt{3}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(2*x*2^(1/2)-3^(1/2))*2^(1/2)+1/2*arctan(2*x*2^(1/2)+3^(1/2))*2^(
1/2)
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(2\sqrt{2}\,x+\sqrt{3}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[Sqrt[3] - 2*Sqrt[2]*x]/Sqrt[2]) + ArcTan[Sqrt[3] + 2*Sqrt[2]*x]/Sq
rt[2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\sqrt{\frac{3}{2}}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\sqrt{\frac{3}{2}}\,x+x^2}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-x^2}\,dx,x,-\sqrt{\frac{3}{2}}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-x^2}\,dx,x,\sqrt{\frac{3}{2}}+2x\right)\\
&= -\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{3}+2\sqrt{2}\,x\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.10, size = 99, normalized size = 2.25

$$\frac{\left(\sqrt{3}-3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{-1-i\sqrt{3}}}\right)}{2\sqrt{3\left(-1-i\sqrt{3}\right)}}+\frac{\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{-1+i\sqrt{3}}}\right)}{2\sqrt{3\left(-1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] ((-3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 - I*Sqrt[3]]])/(2*Sqrt[3*(-1 - I*Sqr
t[3])]) + ((3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 + I*Sqrt[3]]])/(2*Sqrt[3*(-
1 + I*Sqrt[3])])
```

fricas [A] time = 0.40, size = 26, normalized size = 0.59

$$\frac{1}{2}\sqrt{2}\arctan\left(2\sqrt{2}x^{3}\right)+\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(2*sqrt(2)*x^3) + 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

giac [A] time = 0.17, size = 46, normalized size = 1.05

$$\frac{1}{2}\sqrt{2}\arctan\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x+\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x-\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(4*(1/4)^(3/4)*(2*x + sqrt(3)*(1/4)^(1/4))) + 1/2*sqrt(2)
*arctan(4*(1/4)^(3/4)*(2*x - sqrt(3)*(1/4)^(1/4)))
```

maple [A] time = 0.04, size = 40, normalized size = 0.91

$$\frac{\sqrt{2}\arctan\left(\frac{\left(4x-\sqrt{6}\right)\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\arctan\left(\frac{\left(4x+\sqrt{6}\right)\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-2*x^2+1),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/2*(4*x+6^(1/2))*2^(1/2))+1/2*2^(1/2)*arctan(1/2*(4*x-6
^(1/2))*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2x^{2}+1}{4x^{4}-2x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1), x)
```

mupad [B] time = 0.06, size = 21, normalized size = 0.48

$$\frac{\sqrt{2}\ \left(\mathrm{atan}\left(\sqrt{2}\ x\right)+\mathrm{atan}\left(2\ \sqrt{2}\ x^3\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1),x)
[Out] (2^(1/2)*(atan(2^(1/2)*x) + atan(2*2^(1/2)*x^3)))/2
```

sympy [A] time = 0.13, size = 29, normalized size = 0.66

$$\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\sqrt{2}\,x\right)+2\,\mathrm{atan}\left(2\sqrt{2}\,x^3\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-2*x**2+1),x)
[Out] sqrt(2)*(2*atan(sqrt(2)*x) + 2*atan(2*sqrt(2)*x**3))/4
```

3.50 $\int \frac{1+2x^2}{1-3x^2+4x^4} \, dx$

Optimal. Leaf size=23

$$\tan^{-1}\left(4x+\sqrt{7}\right)-\tan^{-1}\left(\sqrt{7}-4x\right)$$

```
[Out] arctan(4*x-7^(1/2))+arctan(4*x+7^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 204}

$$\tan^{-1}\left(4x+\sqrt{7}\right)-\tan^{-1}\left(\sqrt{7}-4x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] -ArcTan[Sqrt[7] - 4*x] + ArcTan[Sqrt[7] + 4*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1-3x^2+4x^4} \, dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{7}x}{2}+x^2} \, dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{7}x}{2}+x^2} \, dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{4}-x^2} \, dx,x,-\frac{\sqrt{7}}{2}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{4}-x^2} \, dx,x,\frac{\sqrt{7}}{2}+2x\right)\\
&= -\tan^{-1}\left(\sqrt{7}-4x\right)+\tan^{-1}\left(\sqrt{7}+4x\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.61

$$-\tan^{-1}\left(\frac{x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] -ArcTan[x/(-1 + 2*x^2)]
```

fricas [A] time = 0.38, size = 15, normalized size = 0.65

$$\arctan\left(4\,x^3-x\right)+\arctan\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(4*x^3 - x) + arctan(2*x)
```

giac [B] time = 0.19, size = 42, normalized size = 1.83

$$\arctan\left(2\,\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x+\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\arctan\left(2\,\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x-\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] arctan(2*sqrt(2)*(1/4)^(3/4)*(4*x + sqrt(14)*(1/4)^(1/4))) + arctan(2*sqrt(
2)*(1/4)^(3/4)*(4*x - sqrt(14)*(1/4)^(1/4)))
```

maple [A] time = 0.04, size = 20, normalized size = 0.87

$$\arctan\left(4x-\sqrt{7}\right)+\arctan\left(4x+\sqrt{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-3*x^2+1),x)
```

```
[Out] arctan(4*x-7^(1/2))+arctan(4*x+7^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2\,x^2+1}{4\,x^4-3\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1), x)
```

mupad [B] time = 4.35, size = 15, normalized size = 0.65

$$\mathrm{atan}\left(2\,x\right)-\mathrm{atan}\left(x-4\,x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1),x)
```

```
[Out] atan(2*x) - atan(x - 4*x^3)
```

sympy [A] time = 0.11, size = 12, normalized size = 0.52

$$\operatorname{atan}{\left(2x\right)}+\operatorname{atan}{\left(4x^{3}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-3*x**2+1),x)
```

```
[Out] atan(2*x) + atan(4*x**3 - x)
```

3.51 $\int \frac{1+2x^2}{1-4x^2+4x^4} \, dx$

Optimal. Leaf size=11

$$\frac{x}{1-2x^2}$$

```
[Out] x/(-2*x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 383}

$$\frac{x}{1-2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 - 2*x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1-4x^2+4x^4} \, dx &= 4 \int \frac{1+2x^2}{\left(-2+4x^2\right)^2} \, dx \\ &= \frac{x}{1-2x^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.09

$$-\frac{x}{2x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] -(x/(-1 + 2*x^2))
```

fricas [A] time = 0.41, size = 12, normalized size = 1.09

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] -x/(2*x^2 - 1)
```

giac [A] time = 0.16, size = 12, normalized size = 1.09

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")
[Out] -x/(2*x^2 - 1)
```

maple [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{x}{2\left(x^2-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-4*x^2+1),x)
[Out] -1/2*x/(x^2-1/2)
```

maxima [A] time = 0.93, size = 12, normalized size = 1.09

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="maxima")
[Out] -x/(2*x^2 - 1)
```

mupad [B] time = 4.30, size = 12, normalized size = 1.09

$$-\frac{x}{2\,\left(x^2-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - 4*x^2 + 1),x)
[Out] -x/(2*(x^2 - 1/2))
```

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$-\frac{x}{2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-4*x**2+1),x)
[Out] -x/(2*x**2 - 1)
```

3.52 $\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(2x+1)$$

```
[Out] -1/2*ln(1-2*x)+1/2*ln(1-x)-1/2*ln(1+x)+1/2*ln(1+2*x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 616, 31}

$$-\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - 2*x]/2 + Log[1 - x]/2 - Log[1 + x]/2 + Log[1 + 2*x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{3x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{3x}{2}+x^2}\,dx\\
&= \frac{1}{2}\int \frac{1}{-1+x}\,dx-\frac{1}{2}\int \frac{1}{-\frac{1}{2}+x}\,dx+\frac{1}{2}\int \frac{1}{\frac{1}{2}+x}\,dx-\frac{1}{2}\int \frac{1}{1+x}\,dx\\
&= -\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(1+x)+\frac{1}{2}\log(1+2x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.74

$$\frac{1}{2}\log\left(-2x^2+x+1\right)-\frac{1}{2}\log\left(-2x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
[Out] -1/2*Log[1 - x - 2*x^2] + Log[1 + x - 2*x^2]/2
```

fricas [A] time = 0.39, size = 25, normalized size = 0.64

$$-\frac{1}{2}\log\left(2x^2+x-1\right)+\frac{1}{2}\log\left(2x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="fricas")
[Out] -1/2*log(2*x^2 + x - 1) + 1/2*log(2*x^2 - x - 1)
```

giac [A] time = 0.17, size = 33, normalized size = 0.85

$$\frac{1}{2}\log\left(|2x+1|\right)-\frac{1}{2}\log\left(|2x-1|\right)-\frac{1}{2}\log\left(|x+1|\right)+\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
[Out] 1/2*log(abs(2*x + 1)) - 1/2*log(abs(2*x - 1)) - 1/2*log(abs(x + 1)) + 1/2*l
og(abs(x - 1))
```

maple [A] time = 0.01, size = 30, normalized size = 0.77

$$-\frac{\ln(x+1)}{2}+\frac{\ln(2x+1)}{2}+\frac{\ln(x-1)}{2}-\frac{\ln(2x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-5*x^2+1),x)
[Out] -1/2*ln(2*x-1)+1/2*ln(2*x+1)-1/2*ln(x+1)+1/2*ln(x-1)
```

maxima [A] time = 1.04, size = 29, normalized size = 0.74

$$\frac{1}{2}\log\left(2x+1\right)-\frac{1}{2}\log\left(2x-1\right)-\frac{1}{2}\log\left(x+1\right)+\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="maxima")
[Out] 1/2*log(2*x + 1) - 1/2*log(2*x - 1) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

mupad [B] time = 0.30, size = 14, normalized size = 0.36

$$-\operatorname{atanh}\left(\frac{x}{2x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - 5*x^2 + 1),x)
[Out] -atanh(x/(2*x^2 - 1))
```

sympy [A] time = 0.13, size = 26, normalized size = 0.67

$$\frac{\log\left(x^2-\frac{x}{2}-\frac{1}{2}\right)}{2}-\frac{\log\left(x^2+\frac{x}{2}-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-5*x**2+1),x)
```

```
[Out] log(x**2 - x/2 - 1/2)/2 - log(x**2 + x/2 - 1/2)/2
```

3.53 $\int \frac{1+2x^2}{1-6x^2+4x^4} \, dx$

Optimal. Leaf size=44

$$\frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(2\sqrt{2}\,x+\sqrt{5}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(2*x*2^(1/2)-5^(1/2))*2^(1/2)-1/2*arctanh(2*x*2^(1/2)+5^(1/2))*
2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(2\sqrt{2}\,x+\sqrt{5}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTanh[Sqrt[5] - 2*Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[5] + 2*Sqrt[2]*x]/Sqr
t[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1-6x^2+4x^4} \, dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\sqrt{\frac{5}{2}}\,x+x^2} \, dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\sqrt{\frac{5}{2}}\,x+x^2} \, dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{2}-x^2} \, dx,x,-\sqrt{\frac{5}{2}}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{2}-x^2} \, dx,x,\sqrt{\frac{5}{2}}+2x\right)\\
&= \frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{5}+2\sqrt{2}\,x\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.95

$$\frac{\log\left(-2x^2+\sqrt{2}x+1\right)-\log\left(2x^2+\sqrt{2}x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
[Out] (Log[1 + Sqrt[2]*x - 2*x^2] - Log[-1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])
```

fricas [A] time = 0.38, size = 47, normalized size = 1.07

$$\frac{1}{4}\sqrt{2}\log\left(\frac{4x^4-2x^2-2\sqrt{2}\left(2x^3-x\right)+1}{4x^4-6x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((4*x^4 - 2*x^2 - 2*sqrt(2)*(2*x^3 - x) + 1)/(4*x^4 - 6*x^2
+ 1))
```

giac [B] time = 0.34, size = 77, normalized size = 1.75

$$-\frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)+\frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right)-\frac{1}{4}\sqrt{2}\log\left(\left|x-\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)+\frac{1}{4}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")
[Out] -1/4*sqrt(2)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/4*sqrt(2)*log(abs
(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10) +
1/4*sqrt(2))) + 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2)))
```

maple [B] time = 0.04, size = 82, normalized size = 1.86

$$-\frac{2\left(-5+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}-\frac{2\left(5+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-6*x^2+1),x)
[Out] -2/5*(-5+5^(1/2))*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctanh(8/(2*10^(1/2)-2*2^
(1/2))*x)-2/5*(5+5^(1/2))*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctanh(8/(2*10^(1
/2)+2*2^(1/2))*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2x^2+1}{4x^4-6x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="maxima")
[Out] integrate((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1), x)
```

mupad [B] time = 0.22, size = 20, normalized size = 0.45

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2\,x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*x)/(2*x^2 - 1)))/2
```

sympy [A] time = 0.12, size = 46, normalized size = 1.05

$$\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}\,x}{2}-\frac{1}{2}\right)}{4}-\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}\,x}{2}-\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x/2 - 1/2)/4 - sqrt(2)*log(x**2 + sqrt(2)*x/2 -
1/2)/4
```

3.54 $\int \frac{1-2x^2}{1+bx^2+4x^4} \, dx$

Optimal. Leaf size=66

$$\frac{\log\left(\sqrt{4-b}\,x+2x^2+1\right)}{2\sqrt{4-b}}-\frac{\log\left(-\sqrt{4-b}\,x+2x^2+1\right)}{2\sqrt{4-b}}$$

```
[Out] -1/2*ln(1+2*x^2-x*(4-b)^(1/2))/(4-b)^(1/2)+1/2*ln(1+2*x^2+x*(4-b)^(1/2))/(4
-b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{\log\left(\sqrt{4-b}\,x+2x^2+1\right)}{2\sqrt{4-b}}-\frac{\log\left(-\sqrt{4-b}\,x+2x^2+1\right)}{2\sqrt{4-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[4 - b]*x + 2*x^2]/(2*Sqrt[4 - b]) + Log[1 + Sqrt[4 - b]*x + 2
*x^2]/(2*Sqrt[4 - b])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+bx^2+4x^4} \, dx &= -\frac{\int \frac{\frac{\sqrt{4-b}}{2}+2x}{-\frac{1}{2}-\frac{1}{2}\sqrt{4-b}\,x-x^2} \, dx}{2\sqrt{4-b}}-\frac{\int \frac{\frac{\sqrt{4-b}}{2}-2x}{-\frac{1}{2}+\frac{1}{2}\sqrt{4-b}\,x-x^2} \, dx}{2\sqrt{4-b}}\\ &= -\frac{\log\left(1-\sqrt{4-b}\,x+2x^2\right)}{2\sqrt{4-b}}+\frac{\log\left(1+\sqrt{4-b}\,x+2x^2\right)}{2\sqrt{4-b}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 127, normalized size = 1.92

$$\frac{\frac{\left(-\sqrt{b^2-16}+b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}\,x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{\sqrt{b-\sqrt{b^2-16}}}-\frac{\left(\sqrt{b^2-16}+b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}\,x}{\sqrt{\sqrt{b^2-16}+b}}\right)}{\sqrt{\sqrt{b^2-16}+b}}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] (((4 + b - Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]]
)/Sqrt[b - Sqrt[-16 + b^2]] - ((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*
x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-16
 + b^2])
```

fricas [A] time = 0.40, size = 109, normalized size = 1.65

$$\left[-\frac{\sqrt{-b+4}\,\log\left(\frac{4\,x^4-(b-8)x^2+2\left(2\,x^3+x\right)\sqrt{-b+4}+1}{4\,x^4+bx^2+1}\right)}{2\,(b-4)},\ \frac{\sqrt{b-4}\,\arctan\left(\frac{4\,x^3+(b-2)x}{\sqrt{b-4}}\right)-\sqrt{b-4}\,\arctan\left(\frac{2\,x}{\sqrt{b-4}}\right)}{b-4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b + 4)*log((4*x^4 - (b - 8)*x^2 + 2*(2*x^3 + x)*sqrt(-b + 4) +
1)/(4*x^4 + b*x^2 + 1))/(b - 4), (sqrt(b - 4)*arctan((4*x^3 + (b - 2)*x)/sq
rt(b - 4)) - sqrt(b - 4)*arctan(2*x/sqrt(b - 4)))/(b - 4)]
```

giac [A] time = 0.31, size = 73, normalized size = 1.11

$$-\frac{\sqrt{b-4}\,b\arctan\left(\frac{4\,\sqrt{\frac{1}{2}}\,x}{\sqrt{b+\sqrt{b^2-16}}}\right)}{b^2-4\,b}-\frac{\sqrt{b-4}\,b\arctan\left(\frac{4\,\sqrt{\frac{1}{2}}\,x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{b^2-4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] -sqrt(b - 4)*b*arctan(4*sqrt(1/2)*x/sqrt(b + sqrt(b^2 - 16)))/(b^2 - 4*b) -
 sqrt(b - 4)*b*arctan(4*sqrt(1/2)*x/sqrt(b - sqrt(b^2 - 16)))/(b^2 - 4*b)
```

maple [B] time = 0.02, size = 279, normalized size = 4.23

$$\frac{b\arctan\left(\frac{4x}{\sqrt{2b-2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)\,(b+4)}\,\sqrt{2b-2\sqrt{(b-4)\,(b+4)}}}-\frac{b\arctan\left(\frac{4x}{\sqrt{2b+2\sqrt{(b-4)(b+4)}}}\right)}{\sqrt{(b-4)\,(b+4)}\,\sqrt{2b+2\sqrt{(b-4)\,(b+4)}}}+\frac{4\arctan\left(\frac{4}{\sqrt{2b-2\sqrt{(}}}\right)}{\sqrt{(b-4)\,(b+4)}\,\sqrt{2b-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+b*x^2+1),x)
```

```
[Out] 4/((b-4)*(b+4))^(1/2)/(2*b-2*((b-4)*(b+4))^(1/2))^(1/2)*arctan(4/(2*b-2*((b
-4)*(b+4))^(1/2))^(1/2)*x)-1/(2*b-2*((b-4)*(b+4))^(1/2))^(1/2)*arctan(4/(2*
b-2*((b-4)*(b+4))^(1/2))^(1/2)*x)+1/((b-4)*(b+4))^(1/2)/(2*b-2*((b-4)*(b+4)
)^(1/2))^(1/2)*b*arctan(4/(2*b-2*((b-4)*(b+4))^(1/2))^(1/2)*x)-4/((b-4)*(b+
4))^(1/2)/(2*b+2*((b-4)*(b+4))^(1/2))^(1/2)*arctan(4/(2*b+2*((b-4)*(b+4))^(
1/2))^(1/2)*x)-1/(2*b+2*((b-4)*(b+4))^(1/2))^(1/2)*arctan(4/(2*b+2*((b-4)*(
b+4))^(1/2))^(1/2)*x)-1/((b-4)*(b+4))^(1/2)/(2*b+2*((b-4)*(b+4))^(1/2))^(1/
2)*b*arctan(4/(2*b+2*((b-4)*(b+4))^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,x^2-1}{4\,x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + b*x^2 + 1), x)
```

mupad [B] time = 0.07, size = 63, normalized size = 0.95

$$-\frac{\operatorname{atan}\left(\frac{2\,x}{\sqrt{b-4}}\right)-\operatorname{atan}\left(\frac{b^3\,x+4\,b^2\,x^3-2\,b^2\,x-16\,b\,x-64\,x^3+32\,x}{(b-4)^{3/2}\,(b+4)}\right)}{\sqrt{b-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(b*x^2 + 4*x^4 + 1),x)
```

```
[Out] -(atan((2*x)/(b - 4)^(1/2)) - atan((32*x - 16*b*x - 2*b^2*x + b^3*x - 64*x^
3 + 4*b^2*x^3)/((b - 4)^(3/2)*(b + 4))))/(b - 4)^(1/2)
```

sympy [A] time = 0.38, size = 94, normalized size = 1.42

$$\frac{\sqrt{-\frac{1}{b-4}}\log\left(x^2+x\left(-\frac{b\sqrt{-\frac{1}{b-4}}}{2}+2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2}-\frac{\sqrt{-\frac{1}{b-4}}\log\left(x^2+x\left(\frac{b\sqrt{-\frac{1}{b-4}}}{2}-2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] sqrt(-1/(b - 4))*log(x**2 + x*(-b*sqrt(-1/(b - 4))/2 + 2*sqrt(-1/(b - 4)))
+ 1/2)/2 - sqrt(-1/(b - 4))*log(x**2 + x*(b*sqrt(-1/(b - 4))/2 - 2*sqrt(-1/
(b - 4))) + 1/2)/2
```

3.55 $\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(2*x/(1/2*10^(1/2)-1/2*2^(1/2)))*2^(1/2)-1/2*arctan(2*x/(1/2*10^(
1/2)+1/2*2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[2] - ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/S
qrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx &= \left(-1-\sqrt{5}\right)\int \frac{1}{3+\sqrt{5}+4x^2}\,dx+\left(-1+\sqrt{5}\right)\int \frac{1}{3-\sqrt{5}+4x^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 84, normalized size = 1.83

$$\frac{-\left(\left(\sqrt{5}-5\right)\sqrt{3+\sqrt{5}}\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)\right)-\sqrt{3-\sqrt{5}}\left(5+\sqrt{5}\right)\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] (-((-5 + Sqrt[5])*Sqrt[3 + Sqrt[5]]*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]) - Sqrt
[3 - Sqrt[5]]*(5 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(4*Sqrt[5])
```

fricas [A] time = 0.39, size = 28, normalized size = 0.61

$$\frac{1}{2}\sqrt{2}\arctan\left(2\sqrt{2}\left(x^3+x\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(2*sqrt(2)*(x^3 + x)) - 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

giac [A] time = 0.18, size = 39, normalized size = 0.85

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x}{\sqrt{10}+\sqrt{2}}\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x}{\sqrt{10}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/2*sqrt(2)*arctan(4*x/(sqr
t(10) - sqrt(2)))
```

maple [B] time = 0.02, size = 136, normalized size = 2.96

$$\frac{2\sqrt{5}\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{2\sqrt{10}-2\sqrt{2}}-\frac{2\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{2\sqrt{10}-2\sqrt{2}}-\frac{2\sqrt{5}\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{2\sqrt{10}+2\sqrt{2}}-\frac{2\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{2\sqrt{10}+2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+6*x^2+1),x)
```

```
[Out] -2*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctan(8/(2*10^(1/2)+2*2^(1/2))*x)-2/(2*1
0^(1/2)+2*2^(1/2))*arctan(8/(2*10^(1/2)+2*2^(1/2))*x)+2*5^(1/2)/(2*10^(1/2)
-2*2^(1/2))*arctan(8/(2*10^(1/2)-2*2^(1/2))*x)-2/(2*10^(1/2)-2*2^(1/2))*arc
tan(8/(2*10^(1/2)-2*2^(1/2))*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2x^2-1}{4x^4+6x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + 6*x^2 + 1), x)
```

mupad [B] time = 4.38, size = 30, normalized size = 0.65

$$\frac{\sqrt{2}\left(\mathrm{atan}\left(2\sqrt{2}x^3+2\sqrt{2}x\right)-\mathrm{atan}\left(\sqrt{2}x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(6*x^2 + 4*x^4 + 1),x)
```

```
[Out] (2^(1/2)*(atan(2*2^(1/2)*x + 2*2^(1/2)*x^3) - atan(2^(1/2)*x)))/2
```

sympy [A] time = 0.13, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2}\,\left(2\,\mathrm{atan}\left(\sqrt{2}\,x\right)-2\,\mathrm{atan}\left(2\sqrt{2}\,x^{3}+2\sqrt{2}\,x\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+6*x**2+1),x)
```

```
[Out] -sqrt(2)*(2*atan(sqrt(2)*x) - 2*atan(2*sqrt(2)*x**3 + 2*sqrt(2)*x))/4
```

3.56 $\int \frac{1-2x^2}{1+5x^2+4x^4} \, dx$

Optimal. Leaf size=9

$$\tan^{-1}(2x) - \tan^{-1}(x)$$

```
[Out] -arctan(x)+arctan(2*x)
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1163, 203}

$$\tan^{-1}(2x) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] -ArcTan[x] + ArcTan[2*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1+5x^2+4x^4} \, dx &= 2 \int \frac{1}{1+4x^2} \, dx - 4 \int \frac{1}{4+4x^2} \, dx \\ &= -\tan^{-1}(x) + \tan^{-1}(2x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.33

$$\tan^{-1}\left(\frac{x}{2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[x/(1 + 2*x^2)]
```

fricas [A] time = 0.41, size = 17, normalized size = 1.89

$$\arctan\left(4\,x^3 + 3\,x\right) - \arctan\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(4*x^3 + 3*x) - arctan(2*x)
```

giac [A] time = 0.17, size = 9, normalized size = 1.00

$$\arctan(2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")
[Out] arctan(2*x) - arctan(x)
```

maple [A] time = 0.01, size = 10, normalized size = 1.11

$$-\arctan(x) + \arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+5*x^2+1),x)
[Out] -arctan(x)+arctan(2*x)
```

maxima [A] time = 2.36, size = 9, normalized size = 1.00

$$\arctan(2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")
[Out] arctan(2*x) - arctan(x)
```

mupad [B] time = 4.36, size = 17, normalized size = 1.89

$$\mathrm{atan}\left(4x^3 + 3x\right) - \mathrm{atan}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(5*x^2 + 4*x^4 + 1),x)
[Out] atan(3*x + 4*x^3) - atan(2*x)
```

sympy [A] time = 0.12, size = 14, normalized size = 1.56

$$-\mathrm{atan}(2x) + \mathrm{atan}\left(4x^3 + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+5*x**2+1),x)
[Out] -atan(2*x) + atan(4*x**3 + 3*x)
```

3.57 $\int \frac{1-2x^2}{1+4x^2+4x^4} dx$

Optimal. Leaf size=11

$$\frac{x}{2x^2+1}$$

```
[Out] x/(2*x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{28, 383\}$

$$\frac{x}{2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 + 2*x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+4x^2+4x^4} dx &= 4\int \frac{1-2x^2}{\left(2+4x^2\right)^2} dx\\ &= \frac{x}{1+2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x}{2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 + 2*x^2)
```

fricas [A] time = 0.38, size = 11, normalized size = 1.00

$$\frac{x}{2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] x/(2*x^2 + 1)
```

giac [A] time = 0.16, size = 11, normalized size = 1.00

$$\frac{x}{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")
[Out] x/(2*x^2 + 1)
```

maple [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{x}{2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+4*x^2+1),x)
[Out] 1/2*x/(x^2+1/2)
```

maxima [A] time = 1.08, size = 11, normalized size = 1.00

$$\frac{x}{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")
[Out] x/(2*x^2 + 1)
```

mupad [B] time = 4.30, size = 11, normalized size = 1.00

$$\frac{x}{2\,\left(x^2+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^2 + 4*x^4 + 1),x)
[Out] x/(2*(x^2 + 1/2))
```

sympy [A] time = 0.09, size = 7, normalized size = 0.64

$$\frac{x}{2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+4*x**2+1),x)
[Out] x/(2*x**2 + 1)
```

3.58 $\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

```
[Out] -1/2*ln(2*x^2-x+1)+1/2*ln(2*x^2+x+1)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - x + 2*x^2]/2 + Log[1 + x + 2*x^2]/2
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx &= -\left(\frac{1}{2}\int \frac{\frac{1}{2}+2x}{-\frac{1}{2}-\frac{x}{2}-x^2}\,dx\right)-\frac{1}{2}\int \frac{\frac{1}{2}-2x}{-\frac{1}{2}+\frac{x}{2}-x^2}\,dx\\ &= -\frac{1}{2}\log\left(1-x+2x^2\right)+\frac{1}{2}\log\left(1+x+2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - x + 2*x^2] + Log[1 + x + 2*x^2]/2
```

fricas [A] time = 0.39, size = 25, normalized size = 0.86

$$\frac{1}{2}\,\log\left(2\,x^2+x+1\right)-\frac{1}{2}\,\log\left(2\,x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="fricas")
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

giac [A] time = 0.15, size = 25, normalized size = 0.86

$$\frac{1}{2}\,\log\left(2\,x^{2}+x+1\right)-\frac{1}{2}\,\log\left(2\,x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

maple [A] time = 0.00, size = 26, normalized size = 0.90

$$-\frac{\ln\left(2x^{2}-x+1\right)}{2}+\frac{\ln\left(2x^{2}+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+3*x^2+1),x)
[Out] -1/2*ln(2*x^2-x+1)+1/2*ln(2*x^2+x+1)
```

maxima [A] time = 1.00, size = 25, normalized size = 0.86

$$\frac{1}{2}\,\log\left(2\,x^{2}+x+1\right)-\frac{1}{2}\,\log\left(2\,x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="maxima")
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

mupad [B] time = 0.06, size = 12, normalized size = 0.41

$$\mathrm{atanh}\left(\frac{x}{2\,x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(3*x^2 + 4*x^4 + 1),x)
[Out] atanh(x/(2*x^2 + 1))
```

sympy [A] time = 0.11, size = 26, normalized size = 0.90

$$-\frac{\log\left(x^{2}-\frac{x}{2}+\frac{1}{2}\right)}{2}+\frac{\log\left(x^{2}+\frac{x}{2}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+3*x**2+1),x)
[Out] -log(x**2 - x/2 + 1/2)/2 + log(x**2 + x/2 + 1/2)/2
```

3.59 $\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{2}\,x+1\right)}{2\sqrt{2}}-\frac{\log\left(2x^2-\sqrt{2}\,x+1\right)}{2\sqrt{2}}$$

```
[Out] -1/4*ln(1+2*x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+2*x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{\log\left(2x^2+\sqrt{2}\,x+1\right)}{2\sqrt{2}}-\frac{\log\left(2x^2-\sqrt{2}\,x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[2]*x + 2*x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + 2*x^2]/(2*Sqr
t[2])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx &= -\frac{\int \frac{\frac{1}{\sqrt{2}}+2x}{-\frac{1}{2}-\frac{x}{\sqrt{2}}-x^2}\,dx}{2\sqrt{2}}-\frac{\int \frac{\frac{1}{\sqrt{2}}-2x}{-\frac{1}{2}+\frac{x}{\sqrt{2}}-x^2}\,dx}{2\sqrt{2}}\\ &= -\frac{\log\left(1-\sqrt{2}\,x+2x^2\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+2x^2\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{2}\,x+1\right)-\log\left(-2x^2+\sqrt{2}\,x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[2]*x - 2*x^2] + Log[1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])
```

fricas [A]   time = 0.40, size = 45, normalized size = 0.90

$$\frac{1}{4}\sqrt{2}\log\left(\frac{4x^4+6x^2+2\sqrt{2}\left(2x^3+x\right)+1}{4x^4+2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((4*x^4 + 6*x^2 + 2*sqrt(2)*(2*x^3 + x) + 1)/(4*x^4 + 2*x^2
+ 1))
```

giac [A]   time = 0.17, size = 34, normalized size = 0.68

$$\frac{1}{4}\sqrt{2}\log\left(x^2+\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(x^2 + (1/4)^(1/4)*x + 1/2) - 1/4*sqrt(2)*log(x^2 - (1/4)^(1
/4)*x + 1/2)
```

maple [A]   time = 0.01, size = 39, normalized size = 0.78

$$-\frac{\sqrt{2}\,\ln\left(2x^2-\sqrt{2}\,x+1\right)}{4}+\frac{\sqrt{2}\,\ln\left(2x^2+\sqrt{2}\,x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+2*x^2+1),x)
```

```
[Out] -1/4*ln(1+2*x^2-2^(1/2)*x)*2^(1/2)+1/4*ln(1+2*x^2+2^(1/2)*x)*2^(1/2)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2x^2-1}{4x^4+2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + 2*x^2 + 1), x)
```

mupad [B]   time = 4.37, size = 20, normalized size = 0.40

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(2*x^2 + 4*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*x)/(2*x^2 + 1)))/2
```

sympy [A]   time = 0.11, size = 46, normalized size = 0.92

$$-\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+2*x**2+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x/2 + 1/2)/4 + sqrt(2)*log(x**2 + sqrt(2)*x/2 +
 1/2)/4
```

3.60 $\int \frac{1-2x^2}{1+x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{3}\,x+1\right)}{2\sqrt{3}}-\frac{\log\left(2x^2-\sqrt{3}\,x+1\right)}{2\sqrt{3}}$$

```
[Out] -1/6*ln(1+2*x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+2*x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1164, 628}

$$\frac{\log\left(2x^2+\sqrt{3}\,x+1\right)}{2\sqrt{3}}-\frac{\log\left(2x^2-\sqrt{3}\,x+1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[3]*x + 2*x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + 2*x^2]/(2*Sqr
t[3])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+x^2+4x^4}\,dx &= -\frac{\int \frac{\frac{\sqrt{3}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{3}x}{2}-x^2}\,dx}{2\sqrt{3}}-\frac{\int \frac{\frac{\sqrt{3}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{3}x}{2}-x^2}\,dx}{2\sqrt{3}}\\ &= -\frac{\log\left(1-\sqrt{3}\,x+2x^2\right)}{2\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+2x^2\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{3}\,x+1\right)-\log\left(-2x^2+\sqrt{3}\,x-1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[3]*x - 2*x^2] + Log[1 + Sqrt[3]*x + 2*x^2])/(2*Sqrt[3])
```

fricas [A] time = 0.39, size = 43, normalized size = 0.86

$$\frac{1}{6}\sqrt{3}\log\left(\frac{4x^4+7x^2+2\sqrt{3}\left(2x^3+x\right)+1}{4x^4+x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((4*x^4 + 7*x^2 + 2*sqrt(3)*(2*x^3 + x) + 1)/(4*x^4 + x^2 +
1))
```

giac [A] time = 0.26, size = 41, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\log\left(x^2+\frac{1}{2}\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{6}\sqrt{3}\log\left(x^2-\frac{1}{2}\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(x^2 + 1/2*sqrt(6)*(1/4)^(1/4)*x + 1/2) - 1/6*sqrt(3)*log(x^
2 - 1/2*sqrt(6)*(1/4)^(1/4)*x + 1/2)
```

maple [A] time = 0.01, size = 39, normalized size = 0.78

$$-\frac{\sqrt{3}\,\ln\left(2x^2-\sqrt{3}\,x+1\right)}{6}+\frac{\sqrt{3}\,\ln\left(2x^2+\sqrt{3}\,x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+x^2+1),x)
```

```
[Out] -1/6*ln(1+2*x^2-3^(1/2)*x)*3^(1/2)+1/6*ln(1+2*x^2+3^(1/2)*x)*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2x^2-1}{4x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + x^2 + 1), x)
```

mupad [B] time = 0.07, size = 20, normalized size = 0.40

$$\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{2\,x^2+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(x^2 + 4*x^4 + 1),x)
```

```
[Out] (3^(1/2)*atanh((3^(1/2)*x)/(2*x^2 + 1)))/3
```

sympy [A] time = 0.12, size = 46, normalized size = 0.92

$$-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x/2 + 1/2)/6 + sqrt(3)*log(x**2 + sqrt(3)*x/2 +
 1/2)/6
```

3.61 $\int \frac{1-2x^2}{1+4x^4} \, dx$

Optimal. Leaf size=31

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

```
[Out] -1/4*ln(2*x^2-2*x+1)+1/4*ln(2*x^2+2*x+1)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1165, 628}

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -Log[1 - 2*x + 2*x^2]/4 + Log[1 + 2*x + 2*x^2]/4
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+4x^4}\,dx &= -\left(\frac{1}{4}\int \frac{1+2x}{-\frac{1}{2}-x-x^2}\,dx\right)-\frac{1}{4}\int \frac{1-2x}{-\frac{1}{2}+x-x^2}\,dx\\ &= -\frac{1}{4}\log\left(1-2x+2x^2\right)+\frac{1}{4}\log\left(1+2x+2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -1/4*Log[1 - 2*x + 2*x^2] + Log[1 + 2*x + 2*x^2]/4
```

fricas [A] time = 0.40, size = 27, normalized size = 0.87

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

giac [A] time = 0.16, size = 34, normalized size = 1.10

$$\frac{1}{4}\log\left(x^2+\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{4}\log\left(x^2-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*log(x^2 + sqrt(2)*(1/4)^(1/4)*x + 1/2) - 1/4*log(x^2 - sqrt(2)*(1/4)^(1
/4)*x + 1/2)
```

maple [A] time = 0.00, size = 28, normalized size = 0.90

$$-\frac{\ln\left(2x^2-2x+1\right)}{4}+\frac{\ln\left(2x^2+2x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+1),x)
```

```
[Out] -1/4*ln(2*x^2-2*x+1)+1/4*ln(2*x^2+2*x+1)
```

maxima [A] time = 1.06, size = 27, normalized size = 0.87

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

mupad [B] time = 0.07, size = 15, normalized size = 0.48

$$\frac{\operatorname{atanh}\left(\frac{2x}{2x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 + 1),x)
```

```
[Out] atanh((2*x)/(2*x^2 + 1))/2
```

sympy [A] time = 0.11, size = 22, normalized size = 0.71

$$-\frac{\log\left(x^2-x+\frac{1}{2}\right)}{4}+\frac{\log\left(x^2+x+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+1),x)
```

```
[Out] -log(x**2 - x + 1/2)/4 + log(x**2 + x + 1/2)/4
```

3.62 $\int \frac{1-2x^2}{1-x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{5}\,x+1\right)}{2\sqrt{5}}-\frac{\log\left(2x^2-\sqrt{5}\,x+1\right)}{2\sqrt{5}}$$

```
[Out] -1/10*ln(1+2*x^2-x*5^(1/2))*5^(1/2)+1/10*ln(1+2*x^2+x*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{\log\left(2x^2+\sqrt{5}\,x+1\right)}{2\sqrt{5}}-\frac{\log\left(2x^2-\sqrt{5}\,x+1\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[5]*x + 2*x^2]/(2*Sqrt[5]) + Log[1 + Sqrt[5]*x + 2*x^2]/(2*Sqr
t[5])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1-x^2+4x^4}\,dx &= -\frac{\int \frac{\frac{\sqrt{5}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{5}x}{2}-x^2}\,dx}{2\sqrt{5}}-\frac{\int \frac{\frac{\sqrt{5}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{5}x}{2}-x^2}\,dx}{2\sqrt{5}}\\ &= -\frac{\log\left(1-\sqrt{5}\,x+2x^2\right)}{2\sqrt{5}}+\frac{\log\left(1+\sqrt{5}\,x+2x^2\right)}{2\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{5}\,x+1\right)-\log\left(-2x^2+\sqrt{5}\,x-1\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[5]*x - 2*x^2] + Log[1 + Sqrt[5]*x + 2*x^2])/(2*Sqrt[5])
```

fricas [A] time = 0.40, size = 45, normalized size = 0.90

$$\frac{1}{10}\sqrt{5}\log\left(\frac{4x^4+9x^2+2\sqrt{5}\left(2x^3+x\right)+1}{4x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*log((4*x^4 + 9*x^2 + 2*sqrt(5)*(2*x^3 + x) + 1)/(4*x^4 - x^2 +
 1))
```

giac [A] time = 0.24, size = 41, normalized size = 0.82

$$\frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*log(x^2 + 1/2*sqrt(10)*(1/4)^(1/4)*x + 1/2) - 1/10*sqrt(5)*log
(x^2 - 1/2*sqrt(10)*(1/4)^(1/4)*x + 1/2)
```

maple [A] time = 0.01, size = 39, normalized size = 0.78

$$-\frac{\sqrt{5}\ \ln\left(2x^2-\sqrt{5}\ x+1\right)}{10}+\frac{\sqrt{5}\ \ln\left(2x^2+\sqrt{5}\ x+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-x^2+1),x)
```

```
[Out] -1/10*ln(1+2*x^2-5^(1/2)*x)*5^(1/2)+1/10*ln(1+2*x^2+5^(1/2)*x)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2x^2-1}{4x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - x^2 + 1), x)
```

mupad [B] time = 4.35, size = 20, normalized size = 0.40

$$\frac{\sqrt{5}\ \mathrm{atanh}\left(\frac{\sqrt{5}\,x}{2\,x^2+1}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - x^2 + 1),x)
```

```
[Out] (5^(1/2)*atanh((5^(1/2)*x)/(2*x^2 + 1)))/5
```

sympy [A] time = 0.12, size = 46, normalized size = 0.92

$$-\frac{\sqrt{5}\log\left(x^2-\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-x**2+1),x)
```

```
[Out] -sqrt(5)*log(x**2 - sqrt(5)*x/2 + 1/2)/10 + sqrt(5)*log(x**2 + sqrt(5)*x/2
+ 1/2)/10
```

3.63 $\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{6}x+1\right)}{2\sqrt{6}}-\frac{\log\left(2x^2-\sqrt{6}x+1\right)}{2\sqrt{6}}$$

```
[Out] -1/12*ln(1+2*x^2-x*6^(1/2))*6^(1/2)+1/12*ln(1+2*x^2+x*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{\log\left(2x^2+\sqrt{6}x+1\right)}{2\sqrt{6}}-\frac{\log\left(2x^2-\sqrt{6}x+1\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[6]*x + 2*x^2]/(2*Sqrt[6]) + Log[1 + Sqrt[6]*x + 2*x^2]/(2*Sqr
t[6])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx &= -\frac{\int \frac{\sqrt{\frac{3}{2}}+2x}{-\frac{1}{2}-\sqrt{\frac{3}{2}}x-x^2}\,dx}{2\sqrt{6}}-\frac{\int \frac{\sqrt{\frac{3}{2}}-2x}{-\frac{1}{2}+\sqrt{\frac{3}{2}}x-x^2}\,dx}{2\sqrt{6}}\\ &= -\frac{\log\left(1-\sqrt{6}x+2x^2\right)}{2\sqrt{6}}+\frac{\log\left(1+\sqrt{6}x+2x^2\right)}{2\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{6}x+1\right)-\log\left(-2x^2+\sqrt{6}x-1\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[6]*x - 2*x^2] + Log[1 + Sqrt[6]*x + 2*x^2])/(2*Sqrt[6])
```

fricas [A] time = 0.39, size = 45, normalized size = 0.90

$$\frac{1}{12}\,\sqrt{6}\log\left(\frac{4\,x^4+10\,x^2+2\,\sqrt{6}\left(2\,x^3+x\right)+1}{4\,x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*log((4*x^4 + 10*x^2 + 2*sqrt(6)*(2*x^3 + x) + 1)/(4*x^4 - 2*x^
2 + 1))
```

giac [A] time = 0.18, size = 40, normalized size = 0.80

$$\frac{1}{12}\,\sqrt{6}\log\left(x^2+\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{12}\,\sqrt{6}\log\left(x^2-\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*log(x^2 + sqrt(3)*(1/4)^(1/4)*x + 1/2) - 1/12*sqrt(6)*log(x^2
- sqrt(3)*(1/4)^(1/4)*x + 1/2)
```

maple [A] time = 0.01, size = 39, normalized size = 0.78

$$-\frac{\sqrt{6}\,\ln\left(2x^2-\sqrt{6}\,x+1\right)}{12}+\frac{\sqrt{6}\,\ln\left(2x^2+\sqrt{6}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-2*x^2+1),x)
```

```
[Out] -1/12*ln(1+2*x^2-6^(1/2)*x)*6^(1/2)+1/12*ln(1+2*x^2+6^(1/2)*x)*6^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2\,x^2-1}{4\,x^4-2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 2*x^2 + 1), x)
```

mupad [B] time = 0.07, size = 20, normalized size = 0.40

$$\frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{2\,x^2+1}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 2*x^2 + 1),x)
```

```
[Out] (6^(1/2)*atanh((6^(1/2)*x)/(2*x^2 + 1)))/6
```

sympy [A] time = 0.12, size = 46, normalized size = 0.92

$$-\frac{\sqrt{6}\log\left(x^2-\frac{\sqrt{6}\,x}{2}+\frac{1}{2}\right)}{12}+\frac{\sqrt{6}\log\left(x^2+\frac{\sqrt{6}\,x}{2}+\frac{1}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-2*x**2+1),x)
```

```
[Out] -sqrt(6)*log(x**2 - sqrt(6)*x/2 + 1/2)/12 + sqrt(6)*log(x**2 + sqrt(6)*x/2
+ 1/2)/12
```

3.64 $\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{7}x+1\right)}{2\sqrt{7}}-\frac{\log\left(2x^2-\sqrt{7}x+1\right)}{2\sqrt{7}}$$

```
[Out] -1/14*ln(1+2*x^2-x*7^(1/2))*7^(1/2)+1/14*ln(1+2*x^2+x*7^(1/2))*7^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{\log\left(2x^2+\sqrt{7}x+1\right)}{2\sqrt{7}}-\frac{\log\left(2x^2-\sqrt{7}x+1\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[7]*x + 2*x^2]/(2*Sqrt[7]) + Log[1 + Sqrt[7]*x + 2*x^2]/(2*Sqr
t[7])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx &= -\frac{\int \frac{\frac{\sqrt{7}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{7}x}{2}-x^2}\,dx}{2\sqrt{7}}-\frac{\int \frac{\frac{\sqrt{7}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{7}x}{2}-x^2}\,dx}{2\sqrt{7}}\\ &= -\frac{\log\left(1-\sqrt{7}x+2x^2\right)}{2\sqrt{7}}+\frac{\log\left(1+\sqrt{7}x+2x^2\right)}{2\sqrt{7}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{7}x+1\right)-\log\left(-2x^2+\sqrt{7}x-1\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[7]*x - 2*x^2] + Log[1 + Sqrt[7]*x + 2*x^2])/(2*Sqrt[7])
```

fricas [A] time = 0.42, size = 45, normalized size = 0.90

$$\frac{1}{14}\sqrt{7}\log\left(\frac{4x^4+11x^2+2\sqrt{7}\left(2x^3+x\right)+1}{4x^4-3x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((4*x^4 + 11*x^2 + 2*sqrt(7)*(2*x^3 + x) + 1)/(4*x^4 - 3*x^
2 + 1))
```

giac [A] time = 0.21, size = 41, normalized size = 0.82

$$\frac{1}{14}\sqrt{7}\log\left(x^2+\frac{1}{2}\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{14}\sqrt{7}\log\left(x^2-\frac{1}{2}\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*log(x^2 + 1/2*sqrt(14)*(1/4)^(1/4)*x + 1/2) - 1/14*sqrt(7)*log
(x^2 - 1/2*sqrt(14)*(1/4)^(1/4)*x + 1/2)
```

maple [A] time = 0.01, size = 39, normalized size = 0.78

$$-\frac{\sqrt{7}\,\ln\left(2x^2-\sqrt{7}\,x+1\right)}{14}+\frac{\sqrt{7}\,\ln\left(2x^2+\sqrt{7}\,x+1\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-3*x^2+1),x)
```

```
[Out] -1/14*ln(1+2*x^2-x*7^(1/2))*7^(1/2)+1/14*ln(1+2*x^2+x*7^(1/2))*7^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2x^2-1}{4x^4-3x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 3*x^2 + 1), x)
```

mupad [B] time = 4.39, size = 20, normalized size = 0.40

$$\frac{\sqrt{7}\,\mathrm{atanh}\left(\frac{\sqrt{7}\,x}{2x^2+1}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 3*x^2 + 1),x)
```

```
[Out] (7^(1/2)*atanh((7^(1/2)*x)/(2*x^2 + 1)))/7
```

sympy [A] time = 0.13, size = 46, normalized size = 0.92

$$-\frac{\sqrt{7}\log\left(x^2-\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-3*x**2+1),x)
```

```
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x/2 + 1/2)/14 + sqrt(7)*log(x**2 + sqrt(7)*x/2
+ 1/2)/14
```

3.65 $\int \frac{1-2x^2}{1-4x^2+4x^4} \, dx$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = $\{28, 21, 206\}$

$$\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTanh[Sqrt[2]*x]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-2x^2}{1-4x^2+4x^4} \, dx &= 4 \int \frac{1-2x^2}{\left(-2+4x^2\right)^2} \, dx \\ &= \int \frac{1}{1-2x^2} \, dx \\ &= \frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [B] time = 0.01, size = 32, normalized size = 2.29

$$\frac{\log\left(2x+\sqrt{2}\right)-\log\left(\sqrt{2}-2x\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
[Out] (-Log[Sqrt[2] - 2*x] + Log[Sqrt[2] + 2*x])/(2*Sqrt[2])
```

fricas [B] time = 0.40, size = 29, normalized size = 2.07

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2x^2+2\sqrt{2}x+1}{2x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((2*x^2 + 2*sqrt(2)*x + 1)/(2*x^2 - 1))
```

giac [B] time = 0.16, size = 29, normalized size = 2.07

$$\frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{4}\sqrt{2}\log\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(abs(x + 1/2*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/2*sqrt(2)
))
```

maple [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-4*x^2+1),x)
[Out] 1/2*arctanh(2^(1/2)*x)*2^(1/2)
```

maxima [B] time = 2.35, size = 25, normalized size = 1.79

$$-\frac{1}{4}\sqrt{2}\log\left(\frac{2x-\sqrt{2}}{2x+\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="maxima")
[Out] -1/4*sqrt(2)*log((2*x - sqrt(2))/(2*x + sqrt(2)))
```

mupad [B] time = 4.33, size = 11, normalized size = 0.79

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 4*x^2 + 1),x)
[Out] (2^(1/2)*atanh(2^(1/2)*x))/2
```

sympy [B] time = 0.11, size = 32, normalized size = 2.29

$$-\frac{\sqrt{2}\log\left(x-\frac{\sqrt{2}}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x+\frac{\sqrt{2}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-4*x**2+1),x)
```

```
[Out] -sqrt(2)*log(x - sqrt(2)/2)/4 + sqrt(2)*log(x + sqrt(2)/2)/4
```

3.66 $\int \frac{1-2x^2}{1-5x^2+4x^4} \, dx$

Optimal. Leaf size=39

$$-\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(x+1)+\frac{1}{6}\log(2x+1)$$

```
[Out] -1/6*ln(1-2*x)-1/6*ln(1-x)+1/6*ln(1+x)+1/6*ln(1+2*x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 616, 31}

$$-\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(x+1)+\frac{1}{6}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - 2*x]/6 - Log[1 - x]/6 + Log[1 + x]/6 + Log[1 + 2*x]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx &= -\left(\frac{1}{4}\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx\right)-\frac{1}{4}\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx\\
&= -\left(\frac{1}{6}\int \frac{1}{-1+x}\,dx\right)-\frac{1}{6}\int \frac{1}{-\frac{1}{2}+x}\,dx+\frac{1}{6}\int \frac{1}{\frac{1}{2}+x}\,dx+\frac{1}{6}\int \frac{1}{1+x}\,dx\\
&= -\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(1+x)+\frac{1}{6}\log(1+2x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.79

$$\frac{1}{6}\log\left(2x^2+3x+1\right)-\frac{1}{6}\log\left(2x^2-3x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

```
[Out] -1/6*Log[1 - 3*x + 2*x^2] + Log[1 + 3*x + 2*x^2]/6
```

fricas [A] time = 0.40, size = 27, normalized size = 0.69

$$\frac{1}{6}\,\log\left(2\,x^{2}+3\,x+1\right)-\frac{1}{6}\,\log\left(2\,x^{2}-3\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*log(2*x^2 + 3*x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

giac [A] time = 0.15, size = 33, normalized size = 0.85

$$\frac{1}{6}\,\log\left(|2\,x+1|\right)-\frac{1}{6}\,\log\left(|2\,x-1|\right)+\frac{1}{6}\,\log\left(|x+1|\right)-\frac{1}{6}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*log(abs(2*x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*l
og(abs(x - 1))
```

maple [A] time = 0.01, size = 30, normalized size = 0.77

$$\frac{\ln\left(x+1\right)}{6}+\frac{\ln\left(2x+1\right)}{6}-\frac{\ln\left(x-1\right)}{6}-\frac{\ln\left(2x-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-5*x^2+1),x)
```

```
[Out] -1/6*ln(2*x-1)+1/6*ln(2*x+1)+1/6*ln(x+1)-1/6*ln(x-1)
```

maxima [A] time = 0.96, size = 29, normalized size = 0.74

$$\frac{1}{6}\,\log\left(2\,x+1\right)-\frac{1}{6}\,\log\left(2\,x-1\right)+\frac{1}{6}\,\log\left(x+1\right)-\frac{1}{6}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*log(2*x + 1) - 1/6*log(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

mupad [B] time = 0.10, size = 15, normalized size = 0.38

$$\frac{\mathrm{atanh}\left(\frac{3\,x}{2\,x^{2}+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 5*x^2 + 1),x)
```

```
[Out] atanh((3*x)/(2*x^2 + 1))/3
```

sympy [A] time = 0.12, size = 29, normalized size = 0.74

$$-\frac{\log\left(x^{2}-\frac{3x}{2}+\frac{1}{2}\right)}{6}+\frac{\log\left(x^{2}+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-5*x**2+1),x)
```

```
[Out] -log(x**2 - 3*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

3.67 $\int \frac{1-2x^2}{1-6x^2+4x^4} \, dx$

Optimal. Leaf size=48

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{5}}\right)}{\sqrt{10}}-\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}$$

```
[Out] -1/10*arctanh(1/5*(1-2*x*2^(1/2))*5^(1/2))*10^(1/2)+1/10*arctanh(1/5*(1+2*x
*2^(1/2))*5^(1/2))*10^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{5}}\right)}{\sqrt{10}}-\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTanh[(1 - 2*Sqrt[2]*x)/Sqrt[5]]/Sqrt[10]) + ArcTanh[(1 + 2*Sqrt[2]*x)/
Sqrt[5]]/Sqrt[10]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1-2x^2}{1-6x^2+4x^4} \, dx &= -\left(\frac{1}{4}\int \frac{1}{-\frac{1}{2}-\frac{x}{\sqrt{2}}+x^2} \, dx\right)-\frac{1}{4}\int \frac{1}{-\frac{1}{2}+\frac{x}{\sqrt{2}}+x^2} \, dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{5}{2}-x^2} \, dx, x, -\frac{1}{\sqrt{2}}+2x\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{5}{2}-x^2} \, dx, x, \frac{1}{\sqrt{2}}+2x\right)\\
&= -\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}+\frac{\tanh^{-1}\left(\frac{1+2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.88

$$\frac{\log\left(2x^2+\sqrt{10}\,x+1\right)-\log\left(-2x^2+\sqrt{10}\,x-1\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[10]*x - 2*x^2] + Log[1 + Sqrt[10]*x + 2*x^2])/(2*Sqrt[10])
```

fricas [A] time = 0.42, size = 45, normalized size = 0.94

$$\frac{1}{20}\,\sqrt{10}\,\log\left(\frac{4\,x^4+14\,x^2+2\,\sqrt{10}\left(2\,x^3+x\right)+1}{4\,x^4-6\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")
[Out] 1/20*sqrt(10)*log((4*x^4 + 14*x^2 + 2*sqrt(10)*(2*x^3 + x) + 1)/(4*x^4 - 6*
x^2 + 1))
```

giac [A] time = 0.32, size = 77, normalized size = 1.60

$$\frac{1}{20}\,\sqrt{10}\,\log\left(\left|x+\frac{1}{4}\,\sqrt{10}+\frac{1}{4}\,\sqrt{2}\right|\right)+\frac{1}{20}\,\sqrt{10}\,\log\left(\left|x+\frac{1}{4}\,\sqrt{10}-\frac{1}{4}\,\sqrt{2}\right|\right)-\frac{1}{20}\,\sqrt{10}\,\log\left(\left|x-\frac{1}{4}\,\sqrt{10}+\frac{1}{4}\,\sqrt{2}\right|\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")
[Out] 1/20*sqrt(10)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/20*sqrt(10)*log(
abs(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt(1
0) + 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2)))
```

maple [B] time = 0.02, size = 82, normalized size = 1.71

$$\frac{2\left(\sqrt{5}-1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}+\frac{2\left(\sqrt{5}+1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-6*x^2+1),x)
[Out] 2/5*(5^(1/2)-1)*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctanh(8/(2*10^(1/2)-2*2^(1
/2))*x)+2/5*(5^(1/2)+1)*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctanh(8/(2*10^(1/2
)+2*2^(1/2))*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,x^2-1}{4\,x^4-6\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="maxima")
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 6*x^2 + 1), x)
```

mupad [B] time = 0.13, size = 20, normalized size = 0.42

$$\frac{\sqrt{10}\,\operatorname{atanh}\left(\frac{\sqrt{10}\,x}{2\,x^2+1}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 6*x^2 + 1),x)
```

```
[Out] (10^(1/2)*atanh((10^(1/2)*x)/(2*x^2 + 1)))/10
```

sympy [A] time = 0.12, size = 46, normalized size = 0.96

$$-\frac{\sqrt{10}\,\log\left(x^2-\frac{\sqrt{10}\,x}{2}+\frac{1}{2}\right)}{20}+\frac{\sqrt{10}\,\log\left(x^2+\frac{\sqrt{10}\,x}{2}+\frac{1}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] -sqrt(10)*log(x**2 - sqrt(10)*x/2 + 1/2)/20 + sqrt(10)*log(x**2 + sqrt(10)*
x/2 + 1/2)/20
```

3.68 $\int \frac{1+x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=62

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

```
[Out] -arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))/(2+b)^(1/2)+arctan((2*x+(2-b)^(1/2)
)/(2+b)^(1/2))/(2+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]) + ArcTan[(Sqrt[2 - b
] + 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+bx^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{2-b}\,x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{2-b}\,x+x^2}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{1}{-2-b-x^2}\,dx,x,-\sqrt{2-b}+2x\right)-\operatorname{Subst}\left(\int \frac{1}{-2-b-x^2}\,dx,x,\sqrt{2-b}+2\right.\\ &= \frac{\tan^{-1}\left(\frac{-\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 124, normalized size = 2.00

$$\frac{\frac{\left(\sqrt{b^2-4}-b+2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}}+\frac{\left(\sqrt{b^2-4}+b-2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2 - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sq
rt[b - Sqrt[-4 + b^2]] + ((-2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt
[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

fricas [A] time = 0.41, size = 101, normalized size = 1.63

$$\left[-\frac{\sqrt{-b-2}\log\left(\frac{x^4-(b+4)x^2-2\left(x^3-x\right)\sqrt{-b-2}+1}{x^4+bx^2+1}\right)}{2(b+2)}, \frac{\sqrt{b+2}\arctan\left(\frac{x^3+(b+1)x}{\sqrt{b+2}}\right)+\sqrt{b+2}\arctan\left(\frac{x}{\sqrt{b+2}}\right)}{b+2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b - 2)*log((x^4 - (b + 4)*x^2 - 2*(x^3 - x)*sqrt(-b - 2) + 1)/(
x^4 + b*x^2 + 1))/(b + 2), (sqrt(b + 2)*arctan((x^3 + (b + 1)*x)/sqrt(b + 2
)) + sqrt(b + 2)*arctan(x/sqrt(b + 2)))/(b + 2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [b]=[0]Precision problem choosing root in common_EXT, current prec
ision 14Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [b]=[0]Precisi
on problem choosing root in common_EXT, current precision 14Undef/Unsigned
Inf encountered in limitLimit: Max order reached or unable to make series e
xpansion Error: Bad Argument Value
```

maple [B] time = 0.04, size = 277, normalized size = 4.47

$$-\frac{b\arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b-2\sqrt{(b-2)(b+2)}}}+\frac{b\arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\sqrt{2b+2\sqrt{(b-2)(b+2)}}}+\frac{2\arctan\left(\frac{}{\sqrt{2b-2}}\right.}{\sqrt{(b-2)(b+2)}\sqrt{2b-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+b*x^2+1),x)
```

```
[Out] -2/((b-2)*(2+b))^(1/2)/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((b-
2)*(2+b))^(1/2)+2*b)^(1/2))+1/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/
```

```
(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))+1/((b-2)*(2+b))^(1/2)/(2*((b-2)*(2+b))^(
1/2)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))*b+2/((b-2)*(2
+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((b-2)*(2+b))^
(1/2)+2*b)^(1/2))+1/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((b-2
)*(2+b))^(1/2)+2*b)^(1/2))-1/((b-2)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*
b)^(1/2)*arctan(2*x/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + b*x^2 + 1), x)
```

mupad [B] time = 0.06, size = 73, normalized size = 1.18

$$\frac{\mathrm{atan}\left(\frac{x}{\sqrt{b+2}}\right)+\mathrm{atan}\left((b+2)\left(x\left(\frac{1}{\sqrt{b+2}}+\frac{\frac{4}{b+2}-1}{(b-2)\sqrt{b+2}}\right)+\frac{x^3\left(\frac{2b}{b+2}-1\right)}{(b-2)\sqrt{b+2}}\right)\right)}{\sqrt{b+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(b*x^2 + x^4 + 1),x)
```

```
[Out] (atan(x/(b + 2)^(1/2)) + atan((b + 2)*(x*(1/(b + 2)^(1/2) + (4/(b + 2) - 1)
/((b - 2)*(b + 2)^(1/2))) + (x^3*((2*b)/(b + 2) - 1))/((b - 2)*(b + 2)^(1/2
)))))/(b + 2)^(1/2)
```

sympy [A] time = 0.38, size = 88, normalized size = 1.42

$$-\frac{\sqrt{-\frac{1}{b+2}}\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b+2}}-2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2}+\frac{\sqrt{-\frac{1}{b+2}}\log\left(x^2+x\left(b\sqrt{-\frac{1}{b+2}}+2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+b*x**2+1),x)
```

```
[Out] -sqrt(-1/(b + 2))*log(x**2 + x*(-b*sqrt(-1/(b + 2)) - 2*sqrt(-1/(b + 2))) -
 1)/2 + sqrt(-1/(b + 2))*log(x**2 + x*(b*sqrt(-1/(b + 2)) + 2*sqrt(-1/(b +
2))) - 1)/2
```

3.69 $\int \frac{1+x^2}{1+5x^2+x^4}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{7}}$$

```
[Out] 1/7*arctan(x*2^(1/2)/(5+21^(1/2))^(1/2))*7^(1/2)+1/7*arctan(x*(1/2*7^(1/2)+
1/2*3^(1/2)))*7^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[7] + ArcTan[Sqrt[(5 + Sqrt[21])/2]*x]
/Sqrt[7]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+5x^2+x^4}\,dx&=\frac{1}{14}\left(7-\sqrt{21}\right)\int \frac{1}{\frac{5}{2}-\frac{\sqrt{21}}{2}+x^2}\,dx+\frac{1}{14}\left(7+\sqrt{21}\right)\int \frac{1}{\frac{5}{2}+\frac{\sqrt{21}}{2}+x^2}\,dx\\&=\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 83, normalized size = 1.69

$$\frac{\left(\sqrt{21}-3\right)\tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}\,x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}}+\frac{\left(3+\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ((-3 + Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sqrt[21])]
+ ((3 + Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[42*(5 + Sqrt[21])]
```

fricas [A] time = 0.42, size = 31, normalized size = 0.63

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(x^3+6x\right)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(x^3 + 6*x)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7
)*x)
```

giac [A] time = 0.18, size = 26, normalized size = 0.53

$$\frac{1}{14}\sqrt{7}\left(\pi\mathrm{sgn}(x)+2\arctan\left(\frac{\sqrt{7}\left(x^2-1\right)}{7x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*(pi*sgn(x) + 2*arctan(1/7*sqrt(7)*(x^2 - 1)/x))
```

maple [B] time = 0.05, size = 136, normalized size = 2.78

$$-\frac{2\sqrt{21}\arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{7\left(2\sqrt{7}-2\sqrt{3}\right)}+\frac{2\arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{2\sqrt{7}-2\sqrt{3}}+\frac{2\sqrt{21}\arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{7\left(2\sqrt{7}+2\sqrt{3}\right)}+\frac{2\arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{2\sqrt{7}+2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+5*x^2+1),x)
```

```
[Out] -2/7*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))+2/(2*
7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))+2/7*21^(1/2)/(2*7^(1/2
)+2*3^(1/2))*arctan(4*x/(2*7^(1/2)+2*3^(1/2)))+2/(2*7^(1/2)+2*3^(1/2))*arct
an(4*x/(2*7^(1/2)+2*3^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+1}{x^4+5x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + 5*x^2 + 1), x)
```

mupad [B] time = 0.08, size = 29, normalized size = 0.59

$$\frac{\sqrt{7}\left(\mathrm{atan}\left(\frac{\sqrt{7}x^3}{7}+\frac{6\sqrt{7}x}{7}\right)+\mathrm{atan}\left(\frac{\sqrt{7}x}{7}\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(5*x^2 + x^4 + 1),x)
```

```
[Out] (7^(1/2)*(atan((6*7^(1/2)*x)/7 + (7^(1/2)*x^3)/7) + atan((7^(1/2)*x)/7)))/7
```

sympy [**A**] time = 0.12, size = 41, normalized size = 0.84

$$\frac{\sqrt{7}\left(2\operatorname{atan}\left(\frac{\sqrt{7}\,x}{7}\right)+2\operatorname{atan}\left(\frac{\sqrt{7}\,x^3}{7}+\frac{6\sqrt{7}\,x}{7}\right)\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+5*x**2+1),x)
```

```
[Out] sqrt(7)*(2*atan(sqrt(7)*x/7) + 2*atan(sqrt(7)*x**3/7 + 6*sqrt(7)*x/7))/14
```

3.70 $\int \frac{1+x^2}{1+4x^2+x^4}\,dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/6*arctan(x/(1/2*6^(1/2)+1
/2*2^(1/2)))*6^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[6] + ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[6]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+4x^2+x^4}\,dx&=\frac{1}{6}\left(3-\sqrt{3}\right)\int \frac{1}{2-\sqrt{3}+x^2}\,dx+\frac{1}{6}\left(3+\sqrt{3}\right)\int \frac{1}{2+\sqrt{3}+x^2}\,dx\\&=\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 1.88

$$\frac{\left(\sqrt{3}-1\right)\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\left(1+\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ((-1 + Sqrt[3])*ArcTan[x/Sqrt[2 - Sqrt[3]]])/(2*Sqrt[3*(2 - Sqrt[3])]) + ((
1 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3*(2 + Sqrt[3])])
```

fricas [A] time = 0.40, size = 31, normalized size = 0.72

$$\frac{1}{6}\,\sqrt{6}\,\arctan\left(\frac{1}{6}\,\sqrt{6}\left(x^3+5\,x\right)\right)+\frac{1}{6}\,\sqrt{6}\,\arctan\left(\frac{1}{6}\,\sqrt{6}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*arctan(1/6*sqrt(6)*(x^3 + 5*x)) + 1/6*sqrt(6)*arctan(1/6*sqrt(6
)*x)
```

giac [A] time = 0.19, size = 26, normalized size = 0.60

$$\frac{1}{12}\,\sqrt{6}\left(\pi\mathrm{sgn}(x)+2\,\arctan\left(\frac{\sqrt{6}\left(x^2-1\right)}{6\,x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*(pi*sgn(x) + 2*arctan(1/6*sqrt(6)*(x^2 - 1)/x))
```

maple [B] time = 0.05, size = 110, normalized size = 2.56

$$-\frac{\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\left(\sqrt{6}-\sqrt{2}\right)}+\frac{\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{\sqrt{6}-\sqrt{2}}+\frac{\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\sqrt{6}+3\sqrt{2}}+\frac{\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{\sqrt{6}+\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+4*x^2+1),x)
```

```
[Out] 1/3*3^(1/2)/(6^(1/2)+2^(1/2))*arctan(2*x/(6^(1/2)+2^(1/2)))+1/(6^(1/2)+2^(1
/2))*arctan(2*x/(6^(1/2)+2^(1/2)))-1/3*3^(1/2)/(6^(1/2)-2^(1/2))*arctan(2*x
/(6^(1/2)-2^(1/2)))+1/(6^(1/2)-2^(1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+1}{x^4+4\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + 4*x^2 + 1), x)
```

mupad [B] time = 0.08, size = 29, normalized size = 0.67

$$\frac{\sqrt{6}\,\left(\mathrm{atan}\left(\frac{\sqrt{6}\,x^3}{6}+\frac{5\,\sqrt{6}\,x}{6}\right)+\mathrm{atan}\left(\frac{\sqrt{6}\,x}{6}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(4*x^2 + x^4 + 1),x)
```

```
[Out] (6^(1/2)*(atan((5*6^(1/2)*x)/6 + (6^(1/2)*x^3)/6) + atan((6^(1/2)*x)/6)))/6
```

sympy [A] time = 0.14, size = 41, normalized size = 0.95

$$\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{6}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x^{3}}{6}+\frac{5\sqrt{6}\,x}{6}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+4*x**2+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/6) + 2*atan(sqrt(6)*x**3/6 + 5*sqrt(6)*x/6))/12
```

3.71 $\int \frac{1+x^2}{1+3x^2+x^4}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{\sqrt{5}}$$

```
[Out] 1/5*arctan(x*2^(1/2)/(3+5^(1/2))^(1/2))*5^(1/2)+1/5*arctan(x*(1/2+1/2*5^(1/
2)))*5^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]/Sqrt[5] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]/S
qrt[5]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+3x^2+x^4}\,dx &= \frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx+\frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx\\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 83, normalized size = 1.69

$$\frac{\left(\sqrt{5}-1\right)\tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}\,x\right)}{\sqrt{10\left(3-\sqrt{5}\right)}}+\frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ((-1 + Sqrt[5])*ArcTan[Sqrt[2/(3 - Sqrt[5])]*x])/Sqrt[10*(3 - Sqrt[5])] + (
(1 + Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/Sqrt[10*(3 + Sqrt[5])]
```

fricas [A] time = 0.40, size = 31, normalized size = 0.63

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\left(x^3+4x\right)\right)+\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(x^3 + 4*x)) + 1/5*sqrt(5)*arctan(1/5*sqrt(5
)*x)
```

giac [A] time = 0.16, size = 26, normalized size = 0.53

$$\frac{1}{10}\sqrt{5}\left(\pi\operatorname{sgn}(x)+2\arctan\left(\frac{\sqrt{5}\left(x^2-1\right)}{5x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*(pi*sgn(x) + 2*arctan(1/5*sqrt(5)*(x^2 - 1)/x))
```

maple [B] time = 0.04, size = 104, normalized size = 2.12

$$-\frac{2\sqrt{5}\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}+\frac{2\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{2\sqrt{5}-2}+\frac{2\sqrt{5}\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}+\frac{2\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{2\sqrt{5}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+3*x^2+1),x)
```

```
[Out] 2/5*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x/(2*5^(1/2)+2))+2/(2*5^(1/2)+2)*arctan(
4*x/(2*5^(1/2)+2))-2/5*5^(1/2)/(2*5^(1/2)-2)*arctan(4*x/(2*5^(1/2)-2))+2/(2
*5^(1/2)-2)*arctan(4*x/(2*5^(1/2)-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+1}{x^4+3x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + 3*x^2 + 1), x)
```

mupad [B] time = 4.39, size = 29, normalized size = 0.59

$$\frac{\sqrt{5}\left(\operatorname{atan}\left(\frac{\sqrt{5}x^3}{5}+\frac{4\sqrt{5}x}{5}\right)+\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(3*x^2 + x^4 + 1),x)
```

```
[Out] (5^(1/2)*(atan((4*5^(1/2)*x)/5 + (5^(1/2)*x^3)/5) + atan((5^(1/2)*x)/5)))/5
```

sympy [A]   time = 0.13, size = 41, normalized size = 0.84

$$\frac{\sqrt{5}\left(2\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)+2\operatorname{atan}\left(\frac{\sqrt{5}x^3}{5}+\frac{4\sqrt{5}x}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+3*x**2+1),x)
[Out] sqrt(5)*(2*atan(sqrt(5)*x/5) + 2*atan(sqrt(5)*x**3/5 + 4*sqrt(5)*x/5))/10
```

3.72 $\int \frac{1+x^2}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=2

$$\tan^{-1}(x)$$

[Out] arctan(x)

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {28, 203}

$$\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + 2*x^2 + x^4),x]

[Out] ArcTan[x]

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+2x^2+x^4}\,dx &= \int \frac{1}{1+x^2}\,dx\\ &= \tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + 2*x^2 + x^4),x]

[Out] ArcTan[x]

fricas [A] time = 0.40, size = 2, normalized size = 1.00

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")

[Out] arctan(x)

giac [A] time = 0.16, size = 2, normalized size = 1.00

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] arctan(x)
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+2*x^2+1),x)
```

```
[Out] arctan(x)
```

maxima [A] time = 2.42, size = 2, normalized size = 1.00

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

mupad [B] time = 4.33, size = 2, normalized size = 1.00

$$\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(2*x^2 + x^4 + 1),x)
```

```
[Out] atan(x)
```

sympy [A] time = 0.10, size = 2, normalized size = 1.00

$$\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+2*x**2+1),x)
```

```
[Out] atan(x)
```

3.73 $\int \frac{1+x^2}{1+x^2+x^4} \, dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{1+x^2+x^4} \, dx &= \frac{1}{2}\int \frac{1}{1-x+x^2} \, dx+\frac{1}{2}\int \frac{1}{1+x+x^2} \, dx\\
&= -\text{Subst}\left(\int \frac{1}{-3-x^2} \, dx,x,-1+2x\right)-\text{Subst}\left(\int \frac{1}{-3-x^2} \, dx,x,1+2x\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.19, size = 99, normalized size = 2.61

$$\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1-i\sqrt{3}\right)}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x^2 + x^4),x]
```

```
[Out] ((-I + Sqrt[3])*ArcTan[x/Sqrt[(1 - I*Sqrt[3])/2]])/Sqrt[6*(1 - I*Sqrt[3])]
+ ((I + Sqrt[3])*ArcTan[x/Sqrt[(1 + I*Sqrt[3])/2]])/Sqrt[6*(1 + I*Sqrt[3])]
```

fricas [A] time = 0.42, size = 31, normalized size = 0.82

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*x)
```

giac [A] time = 0.16, size = 26, normalized size = 0.68

$$\frac{1}{6}\sqrt{3}\left(\pi\mathrm{sgn}(x)+2\arctan\left(\frac{\sqrt{3}\left(x^2-1\right)}{3x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*(x^2 - 1)/x))
```

maple [A] time = 0.01, size = 34, normalized size = 0.89

$$\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+x^2+1),x)
```

```
[Out] 1/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))
```

maxima [A] time = 2.40, size = 33, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1))
```

mupad [B] time = 0.08, size = 29, normalized size = 0.76

$$\frac{\sqrt{3}\,\left(\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\,\sqrt{3}\,x}{3}\right)+\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^2 + x^4 + 1),x)
[Out] (3^(1/2)*(atan((2*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) + atan((3^(1/2)*x)/3)))/3
```

sympy [A] time = 0.12, size = 41, normalized size = 1.08

$$\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\sqrt{3}\,x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+x**2+1),x)
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/6
```

3.74 $\int \frac{1+x^2}{1+x^4} \, dx$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/2*arctan(1+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x^4),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{1+x^4} \, dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{2}x+x^2} \, dx+\frac{1}{2}\int \frac{1}{1+\sqrt{2}x+x^2} \, dx \\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx,x,1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx,x,1+\sqrt{2}x\right)}{\sqrt{2}} \\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.86

$$\frac{\tan^{-1}\left(\sqrt{2}x+1\right)-\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x^4),x]
```

```
[Out] (-ArcTan[1 - Sqrt[2]*x] + ArcTan[1 + Sqrt[2]*x])/Sqrt[2]
```

fricas [A] time = 0.40, size = 29, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^3+x\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + x)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*
x)
```

giac [A] time = 0.19, size = 39, normalized size = 1.11

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)))
```

maple [B] time = 0.00, size = 88, normalized size = 2.51

$$\frac{\sqrt{2}\arctan\left(\sqrt{2}x-1\right)}{2}+\frac{\sqrt{2}\arctan\left(\sqrt{2}x+1\right)}{2}+\frac{\sqrt{2}\ln\left(\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right)}{8}+\frac{\sqrt{2}\ln\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+1),x)
```

```
[Out] 1/2*arctan(-1+2^(1/2)*x)*2^(1/2)+1/8*2^(1/2)*ln((1+x^2+2^(1/2)*x)/(1+x^2-2^
(1/2)*x))+1/2*arctan(1+2^(1/2)*x)*2^(1/2)+1/8*2^(1/2)*ln((1+x^2-2^(1/2)*x)/
(1+x^2+2^(1/2)*x))
```

maxima [A] time = 2.42, size = 39, normalized size = 1.11

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)))
```

mupad [B] time = 4.37, size = 29, normalized size = 0.83

$$\frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}x^3}{2}+\frac{\sqrt{2}x}{2}\right)+\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 + 1),x)
```

```
[Out] (2^(1/2)*(atan((2^(1/2)*x)/2 + (2^(1/2)*x^3)/2) + atan((2^(1/2)*x)/2)))/2
```

sympy [A]   time = 0.12, size = 39, normalized size = 1.11

$$\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^{3}}{2}+\frac{\sqrt{2}\,x}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+1),x)
```

```
[Out] sqrt(2)*(2*atan(sqrt(2)*x/2) + 2*atan(sqrt(2)*x**3/2 + sqrt(2)*x/2))/4
```

3.75 $\int \frac{1+x^2}{1-x^2+x^4}\,dx$

Optimal. Leaf size=23

$$\tan^{-1}\left(2x+\sqrt{3}\right)-\tan^{-1}\left(\sqrt{3}-2x\right)$$

```
[Out] arctan(2*x-3^(1/2))+arctan(2*x+3^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1161, 618, 204}

$$\tan^{-1}\left(2x+\sqrt{3}\right)-\tan^{-1}\left(\sqrt{3}-2x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{1-x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)-\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{3}+2x\right)\\
&= -\tan^{-1}\left(\sqrt{3}-2x\right)+\tan^{-1}\left(\sqrt{3}+2x\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.52

$$-\tan^{-1}\left(\frac{x}{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - x^2 + x^4),x]
[Out] -ArcTan[x/(-1 + x^2)]
```

fricas [A] time = 0.45, size = 7, normalized size = 0.30

$$\arctan\left(x^3\right) + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="fricas")
[Out] arctan(x^3) + arctan(x)
```

giac [A] time = 0.17, size = 30, normalized size = 1.30

$$\frac{1}{4}\,\pi\mathrm{sgn}(x) + \frac{1}{2}\,\arctan\left(\frac{x^4 - 3\,x^2 + 1}{2\left(x^3 - x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="giac")
[Out] 1/4*pi*sgn(x) + 1/2*arctan(1/2*(x^4 - 3*x^2 + 1)/(x^3 - x))
```

maple [A] time = 0.02, size = 20, normalized size = 0.87

$$\arctan\left(2x - \sqrt{3}\right) + \arctan\left(2x + \sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-x^2+1),x)
[Out] arctan(2*x-3^(1/2))+arctan(2*x+3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 1}{x^4 - x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 - x^2 + 1), x)
```

mupad [B] time = 4.31, size = 7, normalized size = 0.30

$$\mathrm{atan}\left(x^3\right) + \mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 - x^2 + 1),x)
[Out] atan(x^3) + atan(x)
```

sympy [A] time = 0.11, size = 7, normalized size = 0.30

$$\mathrm{atan}\,(x) + \mathrm{atan}\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-x**2+1),x)
[Out] atan(x) + atan(x**3)
```

3.76 $\int \frac{1+x^2}{1-2x^2+x^4} \, dx$

Optimal. Leaf size=11

$$\frac{x}{1-x^2}$$

```
[Out] x/(-x^2+1)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {28, 383}

$$\frac{x}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] x/(1 - x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1-2x^2+x^4} \, dx &= \int \frac{1+x^2}{\left(-1+x^2\right)^2} \, dx \\ &= \frac{x}{1-x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] -(x/(-1 + x^2))
```

fricas [A] time = 0.60, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] -x/(x^2 - 1)
```

giac [A] time = 0.15, size = 11, normalized size = 1.00

$$-\frac{1}{x-\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
[Out] -1/(x - 1/x)
```

maple [A] time = 0.00, size = 16, normalized size = 1.45

$$-\frac{1}{2(x+1)}-\frac{1}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-2*x^2+1),x)
[Out] -1/2/(x+1)-1/2/(x-1)
```

maxima [A] time = 1.06, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")
[Out] -x/(x^2 - 1)
```

mupad [B] time = 4.34, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 - 2*x^2 + 1),x)
[Out] -x/(x^2 - 1)
```

sympy [A] time = 0.09, size = 7, normalized size = 0.64

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-2*x**2+1),x)
[Out] -x/(x**2 - 1)
```

3.77 $\int \frac{1+x^2}{1-3x^2+x^4} \, dx$

Optimal. Leaf size=65

$$\frac{1}{2}\log\left(-2x-\sqrt{5}+1\right)+\frac{1}{2}\log\left(-2x+\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x-\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x+\sqrt{5}+1\right)$$

```
[Out] 1/2*ln(1-2*x-5^(1/2))-1/2*ln(1+2*x-5^(1/2))+1/2*ln(1-2*x+5^(1/2))-1/2*ln(1+
2*x+5^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1161, 616, 31}

$$\frac{1}{2}\log\left(-2x-\sqrt{5}+1\right)+\frac{1}{2}\log\left(-2x+\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x-\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] Log[1 - Sqrt[5] - 2*x]/2 + Log[1 + Sqrt[5] - 2*x]/2 - Log[1 - Sqrt[5] + 2*x
]/2 - Log[1 + Sqrt[5] + 2*x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{1-3x^2+x^4} \, dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{5}x+x^2} \, dx+\frac{1}{2}\int \frac{1}{1+\sqrt{5}x+x^2} \, dx \\
&= \frac{1}{2}\int \frac{1}{\frac{1}{2}\left(-1-\sqrt{5}\right)+x} \, dx-\frac{1}{2}\int \frac{1}{\frac{1}{2}\left(1-\sqrt{5}\right)+x} \, dx+\frac{1}{2}\int \frac{1}{\frac{1}{2}\left(-1+\sqrt{5}\right)+x} \, dx-\frac{1}{2}\int \frac{1}{\frac{1}{2}
\\
&= \frac{1}{2}\log\left(1-\sqrt{5}-2x\right)+\frac{1}{2}\log\left(1+\sqrt{5}-2x\right)-\frac{1}{2}\log\left(1-\sqrt{5}+2x\right)-\frac{1}{2}\log\left(1+\sqrt{5}+2\right.
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.45

$$\frac{1}{2}\log\left(-x^2+x+1\right)-\frac{1}{2}\log\left(-x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - x - x^2] + Log[1 + x - x^2]/2
```

fricas [A] time = 0.71, size = 21, normalized size = 0.32

$$-\frac{1}{2}\,\log\left(x^{2}+x-1\right)+\frac{1}{2}\,\log\left(x^{2}-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

giac [A] time = 0.17, size = 43, normalized size = 0.66

$$-\frac{1}{4}\,\log\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}+2\right|\right)+\frac{1}{4}\,\log\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(x + 1/(x - 1/x) - 1/x + 2)) + 1/4*log(abs(x + 1/(x - 1/x) - 1/
x - 2))
```

maple [A] time = 0.01, size = 22, normalized size = 0.34

$$\frac{\ln\left(x^{2}-x-1\right)}{2}-\frac{\ln\left(x^{2}+x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-3*x^2+1),x)
```

```
[Out] -1/2*ln(x^2+x-1)+1/2*ln(x^2-x-1)
```

maxima [A] time = 0.99, size = 21, normalized size = 0.32

$$-\frac{1}{2}\,\log\left(x^{2}+x-1\right)+\frac{1}{2}\,\log\left(x^{2}-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

mupad [B] time = 0.26, size = 12, normalized size = 0.18

$$-\mathrm{atanh}\left(\frac{x}{x^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 - 3*x^2 + 1),x)
```

```
[Out] -atanh(x/(x^2 - 1))
```

sympy [A] time = 0.11, size = 19, normalized size = 0.29

$$\frac{\log\left(x^{2}-x-1\right)}{2}-\frac{\log\left(x^{2}+x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-3*x**2+1),x)
```

```
[Out] log(x**2 - x - 1)/2 - log(x**2 + x - 1)/2
```

3.78 $\int \frac{1+x^2}{1-4x^2+x^4}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}\,x+\sqrt{3}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(x*2^(1/2)-3^(1/2))*2^(1/2)-1/2*arctanh(x*2^(1/2)+3^(1/2))*2^(1
/2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}\,x+\sqrt{3}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] ArcTanh[Sqrt[3] - Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[3] + Sqrt[2]*x]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1-4x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{6}\,x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{6}\,x+x^2}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,-\sqrt{6}+2x\right)-\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{6}+2x\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{3}+\sqrt{2}\,x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.93

$$\frac{\log\left(-x^2+\sqrt{2}\,x+1\right)-\log\left(x^2+\sqrt{2}\,x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] (Log[1 + Sqrt[2]*x - x^2] - Log[-1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])
```

fricas [A] time = 0.62, size = 36, normalized size = 0.84

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4-2\sqrt{2}\left(x^3-x\right)+1}{x^4-4x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 - 2*sqrt(2)*(x^3 - x) + 1)/(x^4 - 4*x^2 + 1))
```

giac [A] time = 0.21, size = 39, normalized size = 0.91

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}-\frac{2}{x}\right|}{\left|2x+2\sqrt{2}-\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2/x)/abs(2*x + 2*sqrt(2) - 2/x))
```

maple [B] time = 0.04, size = 70, normalized size = 1.63

$$-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}\ \operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\left(\sqrt{6}-\sqrt{2}\right)}-\frac{\left(\sqrt{3}+3\right)\sqrt{3}\ \operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-4*x^2+1),x)
```

```
[Out] -1/3*(-3+3^(1/2))*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2/(6^(1/2)-2^(1/2))*x)-
1/3*(3^(1/2)+3)*3^(1/2)/(6^(1/2)+2^(1/2))*arctanh(2/(6^(1/2)+2^(1/2))*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+1}{x^4-4x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 - 4*x^2 + 1), x)
```

mupad [B] time = 4.40, size = 18, normalized size = 0.42

$$-\frac{\sqrt{2}\ \operatorname{atanh}\left(\frac{\sqrt{2}\,x}{x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 - 4*x^2 + 1),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*x)/(x^2 - 1)))/2
```

sympy [**A**] time = 0.11, size = 39, normalized size = 0.91

$$\frac{\sqrt{2}\,\log\left(x^{2}-\sqrt{2}\,x-1\right)}{4}-\frac{\sqrt{2}\,\log\left(x^{2}+\sqrt{2}\,x-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-4*x**2+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x - 1)/4 - sqrt(2)*log(x**2 + sqrt(2)*x - 1)/4
```

3.79 $\int \frac{1+x^2}{1-5x^2+x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctanh(1/3*(-2*x+7^(1/2))*3^(1/2))*3^(1/2)-1/3*arctanh(1/3*(2*x+7^(1/2
))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] ArcTanh[(Sqrt[7] - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTanh[(Sqrt[7] + 2*x)/Sqrt[3]]
/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{1-5x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{7}\,x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{7}\,x+x^2}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{3-x^2}\,dx,x,-\sqrt{7}+2x\right)-\text{Subst}\left(\int \frac{1}{3-x^2}\,dx,x,\sqrt{7}+2x\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}+2x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.87

$$\frac{\log\left(-x^2+\sqrt{3}x+1\right)-\log\left(x^2+\sqrt{3}x-1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 5*x^2 + x^4),x]
[Out] (Log[1 + Sqrt[3]*x - x^2] - Log[-1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])
```

fricas [A] time = 0.59, size = 39, normalized size = 0.85

$$\frac{1}{6}\sqrt{3}\log\left(\frac{x^4+x^2-2\sqrt{3}\left(x^3-x\right)+1}{x^4-5x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log((x^4 + x^2 - 2*sqrt(3)*(x^3 - x) + 1)/(x^4 - 5*x^2 + 1))
```

giac [A] time = 0.24, size = 39, normalized size = 0.85

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2x-2\sqrt{3}-\frac{2}{x}\right|}{\left|2x+2\sqrt{3}-\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) - 2/x)/abs(2*x + 2*sqrt(3) - 2/x))
```

maple [B] time = 0.04, size = 82, normalized size = 1.78

$$-\frac{2\sqrt{21}\left(-7+\sqrt{21}\right)\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}-\frac{2\left(7+\sqrt{21}\right)\sqrt{21}\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-5*x^2+1),x)
[Out] -2/21*(7+21^(1/2))*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctanh(4/(2*7^(1/2)+2*3^
(1/2))*x)-2/21*21^(1/2)*(-7+21^(1/2))/(2*7^(1/2)-2*3^(1/2))*arctanh(4/(2*7^
(1/2)-2*3^(1/2))*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+1}{x^4-5x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 - 5*x^2 + 1), x)
```

mupad [B] time = 4.47, size = 18, normalized size = 0.39

$$-\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}x}{x^2-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 - 5*x^2 + 1),x)
```

```
[Out] -(3^(1/2)*atanh((3^(1/2)*x)/(x^2 - 1)))/3
```

sympy [A] time = 0.12, size = 39, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x-1\right)}{6}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-5*x**2+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x - 1)/6 - sqrt(3)*log(x**2 + sqrt(3)*x - 1)/6
```

3.80 $\int \frac{1-x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=62

$$\frac{\log\left(\sqrt{2-b}\,x+x^2+1\right)}{2\sqrt{2-b}}-\frac{\log\left(-\sqrt{2-b}\,x+x^2+1\right)}{2\sqrt{2-b}}$$

```
[Out] -1/2*ln(1+x^2-x*(2-b)^(1/2))/(2-b)^(1/2)+1/2*ln(1+x^2+x*(2-b)^(1/2))/(2-b)^
(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1164, 628}

$$\frac{\log\left(\sqrt{2-b}\,x+x^2+1\right)}{2\sqrt{2-b}}-\frac{\log\left(-\sqrt{2-b}\,x+x^2+1\right)}{2\sqrt{2-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -Log[1 - Sqrt[2 - b]*x + x^2]/(2*Sqrt[2 - b]) + Log[1 + Sqrt[2 - b]*x + x^2
]/(2*Sqrt[2 - b])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1+bx^2+x^4}\,dx &= -\frac{\int \frac{\sqrt{2-b}+2x}{-1-\sqrt{2-b}\,x-x^2}\,dx}{2\sqrt{2-b}}-\frac{\int \frac{\sqrt{2-b}-2x}{-1+\sqrt{2-b}\,x-x^2}\,dx}{2\sqrt{2-b}}\\ &= -\frac{\log\left(1-\sqrt{2-b}\,x+x^2\right)}{2\sqrt{2-b}}+\frac{\log\left(1+\sqrt{2-b}\,x+x^2\right)}{2\sqrt{2-b}}\end{aligned}$$

Mathematica [B] time = 0.07, size = 125, normalized size = 2.02

$$\frac{\frac{\left(-\sqrt{b^2-4}+b+2\right)\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}}-\frac{\left(\sqrt{b^2-4}+b+2\right)\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2 + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sq
rt[b - Sqrt[-4 + b^2]] - ((2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[
b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

fricas [A] time = 0.65, size = 100, normalized size = 1.61

$$\left[-\frac{\sqrt{-b+2}\log\left(\frac{x^4-(b-4)x^2+2\left(x^3+x\right)\sqrt{-b+2}+1}{x^4+bx^2+1}\right)}{2\,(b-2)}, \frac{\sqrt{b-2}\arctan\left(\frac{x^3+(b-1)x}{\sqrt{b-2}}\right)-\sqrt{b-2}\arctan\left(\frac{x}{\sqrt{b-2}}\right)}{b-2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b + 2)*log((x^4 - (b - 4)*x^2 + 2*(x^3 + x)*sqrt(-b + 2) + 1)/(
x^4 + b*x^2 + 1))/(b - 2), (sqrt(b - 2)*arctan((x^3 + (b - 1)*x)/sqrt(b - 2
)) - sqrt(b - 2)*arctan(x/sqrt(b - 2)))/(b - 2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [b]=[0]Precision problem choosing root in common_EXT, current prec
ision 14Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [b]=[0]Precisi
on problem choosing root in common_EXT, current precision 14Undef/Unsigned
Inf encountered in limitLimit: Max order reached or unable to make series e
xpansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 279, normalized size = 4.50

$$\frac{b\arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\,\sqrt{2b-2\sqrt{(b-2)(b+2)}}}-\frac{b\arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\,\sqrt{2b+2\sqrt{(b-2)(b+2)}}}+\frac{2\arctan\left(\frac{}{\sqrt{2b-}}\right.}{\sqrt{(b-2)(b+2)}\,\sqrt{2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+b*x^2+1),x)
```

```
[Out] -2/((b-2)*(b+2))^(1/2)/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*arctan(2/(2*b+2*((
b-2)*(b+2))^(1/2))^(1/2)*x)-1/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*arctan(2/(2
*b+2*((b-2)*(b+2))^(1/2))^(1/2)*x)-1/((b-2)*(b+2))^(1/2)/(2*b+2*((b-2)*(b+2
))^(1/2))^(1/2)*b*arctan(2/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*x)+2/((b-2)*(b
+2))^(1/2)/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*arctan(2/(2*b-2*((b-2)*(b+2))^
(1/2))^(1/2)*x)-1/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*arctan(2/(2*b-2*((b-2)*
(b+2))^(1/2))^(1/2)*x)+1/((b-2)*(b+2))^(1/2)/(2*b-2*((b-2)*(b+2))^(1/2))^(1
/2)*b*arctan(2/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + b*x^2 + 1), x)
```

mupad [B] time = 4.34, size = 76, normalized size = 1.23

$$-\frac{\operatorname{atan}\left(\frac{x}{\sqrt{b-2}}\right)-\operatorname{atan}\left((b-2)\left(x\left(\frac{1}{\sqrt{b-2}}+\frac{\frac{4}{b-2}+1}{\sqrt{b-2}\,(b+2)}\right)+\frac{x^3\left(\frac{2\,b}{b-2}-1\right)}{\sqrt{b-2}\,(b+2)}\right)\right)}{\sqrt{b-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(b*x^2 + x^4 + 1),x)
```

```
[Out] -(atan(x/(b - 2)^(1/2)) - atan((b - 2)*(x*(1/(b - 2)^(1/2) + (4/(b - 2) + 1
)/((b - 2)^(1/2)*(b + 2))) + (x^3*((2*b)/(b - 2) - 1))/((b - 2)^(1/2)*(b +
2)))))/(b - 2)^(1/2)
```

sympy [A] time = 0.35, size = 87, normalized size = 1.40

$$\frac{\sqrt{-\frac{1}{b-2}}\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b-2}}+2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2}-\frac{\sqrt{-\frac{1}{b-2}}\log\left(x^2+x\left(b\sqrt{-\frac{1}{b-2}}-2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+b*x**2+1),x)
```

```
[Out] sqrt(-1/(b - 2))*log(x**2 + x*(-b*sqrt(-1/(b - 2)) + 2*sqrt(-1/(b - 2))) +
1)/2 - sqrt(-1/(b - 2))*log(x**2 + x*(b*sqrt(-1/(b - 2)) - 2*sqrt(-1/(b - 2
))) + 1)/2
```

3.81 $\int \frac{1-x^2}{1+5x^2+x^4} \, dx$

Optimal. Leaf size=50

$$\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctan(x*2^(1/2)/(5+21^(1/2))^(1/2))*3^(1/2)+1/3*arctan(x*(1/2*7^(1/2)
+1/2*3^(1/2)))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] -(ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[3]) + ArcTan[Sqrt[(5 + Sqrt[21])/2]
*x]/Sqrt[3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1+5x^2+x^4}\,dx &= \frac{1}{6}\left(-3+\sqrt{21}\right)\int \frac{1}{\frac{5}{2}-\frac{\sqrt{21}}{2}+x^2}\,dx-\frac{1}{6}\left(3+\sqrt{21}\right)\int \frac{1}{\frac{5}{2}+\frac{\sqrt{21}}{2}+x^2}\,dx\\ &= -\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 87, normalized size = 1.74

$$\frac{\left(7-\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}\,x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}}+\frac{\left(-7-\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ((7 - Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sqrt[21])] +
 ((-7 - Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[42*(5 + Sqrt[21])]
```

fricas [A] time = 0.65, size = 31, normalized size = 0.62

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+4x\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 4*x)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*x)
```

giac [A] time = 0.17, size = 26, normalized size = 0.52

$$\frac{1}{6}\sqrt{3}\left(\pi\mathrm{sgn}(x)-2\arctan\left(\frac{\sqrt{3}\left(x^2+1\right)}{3x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(pi*sgn(x) - 2*arctan(1/3*sqrt(3)*(x^2 + 1)/x))
```

maple [B] time = 0.02, size = 136, normalized size = 2.72

$$\frac{2\sqrt{21}\arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{3\left(2\sqrt{7}-2\sqrt{3}\right)}-\frac{2\arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{2\sqrt{7}-2\sqrt{3}}-\frac{2\sqrt{21}\arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{3\left(2\sqrt{7}+2\sqrt{3}\right)}-\frac{2\arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{2\sqrt{7}+2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+5*x^2+1),x)
```

```
[Out] 2/3*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctan(4/(2*7^(1/2)-2*3^(1/2))*x)-2/(2*7
^(1/2)-2*3^(1/2))*arctan(4/(2*7^(1/2)-2*3^(1/2))*x)-2/3*21^(1/2)/(2*7^(1/2)
+2*3^(1/2))*arctan(4/(2*7^(1/2)+2*3^(1/2))*x)-2/(2*7^(1/2)+2*3^(1/2))*arcta
n(4/(2*7^(1/2)+2*3^(1/2))*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2-1}{x^4+5x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 5*x^2 + 1), x)
```

mupad [B] time = 0.08, size = 31, normalized size = 0.62

$$\frac{\sqrt{3}\left(\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{4\sqrt{3}x}{3}\right)-\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(5*x^2 + x^4 + 1),x)
[Out] (3^(1/2)*(atan((4*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) - atan((3^(1/2)*x)/3)))/3
```

sympy [**A**] time = 0.13, size = 42, normalized size = 0.84

$$-\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{4\sqrt{3}\,x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+5*x**2+1),x)
[Out] -sqrt(3)*(2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 4*sqrt(3)*x/3))/6
```

3.82 $\int \frac{1-x^2}{1+4x^2+x^4} \, dx$

Optimal. Leaf size=44

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1/2)-1/2*arctan(x/(1/2*6^(1/2)+1
/2*2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{1+4x^2+x^4} \, dx &= \frac{1}{2}\left(-1-\sqrt{3}\right)\int \frac{1}{2+\sqrt{3}+x^2} \, dx+\frac{1}{2}\left(-1+\sqrt{3}\right)\int \frac{1}{2-\sqrt{3}+x^2} \, dx\\
&= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 82, normalized size = 1.86

$$\frac{-\left(\left(\sqrt{3}-3\right)\sqrt{2+\sqrt{3}}\,\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)\right)-\sqrt{2-\sqrt{3}}\,\left(3+\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] (-((-3 + Sqrt[3])*Sqrt[2 + Sqrt[3]]*ArcTan[x/Sqrt[2 - Sqrt[3]]]) - Sqrt[2 -
 Sqrt[3]]*(3 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3])
```

fricas [A] time = 0.65, size = 31, normalized size = 0.70

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^3+3x\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + 3*x)) - 1/2*sqrt(2)*arctan(1/2*sqrt(2
)*x)
```

giac [A] time = 0.16, size = 26, normalized size = 0.59

$$\frac{1}{4}\sqrt{2}\left(\pi\mathrm{sgn}(x)-2\arctan\left(\frac{\sqrt{2}\left(x^2+1\right)}{2x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(pi*sgn(x) - 2*arctan(1/2*sqrt(2)*(x^2 + 1)/x))
```

maple [B] time = 0.02, size = 111, normalized size = 2.52

$$\frac{\sqrt{3}\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{\sqrt{6}-\sqrt{2}}-\frac{\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{\sqrt{6}-\sqrt{2}}-\frac{\sqrt{3}\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{\sqrt{6}+\sqrt{2}}-\frac{\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{\sqrt{6}+\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+4*x^2+1),x)
```

```
[Out] -3^(1/2)/(6^(1/2)+2^(1/2))*arctan(2/(6^(1/2)+2^(1/2))*x)-1/(6^(1/2)+2^(1/2)
)*arctan(2/(6^(1/2)+2^(1/2))*x)+3^(1/2)/(6^(1/2)-2^(1/2))*arctan(2/(6^(1/2)
-2^(1/2))*x)-1/(6^(1/2)-2^(1/2))*arctan(2/(6^(1/2)-2^(1/2))*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2-1}{x^4+4x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 4*x^2 + 1), x)
```

mupad [B] time = 0.08, size = 31, normalized size = 0.70

$$\frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}x^3}{2}+\frac{3\sqrt{2}x}{2}\right)-\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(4*x^2 + x^4 + 1),x)
```

```
[Out] (2^(1/2)*(atan((3*2^(1/2)*x)/2 + (2^(1/2)*x^3)/2) - atan((2^(1/2)*x)/2)))/2
```

sympy [A] time = 0.13, size = 42, normalized size = 0.95

$$-\frac{\sqrt{2}\left(2\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)-2\operatorname{atan}\left(\frac{\sqrt{2}\,x^{3}}{2}+\frac{3\sqrt{2}\,x}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+4*x**2+1),x)
[Out] -sqrt(2)*(2*atan(sqrt(2)*x/2) - 2*atan(sqrt(2)*x**3/2 + 3*sqrt(2)*x/2))/4
```

3.83 $\int \frac{1-x^2}{1+3x^2+x^4} \, dx$

Optimal. Leaf size=39

$$\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

```
[Out] -arctan(x*2^(1/2)/(3+5^(1/2))^(1/2))+arctan(x*(1/2+1/2*5^(1/2)))
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1163, 203}

$$\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[2/(3 + Sqrt[5])]*x] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1+3x^2+x^4}\,dx &= \frac{1}{2}\left(-1-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx+\frac{1}{2}\left(-1+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx\\ &= -\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)+\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 0.26

$$\tan^{-1}\left(\frac{x}{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[x/(1 + x^2)]
```

fricas [A] time = 0.44, size = 13, normalized size = 0.33

$$\arctan\left(x^3+2\,x\right)-\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(x^3 + 2*x) - arctan(x)
```

giac [A] time = 0.18, size = 26, normalized size = 0.67

$$\frac{1}{4}\pi\mathrm{sgn}(x) - \frac{1}{2}\arctan\left(\frac{x^4 + x^2 + 1}{2\left(x^3 + x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) - 1/2*arctan(1/2*(x^4 + x^2 + 1)/(x^3 + x))
```

maple [B] time = 0.02, size = 104, normalized size = 2.67

$$\frac{2\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{2\sqrt{5}-2} - \frac{2\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{2\sqrt{5}-2} - \frac{2\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{2\sqrt{5}+2} - \frac{2\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{2\sqrt{5}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+3*x^2+1),x)
```

```
[Out] -2*5^(1/2)/(2*5^(1/2)+2)*arctan(4/(2*5^(1/2)+2)*x)-2/(2*5^(1/2)+2)*arctan(4
/(2*5^(1/2)+2)*x)+2*5^(1/2)/(2*5^(1/2)-2)*arctan(4/(2*5^(1/2)-2)*x)-2/(2*5^
(1/2)-2)*arctan(4/(2*5^(1/2)-2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 1}{x^4 + 3\,x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 3*x^2 + 1), x)
```

mupad [B] time = 4.31, size = 13, normalized size = 0.33

$$\mathrm{atan}\left(x^3 + 2\,x\right) - \mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(3*x^2 + x^4 + 1),x)
```

```
[Out] atan(2*x + x^3) - atan(x)
```

sympy [A] time = 0.12, size = 10, normalized size = 0.26

$$-\mathrm{atan}\,(x) + \mathrm{atan}\left(x^3 + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+3*x**2+1),x)
```

```
[Out] -atan(x) + atan(x**3 + 2*x)
```

3.84 $\int \frac{1-x^2}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=9

$$\frac{x}{x^2+1}$$

```
[Out] x/(x^2+1)
```

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {28, 383}

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 2*x^2 + x^4),x]
[Out] x/(1 + x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1+2x^2+x^4} \, dx &= \int \frac{1-x^2}{\left(1+x^2\right)^2} \, dx\\ &= \frac{x}{1+x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 2*x^2 + x^4),x]
[Out] x/(1 + x^2)
```

fricas [A] time = 0.38, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] x/(x^2 + 1)
```

giac [A] time = 0.18, size = 7, normalized size = 0.78

$$\frac{1}{x+\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/(x + 1/x)
```

maple [A] time = 0.01, size = 10, normalized size = 1.11

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+2*x^2+1),x)
```

```
[Out] 1/(x^2+1)*x
```

maxima [A] time = 1.00, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1)
```

mupad [B] time = 0.03, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(2*x^2 + x^4 + 1),x)
```

```
[Out] x/(x^2 + 1)
```

sympy [A] time = 0.09, size = 5, normalized size = 0.56

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+2*x**2+1),x)
```

```
[Out] x/(x**2 + 1)
```

3.85 $\int \frac{1-x^2}{1+x^2+x^4} \, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

```
[Out] -1/2*ln(x^2-x+1)+1/2*ln(x^2+x+1)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1164, 628}

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + x^2 + x^4),x]
[Out] -Log[1 - x + x^2]/2 + Log[1 + x + x^2]/2
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1+x^2+x^4} \, dx &= -\left(\frac{1}{2}\int \frac{1+2x}{-1-x-x^2} \, dx\right)-\frac{1}{2}\int \frac{1-2x}{-1+x-x^2} \, dx\\ &= -\frac{1}{2}\log\left(1-x+x^2\right)+\frac{1}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + x^2 + x^4),x]
[Out] -1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/2
```

fricas [A] time = 0.39, size = 21, normalized size = 0.84

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

giac [A] time = 0.15, size = 35, normalized size = 1.40

$$\frac{1}{4}\log\left(\left|x+\frac{1}{x+\frac{1}{x}}+\frac{1}{x}+2\right|\right)-\frac{1}{4}\log\left(\left|x+\frac{1}{x+\frac{1}{x}}+\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(x + 1/(x + 1/x) + 1/x + 2)) - 1/4*log(abs(x + 1/(x + 1/x) + 1/x
 - 2))
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$-\frac{\ln\left(x^2-x+1\right)}{2}+\frac{\ln\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+x^2+1),x)
```

```
[Out] -1/2*ln(x^2-x+1)+1/2*ln(x^2+x+1)
```

maxima [A] time = 1.04, size = 21, normalized size = 0.84

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

mupad [B] time = 0.06, size = 10, normalized size = 0.40

$$\operatorname{atanh}\left(\frac{x}{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^2 + x^4 + 1),x)
```

```
[Out] atanh(x/(x^2 + 1))
```

sympy [A] time = 0.12, size = 19, normalized size = 0.76

$$-\frac{\log\left(x^2-x+1\right)}{2}+\frac{\log\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/2 + log(x**2 + x + 1)/2
```

3.86 $\int \frac{1-x^2}{1+x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{2\sqrt{2}}$$

```
[Out] -1/4*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1165, 628}

$$\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + x^4),x]
```

```
[Out] -Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]
)
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1+x^4}\,dx &= -\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}\,x-x^2}\,dx}{2\sqrt{2}}\\ &= -\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.87

$$\frac{\log\left(x^2+\sqrt{2}\,x+1\right)-\log\left(-x^2+\sqrt{2}\,x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[2]*x - x^2] + Log[1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])
```

fricas [A] time = 0.41, size = 34, normalized size = 0.74

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4+4x^2+2\sqrt{2}\left(x^3+x\right)+1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 4*x^2 + 2*sqrt(2)*(x^3 + x) + 1)/(x^4 + 1))
```

giac [A] time = 0.15, size = 34, normalized size = 0.74

$$\frac{1}{4}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

maple [A] time = 0.00, size = 62, normalized size = 1.35

$$-\frac{\sqrt{2}\,\ln\left(\frac{x^2-\sqrt{2}\,x+1}{x^2+\sqrt{2}\,x+1}\right)}{8}+\frac{\sqrt{2}\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+1),x)
```

```
[Out] 1/8*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))-1/8*2^(1/2)*ln((x^2-2^(
1/2)*x+1)/(x^2+2^(1/2)*x+1))
```

maxima [A] time = 2.26, size = 34, normalized size = 0.74

$$\frac{1}{4}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

mupad [B] time = 0.06, size = 18, normalized size = 0.39

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 + 1),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*x)/(x^2 + 1)))/2
```

sympy [A] time = 0.11, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/4
```

3.87 $\int \frac{1-x^2}{1-x^2+x^4} \, dx$

Optimal. Leaf size=46

$$\frac{\log \left(x^2+\sqrt{3} x+1\right)}{2 \sqrt{3}}-\frac{\log \left(x^2-\sqrt{3} x+1\right)}{2 \sqrt{3}}$$

```
[Out] -1/6*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1164, 628}

$$\frac{\log \left(x^2+\sqrt{3} x+1\right)}{2 \sqrt{3}}-\frac{\log \left(x^2-\sqrt{3} x+1\right)}{2 \sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - x^2 + x^4),x]
```

```
[Out] -Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]
)
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1-x^2+x^4} \, dx &= -\frac{\int \frac{\sqrt{3}+2 x}{-1-\sqrt{3} x-x^2} \, dx}{2 \sqrt{3}}-\frac{\int \frac{\sqrt{3}-2 x}{-1+\sqrt{3} x-x^2} \, dx}{2 \sqrt{3}}\\ &=-\frac{\log \left(1-\sqrt{3} x+x^2\right)}{2 \sqrt{3}}+\frac{\log \left(1+\sqrt{3} x+x^2\right)}{2 \sqrt{3}}\\ \end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.87

$$\frac{\log \left(x^2+\sqrt{3} x+1\right)-\log \left(-x^2+\sqrt{3} x-1\right)}{2 \sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])
```

fricas [A] time = 0.41, size = 39, normalized size = 0.85

$$\frac{1}{6}\sqrt{3}\log\left(\frac{x^4+5x^2+2\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1))
```

giac [A] time = 0.18, size = 39, normalized size = 0.85

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2x-2\sqrt{3}+\frac{2}{x}\right|}{\left|2x+2\sqrt{3}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="giac")
[Out] -1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) + 2/x)/abs(2*x + 2*sqrt(3) + 2/x))
```

maple [A] time = 0.01, size = 35, normalized size = 0.76

$$-\frac{\sqrt{3}\ \ln\left(x^2-\sqrt{3}\ x+1\right)}{6}+\frac{\sqrt{3}\ \ln\left(x^2+\sqrt{3}\ x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-x^2+1),x)
[Out] -1/6*3^(1/2)*ln(x^2-3^(1/2)*x+1)+1/6*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

mupad [B] time = 4.31, size = 18, normalized size = 0.39

$$\frac{\sqrt{3}\ \mathrm{atanh}\left(\frac{\sqrt{3}\,x}{x^2+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 - x^2 + 1),x)
[Out] (3^(1/2)*atanh((3^(1/2)*x)/(x^2 + 1)))/3
```

sympy [A] time = 0.12, size = 39, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-x**2+1),x)
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/6
```

3.88 $\int \frac{1-x^2}{1-2x^2+x^4} \, dx$

Optimal. Leaf size=2

$$\tanh^{-1}(x)$$

```
[Out] arctanh(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {28, 21, 207}

$$\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] ArcTanh[x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1-2x^2+x^4} \, dx &= \int \frac{1-x^2}{\left(-1+x^2\right)^2} \, dx \\ &= -\int \frac{1}{-1+x^2} \, dx \\ &= \tanh^{-1}(x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 19, normalized size = 9.50

$$\frac{1}{2}\log(x+1)-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - x] + Log[1 + x]/2
```

fricas [B]   time = 0.40, size = 13, normalized size = 6.50

$$\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

giac [B]   time = 0.15, size = 15, normalized size = 7.50

$$\frac{1}{2}\log\left(|x+1|\right)-\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [A]   time = 0.00, size = 3, normalized size = 1.50

$$\operatorname{arctanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-2*x^2+1),x)
[Out] arctanh(x)
```

maxima [B]   time = 1.07, size = 13, normalized size = 6.50

$$\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B]   time = 4.30, size = 2, normalized size = 1.00

$$\operatorname{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 - 2*x^2 + 1),x)
[Out] atanh(x)
```

sympy [B]   time = 0.11, size = 12, normalized size = 6.00

$$-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-2*x**2+1),x)
[Out] -log(x - 1)/2 + log(x + 1)/2
```

3.89 $\int \frac{1-x^2}{1-3x^2+x^4}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -1/5*arctanh(1/5*(1-2*x)*5^(1/2))*5^(1/2)+1/5*arctanh(1/5*(1+2*x)*5^(1/2))*
5^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(1 - 2*x)/Sqrt[5]]/Sqrt[5]) + ArcTanh[(1 + 2*x)/Sqrt[5]]/Sqrt[5]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1-3x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-x+x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+x+x^2}\,dx\\ &= \text{Subst}\left(\int \frac{1}{5-x^2}\,dx,x,-1+2x\right)+\text{Subst}\left(\int \frac{1}{5-x^2}\,dx,x,1+2x\right)\\ &= \frac{\tanh^{-1}\left(\frac{-1+2x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\tanh^{-1}\left(\frac{1+2x}{\sqrt{5}}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.05

$$\frac{\log \left(x^2+\sqrt{5} x+1\right)-\log \left(-x^2+\sqrt{5} x-1\right)}{2 \sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 3*x^2 + x^4),x]
[Out] (-Log[-1 + Sqrt[5]*x - x^2] + Log[1 + Sqrt[5]*x + x^2])/(2*Sqrt[5])
```

fricas [A] time = 0.39, size = 39, normalized size = 1.03

$$\frac{1}{10} \sqrt{5} \log \left(\frac{x^4+7 x^2+2 \sqrt{5}\left(x^3+x\right)+1}{x^4-3 x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="fricas")
[Out] 1/10*sqrt(5)*log((x^4 + 7*x^2 + 2*sqrt(5)*(x^3 + x) + 1)/(x^4 - 3*x^2 + 1))
```

giac [A] time = 0.18, size = 39, normalized size = 1.03

$$-\frac{1}{10} \sqrt{5} \log \left(\frac{\left|2 x-2 \sqrt{5}+\frac{2}{x}\right|}{\left|2 x+2 \sqrt{5}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")
[Out] -1/10*sqrt(5)*log(abs(2*x - 2*sqrt(5) + 2/x)/abs(2*x + 2*sqrt(5) + 2/x))
```

maple [A] time = 0.00, size = 34, normalized size = 0.89

$$\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-3*x^2+1),x)
[Out] 1/5*arctanh(1/5*(2*x+1)*5^(1/2))*5^(1/2)+1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^
(1/2))
```

maxima [A] time = 2.46, size = 55, normalized size = 1.45

$$-\frac{1}{10} \sqrt{5} \log \left(\frac{2 x-\sqrt{5}+1}{2 x+\sqrt{5}+1}\right)-\frac{1}{10} \sqrt{5} \log \left(\frac{2 x-\sqrt{5}-1}{2 x+\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="maxima")
[Out] -1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/10*sqrt(5)*l
og((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1))
```

mupad [B] time = 0.11, size = 18, normalized size = 0.47

$$\frac{\sqrt{5} \operatorname{atanh}\left(\frac{\sqrt{5}\, x}{x^2+1}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 - 3*x^2 + 1),x)
```

```
[Out] (5^(1/2)*atanh((5^(1/2)*x)/(x^2 + 1)))/5
```

sympy [**A**] time = 0.12, size = 39, normalized size = 1.03

$$-\frac{\sqrt{5}\log\left(x^2-\sqrt{5}\,x+1\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\sqrt{5}\,x+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-3*x**2+1),x)
```

```
[Out] -sqrt(5)*log(x**2 - sqrt(5)*x + 1)/10 + sqrt(5)*log(x**2 + sqrt(5)*x + 1)/1
0
```

3.90 $\int \frac{1-x^2}{1-4x^2+x^4}\,dx$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{1-\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[Out] -1/6*arctanh(1/3*(1-x*2^(1/2))*3^(1/2))*6^(1/2)+1/6*arctanh(1/3*(1+x*2^(1/2
))*3^(1/2))*6^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{1-\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(1 - Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTanh[(1 + Sqrt[2]*x)/Sqrt[
3]]/Sqrt[6]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{1-4x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{2}x+x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+\sqrt{2}x+x^2}\,dx\\
&= \text{Subst}\left(\int \frac{1}{6-x^2}\,dx,x,-\sqrt{2}+2x\right)+\text{Subst}\left(\int \frac{1}{6-x^2}\,dx,x,\sqrt{2}+2x\right)\\
&= \frac{\tanh^{-1}\left(\frac{-1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}+\frac{\tanh^{-1}\left(\frac{1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.85

$$\frac{\log\left(x^2+\sqrt{6}x+1\right)-\log\left(-x^2+\sqrt{6}x-1\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 4*x^2 + x^4),x]
[Out] (-Log[-1 + Sqrt[6]*x - x^2] + Log[1 + Sqrt[6]*x + x^2])/(2*Sqrt[6])
```

fricas [A] time = 0.39, size = 39, normalized size = 0.83

$$\frac{1}{12}\sqrt{6}\log\left(\frac{x^4+8x^2+2\sqrt{6}\left(x^3+x\right)+1}{x^4-4x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="fricas")
[Out] 1/12*sqrt(6)*log((x^4 + 8*x^2 + 2*sqrt(6)*(x^3 + x) + 1)/(x^4 - 4*x^2 + 1))
```

giac [A] time = 0.32, size = 39, normalized size = 0.83

$$-\frac{1}{12}\sqrt{6}\log\left(\frac{\left|2x-2\sqrt{6}+\frac{2}{x}\right|}{\left|2x+2\sqrt{6}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")
[Out] -1/12*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2/x)/abs(2*x + 2*sqrt(6) + 2/x))
```

maple [A] time = 0.02, size = 70, normalized size = 1.49

$$\frac{\left(\sqrt{3}-1\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}+\frac{\left(1+\sqrt{3}\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\sqrt{6}+3\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-4*x^2+1),x)
[Out] 1/3*(3^(1/2)-1)*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2/(6^(1/2)-2^(1/2))*x)+1/
3*(1+3^(1/2))*3^(1/2)/(6^(1/2)+2^(1/2))*arctanh(2/(6^(1/2)+2^(1/2))*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2-1}{x^4-4x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(x^4 - 4*x^2 + 1), x)
```

mupad [B] time = 4.32, size = 18, normalized size = 0.38

$$\frac{\sqrt{6}\,\operatorname{atanh}\left(\frac{\sqrt{6}\,x}{x^2+1}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 - 4*x^2 + 1),x)
```

```
[Out] (6^(1/2)*atanh((6^(1/2)*x)/(x^2 + 1)))/6
```

sympy [A] time = 0.12, size = 39, normalized size = 0.83

$$-\frac{\sqrt{6}\log\left(x^2-\sqrt{6}\,x+1\right)}{12}+\frac{\sqrt{6}\log\left(x^2+\sqrt{6}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-4*x**2+1),x)
```

```
[Out] -sqrt(6)*log(x**2 - sqrt(6)*x + 1)/12 + sqrt(6)*log(x**2 + sqrt(6)*x + 1)/1
2
```

3.91 $\int \frac{1-x^2}{1-5x^2+x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -1/7*arctanh(1/7*(-2*x+3^(1/2))*7^(1/2))*7^(1/2)+1/7*arctanh(1/7*(2*x+3^(1/
2))*7^(1/2))*7^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[3] - 2*x)/Sqrt[7]]/Sqrt[7]) + ArcTanh[(Sqrt[3] + 2*x)/Sqrt[
7]]/Sqrt[7]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{1-5x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{3}\,x+x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+\sqrt{3}\,x+x^2}\,dx\\
&= \text{Subst}\left(\int \frac{1}{7-x^2}\,dx,x,-\sqrt{3}+2x\right)+\text{Subst}\left(\int \frac{1}{7-x^2}\,dx,x,\sqrt{3}+2x\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3}+2x}{\sqrt{7}}\right)}{\sqrt{7}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.87

$$\frac{\log\left(x^2+\sqrt{7}x+1\right)-\log\left(-x^2+\sqrt{7}x-1\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[7]*x - x^2] + Log[1 + Sqrt[7]*x + x^2])/(2*Sqrt[7])
```

fricas [A] time = 0.40, size = 39, normalized size = 0.85

$$\frac{1}{14}\sqrt{7}\log\left(\frac{x^4+9x^2+2\sqrt{7}\left(x^3+x\right)+1}{x^4-5x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((x^4 + 9*x^2 + 2*sqrt(7)*(x^3 + x) + 1)/(x^4 - 5*x^2 + 1))
```

giac [A] time = 0.22, size = 39, normalized size = 0.85

$$-\frac{1}{14}\sqrt{7}\log\left(\frac{\left|2x-2\sqrt{7}+\frac{2}{x}\right|}{\left|2x+2\sqrt{7}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")
```

```
[Out] -1/14*sqrt(7)*log(abs(2*x - 2*sqrt(7) + 2/x)/abs(2*x + 2*sqrt(7) + 2/x))
```

maple [B] time = 0.02, size = 82, normalized size = 1.78

$$\frac{2\left(-3+\sqrt{21}\right)\sqrt{21}\,\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}+\frac{2\left(3+\sqrt{21}\right)\sqrt{21}\,\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-5*x^2+1),x)
```

```
[Out] 2/21*(3+21^(1/2))*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctanh(4/(2*7^(1/2)+2*3^(
1/2))*x)+2/21*(-3+21^(1/2))*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctanh(4/(2*7^(
1/2)-2*3^(1/2))*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2-1}{x^4-5x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 - 5*x^2 + 1), x)
```

mupad [B] time = 4.39, size = 18, normalized size = 0.39

$$\frac{\sqrt{7}\,\operatorname{atanh}\left(\frac{\sqrt{7}\,x}{x^2+1}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 - 5*x^2 + 1),x)
```

```
[Out] (7^(1/2)*atanh((7^(1/2)*x)/(x^2 + 1)))/7
```

sympy [**A**] time = 0.14, size = 39, normalized size = 0.85

$$-\frac{\sqrt{7}\log\left(x^2-\sqrt{7}\,x+1\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\sqrt{7}\,x+1\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-5*x**2+1),x)
```

```
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x + 1)/14 + sqrt(7)*log(x**2 + sqrt(7)*x + 1)/1
4
```

3.92 $\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(1/2*(1-3*x)*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+3*x)*2^(1/2))*2^(
1/2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[-((1 + 3*x^2)/(1 + 2*x^2 + 9*x^4)),x]
```

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2
])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\int -\frac{1+3x^2}{1+2x^2+9x^4}\,dx &= -\left(\frac{1}{6}\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx\right)-\frac{1}{6}\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,-\frac{2}{3}+2x\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,\frac{2}{3}+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.10, size = 99, normalized size = 2.30

$$-\frac{\left(\sqrt{2}-i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(\sqrt{2}+i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 3*x^2)/(1 + 2*x^2 + 9*x^4),x]
```

```
[Out] -1/2*((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/Sqrt[2*(1 - (2*
I)*Sqrt[2])] - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*Sqr
t[2*(1 + (2*I)*Sqrt[2])])
```

fricas [A] time = 0.39, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(9x^3+5x\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sqr
t(2)*x)
```

giac [A] time = 0.16, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

maple [A] time = 0.01, size = 34, normalized size = 0.79

$$-\frac{\sqrt{2}\arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2-1)/(9*x^4+2*x^2+1),x)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^(
1/2))
```

maxima [A] time = 2.35, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

mupad [B] time = 4.38, size = 29, normalized size = 0.67

$$-\frac{\sqrt{2}\,\left(\mathrm{atan}\left(\frac{9\,\sqrt{2}\,x^3}{4}+\frac{5\,\sqrt{2}\,x}{4}\right)+\mathrm{atan}\left(\frac{3\,\sqrt{2}\,x}{4}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 + 1)/(2*x^2 + 9*x^4 + 1),x)
```

```
[Out] -(2^(1/2)*(atan((5*2^(1/2)*x)/4 + (9*2^(1/2)*x^3)/4) + atan((3*2^(1/2)*x)/4
)))/4
```

sympy [A] time = 0.14, size = 46, normalized size = 1.07

$$-\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{3\sqrt{2}\,x}{4}\right)+2\,\mathrm{atan}\left(\frac{9\sqrt{2}\,x^3}{4}+\frac{5\sqrt{2}\,x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2-1)/(9*x**4+2*x**2+1),x)
```

```
[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4))
/8
```

3.93 $\int \frac{1+3x^2}{-1-2x^2-9x^4} \, dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(1/2*(1-3*x)*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+3*x)*2^(1/2))*2^(
1/2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2
])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x^2}{-1-2x^2-9x^4} \, dx &= -\left(\frac{1}{6}\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2} \, dx\right)-\frac{1}{6}\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2} \, dx\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2} \, dx,x,-\frac{2}{3}+2x\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2} \, dx,x,\frac{2}{3}+2x\right)\\
&= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 99, normalized size = 2.30

$$-\frac{\left(\sqrt{2}-i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(\sqrt{2}+i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

```
[Out] -1/2*((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/Sqrt[2*(1 - (2*
I)*Sqrt[2])] - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*Sqr
t[2*(1 + (2*I)*Sqrt[2])])
```

fricas [A] time = 0.40, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(9x^{3}+5x\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sqr
t(2)*x)
```

giac [A] time = 0.18, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

maple [A] time = 0.00, size = 34, normalized size = 0.79

$$-\frac{\sqrt{2}\arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+1)/(-9*x^4-2*x^2-1),x)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(
1/2))
```

maxima [A] time = 2.49, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

<u>**mupad**</u> **[B]** time = 0.00, size = 29, normalized size = 0.67

$$-\frac{\sqrt{2}\,\left(\mathrm{atan}\left(\frac{9\,\sqrt{2}\,x^3}{4}+\frac{5\,\sqrt{2}\,x}{4}\right)+\mathrm{atan}\left(\frac{3\,\sqrt{2}\,x}{4}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 + 1)/(2*x^2 + 9*x^4 + 1),x)
```

```
[Out] -(2^(1/2)*(atan((5*2^(1/2)*x)/4 + (9*2^(1/2)*x^3)/4) + atan((3*2^(1/2)*x)/4
)))/4
```

<u>**sympy**</u> **[A]** time = 0.15, size = 46, normalized size = 1.07

$$-\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{3\sqrt{2}\,x}{4}\right)+2\,\mathrm{atan}\left(\frac{9\sqrt{2}\,x^3}{4}+\frac{5\sqrt{2}\,x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+1)/(-9*x**4-2*x**2-1),x)
```

```
[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4))
/8
```

3.94 $\int \frac{3+2x^2}{1-2x^2+x^4} \, dx$

Optimal. Leaf size=21

$$\frac{5x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 5/2*x/(-x^2+1)+1/2*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {28, 385, 207}

$$\frac{5x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] (5*x)/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{3+2x^2}{1-2x^2+x^4} \, dx &= \int \frac{3+2x^2}{\left(-1+x^2\right)^2} \, dx \\
&= \frac{5x}{2\left(1-x^2\right)}-\frac{1}{2}\int \frac{1}{-1+x^2} \, dx \\
&= \frac{5x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{10x}{x^2-1}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]
[Out] ((-10*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

fricas [B] time = 0.39, size = 34, normalized size = 1.62

$$\frac{\left(x^2-1\right)\log\left(x+1\right)-\left(x^2-1\right)\log\left(x-1\right)-10\,x}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="fricas")
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 10*x)/(x^2 - 1)
```

giac [A] time = 0.17, size = 25, normalized size = 1.19

$$-\frac{5\,x}{2\left(x^2-1\right)}+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="giac")
[Out] -5/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 28, normalized size = 1.33

$$\frac{\ln\left(x+1\right)}{4}-\frac{\ln\left(x-1\right)}{4}-\frac{5}{4\left(x+1\right)}-\frac{5}{4\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)/(x^4-2*x^2+1),x)
[Out] -5/4/(x+1)+1/4*ln(x+1)-5/4/(x-1)-1/4*ln(x-1)
```

maxima [A] time = 1.10, size = 23, normalized size = 1.10

$$-\frac{5\,x}{2\left(x^2-1\right)}+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="maxima")
[Out] -5/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{\mathrm{atanh}(x)}{2}-\frac{5\,x}{2\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 3)/(x^4 - 2*x^2 + 1),x)
[Out] atanh(x)/2 - (5*x)/(2*(x^2 - 1))
```

sympy [A] time = 0.13, size = 22, normalized size = 1.05

$$-\frac{5x}{2x^2-2}-\frac{\log\left(x-1\right)}{4}+\frac{\log\left(x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)/(x**4-2*x**2+1),x)
```

```
[Out] -5*x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

3.95 $\int \frac{2+3x^2}{5-8x^2+3x^4} \, dx$

Optimal. Leaf size=28

$$\frac{5}{2}\tanh^{-1}(x)-\frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)$$

```
[Out] 5/2*arctanh(x)-7/10*arctanh(1/5*x*15^(1/2))*15^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1166, 207}

$$\frac{5}{2}\tanh^{-1}(x)-\frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (5*ArcTanh[x])/2 - (7*Sqrt[3/5]*ArcTanh[Sqrt[3/5]*x])/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx &= -\left(\frac{15}{2}\int \frac{1}{-3+3x^2}\,dx\right)+\frac{21}{2}\int \frac{1}{-5+3x^2}\,dx\\ &= \frac{5}{2}\tanh^{-1}(x)-\frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.89

$$\frac{1}{20}\left(7\sqrt{15}\log\left(\sqrt{15}-3x\right)-25\log(1-x)+25\log(x+1)-7\sqrt{15}\log\left(3x+\sqrt{15}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (7*Sqrt[15]*Log[Sqrt[15] - 3*x] - 25*Log[1 - x] + 25*Log[1 + x] - 7*Sqrt[15
]*Log[Sqrt[15] + 3*x])/20
```

fricas [B] time = 0.40, size = 49, normalized size = 1.75

$$\frac{7}{20}\,\sqrt{5}\,\sqrt{3}\,\log\left(-\frac{2\,\sqrt{5}\,\sqrt{3}\,x-3\,x^2-5}{3\,x^2-5}\right)+\frac{5}{4}\,\log\left(x+1\right)-\frac{5}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="fricas")
```

```
[Out] 7/20*sqrt(5)*sqrt(3)*log(-(2*sqrt(5)*sqrt(3)*x - 3*x^2 - 5)/(3*x^2 - 5)) +
5/4*log(x + 1) - 5/4*log(x - 1)
```

giac [B] time = 0.17, size = 44, normalized size = 1.57

$$\frac{7}{20}\,\sqrt{15}\,\log\left(\frac{\left|6\,x-2\,\sqrt{15}\right|}{\left|6\,x+2\,\sqrt{15}\right|}\right)+\frac{5}{4}\,\log\left(\left|x+1\right|\right)-\frac{5}{4}\,\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="giac")
```

```
[Out] 7/20*sqrt(15)*log(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15))) + 5/4*log(ab
s(x + 1)) - 5/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{7\sqrt{15}\ \operatorname{arctanh}\left(\frac{\sqrt{15}\,x}{5}\right)}{10}+\frac{5\ln\left(x+1\right)}{4}-\frac{5\ln\left(x-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(3*x^4-8*x^2+5),x)
```

```
[Out] -7/10*arctanh(1/5*x*15^(1/2))*15^(1/2)+5/4*ln(x+1)-5/4*ln(x-1)
```

maxima [B] time = 2.36, size = 38, normalized size = 1.36

$$\frac{7}{20}\,\sqrt{15}\,\log\left(\frac{3\,x-\sqrt{15}}{3\,x+\sqrt{15}}\right)+\frac{5}{4}\,\log\left(x+1\right)-\frac{5}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="maxima")
```

```
[Out] 7/20*sqrt(15)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 5/4*log(x + 1) - 5/4
*log(x - 1)
```

mupad [B] time = 4.39, size = 17, normalized size = 0.61

$$\frac{5\,\mathrm{atanh}(x)}{2}-\frac{7\,\sqrt{15}\,\mathrm{atanh}\left(\frac{\sqrt{15}\,x}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(3*x^4 - 8*x^2 + 5),x)
```

```
[Out] (5*atanh(x))/2 - (7*15^(1/2)*atanh((15^(1/2)*x)/5))/10
```

sympy [B] time = 0.61, size = 53, normalized size = 1.89

$$-\frac{5\log\left(x-1\right)}{4}+\frac{5\log\left(x+1\right)}{4}+\frac{7\sqrt{15}\,\log\left(x-\frac{\sqrt{15}}{3}\right)}{20}-\frac{7\sqrt{15}\,\log\left(x+\frac{\sqrt{15}}{3}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(3*x**4-8*x**2+5),x)
```

```
[Out] -5*log(x - 1)/4 + 5*log(x + 1)/4 + 7*sqrt(15)*log(x - sqrt(15)/3)/20 - 7*sq
rt(15)*log(x + sqrt(15)/3)/20
```

3.96 $\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}(d+e)\tanh^{-1}(x)-\frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}\,x\right)}{2\sqrt{15}}$$

```
[Out] 1/2*(d+e)*arctanh(x)-1/30*(3*d+5*e)*arctanh(1/5*x*15^(1/2))*15^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1166, 207}

$$\frac{1}{2}(d+e)\tanh^{-1}(x)-\frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}\,x\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] ((d + e)*ArcTanh[x])/2 - ((3*d + 5*e)*ArcTanh[Sqrt[3/5]*x])/(2*Sqrt[15])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx &= -\left(\frac{1}{2}(3(d+e))\int \frac{1}{-3+3x^2}\,dx\right)+\frac{1}{2}(3d+5e)\int \frac{1}{-5+3x^2}\,dx\\ &= \frac{1}{2}(d+e)\tanh^{-1}(x)-\frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}\,x\right)}{2\sqrt{15}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 2.00

$$\frac{1}{60}\left(\sqrt{15}\,(3d+5e)\log\left(\sqrt{15}-3x\right)-15(d+e)\log(1-x)+15(d+e)\log(x+1)-\sqrt{15}\,(3d+5e)\log\left(3x+\sqrt{15}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] - 3*x] - 15*(d + e)*Log[1 - x] + 15*(d +
 e)*Log[1 + x] - Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] + 3*x])/60
```

fricas [B] time = 0.45, size = 55, normalized size = 1.53

$$\frac{1}{60}\,\sqrt{15}\,(3\,d+5\,e)\log\left(\frac{3\,x^2-2\,\sqrt{15}\,x+5}{3\,x^2-5}\right)+\frac{1}{4}\,(d+e)\log\left(x+1\right)-\frac{1}{4}\,(d+e)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="fricas")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log((3*x^2 - 2*sqrt(15)*x + 5)/(3*x^2 - 5)) + 1/4
*(d + e)*log(x + 1) - 1/4*(d + e)*log(x - 1)
```

giac [B] time = 0.16, size = 60, normalized size = 1.67

$$\frac{1}{60}\,\sqrt{15}\,(3\,d+5\,e)\log\left(\frac{\left|6\,x-2\,\sqrt{15}\right|}{\left|6\,x+2\,\sqrt{15}\right|}\right)+\frac{1}{4}\,(d+e)\log\left(|x+1|\right)-\frac{1}{4}\,(d+e)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="giac")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15)))
+ 1/4*(d + e)*log(abs(x + 1)) - 1/4*(d + e)*log(abs(x - 1))
```

maple [B] time = 0.01, size = 56, normalized size = 1.56

$$-\frac{\sqrt{15}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{15}\,x}{5}\right)}{10}+\frac{d\ln\left(x+1\right)}{4}-\frac{d\ln\left(x-1\right)}{4}-\frac{\sqrt{15}\,e\,\mathrm{arctanh}\left(\frac{\sqrt{15}\,x}{5}\right)}{6}+\frac{e\ln\left(x+1\right)}{4}-\frac{e\ln\left(x-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(3*x^4-8*x^2+5),x)
```

```
[Out] -1/10*15^(1/2)*arctanh(1/5*15^(1/2)*x)*d-1/6*15^(1/2)*arctanh(1/5*15^(1/2)*
x)*e+1/4*ln(x+1)*d+1/4*ln(x+1)*e-1/4*ln(x-1)*d-1/4*ln(x-1)*e
```

maxima [A] time = 2.41, size = 51, normalized size = 1.42

$$\frac{1}{60}\,\sqrt{15}\,(3\,d+5\,e)\log\left(\frac{3\,x-\sqrt{15}}{3\,x+\sqrt{15}}\right)+\frac{1}{4}\,(d+e)\log\left(x+1\right)-\frac{1}{4}\,(d+e)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 1/4*(d +
 e)*log(x + 1) - 1/4*(d + e)*log(x - 1)
```

mupad [B] time = 4.39, size = 290, normalized size = 8.06

$$\frac{\sqrt{15}\,\mathrm{atanh}\left(\frac{54\,\sqrt{15}\,d^3\,x}{25\left(-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3\right)}-\frac{6\,\sqrt{15}\,e^3\,x}{-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3}-\frac{18\,\sqrt{15}\,d\,e^2\,x}{5\left(-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3\right)}+\frac{18\,\sqrt{15}\,d^2}{5\left(-\frac{54\,d^3}{5}-18\,d^2\,e+1\right.}\right.}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(3*x^4 - 8*x^2 + 5),x)
```

```
[Out] (15^(1/2)*atanh((54*15^(1/2)*d^3*x)/(25*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 +
 30*e^3)) - (6*15^(1/2)*e^3*x)/(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)
```

```
- (18*15^(1/2)*d*e^2*x)/(5*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)) + (
18*15^(1/2)*d^2*e*x)/(5*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)))*(3*d
+ 5*e))/30 - atanh((18*d^3*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3) - (30
*e^3*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3) - (30*d*e^2*x)/(30*d*e^2 -
18*d^2*e - 18*d^3 + 30*e^3) + (18*d^2*e*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 +
30*e^3))*(d/2 + e/2)
```

sympy [B] time = 1.50, size = 474, normalized size = 13.17

$$\frac{(d+e)\log\left(x+\frac{-51d^3(d+e)-180d^2e(d+e)-225de^2(d+e)+60d(d+e)^3-100e^3(d+e)+75e(d+e)^3}{9d^4+24d^3e-40de^3-25e^4}\right)}{4}-\frac{(d+e)\log\left(x+\frac{51d^3(d+e)+180d^2e(d+e)+2\ldots}{9d\ldots}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(3*x**4-8*x**2+5),x)
```

```
[Out] (d + e)*log(x + (-51*d**3*(d + e) - 180*d**2*e*(d + e) - 225*d*e**2*(d + e)
 + 60*d*(d + e)**3 - 100*e**3*(d + e) + 75*e*(d + e)**3)/(9*d**4 + 24*d**3*
e - 40*d*e**3 - 25*e**4))/4 - (d + e)*log(x + (51*d**3*(d + e) + 180*d**2*e
*(d + e) + 225*d*e**2*(d + e) - 60*d*(d + e)**3 + 100*e**3*(d + e) - 75*e*(
d + e)**3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/4 + sqrt(15)*(3*d +
5*e)*log(x + (-17*sqrt(15)*d**3*(3*d + 5*e)/5 - 12*sqrt(15)*d**2*e*(3*d + 5
*e) - 15*sqrt(15)*d*e**2*(3*d + 5*e) + 4*sqrt(15)*d*(3*d + 5*e)**3/15 - 20*
sqrt(15)*e**3*(3*d + 5*e)/3 + sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**
3*e - 40*d*e**3 - 25*e**4))/60 - sqrt(15)*(3*d + 5*e)*log(x + (17*sqrt(15)*
d**3*(3*d + 5*e)/5 + 12*sqrt(15)*d**2*e*(3*d + 5*e) + 15*sqrt(15)*d*e**2*(3
*d + 5*e) - 4*sqrt(15)*d*(3*d + 5*e)**3/15 + 20*sqrt(15)*e**3*(3*d + 5*e)/3
 - sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))
/60
```

3.97 $\int \frac{3+x^2}{1+3x^2+x^4}\,dx$

Optimal. Leaf size=74

$$\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{10}}-\frac{1}{10}\sqrt{180-80\sqrt{5}}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)$$

```
[Out] 1/20*arctan(x*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)-1/10*arctan(x*2
^(1/2)/(3+5^(1/2))^(1/2))*(10-4*5^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1166, 203}

$$\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{10}}-\frac{1}{10}\sqrt{180-80\sqrt{5}}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] -(Sqrt[180 - 80*Sqrt[5]]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/10 + ((3 + Sqrt[5
])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{3+x^2}{1+3x^2+x^4}\,dx &= \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx\\ &= -\frac{1}{5}\sqrt{45-20\sqrt{5}}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{10}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 73, normalized size = 0.99

$$\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)-\left(3-\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] (-((3 - Sqrt[5])^(3/2)*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]) + (3 + Sqrt[5])^(3/
2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

fricas [B] time = 0.44, size = 137, normalized size = 1.85

$$\frac{2}{5}\sqrt{5}\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}+3}\left(\sqrt{5}\sqrt{2}+3\sqrt{2}\right)\sqrt{-4\sqrt{5}+9}-\frac{1}{2}\left(\sqrt{5}x+3x\right)\sqrt{-4\sqrt{5}+9}\right)+\frac{2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(-4*sqrt(5) + 9)*arctan(1/4*sqrt(2*x^2 + sqrt(5) + 3)*(sqrt
(5)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(5) + 9) - 1/2*(sqrt(5)*x + 3*x)*sqrt(
-4*sqrt(5) + 9)) + 2/5*sqrt(5)*sqrt(4*sqrt(5) + 9)*arctan(1/4*(sqrt(2*x^2 -
 sqrt(5) + 3)*(sqrt(5)*sqrt(2) - 3*sqrt(2)) - 2*sqrt(5)*x + 6*x)*sqrt(4*sqr
t(5) + 9))
```

giac [A] time = 0.16, size = 41, normalized size = 0.55

$$\frac{1}{5}\left(2\sqrt{5}-5\right)\arctan\left(\frac{2x}{\sqrt{5}+1}\right)+\frac{1}{5}\left(2\sqrt{5}+5\right)\arctan\left(\frac{2x}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/5*(2*sqrt(5) - 5)*arctan(2*x/(sqrt(5) + 1)) + 1/5*(2*sqrt(5) + 5)*arctan(
2*x/(sqrt(5) - 1))
```

maple [B] time = 0.02, size = 104, normalized size = 1.41

$$\frac{2\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{2\sqrt{5}-2}+\frac{6\sqrt{5}\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}+\frac{2\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{2\sqrt{5}+2}-\frac{6\sqrt{5}\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^4+3*x^2+1),x)
```

```
[Out] 2/(2*5^(1/2)+2)*arctan(4/(2*5^(1/2)+2)*x)-6/5*5^(1/2)/(2*5^(1/2)+2)*arctan(
4/(2*5^(1/2)+2)*x)+2/(2*5^(1/2)-2)*arctan(4/(2*5^(1/2)-2)*x)+6/5*5^(1/2)/(2
*5^(1/2)-2)*arctan(4/(2*5^(1/2)-2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+3}{x^4+3x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 3)/(x^4 + 3*x^2 + 1), x)
```

mupad [B] time = 0.11, size = 117, normalized size = 1.58

$$2\,\mathrm{atanh}\left(\frac{80x\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}-56}-\frac{48\sqrt{5}\,x\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}-56}\right)\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}}-2\,\mathrm{atanh}\left(\frac{80x\sqrt{-\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}+56}+\frac{48\sqrt{5}\,x\sqrt{-\frac{\sqrt{5}}{5}-}}{24\sqrt{5}+56}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3)/(3*x^2 + x^4 + 1),x)
```

```
[Out] 2*atanh((80*x*(5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) - 56) - (48*5^(1/2)*x*(
5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) - 56))*(5^(1/2)/5 - 9/20)^(1/2) - 2*at
anh((80*x*(- 5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) + 56) + (48*5^(1/2)*x*(-
5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) + 56))*(- 5^(1/2)/5 - 9/20)^(1/2)
```

sympy [A] time = 0.21, size = 46, normalized size = 0.62

$$2\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2x}{-1+\sqrt{5}}\right)-2\left(\frac{1}{2}-\frac{\sqrt{5}}{5}\right)\operatorname{atan}\left(\frac{2x}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)/(x**4+3*x**2+1),x)
```

```
[Out] 2*(sqrt(5)/5 + 1/2)*atan(2*x/(-1 + sqrt(5))) - 2*(1/2 - sqrt(5)/5)*atan(2*x
/(1 + sqrt(5)))
```

3.98 $\int \frac{a+bx^2}{1+x^2+x^4}\,dx$

Optimal. Leaf size=83

$$-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/4*(a-b)*ln(x^2-x+1)+1/4*(a-b)*ln(x^2+x+1)-1/6*(a+b)*arctan(1/3*(1-2*x)*3
^(1/2))*3^(1/2)+1/6*(a+b)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -((a + b)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((a + b)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(2*Sqrt[3]) - ((a - b)*Log[1 - x + x^2])/4 + ((a - b)*Log[1 + x
 + x^2])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{1+x^2+x^4}\,dx &= \frac{1}{2}\int \frac{a-(a-b)x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{a+(a-b)x}{1+x+x^2}\,dx\\ &= \frac{1}{4}(a-b)\int \frac{1+2x}{1+x+x^2}\,dx+\frac{1}{4}(-a+b)\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}(a+b)\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}(a\\ &= -\frac{1}{4}(a-b)\log\left(1-x+x^2\right)+\frac{1}{4}(a-b)\log\left(1+x+x^2\right)+\frac{1}{2}(-a-b)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.\\ &= -\frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}(a-b)\log\left(1-x+x^2\right)+\frac{1}{4}(a-b)\log\left(1\right.\end{aligned}$$

Mathematica [C] time = 0.13, size = 97, normalized size = 1.17

$$\frac{\left(2ia+\left(\sqrt{3}-i\right)b\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{6+6i\sqrt{3}}}+\frac{\left(\left(\sqrt{3}+i\right)b-2ia\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] (((2*I)*a + (-I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*
Sqrt[3]] + (((-2*I)*a + (I + Sqrt[3])*b)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[
6 - (6*I)*Sqrt[3]]
```

fricas [A] time = 0.43, size = 69, normalized size = 0.83

$$\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

giac [A] time = 0.15, size = 69, normalized size = 0.83

$$\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

maple [A] time = 0.00, size = 114, normalized size = 1.37

$$\frac{\sqrt{3}\,a\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,a\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{a\ln\left(x^2-x+1\right)}{4}+\frac{a\ln\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}\,b\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+1),x)
```

```
[Out] 1/4*ln(x^2+x+1)*a-1/4*ln(x^2+x+1)*b+1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
*a+1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))*b-1/4*ln(x^2-x+1)*a+1/4*ln(x^2-x
+1)*b+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*a+1/6*3^(1/2)*arctan(1/3*(2*x
-1)*3^(1/2))*b
```

maxima [A] time = 2.43, size = 69, normalized size = 0.83

$$\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-b)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

mupad [B] time = 4.50, size = 827, normalized size = 9.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2 + x^4 + 1),x)
```

```
[Out] - atan(((x*(4*a*b - 4*a^2 + 2*b^2) + (12*a + 24*x*(b/4 - a/4 + (3^(1/2)*a*1
i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)
/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i + (x*(4*a*b -
4*a^2 + 2*b^2) - (12*a - 24*x*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1
i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (
3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i)/((x*(4*a*b - 4*a^2 + 2*b^2) + (12
*a + 24*x*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 +
 (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (
3^(1/2)*b*1i)/12) - (x*(4*a*b - 4*a^2 + 2*b^2) - (12*a - 24*x*(b/4 - a/4 +
(3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3
^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12) - 2*a
*b^2 + 2*a^2*b + 2*b^3))*((a*1i)/2 - (b*1i)/2 + (3^(1/2)*a)/6 + (3^(1/2)*b)
/6) - atan(((x*(4*a*b - 4*a^2 + 2*b^2) + (12*a + 24*x*(a/4 - b/4 + (3^(1/2)
*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b
*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i + (x*(4*a*
b - 4*a^2 + 2*b^2) - (12*a - 24*x*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)
*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4
 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i)/((x*(4*a*b - 4*a^2 + 2*b^2) +
 (12*a + 24*x*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b
/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12
 + (3^(1/2)*b*1i)/12) - (x*(4*a*b - 4*a^2 + 2*b^2) - (12*a - 24*x*(a/4 - b/
4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12
+ (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12) -
 2*a*b^2 + 2*a^2*b + 2*b^3))*((b*1i)/2 - (a*1i)/2 + (3^(1/2)*a)/6 + (3^(1/2
)*b)/6)
```

sympy [C] time = 1.26, size = 740, normalized size = 8.92

$$\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)\log\left(x+\frac{2a^3\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)+6a^2b\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)-12ab^2\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right.}{a^4-a^3b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+1),x)
```

```
[Out] (-a/4 + b/4 - sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(-a/4 + b/4 - sqrt(3)*I
*(a + b)/12) + 6*a**2*b*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12) - 12*a*b**2*(-a
/4 + b/4 - sqrt(3)*I*(a + b)/12) + 24*a*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12)
**3 + 2*b**3*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12) - 48*b*(-a/4 + b/4 - sqrt(
3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4)) + (-a/4 + b/4 + sqrt(
3)*I*(a + b)/12)*log(x + (2*a**3*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) + 6*a*
*2*b*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) - 12*a*b**2*(-a/4 + b/4 + sqrt(3)*
I*(a + b)/12) + 24*a*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(-a/4
+ b/4 + sqrt(3)*I*(a + b)/12) - 48*b*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12)**3
)/(a**4 - a**3*b + a*b**3 - b**4)) + (a/4 - b/4 - sqrt(3)*I*(a + b)/12)*log
(x + (2*a**3*(a/4 - b/4 - sqrt(3)*I*(a + b)/12) + 6*a**2*b*(a/4 - b/4 - sqr
t(3)*I*(a + b)/12) - 12*a*b**2*(a/4 - b/4 - sqrt(3)*I*(a + b)/12) + 24*a*(a
/4 - b/4 - sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(a/4 - b/4 - sqrt(3)*I*(a + b)
/12) - 48*b*(a/4 - b/4 - sqrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3
- b**4)) + (a/4 - b/4 + sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(a/4 - b/4 +
sqrt(3)*I*(a + b)/12) + 6*a**2*b*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) - 12*a*
b**2*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) + 24*a*(a/4 - b/4 + sqrt(3)*I*(a +
b)/12)**3 + 2*b**3*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) - 48*b*(a/4 - b/4 + s
qrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4))
```

3.99 $\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{8}(2a-b)\log\left(x^2-x+1\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)+\frac{x\left(-\left(x^2(a-2b)\right)+a+b\right)}{6\left(x^4+x^2+1\right)}-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b}{}$$

```
[Out] 1/6*x*(a+b-(a-2*b)*x^2)/(x^4+x^2+1)-1/8*(2*a-b)*ln(x^2-x+1)+1/8*(2*a-b)*ln(
x^2+x+1)-1/36*(4*a+b)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/36*(4*a+b)*arct
an(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1178, 1169, 634, 618, 204, 628}

$$\frac{x\left(x^2(-(a-2b))+a+b\right)}{6\left(x^4+x^2+1\right)}-\frac{1}{8}(2a-b)\log\left(x^2-x+1\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\,\mathrm{ta}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(a + b - (a - 2*b)*x^2))/(6*(1 + x^2 + x^4)) - ((4*a + b)*ArcTan[(1 - 2*
x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*a + b)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[
3]) - ((2*a - b)*Log[1 - x + x^2])/8 + ((2*a - b)*Log[1 + x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2}\,dx = \frac{x\left(a+b-(a-2b)x^2\right)}{6\left(1+x^2+x^4\right)}+\frac{1}{6}\int \frac{5a-b+(-a+2b)x^2}{1+x^2+x^4}\,dx$$

$$= \frac{x\left(a+b-(a-2b)x^2\right)}{6\left(1+x^2+x^4\right)}+\frac{1}{12}\int \frac{5a-b-(6a-3b)x}{1-x+x^2}\,dx+\frac{1}{12}\int \frac{5a-b+(6a-3b)x}{1+x+x^2}\,dx$$

$$= \frac{x\left(a+b-(a-2b)x^2\right)}{6\left(1+x^2+x^4\right)}+\frac{1}{8}(2a-b)\int \frac{1+2x}{1+x+x^2}\,dx+\frac{1}{8}(-2a+b)\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}$$

$$= \frac{x\left(a+b-(a-2b)x^2\right)}{6\left(1+x^2+x^4\right)}-\frac{1}{8}(2a-b)\log\left(1-x+x^2\right)+\frac{1}{8}(2a-b)\log\left(1+x+x^2\right)+\frac{1}{12}($$

$$= \frac{x\left(a+b-(a-2b)x^2\right)}{6\left(1+x^2+x^4\right)}-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}}-\frac{1}{8}(2a-b)\log$$

Mathematica [C] time = 0.25, size = 147, normalized size = 1.24

$$\frac{x\left(-ax^2+a+2bx^2+b\right)}{6\left(x^4+x^2+1\right)}-\frac{\left(\left(\sqrt{3}-11i\right)a-2\left(\sqrt{3}-2i\right)b\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{6\sqrt{6+6i\sqrt{3}}}-\frac{\left(\left(\sqrt{3}+11i\right)a-2\left(\sqrt{3}+2i\right)b\right.}{6\sqrt{6-6i\sqrt{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(a + b - a*x^2 + 2*b*x^2))/(6*(1 + x^2 + x^4)) - (((-11*I + Sqrt[3])*a -
 2*(-2*I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/(6*Sqrt[6 + (6*I)*Sqrt
[3]]) - (((11*I + Sqrt[3])*a - 2*(2*I + Sqrt[3])*b)*ArcTan[((I + Sqrt[3])*x
)/2])/(6*Sqrt[6 - (6*I)*Sqrt[3]])
```

fricas [A] time = 0.41, size = 185, normalized size = 1.55

$$-\frac{12\,(a-2\,b)x^3-2\,\sqrt{3}\left((4\,a+b)x^4+(4\,a+b)x^2+4\,a+b\right)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-2\,\sqrt{3}\left((4\,a+b)x^4+(4\,a+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(12*(a - 2*b)*x^3 - 2*sqrt(3)*((4*a + b)*x^4 + (4*a + b)*x^2 + 4*a +
b)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((4*a + b)*x^4 + (4*a + b)*x^2
```

```
+ 4*a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(a + b)*x - 9*((2*a - b)*x^4
 + (2*a - b)*x^2 + 2*a - b)*log(x^2 + x + 1) + 9*((2*a - b)*x^4 + (2*a - b)
*x^2 + 2*a - b)*log(x^2 - x + 1))/(x^4 + x^2 + 1)
```

giac [A] time = 0.16, size = 109, normalized size = 0.92

$$\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(4*a +
b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^2 + x + 1) - 1/8*(2*
a - b)*log(x^2 - x + 1) - 1/6*(a*x^3 - 2*b*x^3 - a*x - b*x)/(x^4 + x^2 + 1)
```

maple [A] time = 0.01, size = 168, normalized size = 1.41

$$\frac{\sqrt{3}\,a\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}+\frac{\sqrt{3}\,a\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}-\frac{a\ln\left(x^2-x+1\right)}{4}+\frac{a\ln\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}\,b\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{36}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*a+2/3*b)*x-2/3*a+1/3*b)/(x^2+x+1)+1/4*a*ln(x^2+x+1)-1/8*b*ln(x^2
+x+1)+1/9*3^(1/2)*a*arctan(1/3*(2*x+1)*3^(1/2))+1/36*3^(1/2)*b*arctan(1/3*(
2*x+1)*3^(1/2))-1/4*((1/3*a-2/3*b)*x-2/3*a+1/3*b)/(x^2-x+1)-1/4*a*ln(x^2-x+
1)+1/8*b*ln(x^2-x+1)+1/9*3^(1/2)*a*arctan(1/3*(2*x-1)*3^(1/2))+1/36*3^(1/2)
*b*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.35, size = 105, normalized size = 0.88

$$\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(4*a +
b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^2 + x + 1) - 1/8*(2*
a - b)*log(x^2 - x + 1) - 1/6*((a - 2*b)*x^3 - (a + b)*x)/(x^4 + x^2 + 1)
```

mupad [B] time = 4.49, size = 897, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2 + x^4 + 1)^2,x)
```

```
[Out] atan((((2*b - 10*a + 24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/7
2))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 -
(19*a*b)/9 + b^2/9))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)*1i
 + ((10*a - 2*b + 24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))
*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19
*a*b)/9 + b^2/9))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)*1i)/(
(19*a*b^2)/36 - (29*a^2*b)/36 + (31*a^3)/108 - (7*b^3)/54 + ((2*b - 10*a +
24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(b/8 - a/4 + (3^(
```

```
1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*
(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - ((10*a - 2*b + 24*x*(
b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(b/8 - a/4 + (3^(1/2)*a
*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*(b/8 -
 a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)))*((a*1i)/2 - (b*1i)/4 + (3^(
1/2)*a)/9 + (3^(1/2)*b)/36) + atan((((2*b - 10*a + 24*x*(a/4 - b/8 + (3^(1/
2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)
*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*
1i)/18 + (3^(1/2)*b*1i)/72)*1i + ((10*a - 2*b + 24*x*(a/4 - b/8 + (3^(1/2)*
a*1i)/18 + (3^(1/2)*b*1i)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*
1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*1i)
/18 + (3^(1/2)*b*1i)/72)*1i)/((19*a*b^2)/36 - (29*a^2*b)/36 + (31*a^3)/108
- (7*b^3)/54 + ((2*b - 10*a + 24*x*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2
)*b*1i)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a
^2)/18 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1
i)/72) - ((10*a - 2*b + 24*x*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i
)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18
 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)
))*((b*1i)/4 - (a*1i)/2 + (3^(1/2)*a)/9 + (3^(1/2)*b)/36) - (x^3*(a/6 - b/3
) - x*(a/6 + b/6))/(x^2 + x^4 + 1)
```

sympy [C] time = 1.89, size = 874, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+1)**2,x)
```

```
[Out] (x**3*(-a + 2*b) + x*(a + b))/(6*x**4 + 6*x**2 + 6) + (-a/4 + b/8 - sqrt(3)
*I*(4*a + b)/72)*log(x + (76*a**3*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) + 9
48*a**2*b*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(-a/4 + b/8 -
sqrt(3)*I*(4*a + b)/72) + 12096*a*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72)**3
+ 148*b**3*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) - 8640*b*(-a/4 + b/8 - sqr
t(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a*b**3
- 7*b**4)) + (-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*(-a/4 +
 b/8 + sqrt(3)*I*(4*a + b)/72) + 948*a**2*b*(-a/4 + b/8 + sqrt(3)*I*(4*a +
b)/72) - 816*a*b**2*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72) + 12096*a*(-a/4 +
 b/8 + sqrt(3)*I*(4*a + b)/72)**3 + 148*b**3*(-a/4 + b/8 + sqrt(3)*I*(4*a +
 b)/72) - 8640*b*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*
a**3*b + 75*a**2*b**2 + 11*a*b**3 - 7*b**4)) + (a/4 - b/8 - sqrt(3)*I*(4*a
+ b)/72)*log(x + (76*a**3*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) + 948*a**2*b
*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(a/4 - b/8 - sqrt(3)*I*(
4*a + b)/72) + 12096*a*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72)**3 + 148*b**3*(
a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 8640*b*(a/4 - b/8 - sqrt(3)*I*(4*a +
b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a*b**3 - 7*b**4)) + (
a/4 - b/8 + sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*(a/4 - b/8 + sqrt(3)*I
*(4*a + b)/72) + 948*a**2*b*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) - 816*a*b*
*2*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) + 12096*a*(a/4 - b/8 + sqrt(3)*I*(4
*a + b)/72)**3 + 148*b**3*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) - 8640*b*(a/
4 - b/8 + sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2
 + 11*a*b**3 - 7*b**4))
```

3.100 $\int \frac{a+bx^2}{2+x^2+x^4} \, dx$

Optimal. Leaf size=234

$$-\frac{\left(a-\sqrt{2}b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(a-\sqrt{2}b\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{\right.$$

```
[Out] -1/28*arctan((-2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(a+b*2^(1/2))
*(-14+28*2^(1/2))^(1/2)+1/28*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2)
)^(1/2))*(a+b*2^(1/2))*(-14+28*2^(1/2))^(1/2)-1/4*ln(x^2+2^(1/2)-x*(-1+2*2^
(1/2))^(1/2))*(a-b*2^(1/2))/(-2+4*2^(1/2))^(1/2)+1/4*ln(x^2+2^(1/2)+x*(-1+2
*2^(1/2))^(1/2))*(a-b*2^(1/2))/(-2+4*2^(1/2))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{\left(a-\sqrt{2}b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(a-\sqrt{2}b\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(2 + x^2 + x^4),x]
```

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] -
2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*A
rcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 - ((a - Sqrt[2]*
b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2])]
) + ((a - Sqrt[2]*b)*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2
*(-1 + 2*Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx = \frac{\int \frac{\sqrt{-1+2\sqrt{2}}\,a-\left(a-\sqrt{2}\,b\right)x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\int \frac{\sqrt{-1+2\sqrt{2}}\,a+\left(a-\sqrt{2}\,b\right)x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

$$=\frac{1}{8}\left(\sqrt{2}\,a+2b\right)\int \frac{1}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx+\frac{1}{8}\left(\sqrt{2}\,a+2b\right)\int \frac{1}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+}$$

$$=-\frac{\left(a-\sqrt{2}\,b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\left(a-\sqrt{2}\,b\right)\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+x^2\right.}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

$$=-\frac{\left(a+\sqrt{2}\,b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\left(a+\sqrt{2}\,b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}-\frac{\left(a-\sqrt{2}\,b\right)\log\left(\sqrt{\right.}}{4\sqrt{2\left(\right.}}$$

Mathematica [C] time = 0.12, size = 111, normalized size = 0.47

$$\frac{\left(\left(\sqrt{7}+i\right)b-2ia\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{14-14i\sqrt{7}}}+\frac{\left(2ia+\left(\sqrt{7}-i\right)b\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{14+14i\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(2 + x^2 + x^4),x]
```

```
[Out] (((-2*I)*a + (I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[14 -
(14*I)*Sqrt[7]] + (((2*I)*a + (-I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 + I*Sqrt[7
])/2]])/Sqrt[14 + (14*I)*Sqrt[7]]
```

fricas [B] time = 0.55, size = 3406, normalized size = 14.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="fricas")
```

```
[Out] 1/112*(28*sqrt(2)*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*
b^4)^(1/4)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a
^2*b^2 + 4*b^4)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sq
rt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^
2))/(a^4 - 4*a^2*b^2 + 4*b^4))*arctan(-1/28*(7*sqrt(1/2)*sqrt(1/7)*(8*a^4 -
 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(3/4)*(sqrt(2)*sqrt(a^4 - 2*a^3
```

```
*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*a - 2*sqrt(
a^4 - 4*a^2*b^2 + 4*b^4)*(a^2*b - a*b^2 + 2*b^3))*sqrt((4*a^4 - 8*a^3*b + 2
0*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*
a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4))*sqrt((2*(a
^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x^2 + sqrt(1/7)*(8*a^4 - 16*a^3
*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(a^4 - 2*a^
3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*b*x - sqrt(7)*(a^3 - a^2*b + 2*a*b^2)*x)
*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4
- 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^
2*b^2 + 4*b^4)) + 2*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^
4)*(a^2 - a*b + 2*b^2))/(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)) + 8*
sqrt(7)*sqrt(2)*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(3/2)*sqrt(a^
4 - 4*a^2*b^2 + 4*b^4) - 7*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*
b^3 + 32*b^4)^(3/4)*(sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b
^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*a*x - 2*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*(a^
2*b - a*b^2 + 2*b^3)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*
b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*
b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)) - 4*sqrt(7)*(a^6 - 3*a^5*b + 9*a^4*b
^2 - 13*a^3*b^3 + 18*a^2*b^4 - 12*a*b^5 + 8*b^6)*sqrt(a^4 - 4*a^2*b^2 + 4*b
^4))/(a^8 - 3*a^7*b + 7*a^6*b^2 - 7*a^5*b^3 + 14*a^3*b^5 - 28*a^2*b^6 + 24*
a*b^7 - 16*b^8)) + 28*sqrt(2)*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32
*a*b^3 + 32*b^4)^(1/4)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sq
rt(a^4 - 4*a^2*b^2 + 4*b^4)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 +
 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 -
8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4))*arctan(-1/28*(7*sqrt(1/2)*sqrt(1
/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(3/4)*(sqrt(2)*sqrt
(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)
*a - 2*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*(a^2*b - a*b^2 + 2*b^3))*sqrt((4*a^4 -
 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*
a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)
)*sqrt((2*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x^2 - sqrt(1/7)*(8*
a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqr
t(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*b*x - sqrt(7)*(a^3 - a^2*b +
 2*a*b^2)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(
2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))
/(a^4 - 4*a^2*b^2 + 4*b^4)) + 2*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*
a*b^3 + 4*b^4)*(a^2 - a*b + 2*b^2))/(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 +
4*b^4)) - 8*sqrt(7)*sqrt(2)*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(
3/2)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4) - 7*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2
*b^2 - 32*a*b^3 + 32*b^4)^(3/4)*(sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4
*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*a*x - 2*sqrt(a^4 - 4*a^2*b^2
+ 4*b^4)*(a^2*b - a*b^2 + 2*b^3)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16
*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)
*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)) + 4*sqrt(7)*(a^6 - 3*a^5
*b + 9*a^4*b^2 - 13*a^3*b^3 + 18*a^2*b^4 - 12*a*b^5 + 8*b^6)*sqrt(a^4 - 4*a
^2*b^2 + 4*b^4))/(a^8 - 3*a^7*b + 7*a^6*b^2 - 7*a^5*b^3 + 14*a^3*b^5 - 28*a
^2*b^6 + 24*a*b^7 - 16*b^8)) - sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 3
2*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4
*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2) + 4*sqrt(7)*(a^4 - 2*a^3*b + 5*a^2*b^
2 - 4*a*b^3 + 4*b^4))*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^
4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b
+ 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4))*log(8*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a
*b^3 + 4*b^4)*x^2 + 4*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 +
 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 +
4*b^4)*b*x - sqrt(7)*(a^3 - a^2*b + 2*a*b^2)*x)*sqrt((4*a^4 - 8*a^3*b + 20*
a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*
b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)) + 8*sqrt(2)*
sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - a*b + 2*b^2)) + sq
rt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*
```

```
sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*
b^2) + 4*sqrt(7)*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*sqrt((4*a^4
 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b +
5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^
4))*log(8*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x^2 - 4*sqrt(1/7)*(
8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*s
qrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*b*x - sqrt(7)*(a^3 - a^2*b
 + 2*a*b^2)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqr
t(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2
))/(a^4 - 4*a^2*b^2 + 4*b^4)) + 8*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 -
4*a*b^3 + 4*b^4)*(a^2 - a*b + 2*b^2)))/(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3
 + 4*b^4)
```

giac [B] time = 0.88, size = 544, normalized size = 2.32

$$-\frac{1}{14336}\sqrt{7}\left(32\sqrt{7}\,2^{\frac{1}{4}}b\left(\sqrt{2}+4\right)^{\frac{3}{2}}+96\sqrt{7}\,2^{\frac{1}{4}}b\sqrt{\sqrt{2}+4}\left(\sqrt{2}-4\right)-24\cdot 2^{\frac{3}{4}}b\left(\sqrt{2}+4\right)\sqrt{-8\sqrt{2}+32}+2^{\frac{3}{4}}b\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="giac")
```

```
[Out] -1/14336*sqrt(7)*(32*sqrt(7)*2^(1/4)*b*(sqrt(2) + 4)^(3/2) + 96*sqrt(7)*2^(
1/4)*b*sqrt(sqrt(2) + 4)*(sqrt(2) - 4) - 24*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-8
*sqrt(2) + 32) + 2^(3/4)*b*(-8*sqrt(2) + 32)^(3/2) - 128*sqrt(7)*2^(3/4)*a*
sqrt(sqrt(2) + 4) + 64*2^(1/4)*a*sqrt(-8*sqrt(2) + 32))*arctan(2*2^(3/4)*sq
rt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)) - 1/14336
*sqrt(7)*(32*sqrt(7)*2^(1/4)*b*(sqrt(2) + 4)^(3/2) + 96*sqrt(7)*2^(1/4)*b*s
qrt(sqrt(2) + 4)*(sqrt(2) - 4) - 24*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-8*sqrt(2)
 + 32) + 2^(3/4)*b*(-8*sqrt(2) + 32)^(3/2) - 128*sqrt(7)*2^(3/4)*a*sqrt(sqr
t(2) + 4) + 64*2^(1/4)*a*sqrt(-8*sqrt(2) + 32))*arctan(2*2^(3/4)*sqrt(1/2)*
(x - 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)) - 1/28672*sqrt(7)
*(24*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-8*sqrt(2) + 32) - sqrt(7)*2^(3/4
)*b*(-8*sqrt(2) + 32)^(3/2) + 32*2^(1/4)*b*(sqrt(2) + 4)^(3/2) + 96*2^(1/4)
*b*sqrt(sqrt(2) + 4)*(sqrt(2) - 4) - 128*2^(3/4)*a*sqrt(sqrt(2) + 4) - 64*s
qrt(7)*2^(1/4)*a*sqrt(-8*sqrt(2) + 32))*log(x^2 + 2*2^(1/4)*x*sqrt(-1/8*sqr
t(2) + 1/2) + sqrt(2)) + 1/28672*sqrt(7)*(24*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4
)*sqrt(-8*sqrt(2) + 32) - sqrt(7)*2^(3/4)*b*(-8*sqrt(2) + 32)^(3/2) + 32*2^
(1/4)*b*(sqrt(2) + 4)^(3/2) + 96*2^(1/4)*b*sqrt(sqrt(2) + 4)*(sqrt(2) - 4)
- 128*2^(3/4)*a*sqrt(sqrt(2) + 4) - 64*sqrt(7)*2^(1/4)*a*sqrt(-8*sqrt(2) +
32))*log(x^2 - 2*2^(1/4)*x*sqrt(-1/8*sqrt(2) + 1/2) + sqrt(2))
```

maple [B] time = 0.10, size = 710, normalized size = 3.03

$$-\frac{\left(-1+2\sqrt{2}\right)\sqrt{2}\,a\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{28\sqrt{1+2\sqrt{2}}}-\frac{\left(-1+2\sqrt{2}\right)a\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{7\sqrt{1+2\sqrt{2}}}+\frac{\sqrt{2}\,a\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{1+2\sqrt{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+2),x)
```

```
[Out] 1/56*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)*a-
1/14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)*b+
1/14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*a-1/28*ln(
x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*b-1/28/(1+2*2^(1/2
))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/
2))*2^(1/2)*a+1/7/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+
```

```
2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*2^(1/2)*b-1/7/(1+2*2^(1/2))^(1/2)*arctan((
2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*a+1/14/(1+2*2
^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*
2^(1/2))*b+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2
^(1/2))^(1/2))*2^(1/2)*a-1/56*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*
2^(1/2))^(1/2)*2^(1/2)*a+1/14*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*
2^(1/2))^(1/2)*2^(1/2)*b-1/14*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*
2^(1/2))^(1/2)*a+1/28*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))
^(1/2)*b-1/28/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^
(1/2))^(1/2))*(-1+2*2^(1/2))*2^(1/2)*a+1/7/(1+2*2^(1/2))^(1/2)*arctan((2*x-
(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*2^(1/2)*b-1/7/(1+
2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1
+2*2^(1/2))*a+1/14/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1
+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*b+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1
+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2 + a}{x^4 + x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2), x)
```

mupad [B] time = 4.49, size = 771, normalized size = 3.29

$$-\mathrm{atan}\left(\frac{a^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}\,7\mathrm{i}}{\frac{\sqrt{7}\,a^3\,1\mathrm{i}}{2}-a\,b^2-2\,a^2\,b+\frac{a^3}{2}+4\,b^3-\sqrt{7}\,a\,b^2\,1\mathrm{i}}-\frac{b^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}\,14\mathrm{i}}{\frac{\sqrt{7}\,a^3\,1\mathrm{i}}{2}-a\,b^2-2\,a^2\,b+\frac{a^3}{2}+4\,b^3-\sqrt{7}\,a\,b^2\,1\mathrm{i}}+\frac{\sqrt{7}\,a^2}{\frac{\sqrt{7}\,a^3\,1\mathrm{i}}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2 + x^4 + 2),x)
```

```
[Out] - atan((a^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^2/
112 + b^2/56)^(1/2)*7i)/((7^(1/2)*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4*b
^3 - 7^(1/2)*a*b^2*1i) - (b^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)
*b^2*1i)/56 + a^2/112 + b^2/56)^(1/2)*14i)/((7^(1/2)*a^3*1i)/2 - a*b^2 - 2*
a^2*b + a^3/2 + 4*b^3 - 7^(1/2)*a*b^2*1i) + (7^(1/2)*a^2*x*((7^(1/2)*a^2*1i
)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^2/112 + b^2/56)^(1/2))/((7^(1/2)
*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4*b^3 - 7^(1/2)*a*b^2*1i) - (2*7^(1/
2)*b^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^2/112 +
 b^2/56)^(1/2))/((7^(1/2)*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4*b^3 - 7^(
1/2)*a*b^2*1i))*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^
2/112 + b^2/56)^(1/2)*2i - 2*atanh((7*a^2*x*((7^(1/2)*b^2*1i)/56 - (7^(1/2)
*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2))/((7^(1/2)*a^3*1i)/2 + a*
b^2 + 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i) - (14*b^2*x*((7^(1/2)*b^2
*1i)/56 - (7^(1/2)*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2))/((7^(1
/2)*a^3*1i)/2 + a*b^2 + 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i) + (7^(1
/2)*a^2*x*((7^(1/2)*b^2*1i)/56 - (7^(1/2)*a^2*1i)/112 - (a*b)/14 + a^2/112
+ b^2/56)^(1/2)*1i)/((7^(1/2)*a^3*1i)/2 + a*b^2 + 2*a^2*b - a^3/2 - 4*b^3 -
 7^(1/2)*a*b^2*1i) - (7^(1/2)*b^2*x*((7^(1/2)*b^2*1i)/56 - (7^(1/2)*a^2*1i)
/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2)*2i)/((7^(1/2)*a^3*1i)/2 + a*b^2 +
 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i))*((7^(1/2)*b^2*1i)/56 - (7^(1/
2)*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2)
```

sympy [A] time = 1.32, size = 122, normalized size = 0.52

$$\mathrm{RootSum}\left(1568t^4 + t^2\left(-28a^2 + 224ab - 56b^2\right) + a^4 - 2a^3b + 5a^2b^2 - 4ab^3 + 4b^4, \left(t \mapsto t\log\left(x + \frac{112t^3a - 4}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+2),x)
```

```
[Out] RootSum(1568*_t**4 + _t**2*(-28*a**2 + 224*a*b - 56*b**2) + a**4 - 2*a**3*b
 + 5*a**2*b**2 - 4*a*b**3 + 4*b**4, Lambda(_t, _t*log(x + (112*_t**3*a - 44
8*_t**3*b + 6*_t*a**3 + 12*_t*a**2*b - 48*_t*a*b**2 + 8*_t*b**3)/(a**4 - a*
*3*b + 2*a*b**3 - 4*b**4))))
```

3.101 $\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx$

Optimal. Leaf size=316

$$-\frac{\left(\sqrt{2}\,(a-4b)+11a-2b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(\left(11+\sqrt{2}\right)a-2\left(2\sqrt{2}\,b+b\right)\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}\,x\right.}{112\sqrt{2\left(2\sqrt{2}-1\right)}}$$

```
[Out] 1/28*x*(3*a+2*b-(a-4*b)*x^2)/(x^4+x^2+2)-1/784*arctan((-2*x+(-1+2*2^(1/2))^
(1/2))/(1+2*2^(1/2))^(1/2))*(-b*(2-4*2^(1/2))+a*(11-2^(1/2)))*(-14+28*2^(1/
2))^(1/2)+1/784*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-b*
(2-4*2^(1/2))+a*(11-2^(1/2)))*(-14+28*2^(1/2))^(1/2)-1/112*ln(x^2+2^(1/2)-x
*(-1+2*2^(1/2))^(1/2))*(11*a-2*b+(a-4*b)*2^(1/2))/(-2+4*2^(1/2))^(1/2)+1/11
2*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(a*(11+2^(1/2))-2*b-4*b*2^(1/2))/(
-2+4*2^(1/2))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1178, 1169, 634, 618, 204, 628}

$$\frac{x\left(x^2(-(a-4b))+3a+2b\right)}{28\left(x^4+x^2+2\right)}-\frac{\left(\sqrt{2}\,(a-4b)+11a-2b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(\left(11+\sqrt{2}\right)a-2\left(2\sqrt{2}\,b+\right.\right.}{112\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```

```
[Out] (x*(3*a + 2*b - (a - 4*b)*x^2))/(28*(2 + x^2 + x^4)) - (Sqrt[(-1 + 2*Sqrt[2
])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]]
- 2*x)/Sqrt[1 + 2*Sqrt[2]]])/56 + (Sqrt[(-1 + 2*Sqrt[2])/14]*((11 - Sqrt[2]
)*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqr
t[2]]])/56 - ((11*a + Sqrt[2]*(a - 4*b) - 2*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sq
rt[2]]*x + x^2])/(112*Sqrt[2*(-1 + 2*Sqrt[2])]) + (((11 + Sqrt[2])*a - 2*(b
 + 2*Sqrt[2]*b))*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(112*Sqrt[2*(
-1 + 2*Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(2+x^2+x^4\right)^2}\,dx = \frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(2+x^2+x^4\right)} + \frac{1}{28}\int \frac{11a-2b+(-a+4b)x^2}{2+x^2+x^4}\,dx$$

$$= \frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(2+x^2+x^4\right)} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}\,(11a-2b)-\left(11a-2b-\sqrt{2}\,(-a+4b)\right)x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{56\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}\,(11a-}{\sqrt{2}+}}{56\sqrt{}}$$

$$= \frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(2+x^2+x^4\right)} - \frac{\left(11a+\sqrt{2}\,(a-4b)-2b\right)\int \frac{-\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{112\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\left(\left(11-\sqrt{}\right.\right.}{}$$

$$= \frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(2+x^2+x^4\right)} - \frac{\left(11a+\sqrt{2}\,(a-4b)-2b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2\right)}{112\sqrt{2\left(-1+2\sqrt{2}\right)}} +$$

$$= \frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(2+x^2+x^4\right)} - \frac{\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)}{56\sqrt{2\left(1+2\sqrt{2}\right)}} + \frac{\left(\left(11-\right.\right.}{}$$

Mathematica [C] time = 0.22, size = 165, normalized size = 0.52

$$\frac{2b\left(2x^3+x\right)-ax\left(x^2-3\right)}{28\left(x^4+x^2+2\right)} - \frac{\left(\left(\sqrt{7}+23i\right)a-4\left(\sqrt{7}+2i\right)b\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{28\sqrt{14-14i\sqrt{7}}} - \frac{\left(\left(\sqrt{7}-23i\right)a-4\left(\sqrt{7}-2i\right)b}{28\sqrt{14+14i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```

```
[Out] (-(a*x*(-3 + x^2)) + 2*b*(x + 2*x^3))/(28*(2 + x^2 + x^4)) - (((23*I + Sqrt
[7])*a - 4*(2*I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/(28*Sqrt[1
4 - (14*I)*Sqrt[7]]) - (((-23*I + Sqrt[7])*a - 4*(-2*I + Sqrt[7])*b)*ArcTan
[x/Sqrt[(1 + I*Sqrt[7])/2]])/(28*Sqrt[14 + (14*I)*Sqrt[7]])
```

fricas [B] time = 0.61, size = 4346, normalized size = 13.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/21952*(196*2^(3/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 233
2*a*b^3 + 484*b^4)^(3/4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3
+ 16*b^4)*(x^4 + x^2 + 2)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 1
8656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 -
 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b
- 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*arctan(1/14*(2^(3/4)*sqrt(2/7)*sqrt(1/1
4)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(3/4)*(sqr
t(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*sqrt
(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(11*a - 2*b) + 2*sq
rt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(67*a^3 - 321*a^2
*b + 234*a*b^2 - 88*b^3))*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 1
8656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 -
 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b
- 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*sqrt((14*(4489*a^4 - 7102*a^3*b + 5757*
a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 + 2^(1/4)*sqrt(2/7)*(4489*a^4 - 7102*a^
3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(4489
*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(a - 4*b)*x + sqrt
(7)*(737*a^3 - 717*a^2*b + 348*a*b^2 - 44*b^3)*x)*sqrt((35912*a^4 - 56816*a
^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 710
2*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2
))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 14*sqrt(2)*sq
rt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(67*a^2 - 5
3*a*b + 22*b^2))/(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b
^4)) - 2^(3/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3
 + 484*b^4)^(3/4)*(sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332
*a*b^3 + 484*b^4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^
4)*(11*a - 2*b)*x + 2*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 1
6*b^4)*(67*a^3 - 321*a^2*b + 234*a*b^2 - 88*b^3)*x)*sqrt((35912*a^4 - 56816
*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7
102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b
^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) - 4*sqrt(7)*s
qrt(2)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(3/2)*
sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4) + 2*sqrt(7)*(30
0763*a^6 - 713751*a^5*b + 860883*a^4*b^2 - 617609*a^3*b^3 + 282678*a^2*b^4
- 76956*a*b^5 + 10648*b^6)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^
3 + 16*b^4))/(5112971*a^8 - 13336819*a^7*b + 16286963*a^6*b^2 - 11087881*a^
5*b^3 + 3832430*a^4*b^4 + 31472*a^3*b^5 - 641872*a^2*b^6 + 265232*a*b^7 - 4
2592*b^8)) + 196*2^(3/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 -
2332*a*b^3 + 484*b^4)^(3/4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b
^3 + 16*b^4)*(x^4 + x^2 + 2)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2
- 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^
2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3
*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*arctan(1/14*(2^(3/4)*sqrt(2/7)*sqrt(
1/14)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(3/4)*(
sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*s
qrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(11*a - 2*b) + 2
```

```
*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(67*a^3 - 321*
a^2*b + 234*a*b^2 - 88*b^3))*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2
- 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^
2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3
*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*sqrt((14*(4489*a^4 - 7102*a^3*b + 57
57*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 - 2^(1/4)*sqrt(2/7)*(4489*a^4 - 7102
*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(4
489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(a - 4*b)*x + s
qrt(7)*(737*a^3 - 717*a^2*b + 348*a*b^2 - 44*b^3)*x)*sqrt((35912*a^4 - 5681
6*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 -
7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*
b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 14*sqrt(2)
*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(67*a^2
- 53*a*b + 22*b^2))/(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 48
4*b^4)) - 2^(3/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*
b^3 + 484*b^4)^(3/4)*(sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2
332*a*b^3 + 484*b^4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16
*b^4)*(11*a - 2*b)*x + 2*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3
+ 16*b^4)*(67*a^3 - 321*a^2*b + 234*a*b^2 - 88*b^3)*x)*sqrt((35912*a^4 - 56
816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4
- 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 10
0*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 4*sqrt(7
)*sqrt(2)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(3/
2)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4) - 2*sqrt(7)*
(300763*a^6 - 713751*a^5*b + 860883*a^4*b^2 - 617609*a^3*b^3 + 282678*a^2*b
^4 - 76956*a*b^5 + 10648*b^6)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a
*b^3 + 16*b^4))/(5112971*a^8 - 13336819*a^7*b + 16286963*a^6*b^2 - 11087881
*a^5*b^3 + 3832430*a^4*b^4 + 31472*a^3*b^5 - 641872*a^2*b^6 + 265232*a*b^7
- 42592*b^8)) + 784*(4489*a^5 - 25058*a^4*b + 34165*a^3*b^2 - 25360*a^2*b^3
 + 9812*a*b^4 - 1936*b^5)*x^3 - 2^(1/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b +
5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*((211*a^2 - 428
*a*b + 100*b^2)*x^4 + (211*a^2 - 428*a*b + 100*b^2)*x^2 + 422*a^2 - 856*a*b
 + 200*b^2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^
4) + 8*sqrt(7)*((4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^
4)*x^4 + 8978*a^4 - 14204*a^3*b + 11514*a^2*b^2 - 4664*a*b^3 + 968*b^4 + (4
489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2))*sqrt((359
12*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqr
t(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 4
28*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4))
*log(32*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 +
 16/7*2^(1/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3
+ 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2
 - 2332*a*b^3 + 484*b^4)*(a - 4*b)*x + sqrt(7)*(737*a^3 - 717*a^2*b + 348*a
*b^2 - 44*b^3)*x)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b
^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*
b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^
2*b^2 + 32*a*b^3 + 16*b^4)) + 32*sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*
a^2*b^2 - 2332*a*b^3 + 484*b^4)*(67*a^2 - 53*a*b + 22*b^2)) + 2^(1/4)*sqrt(
2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(s
qrt(7)*sqrt(2)*((211*a^2 - 428*a*b + 100*b^2)*x^4 + (211*a^2 - 428*a*b + 10
0*b^2)*x^2 + 422*a^2 - 856*a*b + 200*b^2)*sqrt(4489*a^4 - 7102*a^3*b + 5757
*a^2*b^2 - 2332*a*b^3 + 484*b^4) + 8*sqrt(7)*((4489*a^4 - 7102*a^3*b + 5757
*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^4 + 8978*a^4 - 14204*a^3*b + 11514*a^2*b
^2 - 4664*a*b^3 + 968*b^4 + (4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*
b^3 + 484*b^4)*x^2))*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*
a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332
*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120
*a^2*b^2 + 32*a*b^3 + 16*b^4))*log(32*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2
 - 2332*a*b^3 + 484*b^4)*x^2 - 16/7*2^(1/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*
```

```
b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(4489*a
^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(a - 4*b)*x + sqrt(7
)*(737*a^3 - 717*a^2*b + 348*a*b^2 - 44*b^3)*x)*sqrt((35912*a^4 - 56816*a^3
*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*
a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))
/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 32*sqrt(2)*sqrt
(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(67*a^2 - 53*
a*b + 22*b^2)) - 784*(13467*a^5 - 12328*a^4*b + 3067*a^3*b^2 + 4518*a^2*b^3
 - 3212*a*b^4 + 968*b^5)*x)/((4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a
*b^3 + 484*b^4)*x^4 + 8978*a^4 - 14204*a^3*b + 11514*a^2*b^2 - 4664*a*b^3 +
 968*b^4 + (4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^
2)
```

giac [B] time = 0.95, size = 988, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/401408*sqrt(7)*(32*sqrt(7)*2^(1/4)*a*(sqrt(2) + 4)^(3/2) - 128*sqrt(7)*2^
(1/4)*b*(sqrt(2) + 4)^(3/2) + 96*sqrt(7)*2^(1/4)*a*sqrt(sqrt(2) + 4)*(sqrt(
2) - 4) - 384*sqrt(7)*2^(1/4)*b*sqrt(sqrt(2) + 4)*(sqrt(2) - 4) - 24*2^(3/4
)*a*(sqrt(2) + 4)*sqrt(-8*sqrt(2) + 32) + 96*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-
8*sqrt(2) + 32) + 2^(3/4)*a*(-8*sqrt(2) + 32)^(3/2) - 4*2^(3/4)*b*(-8*sqrt(
2) + 32)^(3/2) + 1408*sqrt(7)*2^(3/4)*a*sqrt(sqrt(2) + 4) - 256*sqrt(7)*2^(
3/4)*b*sqrt(sqrt(2) + 4) - 704*2^(1/4)*a*sqrt(-8*sqrt(2) + 32) + 128*2^(1/4
)*b*sqrt(-8*sqrt(2) + 32))*arctan(2*2^(3/4)*sqrt(1/2)*(x + 2^(1/4)*sqrt(-1/
8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)) + 1/401408*sqrt(7)*(32*sqrt(7)*2^(1/4)
*a*(sqrt(2) + 4)^(3/2) - 128*sqrt(7)*2^(1/4)*b*(sqrt(2) + 4)^(3/2) + 96*sqr
t(7)*2^(1/4)*a*sqrt(sqrt(2) + 4)*(sqrt(2) - 4) - 384*sqrt(7)*2^(1/4)*b*sqrt
(sqrt(2) + 4)*(sqrt(2) - 4) - 24*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-8*sqrt(2) +
32) + 96*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-8*sqrt(2) + 32) + 2^(3/4)*a*(-8*sqrt
(2) + 32)^(3/2) - 4*2^(3/4)*b*(-8*sqrt(2) + 32)^(3/2) + 1408*sqrt(7)*2^(3/4
)*a*sqrt(sqrt(2) + 4) - 256*sqrt(7)*2^(3/4)*b*sqrt(sqrt(2) + 4) - 704*2^(1/
4)*a*sqrt(-8*sqrt(2) + 32) + 128*2^(1/4)*b*sqrt(-8*sqrt(2) + 32))*arctan(2*
2^(3/4)*sqrt(1/2)*(x - 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4))
 + 1/802816*sqrt(7)*(24*sqrt(7)*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-8*sqrt(2) + 3
2) - 96*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-8*sqrt(2) + 32) - sqrt(7)*2^(
3/4)*a*(-8*sqrt(2) + 32)^(3/2) + 4*sqrt(7)*2^(3/4)*b*(-8*sqrt(2) + 32)^(3/2
) + 32*2^(1/4)*a*(sqrt(2) + 4)^(3/2) - 128*2^(1/4)*b*(sqrt(2) + 4)^(3/2) +
96*2^(1/4)*a*sqrt(sqrt(2) + 4)*(sqrt(2) - 4) - 384*2^(1/4)*b*sqrt(sqrt(2) +
 4)*(sqrt(2) - 4) + 1408*2^(3/4)*a*sqrt(sqrt(2) + 4) - 256*2^(3/4)*b*sqrt(s
qrt(2) + 4) + 704*sqrt(7)*2^(1/4)*a*sqrt(-8*sqrt(2) + 32) - 128*sqrt(7)*2^(
1/4)*b*sqrt(-8*sqrt(2) + 32))*log(x^2 + 2*2^(1/4)*x*sqrt(-1/8*sqrt(2) + 1/2
) + sqrt(2)) - 1/802816*sqrt(7)*(24*sqrt(7)*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-8
*sqrt(2) + 32) - 96*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-8*sqrt(2) + 32) -
 sqrt(7)*2^(3/4)*a*(-8*sqrt(2) + 32)^(3/2) + 4*sqrt(7)*2^(3/4)*b*(-8*sqrt(2
) + 32)^(3/2) + 32*2^(1/4)*a*(sqrt(2) + 4)^(3/2) - 128*2^(1/4)*b*(sqrt(2) +
 4)^(3/2) + 96*2^(1/4)*a*sqrt(sqrt(2) + 4)*(sqrt(2) - 4) - 384*2^(1/4)*b*sq
rt(sqrt(2) + 4)*(sqrt(2) - 4) + 1408*2^(3/4)*a*sqrt(sqrt(2) + 4) - 256*2^(3
/4)*b*sqrt(sqrt(2) + 4) + 704*sqrt(7)*2^(1/4)*a*sqrt(-8*sqrt(2) + 32) - 128
*sqrt(7)*2^(1/4)*b*sqrt(-8*sqrt(2) + 32))*log(x^2 - 2*2^(1/4)*x*sqrt(-1/8*s
qrt(2) + 1/2) + sqrt(2)) - 1/28*(a*x^3 - 4*b*x^3 - 3*a*x - 2*b*x)/(x^4 + x^
2 + 2)
```

maple [B] time = 0.31, size = 756, normalized size = 2.39

$$\frac{\sqrt{2}\,a\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{16\left(1+2\sqrt{2}\right)^{\frac{3}{2}}}+\frac{3a\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{8\left(1+2\sqrt{2}\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\,a\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{16\left(1+2\sqrt{2}\right)^{\frac{3}{2}}}+\frac{3a\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{8\left(1+2\sqrt{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+2)^2,x)
```

```
[Out] 1/784*((-14*a-28*2^(1/2)*a+112*b*2^(1/2)+56*b)/(1+2*2^(1/2))*x+1/(1+2*2^(1/
2))*(-1+2*2^(1/2))^(1/2)*(-70*a-42*2^(1/2)*a+56*b*2^(1/2)+28*b))/(x^2+(-1+2
*2^(1/2))^(1/2)*x+2^(1/2))+107/1568/(1+2*2^(1/2))*ln(x^2+(-1+2*2^(1/2))^(1/
2)*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)*a-25/784/(1+2*2^(1/2))*ln(x^2+(-
1+2*2^(1/2))^(1/2)*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)*b+53/784/(1+2*2^
(1/2))*ln(x^2+(-1+2*2^(1/2))^(1/2)*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*a-11/196
/(1+2*2^(1/2))*ln(x^2+(-1+2*2^(1/2))^(1/2)*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*
b+1/16/(1+2*2^(1/2))^(3/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^
(1/2))*2^(1/2)*a+3/8/(1+2*2^(1/2))^(3/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/
(1+2*2^(1/2))^(1/2))*a+1/8/(1+2*2^(1/2))^(3/2)*arctan((2*x+(-1+2*2^(1/2))^(
1/2))/(1+2*2^(1/2))^(1/2))*b*2^(1/2)-1/784*(-(-14*a-28*2^(1/2)*a+112*b*2^(1
/2)+56*b)/(1+2*2^(1/2))*x+1/(1+2*2^(1/2))*(-1+2*2^(1/2))^(1/2)*(-70*a-42*2^
(1/2)*a+56*b*2^(1/2)+28*b))/(x^2-(-1+2*2^(1/2))^(1/2)*x+2^(1/2))-107/1568/(
1+2*2^(1/2))*ln(x^2-(-1+2*2^(1/2))^(1/2)*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*2^
(1/2)*a+25/784/(1+2*2^(1/2))*ln(x^2-(-1+2*2^(1/2))^(1/2)*x+2^(1/2))*(-1+2*2
^(1/2))^(1/2)*2^(1/2)*b-53/784/(1+2*2^(1/2))*ln(x^2-(-1+2*2^(1/2))^(1/2)*x+
2^(1/2))*(-1+2*2^(1/2))^(1/2)*a+11/196/(1+2*2^(1/2))*ln(x^2-(-1+2*2^(1/2))^
(1/2)*x+2^(1/2))*(-1+2*2^(1/2))^(1/2)*b+1/16/(1+2*2^(1/2))^(3/2)*arctan((2*
x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*a+3/8/(1+2*2^(1/2))^(3
/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*a+1/8/(1+2*2^(1/
2))^(3/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*b*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(a-4\,b)x^3-(3\,a+2\,b)x}{28\left(x^4+x^2+2\right)}+\frac{1}{28}\int-\frac{(a-4\,b)x^2-11\,a+2\,b}{x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -1/28*((a - 4*b)*x^3 - (3*a + 2*b)*x)/(x^4 + x^2 + 2) + 1/28*integrate(-((a
 - 4*b)*x^2 - 11*a + 2*b)/(x^4 + x^2 + 2), x)
```

mupad [B] time = 4.50, size = 1491, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2 + x^4 + 2)^2,x)
```

```
[Out] atan((b^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3
136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/(
4*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a
^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1
/2)*a^2*b*39i)/3136)) - (a^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 -
 (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*1i
)/3136)^(1/2)*17i)/(16*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3
```

```
*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a
*b^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)) + (a*b*x*((7^(1/2)*a^2*17i)/1254
4 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21
952 - (7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/(4*((7^(1/2)*a^3*187i)/6272 + (7^(1/
2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^
3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)) - (17*7^(1/2)
*a^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 +
 (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2))/(112*((7^
(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a^2*b)/
3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1/2)*a^
2*b*39i)/3136)) + (7^(1/2)*b^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952
 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*
1i)/3136)^(1/2))/(28*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3*a
*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a*b
^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)) + (7^(1/2)*a*b*x*((7^(1/2)*a^2*17i
)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b
^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2))/(28*((7^(1/2)*a^3*187i)/6272 + (7
^(1/2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (
9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)))*((7^(1/2
)*a^2*17i)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/8780
8 + (25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2)*2i - atan((a^2*x*((7^(1/2
)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/8780
8 + (25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2)*17i)/(16*((3*a*b^2)/1568
- (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*
a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/313
6)) - (b^2*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/2
1952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/
(4*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*
a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(
1/2)*a^2*b*39i)/3136)) - (a*b*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/
12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)*a*b*1
i)/3136)^(1/2)*1i)/(4*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3
*187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*
b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/3136)) - (17*7^(1/2)*a^2*x*((7^(1/2)*b^2
*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + (
25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2))/(112*((3*a*b^2)/1568 - (7^(1/
2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/627
2 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/3136)) + (7
^(1/2)*b^2*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/2
1952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2))/(28
*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*a^
2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(1/
2)*a^2*b*39i)/3136)) + (7^(1/2)*a*b*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2
*17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)
*a*b*1i)/3136)^(1/2))/(28*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)
*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2
)*a*b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/3136)))*((7^(1/2)*b^2*1i)/3136 - (7^
(1/2)*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 +
 (7^(1/2)*a*b*1i)/3136)^(1/2)*2i - (x^3*(a/28 - b/7) - x*((3*a)/28 + b/14))
/(x^2 + x^4 + 2)
```

sympy [A] time = 1.80, size = 165, normalized size = 0.52

$$\frac{x^3(-a+4b)+x(3a+2b)}{28x^4+28x^2+56}+\mathrm{RootSum}\left(240945152t^4+t^2\left(-1157968a^2+2348864ab-548800b^2\right)+4489a^4-71\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+2)**2,x)
```

```
[Out] (x**3*(-a + 4*b) + x*(3*a + 2*b))/(28*x**4 + 28*x**2 + 56) + RootSum(240945
152*_t**4 + _t**2*(-1157968*a**2 + 2348864*a*b - 548800*b**2) + 4489*a**4 -
 7102*a**3*b + 5757*a**2*b**2 - 2332*a*b**3 + 484*b**4, Lambda(_t, _t*log(x
 + (2634240*_t**3*a - 3161088*_t**3*b + 11996*_t*a**3 + 12792*_t*a**2*b - 2
1936*_t*a*b**2 + 4384*_t*b**3)/(1139*a**4 - 1169*a**3*b + 318*a**2*b**2 + 1
24*a*b**3 - 88*b**4))))
```

3.102 $\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}\,x^2+x^4}\,dx$

Optimal. Leaf size=160

$$-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x}{\ldots}\right)}{2\sqrt{2}\ldots}$$

```
[Out] -1/8*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(4+2*2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2+2^
(1/2))^(1/2))*(4+2*2^(1/2))^(1/2)-1/2*arctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^
(1/2))^(1/2))/(2+2^(1/2))^(1/2)+1/2*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/
2))^(1/2))/(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x}{\ldots}\right)}{2\sqrt{2}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

```
[Out] -ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
+ ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
 - (Sqrt[1 + 1/Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/4 + (Sqrt[1 + 1
/Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2+\sqrt{2}}} + \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2+\sqrt{2}}}$$

$$= \frac{1}{4}\sqrt{3-2\sqrt{2}}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx + \frac{1}{4}\sqrt{3-2\sqrt{2}}\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx + $$

$$= -\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2+\sqrt{2}}\,x+x^2\right) + \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2+\sqrt{2}}\,x+x^2\right) - $$

$$= -\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)$$

Mathematica [C] time = 0.04, size = 53, normalized size = 0.33

$$\frac{\sqrt{-1-i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1-i}}\right)+\sqrt{-1+i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

```
[Out] (Sqrt[-1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[-1 - I]] + Sqrt[-1 + I]*ArcTan[(2^(1/
4)*x)/Sqrt[-1 + I]])/2^(3/4)
```

fricas [C] time = 0.42, size = 97, normalized size = 0.61

$$\frac{1}{4}\sqrt{(i+1)\sqrt{2}}\log\left(x+\frac{1}{2}\sqrt{2}\sqrt{(i+1)\sqrt{2}}\right)-\frac{1}{4}\sqrt{(i+1)\sqrt{2}}\log\left(x-\frac{1}{2}\sqrt{2}\sqrt{(i+1)\sqrt{2}}\right)+\frac{1}{4}\sqrt{-(i-1)\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((I + 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt((I + 1)*sqrt(2))) - 1/4*
sqrt((I + 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt((I + 1)*sqrt(2))) + 1/4*sqrt
(-(I - 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt(-(I - 1)*sqrt(2))) - 1/4*sqrt(-
(I - 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt(-(I - 1)*sqrt(2)))
```

giac [A] time = 0.38, size = 122, normalized size = 0.76

$$\frac{1}{4}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\log\left(x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2
)) + 1/4*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2
) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/8*
sqrt(2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1)
```

maple [A] time = 0.09, size = 199, normalized size = 1.24

$$\frac{\sqrt{2}\ \arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}-\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\sqrt{2}\ \arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}-\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}-\frac{\sqrt{2}\ \sqrt{2+\sqrt{2}}\ \ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x)
```

```
[Out] 1/8*2^(1/2)*(2+2^(1/2))^(1/2)*ln(1+x^2+x*(2+2^(1/2))^(1/2))+1/2/(2-2^(1/2))
^(1/2)*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*2^(1/2)-1/2/(2-2^(
1/2))^(1/2)*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))-1/8*2^(1/2)*(
2+2^(1/2))^(1/2)*ln(1+x^2-x*(2+2^(1/2))^(1/2))+1/2/(2-2^(1/2))^(1/2)*arctan
((2*x-(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*2^(1/2)-1/2/(2-2^(1/2))^(1/2)*a
rctan((2*x-(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-\sqrt{2}}{x^4-\sqrt{2}\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1), x)
```

mupad [B] time = 4.96, size = 121, normalized size = 0.76

$$-\mathrm{atan}\left(x\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\ 1\mathrm{i}}{32}}\ 2\mathrm{i}-\frac{\sqrt{2}\ \sqrt{8}\ x\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\ 1\mathrm{i}}{32}}}{2}\right)\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\ 1\mathrm{i}}{32}}\ 2\mathrm{i}-\mathrm{atan}\left(x\sqrt{\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\ 1\mathrm{i}}{32}}\ 2\mathrm{i}+\frac{\sqrt{2}\ \sqrt{8}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(1/2) - x^2)/(x^4 - 2^(1/2)*x^2 + 1),x)
```

```
[Out] - atan(x*(2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*2i - (2^(1/2)*8^(1/2)*x*(2^(1
/2)/16 - (8^(1/2)*1i)/32)^(1/2))/2)*(2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*2i
 - atan(x*(2^(1/2)/16 + (8^(1/2)*1i)/32)^(1/2)*2i + (2^(1/2)*8^(1/2)*x*(2^(
1/2)/16 + (8^(1/2)*1i)/32)^(1/2))/2)*(2^(1/2)/16 + (8^(1/2)*1i)/32)^(1/2)*2
i
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2**(1/2))/(1+x**4-x**2*2**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

3.103 $\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}\,x^2+x^4}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\right.}{2\sqrt{}}$$

```
[Out] -1/8*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2-2^
(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-1/2*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2))/(2-2^(1/2))^(1/2)+1/2*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2))/(2-2^(1/2))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\right.}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
+ ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
 - (Sqrt[1 - 1/Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/4 + (Sqrt[1 - 1
/Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2-\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2-\sqrt{2}}}$$
$$=\frac{\left(1-\sqrt{2}\right)\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}}+\frac{\left(-1+\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}}+\frac{1}{4}\sqrt{3+2\sqrt{2}}\int \frac{}{1-\sqrt{2}}$$
$$=-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right)-\frac{1}{2}\sqrt{}$$
$$=-\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)-$$

Mathematica [C] time = 0.04, size = 53, normalized size = 0.31

$$\frac{\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1-i}}\right)+\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

```
[Out] (Sqrt[1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[1 - I]] + Sqrt[1 + I]*ArcTan[(2^(1/4)*
x)/Sqrt[1 + I]])/2^(3/4)
```

fricas [C] time = 0.45, size = 97, normalized size = 0.56

$$\frac{1}{4}\sqrt{(i-1)\sqrt{2}}\log\left(x+\frac{1}{2}\sqrt{2}\sqrt{(i-1)\sqrt{2}}\right)-\frac{1}{4}\sqrt{(i-1)\sqrt{2}}\log\left(x-\frac{1}{2}\sqrt{2}\sqrt{(i-1)\sqrt{2}}\right)+\frac{1}{4}\sqrt{-(i+1)\sqrt{2}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((I - 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt((I - 1)*sqrt(2))) - 1/4*
sqrt((I - 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt((I - 1)*sqrt(2))) + 1/4*sqrt
(-(I + 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt(-(I + 1)*sqrt(2))) - 1/4*sqrt(-
(I + 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt(-(I + 1)*sqrt(2)))
```

giac [A] time = 0.33, size = 126, normalized size = 0.73

$$\frac{1}{4}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-2\sqrt{2}+4}\log\left(x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) + 1/4*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2)
+ 2)) + 1/8*sqrt(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/8*
sqrt(-2*sqrt(2) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

maple [A] time = 0.09, size = 199, normalized size = 1.16

$$\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt{2}\ \arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt{2}\ \arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\sqrt{2}\ \sqrt{2-\sqrt{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x)
```

```
[Out] 1/8*2^(1/2)*(2-2^(1/2))^(1/2)*ln(1+x^2+x*(2-2^(1/2))^(1/2))+1/2/(2+2^(1/2))
^(1/2)*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))+1/2/(2+2^(1/2))^(1
/2)*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*2^(1/2)-1/8*2^(1/2)*(
2-2^(1/2))^(1/2)*ln(1+x^2-x*(2-2^(1/2))^(1/2))+1/2/(2+2^(1/2))^(1/2)*arctan
((2*x-(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))+1/2/(2+2^(1/2))^(1/2)*arctan((2
*x-(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+\sqrt{2}}{x^4+\sqrt{2}\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1), x)
```

mupad [B] time = 4.95, size = 121, normalized size = 0.70

$$\mathrm{atan}\left(x\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\ 1\mathrm{i}}{32}}\ 2\mathrm{i}+\frac{\sqrt{2}\ \sqrt{8}\ x\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\ 1\mathrm{i}}{32}}}{2}\right)\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\ 1\mathrm{i}}{32}}\ 2\mathrm{i}+\mathrm{atan}\left(x\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\ 1\mathrm{i}}{32}}\ 2\mathrm{i}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(1/2) + x^2)/(2^(1/2)*x^2 + x^4 + 1),x)
```

```
[Out] atan(x*(- 2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*2i + (2^(1/2)*8^(1/2)*x*(- 2^
(1/2)/16 - (8^(1/2)*1i)/32)^(1/2))/2)*(- 2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2
)*2i + atan(x*((8^(1/2)*1i)/32 - 2^(1/2)/16)^(1/2)*2i - (2^(1/2)*8^(1/2)*x*
((8^(1/2)*1i)/32 - 2^(1/2)/16)^(1/2))/2)*((8^(1/2)*1i)/32 - 2^(1/2)/16)^(1/
2)*2i
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2**(1/2))/(1+x**4+x**2*2**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

3.104 $\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=160

$$-\frac{\left(1+\sqrt{2}\right)\log\left(-\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1+\sqrt{2}\right)\log\left(\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}-\frac{\left(1-\sqrt{2}\right)\mathrm{t}}{2}$$

```
[Out] -1/4*ln(1+x^2-x*(2-b)^(1/2))*(1+2^(1/2))/(2-b)^(1/2)+1/4*ln(1+x^2+x*(2-b)^(
1/2))*(1+2^(1/2))/(2-b)^(1/2)+1/2*arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1
-2^(1/2))/(2+b)^(1/2)-1/2*arctan((2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1-2^(1/2))
/(2+b)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{\left(1+\sqrt{2}\right)\log\left(-\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1+\sqrt{2}\right)\log\left(\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}-\frac{\left(1-\sqrt{2}\right)\mathrm{t}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - (
(1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - ((
1 + Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) + ((1 + Sqrt[2])
*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
```

```
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{2-b}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2-b}\,x+x^2}\,dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2-b}\,x+x^2}\,dx}{2\sqrt{2-b}}$$

$$= \frac{1}{4}\left(-1+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2-b}\,x+x^2}\,dx + \frac{1}{4}\left(-1+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2-b}\,x+x^2}\,dx - \frac{\left(1+\sqrt{2}\right.}{}$$

$$= -\frac{\left(1+\sqrt{2}\right)\log\left(1-\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}} + \frac{\left(1+\sqrt{2}\right)\log\left(1+\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}} + \frac{1}{2}\left(1-\sqrt{2}\right)\mathrm{Su}$$

$$= \frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{\left(1+\sqrt{2}\right)\log\left(1-\sqrt{2-b}\,x+\right.}{4\sqrt{2-b}}$$

Mathematica [A] time = 0.09, size = 137, normalized size = 0.86

$$\frac{\frac{\left(-\sqrt{b^2-4}+b+2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}} - \frac{\left(\sqrt{b^2-4}+b+2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2*Sqrt[2] + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^
2]]])/Sqrt[b - Sqrt[-4 + b^2]] - ((2*Sqrt[2] + b + Sqrt[-4 + b^2])*ArcTan[(
Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sq
rt[-4 + b^2])
```

fricas [B] time = 0.49, size = 451, normalized size = 2.82

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b+4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\,\log\left(\frac{1}{2}\left(2\,b+3\sqrt{2}\right)x+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-\frac{b^3+\sqrt{2}\,b^2-4\,b-4\sqrt{2}}{\sqrt{b^2-4}}-4\right)\sqrt{-\frac{3\,b\,-}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(
2*b + 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 - (b^3 + sqrt(2)*b^2 - 4*b - 4*sqrt
(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4)))
 + 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2
*(2*b + 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 - (b^3 + sqrt(2)*b^2 - 4*b - 4*sq
rt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4)
)) - 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(1
/2*(2*b + 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 + (b^3 + sqrt(2)*b^2 - 4*b - 4*
sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 -
4))) + 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log
(1/2*(2*b + 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 + (b^3 + sqrt(2)*b^2 - 4*b -
4*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2
- 4)))
```

giac [B] time = 0.32, size = 1501, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b + 2)*b^4 + sqrt(2)*sqrt(b - 2)*b^4 - sqrt(2)*sqrt(b^2 -
 4)*sqrt(b + 2)*b^3 - sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b^3 - sqrt(2)*sqrt(
b + 2)*sqrt(b - 2)*b^3 - 3*sqrt(2)*b^4 + 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2
)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b^3 - sqrt(2)*sqrt(b + 2)*b^3 - s
qrt(2)*sqrt(b - 2)*b^3 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + sqrt(2)*sq
rt(b^2 - 4)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b^2 + 3*sqrt(
2)*b^3 - 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2)*b - sqrt(2)*sqrt(b
^2 - 4)*b^2 - 10*sqrt(2)*sqrt(b + 2)*b^2 - 2*sqrt(b^2 - 4)*sqrt(b + 2)*b^2
- 6*sqrt(2)*sqrt(b - 2)*b^2 - 2*sqrt(b^2 - 4)*sqrt(b - 2)*b^2 - 2*sqrt(b +
2)*sqrt(b - 2)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b + 4*sqrt(2)*sqrt
(b^2 - 4)*sqrt(b - 2)*b + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b + 24*sqrt(2)*
b^2 + 2*sqrt(b^2 - 4)*b^2 - 12*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2
) - 4*sqrt(2)*sqrt(b^2 - 4)*b + 6*sqrt(2)*sqrt(b + 2)*b + 4*sqrt(b^2 - 4)*s
qrt(b + 2)*b + 2*sqrt(2)*sqrt(b - 2)*b + 4*sqrt(b^2 - 4)*sqrt(b - 2)*b + 4*
sqrt(b + 2)*sqrt(b - 2)*b + 6*b^2 + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2) + 4
*sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2) + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2) - 6*
sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2) - 12*sqrt(2)*b - 4*sqrt(b^2 - 4)*b -
2*sqrt(b + 2)*b - 2*sqrt(b - 2)*b - 4*sqrt(2)*sqrt(b^2 - 4) + 20*sqrt(2)*sq
rt(b + 2) + 8*sqrt(b^2 - 4)*sqrt(b + 2) + 4*sqrt(2)*sqrt(b - 2) + 8*sqrt(b^
2 - 4)*sqrt(b - 2) + 8*sqrt(b + 2)*sqrt(b - 2) - 48*sqrt(2) - 8*sqrt(b^2 -
4) + 4*sqrt(b + 2) - 4*sqrt(b - 2) - 24)*arctan(x/sqrt(1/2*b + 1/2*sqrt(b^2
 - 4)))/(b^4 - 2*b^3 - 7*b^2 + 8*b + 12) + 1/4*(sqrt(2)*sqrt(b + 2)*b^4 - s
qrt(2)*sqrt(b - 2)*b^4 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b^3 - sqrt(2)*sq
rt(b^2 - 4)*sqrt(b - 2)*b^3 - sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b^3 + 3*sqrt(
2)*b^4 - 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt
(b^2 - 4)*b^3 - sqrt(2)*sqrt(b + 2)*b^3 + sqrt(2)*sqrt(b - 2)*b^3 - sqrt(2)
*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b^2 + sq
rt(2)*sqrt(b + 2)*sqrt(b - 2)*b^2 - 3*sqrt(2)*b^3 + 3*sqrt(2)*sqrt(b^2 - 4)
*sqrt(b + 2)*sqrt(b - 2)*b - sqrt(2)*sqrt(b^2 - 4)*b^2 - 10*sqrt(2)*sqrt(b
+ 2)*b^2 + 2*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + 6*sqrt(2)*sqrt(b - 2)*b^2 - 2*
sqrt(b^2 - 4)*sqrt(b - 2)*b^2 - 2*sqrt(b + 2)*sqrt(b - 2)*b^2 - 4*sqrt(2)*s
qrt(b^2 - 4)*sqrt(b + 2)*b + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b + 4*sqrt
(2)*sqrt(b + 2)*sqrt(b - 2)*b - 24*sqrt(2)*b^2 + 2*sqrt(b^2 - 4)*b^2 + 12*s
qrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2) - 4*sqrt(2)*sqrt(b^2 - 4)*b +
6*sqrt(2)*sqrt(b + 2)*b - 4*sqrt(b^2 - 4)*sqrt(b + 2)*b - 2*sqrt(2)*sqrt(b
- 2)*b + 4*sqrt(b^2 - 4)*sqrt(b - 2)*b + 4*sqrt(b + 2)*sqrt(b - 2)*b - 6*b^
2 - 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2) + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b -
2) + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2) + 6*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b
 - 2) + 12*sqrt(2)*b - 4*sqrt(b^2 - 4)*b - 2*sqrt(b + 2)*b + 2*sqrt(b - 2)*
b - 4*sqrt(2)*sqrt(b^2 - 4) + 20*sqrt(2)*sqrt(b + 2) - 8*sqrt(b^2 - 4)*sqrt
(b + 2) - 4*sqrt(2)*sqrt(b - 2) + 8*sqrt(b^2 - 4)*sqrt(b - 2) + 8*sqrt(b +
2)*sqrt(b - 2) + 48*sqrt(2) - 8*sqrt(b^2 - 4) + 4*sqrt(b + 2) + 4*sqrt(b -
2) + 24)*arctan(x/sqrt(1/2*b - 1/2*sqrt(b^2 - 4)))/(b^4 - 2*b^3 - 7*b^2 + 8
*b + 12)
```

maple [B] time = 0.02, size = 285, normalized size = 1.78

$$\frac{b \arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\,\sqrt{2b-2\sqrt{(b-2)(b+2)}}}-\frac{b \arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\,\sqrt{2b+2\sqrt{(b-2)(b+2)}}}-\frac{\arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2^(1/2))/(x^4+b*x^2+1),x)
```

```
[Out] -1/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*arctan(2/(2*b+2*((b-2)*(b+2))^(1/2))^(
1/2)*x)-1/((b-2)*(b+2))^(1/2)/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*b*arctan(2/
(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*x)-2/((b-2)*(b+2))^(1/2)/(2*b+2*((b-2)*(b
+2))^(1/2))^(1/2)*arctan(2/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*x)*2^(1/2)-1/(
2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*arctan(2/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)
*x)+1/((b-2)*(b+2))^(1/2)/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*b*arctan(2/(2*b
-2*((b-2)*(b+2))^(1/2))^(1/2)*x)+2/((b-2)*(b+2))^(1/2)/(2*b-2*((b-2)*(b+2))
^(1/2))^(1/2)*arctan(2/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*x)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - \sqrt{2}}{x^4 + bx^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - sqrt(2))/(x^4 + b*x^2 + 1), x)
```

mupad [B] time = 1.07, size = 1227, normalized size = 7.67

$$-\mathrm{atan}\left(\frac{x\sqrt{\frac{12\,b+16\,\sqrt{2}-4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}}\,32\mathrm{i}-b\,x\left(\frac{12\,b+16\,\sqrt{2}-4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}\right)^{3/2}256\mathrm{i}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(1/2) - x^2)/(b*x^2 + x^4 + 1),x)
```

```
[Out] atan((x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b
^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*(-(4*2^(1/2)*b^2 -
16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64
*b^2 + 128))^(3/2)*256i + b^2*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^
3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i -
b^4*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6
 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*(-(4*2^(1/2)*b^2 - 1
6*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(3/2)*128i - b^5*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3
 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3/2)*16i +
2^(1/2)*b*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4
 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - 2^(1/2)*b^3*x*(-(4*
2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(2^(1/2)*b^3 - 4*2^(1/2)*b + 2^(1/2)*(
48*b^2 - 12*b^4 + b^6 - 64)^(1/2) + 2*b^2 - 8))*(-(4*2^(1/2)*b^2 - 16*2^(1/
2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 1
28))^(1/2)*2i - atan((x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b
^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*((12
*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(3/2)*256i + b^2*x*((12*b + 16*2^(1/2) - 4*2^(1/
2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128)
)^(1/2)*8i - b^4*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 -
12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*((12*b +
 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(
8*b^4 - 64*b^2 + 128))^(3/2)*128i - b^5*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b
^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3
/2)*16i + 2^(1/2)*b*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2
 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - 2^(1/2)*b^
```

```
3*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 -
64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(4*2^(1/2)*b - 2^(1/2)*b^3 + 2
^(1/2)*(48*b^2 - 12*b^4 + b^6 - 64)^(1/2) - 2*b^2 + 8))*((12*b + 16*2^(1/2)
 - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(1/2)*2i
```

sympy [B] time = 2.86, size = 1469, normalized size = 9.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] -RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 + 16*sqrt(2)*b**
2 - 48*b - 64*sqrt(2)) + 2*b**2 + 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**3*
(64*b**12/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*
b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729)
 + 672*sqrt(2)*b**11/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b
**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*
b**2 + 729) + 5760*b**10/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(
2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3
402*b**2 + 729) + 12064*sqrt(2)*b**9/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8
+ 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt
(2)*b**3 - 3402*b**2 + 729) + 17744*b**8/(8*b**10 + 88*sqrt(2)*b**9 + 828*b
**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*
sqrt(2)*b**3 - 3402*b**2 + 729) - 27480*sqrt(2)*b**7/(8*b**10 + 88*sqrt(2)*
b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*
b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 154608*b**6/(8*b**10 + 88*sqr
t(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 +
2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 141376*sqrt(2)*b**5/(8*b
**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sq
rt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 69072*b**4/
(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 531
0*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + 61704*s
qrt(2)*b**3/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 647
0*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 72
9) + 78192*b**2/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 +
 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2
+ 729) - 2592*sqrt(2)*b/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2
)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 34
02*b**2 + 729) - 15552/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)
*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 340
2*b**2 + 729)) + _t*(16*b**7/(4*b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqr
t(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) + 116*sqrt(2)*b**6/(4*b**6 + 28*s
qrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) +
 668*b**5/(4*b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**
2 - 27*sqrt(2)*b - 81) + 942*sqrt(2)*b**4/(4*b**6 + 28*sqrt(2)*b**5 + 152*b
**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) + 1226*b**3/(4*b**6
+ 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b -
 81) + 144*sqrt(2)*b**2/(4*b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*
b**3 + 189*b**2 - 27*sqrt(2)*b - 81) - 378*b/(4*b**6 + 28*sqrt(2)*b**5 + 15
2*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) - 108*sqrt(2)/(4*
b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2
)*b - 81)) + x)))
```

3.105 $\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=160

$$\frac{\left(1-\sqrt{2}\right)\log\left(-\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1-\sqrt{2}\right)\log\left(\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}+\frac{\left(1+\sqrt{2}\right)}{2}$$

```
[Out] 1/4*ln(1+x^2-x*(2-b)^(1/2))*(1-2^(1/2))/(2-b)^(1/2)-1/4*ln(1+x^2+x*(2-b)^(1
/2))*(1-2^(1/2))/(2-b)^(1/2)-1/2*arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1+
2^(1/2))/(2+b)^(1/2)+1/2*arctan((2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1+2^(1/2))/
(2+b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1169, 634, 618, 204, 628}

$$\frac{\left(1-\sqrt{2}\right)\log\left(-\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1-\sqrt{2}\right)\log\left(\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}+\frac{\left(1+\sqrt{2}\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) +
((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) + (
(1 - Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) - ((1 - Sqrt[2]
)*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
```

```
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{2-b}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-b}\,x+x^2}\,dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-b}\,x+x^2}\,dx}{2\sqrt{2-b}}$$

$$= \frac{1}{4}\left(1+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2-b}\,x+x^2}\,dx + \frac{1}{4}\left(1+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2-b}\,x+x^2}\,dx + \frac{\left(1-\sqrt{2}\right)\int \frac{-}{1-}}{4\sqrt{2}}$$

$$= \frac{\left(1-\sqrt{2}\right)\log\left(1-\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}} - \frac{\left(1-\sqrt{2}\right)\log\left(1+\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}} + \frac{1}{2}\left(-1-\sqrt{2}\right)\text{Subs}$$

$$= -\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} + \frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} + \frac{\left(1-\sqrt{2}\right)\log\left(1-\sqrt{2-b}\,x+x\right.}{4\sqrt{2-b}}$$

Mathematica [A] time = 0.06, size = 136, normalized size = 0.85

$$\frac{\frac{\left(\sqrt{b^2-4}-b+2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}} + \frac{\left(\sqrt{b^2-4}+b-2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2*Sqrt[2] - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^
2]]])/Sqrt[b - Sqrt[-4 + b^2]] + ((-2*Sqrt[2] + b + Sqrt[-4 + b^2])*ArcTan[
(Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*S
qrt[-4 + b^2])
```

fricas [B] time = 0.49, size = 455, normalized size = 2.84

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3b-4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(\frac{1}{2}\left(2b-3\sqrt{2}\right)x+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-\frac{b^3-\sqrt{2}b^2-4b+4\sqrt{2}}{\sqrt{b^2-4}}-4\right)\sqrt{-\frac{3b-4\sqrt{}}{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(2
*b - 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 - (b^3 - sqrt(2)*b^2 - 4*b + 4*sqrt(
2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4)))
- 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*
(2*b - 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 - (b^3 - sqrt(2)*b^2 - 4*b + 4*sqr
t(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))
) + 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(1/
2*(2*b - 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 + (b^3 - sqrt(2)*b^2 - 4*b + 4*s
qrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4
))) - 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(
1/2*(2*b - 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 + (b^3 - sqrt(2)*b^2 - 4*b + 4
*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 -
 4)))
```

giac [B] time = 0.35, size = 1501, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b + 2)*b^4 + sqrt(2)*sqrt(b - 2)*b^4 - sqrt(2)*sqrt(b^2 -
 4)*sqrt(b + 2)*b^3 - sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b^3 - sqrt(2)*sqrt(
b + 2)*sqrt(b - 2)*b^3 - 3*sqrt(2)*b^4 + 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2
)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b^3 - sqrt(2)*sqrt(b + 2)*b^3 - s
qrt(2)*sqrt(b - 2)*b^3 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + sqrt(2)*sq
rt(b^2 - 4)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b^2 + 3*sqrt(
2)*b^3 - 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2)*b - sqrt(2)*sqrt(b
^2 - 4)*b^2 - 10*sqrt(2)*sqrt(b + 2)*b^2 + 2*sqrt(b^2 - 4)*sqrt(b + 2)*b^2
- 6*sqrt(2)*sqrt(b - 2)*b^2 + 2*sqrt(b^2 - 4)*sqrt(b - 2)*b^2 + 2*sqrt(b +
2)*sqrt(b - 2)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b + 4*sqrt(2)*sqrt
(b^2 - 4)*sqrt(b - 2)*b + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b + 24*sqrt(2)*
b^2 - 2*sqrt(b^2 - 4)*b^2 - 12*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2
) - 4*sqrt(2)*sqrt(b^2 - 4)*b + 6*sqrt(2)*sqrt(b + 2)*b - 4*sqrt(b^2 - 4)*s
qrt(b + 2)*b + 2*sqrt(2)*sqrt(b - 2)*b - 4*sqrt(b^2 - 4)*sqrt(b - 2)*b - 4*
sqrt(b + 2)*sqrt(b - 2)*b - 6*b^2 + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2) + 4
*sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2) + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2) + 6*
sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2) - 12*sqrt(2)*b + 4*sqrt(b^2 - 4)*b +
2*sqrt(b + 2)*b + 2*sqrt(b - 2)*b - 4*sqrt(2)*sqrt(b^2 - 4) + 20*sqrt(2)*sq
rt(b + 2) - 8*sqrt(b^2 - 4)*sqrt(b + 2) + 4*sqrt(2)*sqrt(b - 2) - 8*sqrt(b^
2 - 4)*sqrt(b - 2) - 8*sqrt(b + 2)*sqrt(b - 2) - 48*sqrt(2) + 8*sqrt(b^2 -
4) - 4*sqrt(b + 2) + 4*sqrt(b - 2) + 24)*arctan(x/sqrt(1/2*b + 1/2*sqrt(b^2
 - 4)))/(b^4 - 2*b^3 - 7*b^2 + 8*b + 12) + 1/4*(sqrt(2)*sqrt(b + 2)*b^4 - s
qrt(2)*sqrt(b - 2)*b^4 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b^3 - sqrt(2)*sq
rt(b^2 - 4)*sqrt(b - 2)*b^3 - sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b^3 + 3*sqrt(
2)*b^4 - 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt
(b^2 - 4)*b^3 - sqrt(2)*sqrt(b + 2)*b^3 + sqrt(2)*sqrt(b - 2)*b^3 - sqrt(2)
*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b^2 + sq
rt(2)*sqrt(b + 2)*sqrt(b - 2)*b^2 - 3*sqrt(2)*b^3 + 3*sqrt(2)*sqrt(b^2 - 4)
*sqrt(b + 2)*sqrt(b - 2)*b - sqrt(2)*sqrt(b^2 - 4)*b^2 - 10*sqrt(2)*sqrt(b
+ 2)*b^2 - 2*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + 6*sqrt(2)*sqrt(b - 2)*b^2 + 2*
sqrt(b^2 - 4)*sqrt(b - 2)*b^2 + 2*sqrt(b + 2)*sqrt(b - 2)*b^2 - 4*sqrt(2)*s
qrt(b^2 - 4)*sqrt(b + 2)*b + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b + 4*sqrt
(2)*sqrt(b + 2)*sqrt(b - 2)*b - 24*sqrt(2)*b^2 - 2*sqrt(b^2 - 4)*b^2 + 12*s
qrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2) - 4*sqrt(2)*sqrt(b^2 - 4)*b +
6*sqrt(2)*sqrt(b + 2)*b + 4*sqrt(b^2 - 4)*sqrt(b + 2)*b - 2*sqrt(2)*sqrt(b
- 2)*b - 4*sqrt(b^2 - 4)*sqrt(b - 2)*b - 4*sqrt(b + 2)*sqrt(b - 2)*b + 6*b^
2 - 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2) + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b -
2) + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2) - 6*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b
 - 2) + 12*sqrt(2)*b + 4*sqrt(b^2 - 4)*b + 2*sqrt(b + 2)*b - 2*sqrt(b - 2)*
b - 4*sqrt(2)*sqrt(b^2 - 4) + 20*sqrt(2)*sqrt(b + 2) + 8*sqrt(b^2 - 4)*sqrt
(b + 2) - 4*sqrt(2)*sqrt(b - 2) - 8*sqrt(b^2 - 4)*sqrt(b - 2) - 8*sqrt(b +
2)*sqrt(b - 2) + 48*sqrt(2) + 8*sqrt(b^2 - 4) - 4*sqrt(b + 2) - 4*sqrt(b -
2) - 24)*arctan(x/sqrt(1/2*b - 1/2*sqrt(b^2 - 4)))/(b^4 - 2*b^3 - 7*b^2 + 8
*b + 12)
```

maple [B] time = 0.02, size = 283, normalized size = 1.77

$$-\frac{b \arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\,\sqrt{2b-2\sqrt{(b-2)(b+2)}}}+\frac{b \arctan\left(\frac{2x}{\sqrt{2b+2\sqrt{(b-2)(b+2)}}}\right)}{\sqrt{(b-2)(b+2)}\,\sqrt{2b+2\sqrt{(b-2)(b+2)}}}+\frac{\arctan\left(\frac{2x}{\sqrt{2b-2\sqrt{(b-2)(}}}\right.}{\sqrt{2b-2\sqrt{(b-2)(b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2^(1/2))/(x^4+b*x^2+1),x)
```

```
[Out] 1/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*arctan(2/(2*b+2*((b-2)*(b+2))^(1/2))^(1
/2)*x)+1/((b-2)*(b+2))^(1/2)/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*b*arctan(2/(
2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*x)-2/((b-2)*(b+2))^(1/2)/(2*b+2*((b-2)*(b+
2))^(1/2))^(1/2)*2^(1/2)*arctan(2/(2*b+2*((b-2)*(b+2))^(1/2))^(1/2)*x)+1/(2
*b-2*((b-2)*(b+2))^(1/2))^(1/2)*arctan(2/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*
x)-1/((b-2)*(b+2))^(1/2)/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*b*arctan(2/(2*b-
2*((b-2)*(b+2))^(1/2))^(1/2)*x)+2/((b-2)*(b+2))^(1/2)/(2*b-2*((b-2)*(b+2))^
(1/2))^(1/2)*2^(1/2)*arctan(2/(2*b-2*((b-2)*(b+2))^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+\sqrt{2}}{x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + b*x^2 + 1), x)
```

mupad [B] time = 5.25, size = 1227, normalized size = 7.67

$$-\mathrm{atan}\left(\frac{x\sqrt{\frac{12\,b-16\,\sqrt{2}+4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}}\,32\mathrm{i}-b\,x\left(\frac{12\,b-16\,\sqrt{2}+4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}\right)^{3/2}256\mathrm{i}+b^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(1/2) + x^2)/(b*x^2 + x^4 + 1),x)
```

```
[Out] atan((x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b
^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*(-(16*2^(1/2) - 12*
b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64
*b^2 + 128))^(3/2)*256i + b^2*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^
3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i -
b^4*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6
 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*(-(16*2^(1/2) - 12*b
 - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(3/2)*128i - b^5*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3
 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3/2)*16i -
2^(1/2)*b*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4
 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i + 2^(1/2)*b^3*x*(-(16
*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(2^(1/2)*b^3 - 4*2^(1/2)*b + 2^(1/2)*(
48*b^2 - 12*b^4 + b^6 - 64)^(1/2) - 2*b^2 + 8))*(-(16*2^(1/2) - 12*b - 4*2^
(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 1
28))^(1/2)*2i - atan((x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b
^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*((12
*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(3/2)*256i + b^2*x*((12*b - 16*2^(1/2) + 4*2^(1/
2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128)
)^(1/2)*8i - b^4*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 -
12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*((12*b -
 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(
8*b^4 - 64*b^2 + 128))^(3/2)*128i - b^5*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b
^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3
/2)*16i - 2^(1/2)*b*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2
 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i + 2^(1/2)*b^
```

```
3*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 -
64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(4*2^(1/2)*b - 2^(1/2)*b^3 + 2
^(1/2)*(48*b^2 - 12*b^4 + b^6 - 64)^(1/2) + 2*b^2 - 8))*((12*b - 16*2^(1/2)
 + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(1/2)*2i
```

sympy [B] time = 2.73, size = 1467, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 - 16*sqrt(2)*b**2
 - 48*b + 64*sqrt(2)) + 2*b**2 - 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**3*(
64*b**12/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b
**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729)
- 672*sqrt(2)*b**11/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b*
*7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b
**2 + 729) + 5760*b**10/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2
)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 34
02*b**2 + 729) - 12064*sqrt(2)*b**9/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 -
 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(
2)*b**3 - 3402*b**2 + 729) + 17744*b**8/(8*b**10 - 88*sqrt(2)*b**9 + 828*b*
*8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*s
qrt(2)*b**3 - 3402*b**2 + 729) + 27480*sqrt(2)*b**7/(8*b**10 - 88*sqrt(2)*b
**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b
**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 154608*b**6/(8*b**10 - 88*sqrt
(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2
781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + 141376*sqrt(2)*b**5/(8*b*
*10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqr
t(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 69072*b**4/(
8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310
*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 61704*sq
rt(2)*b**3/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470
*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729
) + 78192*b**2/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 +
6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 +
 729) + 2592*sqrt(2)*b/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)
*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 340
2*b**2 + 729) - 15552/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*
b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402
*b**2 + 729)) + _t*(16*b**7/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt
(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) - 116*sqrt(2)*b**6/(4*b**6 - 28*sq
rt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) +
668*b**5/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2
 + 27*sqrt(2)*b - 81) - 942*sqrt(2)*b**4/(4*b**6 - 28*sqrt(2)*b**5 + 152*b*
*4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) + 1226*b**3/(4*b**6 -
 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b -
81) - 144*sqrt(2)*b**2/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b
**3 + 189*b**2 + 27*sqrt(2)*b - 81) - 378*b/(4*b**6 - 28*sqrt(2)*b**5 + 152
*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) + 108*sqrt(2)/(4*b
**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)
*b - 81)) + x)))
```

3.106 $\int \frac{2a-x^2}{a^2-ax^2+x^4} \, dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2\sqrt{a}}$$

```
[Out] 1/2*arctan(-3^(1/2)+2*x/a^(1/2))/a^(1/2)+1/2*arctan(3^(1/2)+2*x/a^(1/2))/a^
(1/2)-1/4*ln(a+x^2-x*3^(1/2)*a^(1/2))*3^(1/2)/a^(1/2)+1/4*ln(a+x^2+x*3^(1/2
)*a^(1/2))*3^(1/2)/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - (2*x)/Sqrt[a]]/(2*Sqrt[a]) + ArcTan[Sqrt[3] + (2*x)/Sqrt[
a]]/(2*Sqrt[a]) - (Sqrt[3]*Log[a - Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a]) +
(Sqrt[3]*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = \frac{\int \frac{2\sqrt{3}a^{3/2}-3ax}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}} + \frac{\int \frac{2\sqrt{3}a^{3/2}+3ax}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}}$$

$$= \frac{1}{4}\int \frac{1}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx - \frac{\sqrt{3}\int \frac{-\sqrt{3}\sqrt{a}+2x}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{a}} + \frac{\sqrt{3}\int$$

$$= -\frac{\sqrt{3}\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}} + \frac{\sqrt{3}\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\right.}{2\sqrt{3}\sqrt{a}}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{3}\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}} + \frac{\sqrt{3}\log\left(a+\right.}{4}$$

Mathematica [C] time = 0.17, size = 115, normalized size = 1.01

$$\frac{\sqrt[4]{-1}\left(\sqrt{\sqrt{3}-i}\left(\sqrt{3}-3i\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt{a}}\right)-\sqrt{\sqrt{3}+i}\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt{a}}\right)\right)}{2\sqrt{6}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[-
I + Sqrt[3]]*Sqrt[a])]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1 +
 I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])]))/(2*Sqrt[6]*Sqrt[a])
```

fricas [B] time = 0.43, size = 517, normalized size = 4.54

$$\frac{1}{24}\left(\sqrt{3}a\sqrt{\frac{1}{a^2}}+2\sqrt{3}\right)\sqrt{-4a\sqrt{\frac{1}{a^2}}+8}\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\log\left(6a^2\sqrt{\frac{1}{a^2}}+6x^2+\left(\sqrt{3}a^2\sqrt{\frac{1}{a^2}}x+2\sqrt{3}ax\right)\sqrt{-4a\sqrt{\frac{1}{a^2}}+8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(3)*a*sqrt(a^(-2)) + 2*sqrt(3))*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-
2))^(1/4)*log(6*a^2*sqrt(a^(-2)) + 6*x^2 + (sqrt(3)*a^2*sqrt(a^(-2))*x + 2*
sqrt(3)*a*x)*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4)) - 1/24*(sqrt(3)*a*
sqrt(a^(-2)) + 2*sqrt(3))*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4)*log(6*
a^2*sqrt(a^(-2)) + 6*x^2 - (sqrt(3)*a^2*sqrt(a^(-2))*x + 2*sqrt(3)*a*x)*sqr
t(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4)) - 1/2*sqrt(-4*a*sqrt(a^(-2)) + 8)*
(a^(-2))^(1/4)*arctan(1/18*(sqrt(6)*a^2*sqrt(a^(-2)) + 2*sqrt(6)*a)*sqrt(6*
a^2*sqrt(a^(-2)) + 6*x^2 + (sqrt(3)*a^2*sqrt(a^(-2))*x + 2*sqrt(3)*a*x)*sqr
t(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4))*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2
))^(3/4) - 1/3*(a^2*sqrt(a^(-2))*x + 2*a*x)*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^
(-2))^(3/4) - 1/3*sqrt(3)*a*sqrt(a^(-2)) - 2/3*sqrt(3)) - 1/2*sqrt(-4*a*sqr
t(a^(-2)) + 8)*(a^(-2))^(1/4)*arctan(1/18*(sqrt(6)*a^2*sqrt(a^(-2)) + 2*sqr
t(6)*a)*sqrt(6*a^2*sqrt(a^(-2)) + 6*x^2 - (sqrt(3)*a^2*sqrt(a^(-2))*x + 2*s
qrt(3)*a*x)*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4))*sqrt(-4*a*sqrt(a^(-
```

```
2)) + 8)*(a^(-2))^(3/4) - 1/3*(a^2*sqrt(a^(-2))*x + 2*a*x)*sqrt(-4*a*sqrt(a
^(-2)) + 8)*(a^(-2))^(3/4) + 1/3*sqrt(3)*a*sqrt(a^(-2)) + 2/3*sqrt(3))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-16,[2,0]%%%}+%%%{-4,[0,1]%%%},0,%%%{64,[4,0]
%%%}+%%%{8,[2,2]%%%}+%%%{16,[2,1]%%%}+%%%{6,[0,2]%%%},0,%%%{-64,[4,2]%%%}+%
%%{-128,[4,1]%%%}+%%%{48,[2,3]%%%}+%%%{16,[2,2]%%%}+%%%{-4,[0,3]%%%},0,%%%{
16,[4,4]%%%}+%%%{-64,[4,3]%%%}+%%%{64,[4,2]%%%}+%%%{8,[2,4]%%%}+%%%{-16,[2,
3]%%%}+%%%{1,[0,4]%%%}] at parameters values [16,-63]Warning, choosing root
 of [1,0,%%%{-16,[2,0]%%%}+%%%{-4,[0,1]%%%},0,%%%{64,[4,0]%%%}+%%%{8,[2,2]%
%%}+%%%{16,[2,1]%%%}+%%%{6,[0,2]%%%},0,%%%{-64,[4,2]%%%}+%%%{-128,[4,1]%%%}
+%%%{48,[2,3]%%%}+%%%{16,[2,2]%%%}+%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%{
-64,[4,3]%%%}+%%%{64,[4,2]%%%}+%%%{8,[2,4]%%%}+%%%{-16,[2,3]%%%}+%%%{1,[0,4
]%%%}] at parameters values [39,13]-((-32*a^5-40*a^4*abs(a)+8*sqrt(3)*a^4*s
qrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*im(sign(cos(acos(a/2/abs(a))/2)))^3-1/1
2*(-864*sqrt(3)*a^5+864*a^4*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*im(sign(co
s(acos(a/2/abs(a))/2)))^2*im(sign(sin(acos(a/2/abs(a))/2)))-1/24*(-2880*sqr
t(3)*a^5+1728*a^4*sqrt(5*a^2+4*a*abs(a))-2304*sqrt(3)*a^4*abs(a))/sqrt(abs(
a))*im(sign(cos(acos(a/2/abs(a))/2)))^2*re(sign(cos(acos(a/2/abs(a))/2)))-(
-72*a^4*abs(a)+24*sqrt(3)*a^4*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*im(sign(
cos(acos(a/2/abs(a))/2)))^2*re(sign(sin(acos(a/2/abs(a))/2)))-(-72*a^4*abs(
a)+24*sqrt(3)*a^4*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*im(sign(cos(acos(a/2
/abs(a))/2)))*im(sign(sin(acos(a/2/abs(a))/2)))^2-(-144*a^4*abs(a)+48*sqrt(
3)*a^4*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*im(sign(cos(acos(a/2/abs(a))/2)
))*im(sign(sin(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))+1/24
*(-3456*sqrt(3)*a^5+3456*a^4*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*im(sign(c
os(acos(a/2/abs(a))/2)))*im(sign(sin(acos(a/2/abs(a))/2)))*re(sign(sin(acos
(a/2/abs(a))/2)))-(-96*a^5-120*a^4*abs(a)+24*sqrt(3)*a^4*sqrt(5*a^2+4*a*abs
(a)))/sqrt(abs(a))*im(sign(cos(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/a
bs(a))/2)))^2+1/24*(-3456*sqrt(3)*a^5+3456*a^4*sqrt(5*a^2+4*a*abs(a)))/sqrt
(abs(a))*im(sign(cos(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2))
)*re(sign(sin(acos(a/2/abs(a))/2)))+(-72*a^4*abs(a)+24*sqrt(3)*a^4*sqrt(5*a
^2-4*a*abs(a)))/sqrt(abs(a))*im(sign(cos(acos(a/2/abs(a))/2)))*re(sign(sin(
acos(a/2/abs(a))/2)))^2+(64*sqrt(3)*a^5-128*a^5-64*a^4*abs(a))/sqrt(abs(a))
*im(sign(cos(acos(a/2/abs(a))/2)))+1/8*(-320*sqrt(3)*a^5+192*a^4*sqrt(5*a^2
-4*a*abs(a))+256*sqrt(3)*a^4*abs(a))/sqrt(abs(a))*im(sign(sin(acos(a/2/abs(
a))/2)))^3+1/12*(-864*sqrt(3)*a^5+864*a^4*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(
a))*im(sign(sin(acos(a/2/abs(a))/2)))^2*re(sign(cos(acos(a/2/abs(a))/2)))+(
96*a^5-120*a^4*abs(a)+24*sqrt(3)*a^4*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*i
m(sign(sin(acos(a/2/abs(a))/2)))^2*re(sign(sin(acos(a/2/abs(a))/2)))+1/12*(
-864*sqrt(3)*a^5+864*a^4*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*im(sign(sin(a
cos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))^2+(-144*a^4*abs(a)+4
8*sqrt(3)*a^4*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*im(sign(sin(acos(a/2/abs
(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/2/abs(a))/2)
))-1/24*(-2880*sqrt(3)*a^5+1728*a^4*sqrt(5*a^2-4*a*abs(a))+2304*sqrt(3)*a^4
*abs(a))/sqrt(abs(a))*im(sign(sin(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/
2/abs(a))/2)))^2+(-128*sqrt(3)*a^5+384*abs(a)*a^4+256*sqrt(3)*a^4*abs(a))*1
/2/sqrt(abs(a))*im(sign(sin(acos(a/2/abs(a))/2)))+1/8*(-320*sqrt(3)*a^5+192
*a^4*sqrt(5*a^2+4*a*abs(a))-256*sqrt(3)*a^4*abs(a))/sqrt(abs(a))*re(sign(co
s(acos(a/2/abs(a))/2)))^3+(-72*a^4*abs(a)+24*sqrt(3)*a^4*sqrt(5*a^2+4*a*abs
(a)))/sqrt(abs(a))*re(sign(cos(acos(a/2/abs(a))/2)))^2*re(sign(sin(acos(a/2
/abs(a))/2)))-1/12*(-864*sqrt(3)*a^5+864*a^4*sqrt(5*a^2-4*a*abs(a)))/sqrt(a
bs(a))*re(sign(cos(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/2/abs(a))/2)))^
```

```
2+(128*sqrt(3)*a^5+384*abs(a)*a^4+256*sqrt(3)*a^4*abs(a))*1/2/sqrt(abs(a))*
re(sign(cos(acos(a/2/abs(a))/2)))-(32*a^5-40*a^4*abs(a)+8*sqrt(3)*a^4*sqrt(
5*a^2-4*a*abs(a)))/sqrt(abs(a))*re(sign(sin(acos(a/2/abs(a))/2)))^3+(128*a^
3*sqrt(abs(a))*abs(a)+64*sqrt(3)*a^4*sqrt(abs(a))-64*a^4*sqrt(abs(a)))*re(s
ign(sin(acos(a/2/abs(a))/2))))/(256*a^3*sqrt(2*a^2+a*abs(a))*sqrt(3)*abs(a)
-256*a^3*sqrt(2*a^2-a*abs(a))*sqrt(3)*abs(a))*ln(x^2-2*sqrt((1+a*1/2/abs(a)
)/2)*sqrt(abs(a))*sign(cos(acos(a*1/2/abs(a))/2))*x+sqrt(abs(a))*sqrt(abs(a
)))-(1/8*(-320*sqrt(3)*a^5+192*a^4*sqrt(5*a^2+4*a*abs(a))-256*sqrt(3)*a^4*a
bs(a))/sqrt(abs(a))*im(sign(cos(acos(a/2/abs(a))/2)))^3+(-72*a^4*abs(a)+24*
sqrt(3)*a^4*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*im(sign(cos(acos(a/2/abs(a
))/2)))^2*im(sign(sin(acos(a/2/abs(a))/2)))+(-96*a^5-120*a^4*abs(a)+24*sqrt
(3)*a^4*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*im(sign(cos(acos(a/2/abs(a))/2
)))^2*re(sign(cos(acos(a/2/abs(a))/2)))-1/12*(-864*sqrt(3)*a^5+864*a^4*sqrt
(5*a^2+4*a*abs(a)))/sqrt(abs(a))*im(sign(cos(acos(a/2/abs(a))/2)))^2*re(sig
n(sin(acos(a/2/abs(a))/2)))-1/12*(-864*sqrt(3)*a^5+864*a^4*sqrt(5*a^2-4*a*a
bs(a)))/sqrt(abs(a))*im(sign(cos(acos(a/2/abs(a))/2)))*im(sign(sin(acos(a/2
/abs(a))/2)))^2-1/24*(-3456*sqrt(3)*a^5+3456*a^4*sqrt(5*a^2+4*a*abs(a)))/sq
rt(abs(a))*im(sign(cos(acos(a/2/abs(a))/2)))*im(sign(sin(acos(a/2/abs(a))/2
)))*re(sign(cos(acos(a/2/abs(a))/2)))-(-144*a^4*abs(a)+48*sqrt(3)*a^4*sqrt(
5*a^2-4*a*abs(a)))/sqrt(abs(a))*im(sign(cos(acos(a/2/abs(a))/2)))*im(sign(s
in(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/2/abs(a))/2)))-1/24*(-2880*sqrt
(3)*a^5+1728*a^4*sqrt(5*a^2+4*a*abs(a))-2304*sqrt(3)*a^4*abs(a))/sqrt(abs(a
))*im(sign(cos(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))^2-(-
144*a^4*abs(a)+48*sqrt(3)*a^4*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*im(sign(
cos(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))*re(sign(sin(aco
s(a/2/abs(a))/2)))+1/12*(-864*sqrt(3)*a^5+864*a^4*sqrt(5*a^2-4*a*abs(a)))/s
qrt(abs(a))*im(sign(cos(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/2/abs(a))/
2)))^2+(-128*sqrt(3)*a^5+384*abs(a)*a^4-256*sqrt(3)*a^4*abs(a))*1/2/sqrt(ab
s(a))*im(sign(cos(acos(a/2/abs(a))/2)))-(32*a^5-40*a^4*abs(a)+8*sqrt(3)*a^4
*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*im(sign(sin(acos(a/2/abs(a))/2)))^3-(
-72*a^4*abs(a)+24*sqrt(3)*a^4*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*im(sign(
sin(acos(a/2/abs(a))/2)))^2*re(sign(cos(acos(a/2/abs(a))/2)))+1/24*(-2880*s
qrt(3)*a^5+1728*a^4*sqrt(5*a^2-4*a*abs(a))+2304*sqrt(3)*a^4*abs(a))/sqrt(ab
s(a))*im(sign(sin(acos(a/2/abs(a))/2)))^2*re(sign(sin(acos(a/2/abs(a))/2)))
-(-72*a^4*abs(a)+24*sqrt(3)*a^4*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*im(sig
n(sin(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))^2+1/24*(-3456
*sqrt(3)*a^5+3456*a^4*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*im(sign(sin(acos
(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/2/ab
s(a))/2)))+(96*a^5-120*a^4*abs(a)+24*sqrt(3)*a^4*sqrt(5*a^2-4*a*abs(a)))/sq
rt(abs(a))*im(sign(sin(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/2/abs(a))/2
)))^2+(64*sqrt(3)*a^5-128*a^5+64*a^4*abs(a))/sqrt(abs(a))*im(sign(sin(acos(
a/2/abs(a))/2)))-(-32*a^5-40*a^4*abs(a)+8*sqrt(3)*a^4*sqrt(5*a^2+4*a*abs(a)
))/sqrt(abs(a))*re(sign(cos(acos(a/2/abs(a))/2)))^3+1/12*(-864*sqrt(3)*a^5+
864*a^4*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*re(sign(cos(acos(a/2/abs(a))/2
)))^2*re(sign(sin(acos(a/2/abs(a))/2)))+(-72*a^4*abs(a)+24*sqrt(3)*a^4*sqrt
(5*a^2-4*a*abs(a)))/sqrt(abs(a))*re(sign(cos(acos(a/2/abs(a))/2)))*re(sign(
sin(acos(a/2/abs(a))/2)))^2+(64*sqrt(3)*a^5-128*a^5-64*a^4*abs(a))/sqrt(abs
(a))*re(sign(cos(acos(a/2/abs(a))/2)))-1/8*(-320*sqrt(3)*a^5+192*a^4*sqrt(5
*a^2-4*a*abs(a))+256*sqrt(3)*a^4*abs(a))/sqrt(abs(a))*re(sign(sin(acos(a/2/
abs(a))/2)))^3+(-128*sqrt(3)*a^5+384*abs(a)*a^4+256*sqrt(3)*a^4*abs(a))*1/2
/sqrt(abs(a))*re(sign(sin(acos(a/2/abs(a))/2))))/(128*a^3*sqrt(2*a^2+a*abs(
a))*sqrt(3)*abs(a)-128*a^3*sqrt(2*a^2-a*abs(a))*sqrt(3)*abs(a))*atan((x-sig
n(cos(acos(a*1/2/abs(a))/2))*sqrt((1+a*1/2/abs(a))/2)*sqrt(abs(a)))/sign(si
n(acos(a*1/2/abs(a))/2))/sqrt((1-a*1/2/abs(a))/2)/sqrt(abs(a)))-(2*abs(a)*s
qrt(abs(a))*a^2*cosh(im(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)-2*
abs(a)*sqrt(abs(a))*a^2*sin(re(acos(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)
))/2)-3*a^2*sqrt(abs(a))*a*cos(re(acos(a/2/abs(a)))/2)^2*cosh(im(acos(a/2/a
bs(a)))/2)^3*sin(re(acos(a/2/abs(a)))/2)+9*a^2*sqrt(abs(a))*a*cos(re(acos(a
/2/abs(a)))/2)^2*cosh(im(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/abs(a)))/2)
```

```
*sinh(im(acos(a/2/abs(a)))/2)-9*a^2*sqrt(abs(a))*a*cos(re(acos(a/2/abs(a)))
/2)^2*cosh(im(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)*sinh(im(acos
(a/2/abs(a)))/2)^2+3*a^2*sqrt(abs(a))*a*cos(re(acos(a/2/abs(a)))/2)^2*sin(r
e(acos(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)^3-2*sqrt(3)*a^2*sqrt(ab
s(a))*a*cos(re(acos(a/2/abs(a)))/2)*cosh(im(acos(a/2/abs(a)))/2)+2*sqrt(3)*
a^2*sqrt(abs(a))*a*cos(re(acos(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)
+a^2*sqrt(abs(a))*a*cosh(im(acos(a/2/abs(a)))/2)^3*sin(re(acos(a/2/abs(a)))
/2)^3-3*a^2*sqrt(abs(a))*a*cosh(im(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/a
bs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)+3*a^2*sqrt(abs(a))*a*cosh(im(acos
(a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)
^2-a^2*sqrt(abs(a))*a*sin(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)
))/2)^3+sqrt(3)*abs(a)*a^2*sqrt(abs(a))*cos(re(acos(a/2/abs(a)))/2)^3*cosh(
im(acos(a/2/abs(a)))/2)^3-3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*cos(re(acos(a/2
/abs(a)))/2)^3*cosh(im(acos(a/2/abs(a)))/2)^2*sinh(im(acos(a/2/abs(a)))/2)+
3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*cos(re(acos(a/2/abs(a)))/2)^3*cosh(im(aco
s(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)^2-sqrt(3)*abs(a)*a^2*sqrt(ab
s(a))*cos(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)^3-3*sqrt(3
)*abs(a)*a^2*sqrt(abs(a))*cos(re(acos(a/2/abs(a)))/2)*cosh(im(acos(a/2/abs(
a)))/2)^3*sin(re(acos(a/2/abs(a)))/2)^2+9*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*c
os(re(acos(a/2/abs(a)))/2)*cosh(im(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/a
bs(a)))/2)^2*sinh(im(acos(a/2/abs(a)))/2)-9*sqrt(3)*abs(a)*a^2*sqrt(abs(a))
*cos(re(acos(a/2/abs(a)))/2)*cosh(im(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/a
bs(a)))/2)^2*sinh(im(acos(a/2/abs(a)))/2)^2+3*sqrt(3)*abs(a)*a^2*sqrt(abs(a
))*cos(re(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)^2*sinh(im(acos(a
/2/abs(a)))/2)^3)*1/4/sqrt(3)/a^4*ln(x^2+2*sqrt(abs(a))*cos(acos(a*1/2/abs(
a))/2)*x+sqrt(abs(a))*sqrt(abs(a)))+(2*abs(a)*sqrt(abs(a))*a^2*cos(re(acos(
a/2/abs(a)))/2)*cosh(im(acos(a/2/abs(a)))/2)-2*abs(a)*sqrt(abs(a))*a^2*cos(
re(acos(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)-a^2*sqrt(abs(a))*a*cos
(re(acos(a/2/abs(a)))/2)^3*cosh(im(acos(a/2/abs(a)))/2)^3+3*a^2*sqrt(abs(a)
)*a*cos(re(acos(a/2/abs(a)))/2)^3*cosh(im(acos(a/2/abs(a)))/2)^2*sinh(im(ac
os(a/2/abs(a)))/2)-3*a^2*sqrt(abs(a))*a*cos(re(acos(a/2/abs(a)))/2)^3*cosh(
im(acos(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)^2+a^2*sqrt(abs(a))*a*c
os(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)^3+3*a^2*sqrt(abs(
a))*a*cos(re(acos(a/2/abs(a)))/2)*cosh(im(acos(a/2/abs(a)))/2)^3*sin(re(aco
s(a/2/abs(a)))/2)^2-9*a^2*sqrt(abs(a))*a*cos(re(acos(a/2/abs(a)))/2)*cosh(i
m(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/abs(a)))/2)^2*sinh(im(acos(a/2/abs
(a)))/2)+9*a^2*sqrt(abs(a))*a*cos(re(acos(a/2/abs(a)))/2)*cosh(im(acos(a/2/
abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)^2*sinh(im(acos(a/2/abs(a)))/2)^2-3*
a^2*sqrt(abs(a))*a*cos(re(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)^
2*sinh(im(acos(a/2/abs(a)))/2)^3+2*sqrt(3)*a^2*sqrt(abs(a))*a*cosh(im(acos(
a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)-2*sqrt(3)*a^2*sqrt(abs(a))*a*si
n(re(acos(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)-3*sqrt(3)*abs(a)*a^2
*sqrt(abs(a))*cos(re(acos(a/2/abs(a)))/2)^2*cosh(im(acos(a/2/abs(a)))/2)^3*
sin(re(acos(a/2/abs(a)))/2)+9*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*cos(re(acos(a
/2/abs(a)))/2)^2*cosh(im(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/abs(a)))/2)
*sinh(im(acos(a/2/abs(a)))/2)-9*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*cos(re(acos
(a/2/abs(a)))/2)^2*cosh(im(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)
*sinh(im(acos(a/2/abs(a)))/2)^2+3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*cos(re(ac
os(a/2/abs(a)))/2)^2*sin(re(acos(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/
2)^3+sqrt(3)*abs(a)*a^2*sqrt(abs(a))*cosh(im(acos(a/2/abs(a)))/2)^3*sin(re(
acos(a/2/abs(a)))/2)^3-3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*cosh(im(acos(a/2/a
bs(a)))/2)^2*sin(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)+3*s
qrt(3)*abs(a)*a^2*sqrt(abs(a))*cosh(im(acos(a/2/abs(a)))/2)*sin(re(acos(a/2
/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)^2-sqrt(3)*abs(a)*a^2*sqrt(abs(a
))*sin(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)^3)*1/2/sqrt(3
)/a^4*atan((x+cos(acos(a*1/2/abs(a))/2)*sqrt(abs(a)))/sin(acos(a*1/2/abs(a)
)/2)/sqrt(abs(a)))
```

maple [A] time = 0.04, size = 92, normalized size = 0.81

$$\frac{\arctan\left(\frac{2x+\sqrt{3}\,\sqrt{a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\arctan\left(\frac{-2x+\sqrt{3}\,\sqrt{a}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,\sqrt{a}\,x+a\right)}{4\sqrt{a}}-\frac{\sqrt{3}\,\ln\left(-x^2+\sqrt{3}\,\sqrt{a}\,x-a\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*a)/(x^4-a*x^2+a^2),x)
```

```
[Out] -1/4/a^(1/2)*3^(1/2)*ln(x*3^(1/2)*a^(1/2)-x^2-a)-1/2/a^(1/2)*arctan((3^(1/2
)*a^(1/2)-2*x)/a^(1/2))+1/4*ln(a+x^2+x*3^(1/2)*a^(1/2))*3^(1/2)/a^(1/2)+1/2
/a^(1/2)*arctan((2*x+3^(1/2)*a^(1/2))/a^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-2\,a}{x^4-ax^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*a)/(x^4 - a*x^2 + a^2), x)
```

mupad [B] time = 4.48, size = 133, normalized size = 1.17

$$-\frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{\frac{1}{8\,a}+\frac{\sqrt{3}\,1\mathrm{i}}{8\,a}}\,1\mathrm{i}+\sqrt{3}\,x\,\sqrt{\frac{1}{8\,a}+\frac{\sqrt{3}\,1\mathrm{i}}{8\,a}}\right)\sqrt{\frac{1+\sqrt{3}\,1\mathrm{i}}{a}}\,1\mathrm{i}}{4}-\frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{\frac{1}{8\,a}-\frac{\sqrt{3}\,1\mathrm{i}}{8\,a}}\,1\mathrm{i}-\sqrt{3}\,x\,\sqrt{\frac{1}{8\,a}-\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a - x^2)/(a^2 - a*x^2 + x^4),x)
```

```
[Out] - (8^(1/2)*atan(x*((3^(1/2)*1i)/(8*a) + 1/(8*a))^(1/2)*1i + 3^(1/2)*x*((3^(
1/2)*1i)/(8*a) + 1/(8*a))^(1/2))*((3^(1/2)*1i + 1)/a)^(1/2)*1i)/4 - (8^(1/2
)*atan(x*(1/(8*a) - (3^(1/2)*1i)/(8*a))^(1/2)*1i - 3^(1/2)*x*(1/(8*a) - (3^
(1/2)*1i)/(8*a))^(1/2))*(-(3^(1/2)*1i - 1)/a)^(1/2)*1i)/4
```

sympy [A] time = 0.25, size = 27, normalized size = 0.24

$$-\operatorname{RootSum}\left(16t^4a^2-4t^2a+1,\left(t\mapsto t\log\left(-2ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*a)/(x**4-a*x**2+a**2),x)
```

```
[Out] -RootSum(16*_t**4*a**2 - 4*_t**2*a + 1, Lambda(_t, _t*log(-2*_t*a + x)))
```

3.107 $\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2\sqrt[4]{a}}$$

```
[Out] 1/2*arctan(2*x/a^(1/4)-3^(1/2))/a^(1/4)+1/2*arctan(2*x/a^(1/4)+3^(1/2))/a^(
1/4)-1/4*ln(x^2-a^(1/4)*x*3^(1/2)+a^(1/2))*3^(1/2)/a^(1/4)+1/4*ln(x^2+a^(1/
4)*x*3^(1/2)+a^(1/2))*3^(1/2)/a^(1/4)
```

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - (2*x)/a^(1/4)]/(2*a^(1/4)) + ArcTan[Sqrt[3] + (2*x)/a^(1/
4)]/(2*a^(1/4)) - (Sqrt[3]*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/
4)) + (Sqrt[3]*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = \frac{\int \frac{2\sqrt{3}a^{3/4}-3\sqrt{a}x}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}a^{3/4}} + \frac{\int \frac{2\sqrt{3}a^{3/4}+3\sqrt{a}x}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}a^{3/4}}$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx - \frac{\sqrt{3}\int \frac{-\sqrt{3}\sqrt[4]{a}+2x}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{4\sqrt[4]{a}}$$

$$= -\frac{\sqrt{3}\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}} + \frac{\sqrt{3}\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx\right.}{2\sqrt{3}\sqrt[4]{}}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} - \frac{\sqrt{3}\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}} + \frac{\sqrt{3}\log\left(\right.}{}$$

Mathematica [C] time = 0.15, size = 115, normalized size = 0.94

$$\frac{\sqrt[4]{-1}\left(\sqrt{\sqrt{3}-i}\left(\sqrt{3}-3i\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)-\sqrt{\sqrt{3}+i}\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)\right)}{2\sqrt{6}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[-
I + Sqrt[3]]*a^(1/4))]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1 +
 I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))]))/(2*Sqrt[6]*a^(1/4))
```

fricas [B] time = 0.42, size = 251, normalized size = 2.06

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(-\sqrt{\frac{1}{2}}\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a)*log(sqrt(1/2)*sqrt(a
)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a) + x) - 1/2*sqrt(1/2)*sqrt((sqrt(
3)*a*sqrt(-1/a) + sqrt(a))/a)*log(-sqrt(1/2)*sqrt(a)*sqrt((sqrt(3)*a*sqrt(-
1/a) + sqrt(a))/a) + x) + 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(
a))/a)*log(sqrt(1/2)*sqrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + x)
 - 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a)*log(-sqrt(1/2)*s
qrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 96, normalized size = 0.79

$$\frac{\arctan\left(\frac{2x+\sqrt{3}\,a^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{1}{4}}}-\frac{\arctan\left(\frac{-2x+\sqrt{3}\,a^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{1}{4}}}+\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,a^{\frac{1}{4}}x+\sqrt{a}\right)}{4a^{\frac{1}{4}}}-\frac{\sqrt{3}\,\ln\left(-x^{2}+\sqrt{3}\,a^{\frac{1}{4}}x-\sqrt{a}\right)}{4a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*a^(1/2))/(a+x^4-a^(1/2)*x^2),x)
```

```
[Out] 1/4*ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))*3^(1/2)/a^(1/4)+1/2/a^(1/4)*arctan((2
*x+3^(1/2)*a^(1/4))/a^(1/4))-1/4/a^(1/4)*3^(1/2)*ln(a^(1/4)*x*3^(1/2)-x^2-a
^(1/2))-1/2/a^(1/4)*arctan((3^(1/2)*a^(1/4)-2*x)/a^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}-2\sqrt{a}}{x^{4}-\sqrt{a}\,x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*sqrt(a))/(x^4 - sqrt(a)*x^2 + a), x)
```

mupad [B] time = 5.06, size = 159, normalized size = 1.30

$$2\,\mathrm{atanh}\left(x\,\sqrt{\frac{1}{8\,\sqrt{a}}-\frac{\sqrt{-27\,a^{3}}}{24\,a^{2}}}-\frac{9\,a^{3/2}\,x\,\sqrt{\frac{1}{8\,\sqrt{a}}-\frac{\sqrt{-27\,a^{3}}}{24\,a^{2}}}}{\sqrt{-27\,a^{3}}}\right)\sqrt{\frac{1}{8\,\sqrt{a}}-\frac{\sqrt{-27\,a^{3}}}{24\,a^{2}}}+2\,\mathrm{atanh}\left(x\,\sqrt{\frac{\sqrt{-27\,a^{3}}}{24\,a^{2}}+\frac{1}{8\,\sqrt{a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a^(1/2) - x^2)/(a + x^4 - a^(1/2)*x^2),x)
```

```
[Out] 2*atanh(x*(1/(8*a^(1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2) - (9*a^(3/2)*x*(
1/(8*a^(1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2))/(-27*a^3)^(1/2))*(1/(8*a^(
1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2) + 2*atanh(x*((-27*a^3)^(1/2)/(24*a^
2) + 1/(8*a^(1/2)))^(1/2) + (9*a^(3/2)*x*((-27*a^3)^(1/2)/(24*a^2) + 1/(8*a
^(1/2)))^(1/2))/(-27*a^3)^(1/2))*((-27*a^3)^(1/2)/(24*a^2) + 1/(8*a^(1/2)))
^(1/2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*a**(1/2))/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

3.108 $\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx$

Optimal. Leaf size=124

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}$$

```
[Out] -1/4*ln(b^(2/3)-b^(1/3)*x+x^2)/b^(1/3)+1/4*ln(b^(2/3)+b^(1/3)*x+x^2)/b^(1/3
)-1/2*arctan(1/3*(b^(1/3)-2*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)+1/2*arctan(
1/3*(b^(1/3)+2*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

Rubi [A] time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(b^(1/3) - 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3)) + (Sqrt[3]
*ArcTan[(b^(1/3) + 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3)) - Log[b^(2/3) - b^(
1/3)*x + x^2]/(4*b^(1/3)) + Log[b^(2/3) + b^(1/3)*x + x^2]/(4*b^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
```

```
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = \frac{\int \frac{2b-b^{2/3}x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx}{2b}+\frac{\int \frac{2b+b^{2/3}x}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx}{2b}$$

$$= \frac{3}{4}\int \frac{1}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx+\frac{3}{4}\int \frac{1}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx-\frac{\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx}{4\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}+2x}{b^{2/3}+\sqrt[3]{b}x+x}}{4\sqrt[3]{b}}$$

$$= -\frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}-$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}-\frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x\right.}{4\sqrt[3]{b}}$$

Mathematica [C] time = 0.13, size = 115, normalized size = 0.93

$$\frac{\sqrt[4]{-1}\left(\sqrt{\sqrt{3}-i}\left(\sqrt{3}-3i\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[3]{b}}\right)-\sqrt{\sqrt{3}+i}\left(\sqrt{3}+3i\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[3]{b}}\right)\right)}{2\sqrt{6}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[I
 + Sqrt[3]]*b^(1/3))] - Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTanh[((1 + I)*
x)/(Sqrt[-I + Sqrt[3]]*b^(1/3))]))/(2*Sqrt[6]*b^(1/3))
```

fricas [A] time = 0.46, size = 264, normalized size = 2.13

$$\left[\frac{\sqrt{3}\,b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2x^3+\sqrt{3}\left(2b^{\frac{2}{3}}x^2+bx-b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3b^{\frac{2}{3}}x-b}{x^3+b}\right)+\sqrt{3}\,b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2x^3+\sqrt{3}\left(2b^{\frac{2}{3}}x^2-bx-b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3b^{\frac{2}{3}}x+b}{x^3-b}\right)+b^{\frac{2}{3}}}{4b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*x^3 + sqrt(3)*(2*b^(2/3)*x^2 + b*x
- b^(4/3))*sqrt(-1/b^(2/3)) - 3*b^(2/3)*x - b)/(x^3 + b)) + sqrt(3)*b*sqrt(
-1/b^(2/3))*log((2*x^3 + sqrt(3)*(2*b^(2/3)*x^2 - b*x - b^(4/3))*sqrt(-1/b^
(2/3)) - 3*b^(2/3)*x + b)/(x^3 - b)) + b^(2/3)*log(x^2 + b^(1/3)*x + b^(2/3
)) - b^(2/3)*log(x^2 - b^(1/3)*x + b^(2/3)))/b, 1/4*(2*sqrt(3)*b^(2/3)*arct
an(1/3*sqrt(3)*(2*x + b^(1/3))/b^(1/3)) - 2*sqrt(3)*b^(2/3)*arctan(-1/3*sqr
t(3)*(2*x - b^(1/3))/b^(1/3)) + b^(2/3)*log(x^2 + b^(1/3)*x + b^(2/3)) - b^
(2/3)*log(x^2 - b^(1/3)*x + b^(2/3)))/b]
```

giac [A] time = 0.18, size = 92, normalized size = 0.74

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}}+\frac{\log\left(x^2+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}-\frac{\log\left(x^2-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3) +
 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3)
+ 1/4*log(x^2 + b^(1/3)*x + b^(2/3))/b^(1/3) - 1/4*log(x^2 - b^(1/3)*x + b^
(2/3))/b^(1/3)
```

maple [A] time = 0.03, size = 89, normalized size = 0.72

$$\frac{\sqrt{3}\arctan\left(\frac{\left(2x-b^{\frac{1}{3}}\right)\sqrt{3}}{3b^{\frac{1}{3}}}\right)}{2b^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\left(2x+b^{\frac{1}{3}}\right)\sqrt{3}}{3b^{\frac{1}{3}}}\right)}{2b^{\frac{1}{3}}}-\frac{\ln\left(x^2-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}+\frac{\ln\left(x^2+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x)
```

```
[Out] -1/4*ln(b^(2/3)-b^(1/3)*x+x^2)/b^(1/3)+1/2*3^(1/2)/b^(1/3)*arctan(1/3*(-b^(
1/3)+2*x)*3^(1/2)/b^(1/3))+1/4*ln(b^(2/3)+b^(1/3)*x+x^2)/b^(1/3)+1/2*arctan
(1/3*(b^(1/3)+2*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

maxima [A] time = 2.29, size = 88, normalized size = 0.71

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{2b^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{2b^{\frac{1}{3}}}+\frac{\log\left(x^2+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}-\frac{\log\left(x^2-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/b^(1/3))/b^(1/3) + 1/2*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - b^(1/3))/b^(1/3))/b^(1/3) + 1/4*log(x^2 + b^(1
/3)*x + b^(2/3))/b^(1/3) - 1/4*log(x^2 - b^(1/3)*x + b^(2/3))/b^(1/3)
```

mupad [B] time = 0.24, size = 133, normalized size = 1.07

$$\frac{\sqrt{8}\,\mathrm{atan}\left(x\sqrt{-\frac{1}{8\,b^{2/3}}-\frac{\sqrt{3}\,1\mathrm{i}}{8\,b^{2/3}}}\,1\mathrm{i}+\sqrt{3}\,x\sqrt{-\frac{1}{8\,b^{2/3}}-\frac{\sqrt{3}\,1\mathrm{i}}{8\,b^{2/3}}}\right)\sqrt{-\frac{1+\sqrt{3}\,1\mathrm{i}}{b^{2/3}}}\,1\mathrm{i}}{4}+\frac{\sqrt{8}\,\mathrm{atan}\left(x\sqrt{-\frac{1}{8\,b^{2/3}}+\frac{\sqrt{3}\,1\mathrm{i}}{8\,b^{2/3}}}\,1\mathrm{i}-\sqrt{3}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b^(2/3) + x^2)/(b^(4/3) + x^4 + b^(2/3)*x^2),x)
```

```
[Out] (8^(1/2)*atan(x*(- (3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)))^(1/2)*1i + 3^(
1/2)*x*(- (3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)))^(1/2))*(-(3^(1/2)*1i +
1)/b^(2/3))^(1/2)*1i)/4 + (8^(1/2)*atan(x*((3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*
```

```
b^(2/3)))^(1/2)*1i - 3^(1/2)*x*((3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)))^(
1/2))*((3^(1/2)*1i - 1)/b^(2/3))^(1/2)*1i)/4
```

sympy [C] time = 0.31, size = 143, normalized size = 1.15

$$\frac{\left(-\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)+x\right)+\left(-\frac{1}{4}+\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}+\frac{\sqrt{3}i}{4}\right)+x\right)+\left(\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(\frac{1}{4}-\frac{\sqrt{3}i}{4}\right.\right.}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b**(2/3)+x**2)/(b**(4/3)+b**(2/3)*x**2+x**4),x)
```

```
[Out] ((-1/4 - sqrt(3)*I/4)*log(2*b**(1/3)*(-1/4 - sqrt(3)*I/4) + x) + (-1/4 + sq
rt(3)*I/4)*log(2*b**(1/3)*(-1/4 + sqrt(3)*I/4) + x) + (1/4 - sqrt(3)*I/4)*l
og(2*b**(1/3)*(1/4 - sqrt(3)*I/4) + x) + (1/4 + sqrt(3)*I/4)*log(2*b**(1/3)
*(1/4 + sqrt(3)*I/4) + x))/b**(1/3)
```

3.109 $\int \frac{A+Bx^2}{a^2-ax^2+x^4} dx$

Optimal. Leaf size=136

$$-\frac{(A-aB)\log\left(-\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{3}\,a^{3/2}}+\frac{(A-aB)\log\left(\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{3}\,a^{3/2}}-\frac{(aB+A)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(aB+A}{}$$

```
[Out] 1/2*(B*a+A)*arctan(-3^(1/2)+2*x/a^(1/2))/a^(3/2)+1/2*(B*a+A)*arctan(3^(1/2)
+2*x/a^(1/2))/a^(3/2)-1/12*(-B*a+A)*ln(a+x^2-x*3^(1/2)*a^(1/2))/a^(3/2)*3^(
1/2)+1/12*(-B*a+A)*ln(a+x^2+x*3^(1/2)*a^(1/2))/a^(3/2)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{(A-aB)\log\left(-\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{3}\,a^{3/2}}+\frac{(A-aB)\log\left(\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{3}\,a^{3/2}}-\frac{(aB+A)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(aB+A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] -((A + a*B)*ArcTan[Sqrt[3] - (2*x)/Sqrt[a]])/(2*a^(3/2)) + ((A + a*B)*ArcTa
n[Sqrt[3] + (2*x)/Sqrt[a]])/(2*a^(3/2)) - ((A - a*B)*Log[a - Sqrt[3]*Sqrt[a
]*x + x^2])/(4*Sqrt[3]*a^(3/2)) + ((A - a*B)*Log[a + Sqrt[3]*Sqrt[a]*x + x^
2])/(4*Sqrt[3]*a^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
```

```
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx = \frac{\int \frac{\sqrt{3}\sqrt{a}A-(A-aB)x}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}} + \frac{\int \frac{\sqrt{3}\sqrt{a}A+(A-aB)x}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}}$$

$$= -\frac{(A-aB)\int \frac{-\sqrt{3}\sqrt{a}+2x}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{3}a^{3/2}} + \frac{(A-aB)\int \frac{\sqrt{3}\sqrt{a}+2x}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{3}a^{3/2}} + \frac{(A+aB)\int \frac{1}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{4a} +$$

$$= -\frac{(A-aB)\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}} + \frac{(A-aB)\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}} + \frac{(A+aB)\,\mathrm{Subst}\left(\right.}{2}$$

$$= -\frac{(A+aB)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{(A+aB)\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{(A-aB)\log\left(a-\sqrt{3}\sqrt{a}x+\right.}{4\sqrt{3}a^{3/2}}$$

Mathematica [C] time = 0.15, size = 130, normalized size = 0.96

$$\frac{\sqrt[4]{-1}\left(\frac{\left(\left(\sqrt{3}-i\right)aB-2iA\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt{a}}\right)}{\sqrt{\sqrt{3}-i}}-\frac{\left(\left(\sqrt{3}+i\right)aB+2iA\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt{a}}\right)}{\sqrt{\sqrt{3}+i}}\right)}{\sqrt{6}a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*a*B)*ArcTan[((1 + I)*x)/(Sqrt[-I +
 Sqrt[3]]*Sqrt[a])])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (I + Sqrt[3])*a*B)*Ar
cTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])])/Sqrt[I + Sqrt[3]]))/(Sqrt[6
]*a^(3/2))
```

fricas [B] time = 1.09, size = 4551, normalized size = 33.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(1/9)^(1/4)*a^6*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^
3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3
 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*
((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4)*sqrt(
(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6)*arctan((18*sqrt(1/3)*(1/9)^(3/4)*(sqrt
(1/3)*A*a^10*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)
/a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) - sqrt(1/3)*(B^3*a^10 + A*B
^2*a^9 + A^2*B*a^8)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt((2*B^4*
a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^
4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)
/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*sqrt(((B^4*a^4 + 2*A*B^3*a^3 + 3*A^
2*B^2*a^2 + 2*A^3*B*a + A^4)*x^2 + 3*sqrt(1/3)*(1/9)^(1/4)*(B*a^6*x*sqrt((B
^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6) - (A*B^2*a^4 +
```

```
A^2*B*a^3 + A^3*a^2)*x)*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*
A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a
^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)
)*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4) + (
B^2*a^6 + A*B*a^5 + A^2*a^4)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 +
2*A^3*B*a + A^4)/a^6))/(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a +
 A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4
) - 18*sqrt(1/3)*(1/9)^(3/4)*(sqrt(1/3)*A*a^10*x*sqrt((B^4*a^4 + 2*A*B^3*a^
3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A
^4)/a^6) - sqrt(1/3)*(B^3*a^10 + A*B^2*a^9 + A^2*B*a^8)*x*sqrt((B^4*a^4 - 2
*A^2*B^2*a^2 + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4
*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*
a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4
))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4) +
2*sqrt(1/3)*(B^4*a^10 + 2*A*B^3*a^9 + 3*A^2*B^2*a^8 + 2*A^3*B*a^7 + A^4*a^6
)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt(
(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) + sqrt(1/3)*(B^6*a^9 + 3*A*B^5*a^8 + 6
*A^2*B^4*a^7 + 7*A^3*B^3*a^6 + 6*A^4*B^2*a^5 + 3*A^5*B*a^4 + A^6*a^3)*sqrt(
(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))/(B^8*a^8 + 3*A*B^7*a^7 + 5*A^2*B^6*a^
6 + 4*A^3*B^5*a^5 - 4*A^5*B^3*a^3 - 5*A^6*B^2*a^2 - 3*A^7*B*a - A^8)) + 4*(
1/9)^(1/4)*a^6*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a +
2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2
*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^
4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4)*sqrt((B^4*a^4
 - 2*A^2*B^2*a^2 + A^4)/a^6)*arctan((18*sqrt(1/3)*(1/9)^(3/4)*(sqrt(1/3)*A*
a^10*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sq
rt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) - sqrt(1/3)*(B^3*a^10 + A*B^2*a^9 +
 A^2*B*a^8)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*
A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*
a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(
B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*sqrt(((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^
2 + 2*A^3*B*a + A^4)*x^2 - 3*sqrt(1/3)*(1/9)^(1/4)*(B*a^6*x*sqrt((B^4*a^4 +
 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6) - (A*B^2*a^4 + A^2*B*a
^3 + A^3*a^2)*x)*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a
+ 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A
^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*
a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4) + (B^2*a^6
+ A*B*a^5 + A^2*a^4)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*
a + A^4)/a^6))/(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4))*(
(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4) - 18*s
qrt(1/3)*(1/9)^(3/4)*(sqrt(1/3)*A*a^10*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^
2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6)
 - sqrt(1/3)*(B^3*a^10 + A*B^2*a^9 + A^2*B*a^8)*x*sqrt((B^4*a^4 - 2*A^2*B^2
*a^2 + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a
 + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*
A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4
*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4) - 2*sqrt(1
/3)*(B^4*a^10 + 2*A*B^3*a^9 + 3*A^2*B^2*a^8 + 2*A^3*B*a^7 + A^4*a^6)*sqrt((
B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt((B^4*a^4
 - 2*A^2*B^2*a^2 + A^4)/a^6) - sqrt(1/3)*(B^6*a^9 + 3*A*B^5*a^8 + 6*A^2*B^4
*a^7 + 7*A^3*B^3*a^6 + 6*A^4*B^2*a^5 + 3*A^5*B*a^4 + A^6*a^3)*sqrt((B^4*a^4
 - 2*A^2*B^2*a^2 + A^4)/a^6))/(B^8*a^8 + 3*A*B^7*a^7 + 5*A^2*B^6*a^6 + 4*A^
3*B^5*a^5 - 4*A^5*B^3*a^3 - 5*A^6*B^2*a^2 - 3*A^7*B*a - A^8)) - sqrt(1/3)*(
1/9)^(1/4)*(2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 - (
B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2
+ 2*A^3*B*a + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*
A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a
^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)
)*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4)*log
```

```
(2*(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)*x^2 + 6*sqrt(1
/3)*(1/9)^(1/4)*(B*a^6*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^
3*B*a + A^4)/a^6) - (A*B^2*a^4 + A^2*B*a^3 + A^3*a^2)*x)*sqrt((2*B^4*a^4 +
4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^
2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))
/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 +
 2*A^3*B*a + A^4)/a^6)^(1/4) + 2*(B^2*a^6 + A*B*a^5 + A^2*a^4)*sqrt((B^4*a^
4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)) + sqrt(1/3)*(1/9)^
(1/4)*(2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 - (B^2*a
^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A
^3*B*a + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B
*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 +
3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B
^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4)*log(2*(B
^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)*x^2 - 6*sqrt(1/3)*(
1/9)^(1/4)*(B*a^6*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a
 + A^4)/a^6) - (A*B^2*a^4 + A^2*B*a^3 + A^3*a^2)*x)*sqrt((2*B^4*a^4 + 4*A*B
^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3
)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4
*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^
3*B*a + A^4)/a^6)^(1/4) + 2*(B^2*a^6 + A*B*a^5 + A^2*a^4)*sqrt((B^4*a^4 + 2
*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)))/(B^4*a^4 + 2*A*B^3*a^3
 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-16,[2,0]%%%}+%%%{-4,[0,1]%%%},0,%%%{64,[4,0]
%%%}+%%%{8,[2,2]%%%}+%%%{16,[2,1]%%%}+%%%{6,[0,2]%%%},0,%%%{-64,[4,2]%%%}+%
%%{-128,[4,1]%%%}+%%%{48,[2,3]%%%}+%%%{16,[2,2]%%%}+%%%{-4,[0,3]%%%},0,%%%{
16,[4,4]%%%}+%%%{-64,[4,3]%%%}+%%%{64,[4,2]%%%}+%%%{8,[2,4]%%%}+%%%{-16,[2,
3]%%%}+%%%{1,[0,4]%%%}] at parameters values [71,-96]Warning, choosing root
 of [1,0,%%%{-16,[2,0]%%%}+%%%{-4,[0,1]%%%},0,%%%{64,[4,0]%%%}+%%%{8,[2,2]%
%%}+%%%{16,[2,1]%%%}+%%%{6,[0,2]%%%},0,%%%{-64,[4,2]%%%}+%%%{-128,[4,1]%%%}
+%%%{48,[2,3]%%%}+%%%{16,[2,2]%%%}+%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%{
-64,[4,3]%%%}+%%%{64,[4,2]%%%}+%%%{8,[2,4]%%%}+%%%{-16,[2,3]%%%}+%%%{1,[0,4
]%%%}] at parameters values [72,-72]((64*a^3*sqrt(abs(a))*abs(a)+32*sqrt(3)
*a^4*sqrt(abs(a))+32*a^4*sqrt(abs(a)))*A*im(sign(cos(acos(a/2/abs(a))/2)))+
(64*sqrt(3)*a^5+192*abs(a)*a^4-128*sqrt(3)*a^4*abs(a))*1/2/sqrt(abs(a))*A*i
m(sign(sin(acos(a/2/abs(a))/2)))+(-64*sqrt(3)*a^5+192*abs(a)*a^4-128*sqrt(3
)*a^4*abs(a))*1/2/sqrt(abs(a))*A*re(sign(cos(acos(a/2/abs(a))/2)))+(32*sqrt
(3)*a^5-64*a^5+32*a^4*abs(a))/sqrt(abs(a))*A*re(sign(sin(acos(a/2/abs(a))/2
)))+(-32*a^6-40*a^5*abs(a)+8*sqrt(3)*a^5*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a
))*B*im(sign(cos(acos(a/2/abs(a))/2)))^3-1/12*(-864*sqrt(3)*a^6+864*a^5*sqr
t(5*a^2+4*a*abs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))^2*im(
sign(sin(acos(a/2/abs(a))/2)))-1/24*(-2880*sqrt(3)*a^6+1728*abs(a)*a^4*sqrt
(5*a^2+4*a*abs(a))-2304*sqrt(3)*a^5*abs(a))/sqrt(abs(a))*B*im(sign(cos(acos
(a/2/abs(a))/2)))^2*re(sign(cos(acos(a/2/abs(a))/2)))-(-72*a^5*abs(a)+24*sq
rt(3)*a^5*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a
))/2)))^2*re(sign(sin(acos(a/2/abs(a))/2)))-(-72*a^5*abs(a)+24*sqrt(3)*a^5*
sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))*im
(sign(sin(acos(a/2/abs(a))/2)))^2-(-144*a^5*abs(a)+48*sqrt(3)*a^5*sqrt(5*a^
2+4*a*abs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))*im(sign(sin
(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))+1/24*(-3456*sqrt(3
)*a^6+3456*a^5*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/
```

```
abs(a))/2)))*im(sign(sin(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/2/abs(a))
/2)))-(-96*a^6-120*a^5*abs(a)+24*sqrt(3)*a^5*sqrt(5*a^2+4*a*abs(a)))/sqrt(a
bs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2))
)^2+1/24*(-3456*sqrt(3)*a^6+3456*a^5*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*B
*im(sign(cos(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))*re(sig
n(sin(acos(a/2/abs(a))/2)))+(-72*a^5*abs(a)+24*sqrt(3)*a^5*sqrt(5*a^2-4*a*a
bs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a
/2/abs(a))/2)))^2+1/8*(-320*sqrt(3)*a^6+192*abs(a)*a^4*sqrt(5*a^2-4*a*abs(a
))+256*sqrt(3)*a^5*abs(a))/sqrt(abs(a))*B*im(sign(sin(acos(a/2/abs(a))/2)))
^3+1/12*(-864*sqrt(3)*a^6+864*a^5*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*im
(sign(sin(acos(a/2/abs(a))/2)))^2*re(sign(cos(acos(a/2/abs(a))/2)))+(96*a^6
-120*a^5*abs(a)+24*sqrt(3)*a^5*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*im(si
gn(sin(acos(a/2/abs(a))/2)))^2*re(sign(sin(acos(a/2/abs(a))/2)))+1/12*(-864
*sqrt(3)*a^6+864*a^5*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*B*im(sign(sin(aco
s(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))^2+(-144*a^5*abs(a)+48*
sqrt(3)*a^5*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*im(sign(sin(acos(a/2/abs
(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/2/abs(a))/2)
))-1/24*(-2880*sqrt(3)*a^6+1728*abs(a)*a^4*sqrt(5*a^2-4*a*abs(a))+2304*sqrt
(3)*a^5*abs(a))/sqrt(abs(a))*B*im(sign(sin(acos(a/2/abs(a))/2)))*re(sign(si
n(acos(a/2/abs(a))/2)))^2+1/8*(-320*sqrt(3)*a^6+192*abs(a)*a^4*sqrt(5*a^2+4
*a*abs(a))-256*sqrt(3)*a^5*abs(a))/sqrt(abs(a))*B*re(sign(cos(acos(a/2/abs(
a))/2)))^3+(-72*a^5*abs(a)+24*sqrt(3)*a^5*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(
a))*B*re(sign(cos(acos(a/2/abs(a))/2)))^2*re(sign(sin(acos(a/2/abs(a))/2)))
-1/12*(-864*sqrt(3)*a^6+864*a^5*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*re(s
ign(cos(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/2/abs(a))/2)))^2-(32*a^6-4
0*a^5*abs(a)+8*sqrt(3)*a^5*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*re(sign(s
in(acos(a/2/abs(a))/2)))^3)/(256*a^4*sqrt(2*a^2+a*abs(a))*sqrt(3)*abs(a)-25
6*a^4*sqrt(2*a^2-a*abs(a))*sqrt(3)*abs(a))*ln(x^2-2*sqrt((1+a*1/2/abs(a))/2
)*sqrt(abs(a))*sign(cos(acos(a*1/2/abs(a))/2))*x+sqrt(abs(a))*sqrt(abs(a)))
+((64*sqrt(3)*a^5+192*abs(a)*a^4+128*sqrt(3)*a^4*abs(a))*1/2/sqrt(abs(a))*A
*im(sign(cos(acos(a/2/abs(a))/2)))+(64*a^3*sqrt(abs(a))*abs(a)+32*sqrt(3)*a
^4*sqrt(abs(a))-32*a^4*sqrt(abs(a)))*A*im(sign(sin(acos(a/2/abs(a))/2)))+(6
4*a^3*sqrt(abs(a))*abs(a)+32*sqrt(3)*a^4*sqrt(abs(a))+32*a^4*sqrt(abs(a)))*
A*re(sign(cos(acos(a/2/abs(a))/2)))+(64*sqrt(3)*a^5+192*abs(a)*a^4-128*sqrt
(3)*a^4*abs(a))*1/2/sqrt(abs(a))*A*re(sign(sin(acos(a/2/abs(a))/2)))+1/8*(-
320*sqrt(3)*a^6+192*abs(a)*a^4*sqrt(5*a^2+4*a*abs(a))-256*sqrt(3)*a^5*abs(a
))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))^3+(-72*a^5*abs(a)+24*sq
rt(3)*a^5*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a
))/2)))^2*im(sign(sin(acos(a/2/abs(a))/2)))+(-96*a^6-120*a^5*abs(a)+24*sqrt
(3)*a^5*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))
/2)))^2*re(sign(cos(acos(a/2/abs(a))/2)))-1/12*(-864*sqrt(3)*a^6+864*a^5*sq
rt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))^2*re
(sign(sin(acos(a/2/abs(a))/2)))-1/12*(-864*sqrt(3)*a^6+864*a^5*sqrt(5*a^2-4
*a*abs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))*im(sign(sin(ac
os(a/2/abs(a))/2)))^2-1/24*(-3456*sqrt(3)*a^6+3456*a^5*sqrt(5*a^2+4*a*abs(a
)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))*im(sign(sin(acos(a/2/a
bs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))-(-144*a^5*abs(a)+48*sqrt(3)*a
^5*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))
*im(sign(sin(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/2/abs(a))/2)))-1/24*(
-2880*sqrt(3)*a^6+1728*abs(a)*a^4*sqrt(5*a^2+4*a*abs(a))-2304*sqrt(3)*a^5*a
bs(a))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/
2/abs(a))/2)))^2-(-144*a^5*abs(a)+48*sqrt(3)*a^5*sqrt(5*a^2+4*a*abs(a)))/sq
rt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))
/2)))*re(sign(sin(acos(a/2/abs(a))/2)))+1/12*(-864*sqrt(3)*a^6+864*a^5*sqrt
(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*im(sign(cos(acos(a/2/abs(a))/2)))*re(sig
n(sin(acos(a/2/abs(a))/2)))^2-(32*a^6-40*a^5*abs(a)+8*sqrt(3)*a^5*sqrt(5*a^
2-4*a*abs(a)))/sqrt(abs(a))*B*im(sign(sin(acos(a/2/abs(a))/2)))^3-(-72*a^5*
abs(a)+24*sqrt(3)*a^5*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*im(sign(sin(ac
os(a/2/abs(a))/2)))^2*re(sign(cos(acos(a/2/abs(a))/2)))+1/24*(-2880*sqrt(3)
```

```
*a^6+1728*abs(a)*a^4*sqrt(5*a^2-4*a*abs(a))+2304*sqrt(3)*a^5*abs(a))/sqrt(a
bs(a))*B*im(sign(sin(acos(a/2/abs(a))/2)))^2*re(sign(sin(acos(a/2/abs(a))/2
)))-(-72*a^5*abs(a)+24*sqrt(3)*a^5*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*B*i
m(sign(sin(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))^2+1/24*(
-3456*sqrt(3)*a^6+3456*a^5*sqrt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*im(sign(s
in(acos(a/2/abs(a))/2)))*re(sign(cos(acos(a/2/abs(a))/2)))*re(sign(sin(acos
(a/2/abs(a))/2)))+(96*a^6-120*a^5*abs(a)+24*sqrt(3)*a^5*sqrt(5*a^2-4*a*abs(
a)))/sqrt(abs(a))*B*im(sign(sin(acos(a/2/abs(a))/2)))*re(sign(sin(acos(a/2/
abs(a))/2)))^2-(-32*a^6-40*a^5*abs(a)+8*sqrt(3)*a^5*sqrt(5*a^2+4*a*abs(a)))
/sqrt(abs(a))*B*re(sign(cos(acos(a/2/abs(a))/2)))^3+1/12*(-864*sqrt(3)*a^6+
864*a^5*sqrt(5*a^2+4*a*abs(a)))/sqrt(abs(a))*B*re(sign(cos(acos(a/2/abs(a))
/2)))^2*re(sign(sin(acos(a/2/abs(a))/2)))+(-72*a^5*abs(a)+24*sqrt(3)*a^5*sq
rt(5*a^2-4*a*abs(a)))/sqrt(abs(a))*B*re(sign(cos(acos(a/2/abs(a))/2)))*re(s
ign(sin(acos(a/2/abs(a))/2)))^2-1/8*(-320*sqrt(3)*a^6+192*abs(a)*a^4*sqrt(5
*a^2-4*a*abs(a))+256*sqrt(3)*a^5*abs(a))/sqrt(abs(a))*B*re(sign(sin(acos(a/
2/abs(a))/2)))^3)/(128*a^4*sqrt(2*a^2+a*abs(a))*sqrt(3)*abs(a)-128*a^4*sqrt
(2*a^2-a*abs(a))*sqrt(3)*abs(a))*atan((x-sign(cos(acos(a*1/2/abs(a))/2))*sq
rt((1+a*1/2/abs(a))/2)*sqrt(abs(a)))/sign(sin(acos(a*1/2/abs(a))/2))/sqrt((
1-a*1/2/abs(a))/2)/sqrt(abs(a)))+(-abs(a)*sqrt(abs(a))*A*a*cosh(im(acos(a/2
/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)+abs(a)*sqrt(abs(a))*A*a*sin(re(aco
s(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)+sqrt(3)*a^2*sqrt(abs(a))*A*c
os(re(acos(a/2/abs(a)))/2)*cosh(im(acos(a/2/abs(a)))/2)-sqrt(3)*a^2*sqrt(ab
s(a))*A*cos(re(acos(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)-3*a^2*sqrt
(abs(a))*B*a*cos(re(acos(a/2/abs(a)))/2)^2*cosh(im(acos(a/2/abs(a)))/2)^3*s
in(re(acos(a/2/abs(a)))/2)+9*a^2*sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(a)))/
2)^2*cosh(im(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/abs(a)))/2)*sinh(im(aco
s(a/2/abs(a)))/2)-9*a^2*sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(a)))/2)^2*cosh
(im(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a
)))/2)^2+3*a^2*sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(a)))/2)^2*sin(re(acos(a
/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)^3+a^2*sqrt(abs(a))*B*a*cosh(im(
acos(a/2/abs(a)))/2)^3*sin(re(acos(a/2/abs(a)))/2)^3-3*a^2*sqrt(abs(a))*B*a
*cosh(im(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(
a/2/abs(a)))/2)+3*a^2*sqrt(abs(a))*B*a*cosh(im(acos(a/2/abs(a)))/2)*sin(re(
acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)^2-a^2*sqrt(abs(a))*B*a*
sin(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)^3+sqrt(3)*abs(a)
*a^2*sqrt(abs(a))*B*cos(re(acos(a/2/abs(a)))/2)^3*cosh(im(acos(a/2/abs(a)))
/2)^3-3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cos(re(acos(a/2/abs(a)))/2)^3*cos
h(im(acos(a/2/abs(a)))/2)^2*sinh(im(acos(a/2/abs(a)))/2)+3*sqrt(3)*abs(a)*a
^2*sqrt(abs(a))*B*cos(re(acos(a/2/abs(a)))/2)^3*cosh(im(acos(a/2/abs(a)))/2
)*sinh(im(acos(a/2/abs(a)))/2)^2-sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cos(re(a
cos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)^3-3*sqrt(3)*abs(a)*a^2*s
qrt(abs(a))*B*cos(re(acos(a/2/abs(a)))/2)*cosh(im(acos(a/2/abs(a)))/2)^3*si
n(re(acos(a/2/abs(a)))/2)^2+9*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cos(re(acos
(a/2/abs(a)))/2)*cosh(im(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/abs(a)))/2)
^2*sinh(im(acos(a/2/abs(a)))/2)-9*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cos(re(
acos(a/2/abs(a)))/2)*cosh(im(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/
2)^2*sinh(im(acos(a/2/abs(a)))/2)^2+3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cos
(re(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)^2*sinh(im(acos(a/2/abs
(a)))/2)^3)*1/4/sqrt(3)/a^4*ln(x^2+2*sqrt(abs(a))*cos(acos(a*1/2/abs(a))/2)
*x+sqrt(abs(a))*sqrt(abs(a)))-(-abs(a)*sqrt(abs(a))*A*a*cos(re(acos(a/2/abs
(a)))/2)*cosh(im(acos(a/2/abs(a)))/2)+abs(a)*sqrt(abs(a))*A*a*cos(re(acos(a
/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)-sqrt(3)*a^2*sqrt(abs(a))*A*cosh
(im(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)+sqrt(3)*a^2*sqrt(abs(a
))*A*sin(re(acos(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)-a^2*sqrt(abs(
a))*B*a*cos(re(acos(a/2/abs(a)))/2)^3*cosh(im(acos(a/2/abs(a)))/2)^3+3*a^2*
sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(a)))/2)^3*cosh(im(acos(a/2/abs(a)))/2)
^2*sinh(im(acos(a/2/abs(a)))/2)-3*a^2*sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(
a)))/2)^3*cosh(im(acos(a/2/abs(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)^2+a^2*s
qrt(abs(a))*B*a*cos(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)^
```

```
3+3*a^2*sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(a)))/2)*cosh(im(acos(a/2/abs(a
)))/2)^3*sin(re(acos(a/2/abs(a)))/2)^2-9*a^2*sqrt(abs(a))*B*a*cos(re(acos(a
/2/abs(a)))/2)*cosh(im(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/abs(a)))/2)^2
*sinh(im(acos(a/2/abs(a)))/2)+9*a^2*sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(a)
))/2)*cosh(im(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/abs(a)))/2)^2*sinh(im(ac
os(a/2/abs(a)))/2)^2-3*a^2*sqrt(abs(a))*B*a*cos(re(acos(a/2/abs(a)))/2)*sin
(re(acos(a/2/abs(a)))/2)^2*sinh(im(acos(a/2/abs(a)))/2)^3-3*sqrt(3)*abs(a)*
a^2*sqrt(abs(a))*B*cos(re(acos(a/2/abs(a)))/2)^2*cosh(im(acos(a/2/abs(a)))/
2)^3*sin(re(acos(a/2/abs(a)))/2)+9*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cos(re
(acos(a/2/abs(a)))/2)^2*cosh(im(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/abs(
a)))/2)*sinh(im(acos(a/2/abs(a)))/2)-9*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*co
s(re(acos(a/2/abs(a)))/2)^2*cosh(im(acos(a/2/abs(a)))/2)*sin(re(acos(a/2/ab
s(a)))/2)*sinh(im(acos(a/2/abs(a)))/2)^2+3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*
B*cos(re(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/abs(a)))/2)*sinh(im(acos(a/
2/abs(a)))/2)^3+sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cosh(im(acos(a/2/abs(a)))
/2)^3*sin(re(acos(a/2/abs(a)))/2)^3-3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cos
h(im(acos(a/2/abs(a)))/2)^2*sin(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/
abs(a)))/2)+3*sqrt(3)*abs(a)*a^2*sqrt(abs(a))*B*cosh(im(acos(a/2/abs(a)))/2
)*sin(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)))/2)^2-sqrt(3)*abs(
a)*a^2*sqrt(abs(a))*B*sin(re(acos(a/2/abs(a)))/2)^3*sinh(im(acos(a/2/abs(a)
))/2)^3)*1/2/sqrt(3)/a^4*atan((x+cos(acos(a*1/2/abs(a))/2)*sqrt(abs(a)))/si
n(acos(a*1/2/abs(a))/2)/sqrt(abs(a)))
```

maple [A] time = 0.03, size = 190, normalized size = 1.40

$$\frac{B\arctan\left(\frac{2x+\sqrt{3}\,\sqrt{a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{B\arctan\left(\frac{-2x+\sqrt{3}\,\sqrt{a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{3}\,B\ln\left(x^2+\sqrt{3}\,\sqrt{a}\,x+a\right)}{12\sqrt{a}}+\frac{\sqrt{3}\,B\ln\left(-x^2+\sqrt{3}\,\sqrt{a}\,x-a\right)}{12\sqrt{a}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(x^4-a*x^2+a^2),x)
```

```
[Out] 1/12/a^(1/2)*ln(-x^2+3^(1/2)*a^(1/2)*x-a)*B*3^(1/2)-1/12/a^(3/2)*ln(-x^2+3^
(1/2)*a^(1/2)*x-a)*A*3^(1/2)-1/2/a^(1/2)*arctan((-2*x+3^(1/2)*a^(1/2))/a^(1
/2))*B-1/2/a^(3/2)*arctan((-2*x+3^(1/2)*a^(1/2))/a^(1/2))*A-1/12/a^(1/2)*ln
(x^2+3^(1/2)*a^(1/2)*x+a)*B*3^(1/2)+1/12/a^(3/2)*ln(x^2+3^(1/2)*a^(1/2)*x+a
)*A*3^(1/2)+1/2/a^(1/2)*arctan((2*x+3^(1/2)*a^(1/2))/a^(1/2))*B+1/2/a^(3/2)
*arctan((2*x+3^(1/2)*a^(1/2))/a^(1/2))*A
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{Bx^2+A}{x^4-ax^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2), x)
```

mupad [B] time = 4.59, size = 1007, normalized size = 7.40

$$\mathrm{atan}\left(\frac{A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}\,6\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}}+\frac{2\,\sqrt{3}\,A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a^2 - a*x^2 + x^4),x)
```

```
[Out] atan((A^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^
3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 +
 (3^(1/2)*A^3*1i)/a - A*B^2*a - 3^(1/2)*A*B^2*a*1i) + (2*3^(1/2)*A^2*x*((3^
(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24*a^3
) - (A*B)/(6*a^2))^(1/2))/(2*A^2*B + A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*1i)/a
 - A*B^2*a - 3^(1/2)*A*B^2*a*1i) - (B^2*a^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^
2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2)*
6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*1i)/a - A*B^2*a - 3^(1/2)*A
*B^2*a*1i) - (2*3^(1/2)*B^2*a^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (
3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2))/(2*A^2*B +
A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*1i)/a - A*B^2*a - 3^(1/2)*A*B^2*a*1i))*(-(
3^(1/2)*A^2*1i + A^2 + B^2*a^2 - 3^(1/2)*B^2*a^2*1i + 4*A*B*a)/(24*a^3))^(1
/2)*2i + atan((A^2*x*((3^(1/2)*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3)
 - (3^(1/2)*B^2*1i)/(24*a) - (A*B)/(6*a^2))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*
B^3*a^2 - (3^(1/2)*A^3*1i)/a - A*B^2*a + 3^(1/2)*A*B^2*a*1i) - (2*3^(1/2)*A
^2*x*((3^(1/2)*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3) - (3^(1/2)*B^2*
1i)/(24*a) - (A*B)/(6*a^2))^(1/2))/(2*A^2*B + A^3/a - 2*B^3*a^2 - (3^(1/2)*
A^3*1i)/a - A*B^2*a + 3^(1/2)*A*B^2*a*1i) - (B^2*a^2*x*((3^(1/2)*A^2*1i)/(2
4*a^3) - B^2/(24*a) - A^2/(24*a^3) - (3^(1/2)*B^2*1i)/(24*a) - (A*B)/(6*a^2
))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 - (3^(1/2)*A^3*1i)/a - A*B^2*a +
3^(1/2)*A*B^2*a*1i) + (2*3^(1/2)*B^2*a^2*x*((3^(1/2)*A^2*1i)/(24*a^3) - B^2
/(24*a) - A^2/(24*a^3) - (3^(1/2)*B^2*1i)/(24*a) - (A*B)/(6*a^2))^(1/2))/(2
*A^2*B + A^3/a - 2*B^3*a^2 - (3^(1/2)*A^3*1i)/a - A*B^2*a + 3^(1/2)*A*B^2*a
*1i))*(-(A^2 - 3^(1/2)*A^2*1i + B^2*a^2 + 3^(1/2)*B^2*a^2*1i + 4*A*B*a)/(24
*a^3))^(1/2)*2i
```

sympy [A] time = 1.91, size = 172, normalized size = 1.26

$$\operatorname{RootSum}\left(144t^4a^6 + t^2\left(12A^2a^3 + 48ABa^4 + 12B^2a^5\right) + A^4 + 2A^3Ba + 3A^2B^2a^2 + 2AB^3a^3 + B^4a^4, \left(t \mapsto t\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(x**4-a*x**2+a**2),x)
```

```
[Out] RootSum(144*_t**4*a**6 + _t**2*(12*A**2*a**3 + 48*A*B*a**4 + 12*B**2*a**5)
+ A**4 + 2*A**3*B*a + 3*A**2*B**2*a**2 + 2*A*B**3*a**3 + B**4*a**4, Lambda(
_t, _t*log(x + (24*_t**3*A*a**5 + 48*_t**3*B*a**6 - 2*_t*A**3*a**2 + 6*_t*A
**2*B*a**3 + 12*_t*A*B**2*a**4 + 2*_t*B**3*a**5)/(-A**4 - A**3*B*a + A*B**3
*a**3 + B**4*a**4))))
```

3.110 $\int \frac{A+Bx^2}{a-\sqrt{a}\,x^2+x^4}\,dx$

Optimal. Leaf size=160

$$-\frac{\left(A-\sqrt{a}\,B\right)\log\left(-\sqrt{3}\,\sqrt[4]{a}\,x+\sqrt{a}+x^2\right)}{4\sqrt{3}\,a^{3/4}}+\frac{\left(A-\sqrt{a}\,B\right)\log\left(\sqrt{3}\,\sqrt[4]{a}\,x+\sqrt{a}+x^2\right)}{4\sqrt{3}\,a^{3/4}}-\frac{\left(\sqrt{a}\,B+A\right)\tan^{-1}\left(\sqrt{3}-\frac{2}{\sqrt[4]{}}\right.}{2a^{3/4}}$$

```
[Out] -1/12*ln(x^2-a^(1/4)*x*3^(1/2)+a^(1/2))*(A-B*a^(1/2))/a^(3/4)*3^(1/2)+1/12*
ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))*(A-B*a^(1/2))/a^(3/4)*3^(1/2)+1/2*arctan(
2*x/a^(1/4)-3^(1/2))*(A+B*a^(1/2))/a^(3/4)+1/2*arctan(2*x/a^(1/4)+3^(1/2))*
(A+B*a^(1/2))/a^(3/4)
```

Rubi [A] time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\left(A-\sqrt{a}\,B\right)\log\left(-\sqrt{3}\,\sqrt[4]{a}\,x+\sqrt{a}+x^2\right)}{4\sqrt{3}\,a^{3/4}}+\frac{\left(A-\sqrt{a}\,B\right)\log\left(\sqrt{3}\,\sqrt[4]{a}\,x+\sqrt{a}+x^2\right)}{4\sqrt{3}\,a^{3/4}}-\frac{\left(\sqrt{a}\,B+A\right)\tan^{-1}\left(\sqrt{3}-\frac{2}{\sqrt[4]{}}\right.}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] -((A + Sqrt[a]*B)*ArcTan[Sqrt[3] - (2*x)/a^(1/4)])/(2*a^(3/4)) + ((A + Sqrt
[a]*B)*ArcTan[Sqrt[3] + (2*x)/a^(1/4)])/(2*a^(3/4)) - ((A - Sqrt[a]*B)*Log[
Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3]*a^(3/4)) + ((A - Sqrt[a]*B)*
Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3]*a^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
```

```
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^2}{a-\sqrt{a}\,x^2+x^4}\,dx = \frac{\int \frac{\sqrt{3}\sqrt[4]{a}A-\left(A-\sqrt{a}B\right)x}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}\,a^{3/4}} + \frac{\int \frac{\sqrt{3}\sqrt[4]{a}A+\left(A-\sqrt{a}B\right)x}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}\,a^{3/4}}$$

$$= \frac{1}{4}\left(\frac{A}{\sqrt{a}}+B\right)\int \frac{1}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}\,x+x^2}\,dx + \frac{1}{4}\left(\frac{A}{\sqrt{a}}+B\right)\int \frac{1}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}\,x+x^2}\,dx - \frac{\left(A-\right.}{}$$

$$= -\frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}\,x+x^2\right)}{4\sqrt{3}\,a^{3/4}} + \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}\,x+x^2\right)}{4\sqrt{3}\,a^{3/4}} + \frac{\left(A+\right.}{}$$

$$= -\frac{\left(A+\sqrt{a}B\right)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} + \frac{\left(A+\sqrt{a}B\right)\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} - \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}\right.}{4\sqrt{3}\,a^{3}}$$

Mathematica [C] time = 0.13, size = 138, normalized size = 0.86

$$\frac{\sqrt[4]{-1}\left(\frac{\left(\left(\sqrt{3}-i\right)\sqrt{a}B-2iA\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}-i}} - \frac{\left(\left(\sqrt{3}+i\right)\sqrt{a}B+2iA\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}+i}}\right)}{\sqrt{6}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*Sqrt[a]*B)*ArcTan[((1 + I)*x)/(Sqr
t[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (I + Sqrt[3])*S
qrt[a]*B)*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))])/Sqrt[I + Sqrt[3
]]))/(Sqrt[6]*a^(3/4))
```

fricas [B] time = 0.54, size = 1141, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a
 + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^3 - A^6)*x + 3*sqrt
(1/6)*(A*B^4*a^3 - A^5*a - sqrt(1/3)*(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2
 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - sqrt(1/3)*(A*B^2*a^3
- A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*
a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2
)*sqrt(a))/a^2)) - 1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^
4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^
3 - A^6)*x - 3*sqrt(1/6)*(A*B^4*a^3 - A^5*a - sqrt(1/3)*(2*B^3*a^4 + A^2*B*
a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - sq
rt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqr
t(a))*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/
```

```
a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) + 1/2*sqrt(1/6)*sqrt(-(4*A*B*a - 3*sqrt
(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))
/a^2)*log(2*(B^6*a^3 - A^6)*x + 3*sqrt(1/6)*(A*B^4*a^3 - A^5*a + sqrt(1/3)*
(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3
*a^2 - A^4*B*a + sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2
*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 -
2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) - 1/2*sqrt(1/6)*sqrt
(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^
2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^3 - A^6)*x - 3*sqrt(1/6)*(A*B^4*a^3 -
 A^5*a + sqrt(1/3)*(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A
^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a + sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(
B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^
2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 198, normalized size = 1.24

$$\frac{B\arctan\left(\frac{2x+\sqrt{3}\,a^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{1}{4}}}-\frac{B\arctan\left(\frac{-2x+\sqrt{3}\,a^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{1}{4}}}-\frac{\sqrt{3}\,B\ln\left(x^{2}+\sqrt{3}\,a^{\frac{1}{4}}x+\sqrt{a}\right)}{12a^{\frac{1}{4}}}+\frac{\sqrt{3}\,B\ln\left(-x^{2}+\sqrt{3}\,a^{\frac{1}{4}}x-\sqrt{a}\right)}{12a^{\frac{1}{4}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(a+x^4-a^(1/2)*x^2),x)
```

```
[Out] 1/12/a^(3/4)*ln(x^2+3^(1/2)*a^(1/4)*x+a^(1/2))*A*3^(1/2)-1/12/a^(1/4)*ln(x^
2+3^(1/2)*a^(1/4)*x+a^(1/2))*B*3^(1/2)+1/2/a^(3/4)*arctan((2*x+3^(1/2)*a^(1
/4))/a^(1/4))*A+1/2/a^(1/4)*arctan((2*x+3^(1/2)*a^(1/4))/a^(1/4))*B-1/12/a^
(3/4)*ln(-x^2+3^(1/2)*a^(1/4)*x-a^(1/2))*A*3^(1/2)+1/12/a^(1/4)*ln(-x^2+3^(
1/2)*a^(1/4)*x-a^(1/2))*B*3^(1/2)-1/2/a^(3/4)*arctan((-2*x+3^(1/2)*a^(1/4))
/a^(1/4))*A-1/2/a^(1/4)*arctan((-2*x+3^(1/2)*a^(1/4))/a^(1/4))*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^{2}+A}{x^{4}-\sqrt{a}\,x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - sqrt(a)*x^2 + a), x)
```

mupad [B] time = 4.99, size = 1155, normalized size = 7.22

$$-2\,\mathrm{atanh}\left(\frac{6\,A^2\,x\,\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-\frac{A^2\,\sqrt{-27\,a^3}}{72\,a^3}-\frac{A^2}{24\,a^{3/2}}-\frac{A\,B}{6\,a}}}{2\,A^2\,B-2\,B^3\,a+\frac{A^3}{\sqrt{a}}-A\,B^2\,\sqrt{a}+\frac{A^3\,\sqrt{-27\,a^3}}{3\,a^2}-\frac{A\,B^2\,\sqrt{-27\,a^3}}{3\,a}}-\frac{6\,B^2\,a\,x\,\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-\frac{A^2\,\sqrt{-}}{72}}}{2\,A^2\,B-2\,B^3\,a+\frac{A^3}{\sqrt{a}}-A\,B^2\,\sqrt{a}+\frac{A^3}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + x^4 - a^(1/2)*x^2),x)
```

```
[Out] - 2*atanh((6*A^2*x*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^
2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(2*A^2
*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^2)
- (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (6*B^2*a*x*((B^2*(-27*a^3)^(1/2))/(72*a^
2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) -
 (A*B)/(6*a))^(1/2))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^
3*(-27*a^3)^(1/2))/(3*a^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (2*A^2*x*(-27
*a^3)^(1/2)*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*
a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(3*a^(3/2)*(2
*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a
^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a))) + (2*B^2*x*(-27*a^3)^(1/2)*((B^2*(-27
*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) -
 A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(3*a^(1/2)*(2*A^2*B - 2*B^3*a + A^3
/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^2) - (A*B^2*(-27*a^3)
^(1/2))/(3*a))))*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*
(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2) - 2*atanh
((6*A^2*x*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3
/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2))/(2*A^2*B - 2*B^
3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*B^2*
(-27*a^3)^(1/2))/(3*a)) - (6*B^2*a*x*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/
(24*a^(1/2)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6
*a))^(1/2))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^
3)^(1/2))/(3*a^2) + (A*B^2*(-27*a^3)^(1/2))/(3*a)) + (2*A^2*x*(-27*a^3)^(1/
2)*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2)) -
(B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2))/(3*a^(3/2)*(2*A^2*B -
2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*
B^2*(-27*a^3)^(1/2))/(3*a))) - (2*B^2*x*(-27*a^3)^(1/2)*((A^2*(-27*a^3)^(1/
2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/
(72*a^2) - (A*B)/(6*a))^(1/2))/(3*a^(1/2)*(2*A^2*B - 2*B^3*a + A^3/a^(1/2)
- A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*B^2*(-27*a^3)^(1/2))/(
3*a))))*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2
)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

3.111 $\int \frac{A+Bx^2}{a-\sqrt{ac}\,x^2+cx^4}\,dx$

Optimal. Leaf size=414

$$-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}-$$

```
[Out] -1/2*arctan((-2*x*c^(1/2)+(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))/(2*a^(1/2)
*c^(1/2)-(a*c)^(1/2))^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(1/2)/c^(1/2)/(2*a^(1/
2)*c^(1/2)-(a*c)^(1/2))^(1/2)+1/2*arctan((2*x*c^(1/2)+(2*a^(1/2)*c^(1/2)+(a
*c)^(1/2))^(1/2))/(2*a^(1/2)*c^(1/2)-(a*c)^(1/2))^(1/2))*(B*a^(1/2)+A*c^(1/
2))/a^(1/2)/c^(1/2)/(2*a^(1/2)*c^(1/2)-(a*c)^(1/2))^(1/2)-1/4*ln(a^(1/2)+x^
2*c^(1/2)-x*(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))*(A-B*a^(1/2)/c^(1/2))/a^
(1/2)/(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2)+1/4*ln(a^(1/2)+x^2*c^(1/2)+x*(2
*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))*(A-B*a^(1/2)/c^(1/2))/a^(1/2)/(2*a^(1/
2)*c^(1/2)+(a*c)^(1/2))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]] - 2*S
qrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*S
qrt[a]*Sqrt[c] - Sqrt[a*c]]) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt
[a]*Sqrt[c] + Sqrt[a*c]] + 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]
])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) - ((A - (Sqrt[a]
*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*
x^2])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]) + ((A - (Sqrt[a]*B)/S
qrt[c])*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2])
/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^2}{a-\sqrt{ac}\,x^2+cx^4}\,dx = \frac{\int \frac{\frac{A\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}-\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\,x}{\sqrt{c}}+x^2}\,dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\int \frac{\frac{A\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}+\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\,x}{\sqrt{c}}+x^2}\,dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

$$=\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\,x}{\sqrt{c}}+x^2}\,dx}{4c}+\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\,x}{\sqrt{c}}+x^2}\,dx}{4c}-\frac{\left(A-\frac{\sqrt{a}\cdots}{\sqrt{c}\cdots}\right.\cdots}{4\cdots}$$

$$=-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\,x+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2\sqrt{a}\cdots}\right.}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\cdots}}$$

$$=-\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}+\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}-\frac{(\cdots}{\cdots}$$

Mathematica [C] time = 0.20, size = 247, normalized size = 0.60

$$\frac{\frac{\left(\sqrt{3}\sqrt{a}B\sqrt{c}-i\left(B\sqrt{ac}+2Ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{ac}-i\sqrt{3}\sqrt{a}\sqrt{c}}}\right)}{\sqrt{-\sqrt{ac}-i\sqrt{3}\sqrt{a}\sqrt{c}}}+\frac{\left(\sqrt{3}\sqrt{a}B\sqrt{c}+i\left(B\sqrt{ac}+2Ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{ac}+i\sqrt{3}\sqrt{a}\sqrt{c}}}\right)}{\sqrt{-\sqrt{ac}+i\sqrt{3}\sqrt{a}\sqrt{c}}}}{\sqrt{6}\sqrt{a}c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```

```
[Out] (((Sqrt[3]*Sqrt[a]*B*Sqrt[c] - I*(2*A*c + B*Sqrt[a*c]))*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[(-I)*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/Sqrt[(-I)*Sqrt[3]*
Sqrt[a]*Sqrt[c] - Sqrt[a*c]] + ((Sqrt[3]*Sqrt[a]*B*Sqrt[c] + I*(2*A*c + B*S
qrt[a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[I*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt
[a*c]]])/Sqrt[I*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt[a*c]])/(Sqrt[6]*Sqrt[a]*c)
```

fricas [B] time = 0.66, size = 1457, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c +
A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log
(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3
)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3
*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*
sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c
^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) + 1/2*sqrt(1/6)*s
qrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^
3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A
^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3)*(2*B^3*a^4*c^2
 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A
^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt(-(3*sqrt(1/
3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c
 + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) - 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)
*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c -
 (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqr
t(1/6)*(A*B^4*a^3*c - A^5*a*c^3 + sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)
*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^
4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*
B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(
B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)
*sqrt(a*c))/(a^2*c^2))) + 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*s
qrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*
c - A^5*a*c^3 + sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 -
2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1
/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^3*c^3)))*sqrt(a*c))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c
^2)))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 404, normalized size = 0.98

$$-\frac{A\arctan\left(\frac{-2\sqrt{c}\,x+\sqrt{3}\,(ac)^{\frac{1}{4}}}{\sqrt{4\sqrt{a}\,\sqrt{c}-3\sqrt{ac}}}\right)}{2\sqrt{4\sqrt{a}\,\sqrt{c}-3\sqrt{ac}}\,\sqrt{a}}+\frac{A\arctan\left(\frac{2\sqrt{c}\,x+\sqrt{3}\,(ac)^{\frac{1}{4}}}{\sqrt{4\sqrt{a}\,\sqrt{c}-3\sqrt{ac}}}\right)}{2\sqrt{4\sqrt{a}\,\sqrt{c}-3\sqrt{ac}}\,\sqrt{a}}-\frac{B\arctan\left(\frac{-2\sqrt{c}\,x+\sqrt{3}\,(ac)^{\frac{1}{4}}}{\sqrt{4\sqrt{a}\,\sqrt{c}-3\sqrt{ac}}}\right)}{2\sqrt{4\sqrt{a}\,\sqrt{c}-3\sqrt{ac}}\,\sqrt{c}}+\frac{B\arctan\left(\frac{2\sqrt{c}\,x+\sqrt{3}\,(ac)^{\frac{1}{4}}}{\sqrt{4\sqrt{a}\,\sqrt{c}-3\sqrt{ac}}}\right)}{2\sqrt{4\sqrt{a}\,\sqrt{c}-3\sqrt{ac}}\,\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x)
```

```
[Out] 1/12/a/c^(3/2)*ln(x*3^(1/2)*(a*c)^(1/4)-c^(1/2)*x^2-a^(1/2))*B*3^(1/2)*(a*c
)^(3/4)-1/12/a^(3/2)/c*ln(x*3^(1/2)*(a*c)^(1/4)-c^(1/2)*x^2-a^(1/2))*A*3^(1
/2)*(a*c)^(3/4)-1/2/a^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan(
(3^(1/2)*(a*c)^(1/4)-2*c^(1/2)*x)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*
A-1/2/c^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((3^(1/2)*(a*c)
^(1/4)-2*c^(1/2)*x)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*B-1/12/a/c^(3/
2)*ln(c^(1/2)*x^2+x*3^(1/2)*(a*c)^(1/4)+a^(1/2))*B*3^(1/2)*(a*c)^(3/4)+1/12
/a^(3/2)/c*ln(c^(1/2)*x^2+x*3^(1/2)*(a*c)^(1/4)+a^(1/2))*A*3^(1/2)*(a*c)^(3
/4)+1/2/a^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((2*c^(1/2)*x
+3^(1/2)*(a*c)^(1/4))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*A+1/2/c^(1/2
)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((2*c^(1/2)*x+3^(1/2)*(a*c)
^(1/4))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{cx^4 - \sqrt{ac}\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 - sqrt(a*c)*x^2 + a), x)
```

mupad [B] time = 5.22, size = 3285, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + c*x^4 - x^2*(a*c)^(1/2)),x)
```

```
[Out] - atan(((((12*A*a)/c^2 - (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(B
^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) -
A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*
(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c
*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c
^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)
 + (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3
*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(
3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/
(72*a^3*c^3))^(1/2)*1i - (((12*A*a)/c^2 + (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*
(a*c)^(1/2))*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2
*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/
2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(B^2*a*(-27*a^3*c
^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/
2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(7
2*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*
(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2
) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^
2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)*1i)/((((12*A*a)/c^2 - (2*x*(4*c*(a*c)^
(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3
*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^
2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-
(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2)
- A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*
c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a
*c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) -
 B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)
^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (((12*A*a)/c^2 + (2
```

```
*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(B^2*a*(-27*a^3*c^3)^(1/2) -
A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*
a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^
(1/2))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*
a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2
) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*
c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^
3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A
^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (2*(B
^3*a + A^2*B*c + A*B^2*(a*c)^(1/2)))/c^4))*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A
^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a
^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(
1/2)*2i - atan(((((12*A*a)/c^2 - (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/
2))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^
(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^
2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2)
 - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A
*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3
))^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(A^2*c*
(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c
*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)
^(1/2))/(72*a^3*c^3))^(1/2)*1i - (((12*A*a)/c^2 + (2*x*(4*c*(a*c)^(3/2) - 1
6*a*c^2*(a*c)^(1/2))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/
2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(
a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(A^2*c*(-
27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(
a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(
1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)
))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a
*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) +
4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)*1i)/((((12*A*a)/c^2 - (2*x*(4*
c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*
(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^
2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))
/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c
)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*
B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*
A*B*c*(a*c)^(1/2)))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)
^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c
^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (((12*A*a)/
c^2 + (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(A^2*c*(-27*a^3*c^3)^
(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) +
 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^
3*c^3))^(1/2))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2
) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a
*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2
- B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a
*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c
^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)
 + (2*(B^3*a + A^2*B*c + A*B^2*(a*c)^(1/2)))/c^4))*(-(A^2*c*(-27*a^3*c^3)^(
1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) +
12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3
*c^3))^(1/2)*2i
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+c*x**4-x**2*(a*c)**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

3.112 $\int \frac{A+Bx^2}{a-\sqrt{a}\,\sqrt{c}\,x^2+cx^4}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\tan^{-1}\left(\frac{2\sqrt[4]{c}\,x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}c^{3/4}}-\frac{\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\log\left(-\sqrt{3}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{\right.}}{4\sqrt{3}\,a^{3/4}\sqrt[4]{c}}$$

```
[Out] -1/12*ln(-a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c^(1/2))*(A-B*a^(1/2)/c^(1/
2))/a^(3/4)/c^(1/4)*3^(1/2)+1/12*ln(a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c
^(1/2))*(A-B*a^(1/2)/c^(1/2))/a^(3/4)/c^(1/4)*3^(1/2)+1/2*arctan(2*c^(1/4)*
x/a^(1/4)-3^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(3/4)/c^(3/4)+1/2*arctan(2*c^(1/
4)*x/a^(1/4)+3^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(3/4)/c^(3/4)
```

Rubi [A] time = 0.17, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\tan^{-1}\left(\frac{2\sqrt[4]{c}\,x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}c^{3/4}}-\frac{\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\log\left(-\sqrt{3}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{\right.}}{4\sqrt{3}\,a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] - (2*c^(1/4)*x)/a^(1/4)])/(2*a^(3/
4)*c^(3/4)) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] + (2*c^(1/4)*x)/a^(1/
4)])/(2*a^(3/4)*c^(3/4)) - ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[3]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[3]*a^(3/4)*c^(1/4)) + ((A - (Sqr
t[a]*B)/Sqrt[c])*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
*Sqrt[3]*a^(3/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}\,x^2+cx^4}\,dx = \frac{\int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}}-\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{2\sqrt{3}\,a^{3/4}\sqrt[4]{c}} + \frac{\int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}}+\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{2\sqrt{3}\,a^{3/4}\sqrt[4]{c}}$$

$$= \frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c} + \frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c} + \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \cdots}{4\sqrt{3}\,a^{3/\cdots}}$$

$$= \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{3}\,a^{3/4}c^{3/4}} + \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}\,x\cdots\right.}{4\sqrt{3}\,a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{\left(\sqrt{a}B\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.19, size = 163, normalized size = 0.70

$$\frac{\sqrt[4]{-1}\left(\frac{\left(\left(\sqrt{3}-i\right)\sqrt{a}B-2iA\sqrt{c}\right)\tan^{-1}\left(\frac{(1+i)\sqrt[4]{c}\,x}{\sqrt{\sqrt{3}-i}\,\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}-i}}-\frac{\left(\left(\sqrt{3}+i\right)\sqrt{a}B+2iA\sqrt{c}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt[4]{c}\,x}{\sqrt{\sqrt{3}+i}\,\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}+i}}\right)}{\sqrt{6}\,a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-I + Sqrt[3])*Sqrt[a]*B - (2*I)*A*Sqrt[c])*ArcTan[((1 + I)*
c^(1/4)*x)/(Sqrt[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((I + Sqrt[
3])*Sqrt[a]*B + (2*I)*A*Sqrt[c])*ArcTanh[((1 + I)*c^(1/4)*x)/(Sqrt[I + Sqrt
[3]]*a^(1/4))])/Sqrt[I + Sqrt[3]]))/(Sqrt[6]*a^(3/4)*c^(3/4))
```

fricas [B] time = 1.17, size = 1469, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c +
A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2
))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - (A
^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) - sqrt(1/3)
```

```
*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/
(a^3*c^3)))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)))
 + 1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c
^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
(A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt
(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) - sqrt(1/
3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2
)/(a^3*c^3)))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
)) - 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
 + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*
c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
 (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqr
t(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) + sqrt(1
/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)/(a^3*c^3)))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
)) + 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
 + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*
c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
 (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqr
t(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) + sqrt(1
/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)/(a^3*c^3)))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 320, normalized size = 1.37

$$\frac{A\arctan\left(\frac{2\sqrt{c}\,x+\sqrt{3}\,a^{\frac{1}{4}}c^{\frac{1}{4}}}{\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{a}}-\frac{A\arctan\left(\frac{-2\sqrt{c}\,x+\sqrt{3}\,a^{\frac{1}{4}}c^{\frac{1}{4}}}{\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{a}}+\frac{B\arctan\left(\frac{2\sqrt{c}\,x+\sqrt{3}\,a^{\frac{1}{4}}c^{\frac{1}{4}}}{\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}-\frac{B\arctan\left(\frac{-2\sqrt{c}\,x+\sqrt{3}\,a^{\frac{1}{4}}c^{\frac{1}{4}}}{\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x)
```

```
[Out] -1/12/c^(1/4)/a^(3/4)*ln(a^(1/4)*c^(1/4)*x*3^(1/2)-c^(1/2)*x^2-a^(1/2))*A*3
^(1/2)+1/12/c^(3/4)/a^(1/4)*ln(a^(1/4)*c^(1/4)*x*3^(1/2)-c^(1/2)*x^2-a^(1/2
))*B*3^(1/2)-1/2/a^(1/2)/(a^(1/2)*c^(1/2))^(1/2)*arctan((3^(1/2)*c^(1/4)*a^
(1/4)-2*c^(1/2)*x)/(a^(1/2)*c^(1/2))^(1/2))*A-1/2/c^(1/2)/(a^(1/2)*c^(1/2))
^(1/2)*arctan((3^(1/2)*c^(1/4)*a^(1/4)-2*c^(1/2)*x)/(a^(1/2)*c^(1/2))^(1/2)
)*B+1/12/c^(1/4)/a^(3/4)*ln(a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+c^(1/2)*x^2)*
A*3^(1/2)-1/12/c^(3/4)/a^(1/4)*ln(a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+c^(1/2)
*x^2)*B*3^(1/2)+1/2/a^(1/2)/(a^(1/2)*c^(1/2))^(1/2)*arctan((2*c^(1/2)*x+3^(
1/2)*c^(1/4)*a^(1/4))/(a^(1/2)*c^(1/2))^(1/2))*A+1/2/c^(1/2)/(a^(1/2)*c^(1/
```

```
2))^(1/2)*arctan((2*c^(1/2)*x+3^(1/2)*c^(1/4)*a^(1/4))/(a^(1/2)*c^(1/2))^(1
/2))*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{cx^4 - \sqrt{a}\,\sqrt{c}\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 - sqrt(a)*sqrt(c)*x^2 + a), x)
```

mupad [B] time = 5.29, size = 1575, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + c*x^4 - a^(1/2)*c^(1/2)*x^2),x)
```

```
[Out] - 2*atanh((6*A^2*x*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2
)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(
24*a^(3/2)*c^(1/2)))^(1/2))/((2*A^2*B)/c - (2*B^3*a)/c^2 + A^3/(a^(1/2)*c^(
1/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^2) - (A*B^2*a^(1/2))/c^(3/2) - (
A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^3)) - (6*B^2*a*x*((B^2*(-27*a^3*c^3)^(1/2
))/(72*a^2*c^3) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*
c^3)^(1/2))/(72*a^3*c^2) - A^2/(24*a^(3/2)*c^(1/2)))^(1/2))/(2*A^2*B - (2*B
^3*a)/c + (A^3*c^(1/2))/a^(1/2) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c) - (A*
B^2*a^(1/2))/c^(1/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^2)) - (2*A^2*x*(-
27*a^3*c^3)^(1/2)*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2)
*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(2
4*a^(3/2)*c^(1/2)))^(1/2))/(3*a^(3/2)*c^(7/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^
4 + A^3/(a^(1/2)*c^(5/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*
a^(1/2))/c^(7/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))) + (2*B^2*x*(-27*
a^3*c^3)^(1/2)*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2)*c^
(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(24*a
^(3/2)*c^(1/2)))^(1/2))/(3*a^(1/2)*c^(9/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^4 +
 A^3/(a^(1/2)*c^(5/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*a^(
1/2))/c^(7/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))))*((B^2*(-27*a^3*c^3
)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-2
7*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(24*a^(3/2)*c^(1/2)))^(1/2) - 2*atanh(
(6*A^2*x*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2))
 - (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72
*a^2*c^3))^(1/2))/((2*A^2*B)/c - (2*B^3*a)/c^2 + A^3/(a^(1/2)*c^(1/2)) - (A
^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^2) - (A*B^2*a^(1/2))/c^(3/2) + (A*B^2*(-27
*a^3*c^3)^(1/2))/(3*a*c^3)) - (6*B^2*a*x*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3
*c^2) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2))
 - (B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3))^(1/2))/(2*A^2*B - (2*B^3*a)/c +
(A^3*c^(1/2))/a^(1/2) - (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c) - (A*B^2*a^(1/2
))/c^(1/2) + (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^2)) + (2*A^2*x*(-27*a^3*c^3
)^(1/2)*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2))
- (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72*
a^2*c^3))^(1/2))/(3*a^(3/2)*c^(7/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^4 + A^3/(a
^(1/2)*c^(5/2)) - (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*a^(1/2))/c
^(7/2) + (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))) - (2*B^2*x*(-27*a^3*c^3)^(
1/2)*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2)) - (
A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72*a^2
*c^3))^(1/2))/(3*a^(1/2)*c^(9/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^4 + A^3/(a^(1
/2)*c^(5/2)) - (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*a^(1/2))/c^(7
```

```
/2) + (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))))*((A^2*(-27*a^3*c^3)^(1/2))/(
72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^
(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3))^(1/2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+c*x**4-x**2*a**(1/2)*c**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

3.113 $\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx$

Optimal. Leaf size=96

$$\sqrt{7+2\sqrt{13}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{13}-1\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(1+13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))*(
-2+2*13^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(1+13^(1/2))^(1/2),1/6*I*3^(1/2)+1
/6*I*39^(1/2))*(7+2*13^(1/2))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1180, 524, 424, 419}

$$\sqrt{7+2\sqrt{13}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{13}-1\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
```

```
[Out] -(Sqrt[(-1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(1 + Sqrt[13])]*x], (-7 -
 Sqrt[13])/6]) + Sqrt[7 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[13]
)]*x], (-7 - Sqrt[13])/6]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = 2\int \frac{3-x^2}{\sqrt{1+\sqrt{13}-2x^2}\,\sqrt{-1+\sqrt{13}+2x^2}}\,dx$$

$$= \left(5+\sqrt{13}\right)\int \frac{1}{\sqrt{1+\sqrt{13}-2x^2}\,\sqrt{-1+\sqrt{13}+2x^2}}\,dx - \int \frac{\sqrt{-1+\sqrt{13}+2x^2}}{\sqrt{1+\sqrt{13}-2x^2}}\,dx$$

$$= -\sqrt{\frac{1}{2}\left(-1+\sqrt{13}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)\middle|\frac{1}{6}\left(-7-\sqrt{13}\right)\right) + \sqrt{7+2\sqrt{13}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\cdot}{1\cdot}}\right.\right.$$

Mathematica [C] time = 0.14, size = 103, normalized size = 1.07

$$-\frac{i\left(\left(1+\sqrt{13}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)\middle|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)-\left(\sqrt{13}-5\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)\middle|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)\right)}{\sqrt{2\left(1+\sqrt{13}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
[Out] ((-I)*((1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]*x], (-7 +
 Sqrt[13])/6] - (-5 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]
*x], (-7 + Sqrt[13])/6]))/Sqrt[2*(1 + Sqrt[13])]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+3}\left(x^2-3\right)}{x^4-x^2-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + x^2 + 3)*(x^2 - 3)/(x^4 - x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

maple [B] time = 0.11, size = 200, normalized size = 2.08

$$\frac{18\sqrt{-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{-6+6\sqrt{13}}\,x}{6}, \frac{i\sqrt{3}}{6}+\frac{i\sqrt{39}}{6}\right)}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x^4+x^2+3}} + \frac{36\sqrt{-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4+x^2+3)^(1/2),x)
```

```
[Out] 36/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*13^
(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)/(1+13^(1/2))*(EllipticF(1/6*x*(-6+6*13
^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))-EllipticE(1/6*x*(-6+6*13^(1/2))
^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2)))+18/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6
*13^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)*
EllipticF(1/6*x*(-6+6*13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(x^2 - x^4 + 3)^(1/2),x)
[Out] -int((x^2 - 3)/(x^2 - x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{-x^4+x^2+3}}\,dx-\int \left(-\frac{3}{\sqrt{-x^4+x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4+x**2+3)**(1/2),x)
[Out] -Integral(x**2/sqrt(-x**4 + x**2 + 3), x) - Integral(-3/sqrt(-x**4 + x**2 +
 3), x)
```

3.114 $\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx$

Optimal. Leaf size=25

$$4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

```
[Out] -EllipticE(1/3*x*3^(1/2),I*3^(1/2))+4*EllipticF(1/3*x*3^(1/2),I*3^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1180, 21, 423, 424, 419}

$$4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
```

```
[Out] -EllipticE[ArcSin[x/Sqrt[3]], -3] + 4*EllipticF[ArcSin[x/Sqrt[3]], -3]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx &= 2\int \frac{3-x^2}{\sqrt{6-2x^2}\,\sqrt{2+2x^2}}\,dx\\ &= \int \frac{\sqrt{6-2x^2}}{\sqrt{2+2x^2}}\,dx\\ &= 8\int \frac{1}{\sqrt{6-2x^2}\,\sqrt{2+2x^2}}\,dx - \int \frac{\sqrt{2+2x^2}}{\sqrt{6-2x^2}}\,dx\\ &= -E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle| -3\right) + 4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle| -3\right)\end{aligned}$$

Mathematica [C] time = 0.06, size = 19, normalized size = 0.76

$$-i\sqrt{3}\,E\left(i\sinh^{-1}(x)\middle| -\frac{1}{3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
[Out] (-I)*Sqrt[3]*EllipticE[I*ArcSinh[x], -1/3]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-x^4+2\,x^2+3}}{x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 2*x^2 + 3)/(x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-3}{\sqrt{-x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

maple [B] time = 0.02, size = 113, normalized size = 4.52

$$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,x}{3}, i\sqrt{3}\right)}{\sqrt{-x^4+2x^2+3}} + \frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{x^2+1}\ \left(-\mathrm{EllipticE}\left(\frac{\sqrt{3}\,x}{3}, i\sqrt{3}\right)+\mathrm{EllipticF}\left(\right.\right.}{3\sqrt{-x^4+2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x)
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)*(EllipticF(
1/3*3^(1/2)*x,I*3^(1/2))-EllipticE(1/3*3^(1/2)*x,I*3^(1/2)))+3^(1/2)*(-3*x^
2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)*EllipticF(1/3*3^(1/2)*x,I*3^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-3}{\sqrt{-x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^2-3}{\sqrt{-x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(2*x^2 - x^4 + 3)^(1/2),x)
```

```
[Out] int(-(x^2 - 3)/(2*x^2 - x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{-x^4+2x^2+3}}\,dx-\int \left(-\frac{3}{\sqrt{-x^4+2x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4+2*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 2*x*
*2 + 3), x)
```

3.115 $\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx$

Optimal. Leaf size=96

$$\sqrt{9+2\sqrt{21}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{21}-3\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(3+21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))*(-
6+2*21^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(3+21^(1/2))^(1/2),1/2*I*3^(1/2)+1/
2*I*7^(1/2))*(9+2*21^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1180, 524, 424, 419}

$$\sqrt{9+2\sqrt{21}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{21}-3\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
```

```
[Out] -(Sqrt[(-3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(3 + Sqrt[21])]*x], (-5 -
 Sqrt[21])/2]) + Sqrt[9 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(3 + Sqrt[21]
)]*x], (-5 - Sqrt[21])/2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx = 2\int \frac{3-x^2}{\sqrt{3+\sqrt{21}-2x^2}\,\sqrt{-3+\sqrt{21}+2x^2}}\,dx$$
$$= \left(3+\sqrt{21}\right)\int \frac{1}{\sqrt{3+\sqrt{21}-2x^2}\,\sqrt{-3+\sqrt{21}+2x^2}}\,dx - \int \frac{\sqrt{-3+\sqrt{21}+2x^2}}{\sqrt{3+\sqrt{21}-2x^2}}\,dx$$
$$= -\sqrt{\frac{1}{2}\left(-3+\sqrt{21}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)\middle|\frac{1}{2}\left(-5-\sqrt{21}\right)\right) + \frac{1}{2}\sqrt{36+8\sqrt{21}}\,F\left(\sin^{-1}\right.$$

Mathematica [C] time = 0.17, size = 103, normalized size = 1.07

$$-\frac{i\left(\left(3+\sqrt{21}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)\middle|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)-\left(\sqrt{21}-3\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)\middle|\frac{1}{2}\left(-5+\sqrt{21}\right)\right.\right.}{\sqrt{2\left(3+\sqrt{21}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
```

```
[Out] ((-I)*((3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21])]*x], (-5 +
 Sqrt[21])/2] - (-3 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21])]
*x], (-5 + Sqrt[21])/2]))/Sqrt[2*(3 + Sqrt[21])]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+3\,x^2+3}\left(x^2-3\right)}{x^4-3\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 3*x^2 + 3)*(x^2 - 3)/(x^4 - 3*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-3}{\sqrt{-x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

maple [B] time = 0.10, size = 204, normalized size = 2.12

$$\frac{18\sqrt{-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-18+6\sqrt{21}}\,x}{6},\frac{i\sqrt{3}}{2}+\frac{i\sqrt{7}}{2}\right)}{\sqrt{-18+6\sqrt{21}}\,\sqrt{-x^4+3x^2+3}}+\frac{36\sqrt{-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x)
```

```
[Out] 36/(-18+6*21^(1/2))^(1/2)*(1-(-1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*21
^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^(1/2)/(3+21^(1/2))*(EllipticF(1/6*x*(-18+
6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))-EllipticE(1/6*x*(-18+6*21^(1
/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2)))+18/(-18+6*21^(1/2))^(1/2)*(1-(-1/2
+1/6*21^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^
(1/2)*EllipticF(1/6*x*(-18+6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 3}{\sqrt{-x^4 + 3\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2 - 3}{\sqrt{-x^4 + 3\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(3*x^2 - x^4 + 3)^(1/2),x)
```

```
[Out] int(-(x^2 - 3)/(3*x^2 - x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{-x^4 + 3x^2 + 3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4 + 3x^2 + 3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4+3*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 3*x*
*2 + 3), x)
```

3.116 $\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx$

Optimal. Leaf size=92

$$\sqrt{5+2\sqrt{13}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)\middle|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)\middle|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(-1+13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))*
(2+2*13^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(-1+13^(1/2))^(1/2),1/6*I*39^(1/2)
-1/6*I*3^(1/2))*(5+2*13^(1/2))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1180, 524, 424, 419}

$$\sqrt{5+2\sqrt{13}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)\middle|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)\middle|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```

```
[Out] -(Sqrt[(1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(-1 + Sqrt[13])]*x], (-7 +
 Sqrt[13])/6]) + Sqrt[5 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[13
])]*x], (-7 + Sqrt[13])/6]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = 2\int \frac{3-x^2}{\sqrt{-1+\sqrt{13}-2x^2}\sqrt{1+\sqrt{13}+2x^2}}\,dx$$
$$= \left(7+\sqrt{13}\right)\int \frac{1}{\sqrt{-1+\sqrt{13}-2x^2}\sqrt{1+\sqrt{13}+2x^2}}\,dx - \int \frac{\sqrt{1+\sqrt{13}+2x^2}}{\sqrt{-1+\sqrt{13}-2x^2}}\,dx$$
$$= -\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)\middle|\frac{1}{6}\left(-7+\sqrt{13}\right)\right) + \sqrt{5+2\sqrt{13}}\,F\left(\sin^{-1}\left(\sqrt{\frac{}{-1}}\right.\right.$$

Mathematica [C] time = 0.13, size = 107, normalized size = 1.16

$$-\frac{i\left(\left(\sqrt{13}-1\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)\middle|-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)-\left(\sqrt{13}-7\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)\middle|-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)\right)}{\sqrt{2\left(\sqrt{13}-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```

```
[Out] ((-I)*((-1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]*x], -7/6
- Sqrt[13]/6] - (-7 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]*
x], -7/6 - Sqrt[13]/6]))/Sqrt[2*(-1 + Sqrt[13])]
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4-x^2+3}\left(x^2-3\right)}{x^4+x^2-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - x^2 + 3)*(x^2 - 3)/(x^4 + x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

maple [B] time = 0.10, size = 204, normalized size = 2.22

$$\frac{18\sqrt{-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{6+6\sqrt{13}}\,x}{6}, \frac{i\sqrt{39}}{6}-\frac{i\sqrt{3}}{6}\right)}{\sqrt{6+6\sqrt{13}}\,\sqrt{-x^4-x^2+3}}+\frac{36\sqrt{-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4-x^2+3)^(1/2),x)
```

```
[Out] 36/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*13^(1/
2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)/(-1+13^(1/2))*(EllipticF(1/6*x*(6+6*13^(1
/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))-EllipticE(1/6*x*(6+6*13^(1/2))^(1/
2),1/6*I*39^(1/2)-1/6*I*3^(1/2)))+18/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1
/2))*x^2)^(1/2)*(1-(1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)*Ellipti
cF(1/6*x*(6+6*13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(3 - x^4 - x^2)^(1/2),x)
```

```
[Out] int(-(x^2 - 3)/(3 - x^4 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{-x^4-x^2+3}}\,dx-\int \left(-\frac{3}{\sqrt{-x^4-x^2+3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4-x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - x**2 + 3), x) - Integral(-3/sqrt(-x**4 - x**2 +
 3), x)
```

3.117 $\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx$

Optimal. Leaf size=27

$$2\sqrt{3}F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)-\sqrt{3}E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)$$

```
[Out] -EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+2*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1180, 524, 424, 419}

$$2\sqrt{3}F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)-\sqrt{3}E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] -(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) + 2*Sqrt[3]*EllipticF[ArcSin[x], -1/3
]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx &= 2\int \frac{3-x^2}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx \\ &= 12\int \frac{1}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx - \int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx \\ &= -\sqrt{3}\,E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)+2\sqrt{3}\,F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)\end{aligned}$$

Mathematica [C] time = 0.06, size = 35, normalized size = 1.30

$$-i\left(2F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)+E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
[Out] (-I)*(EllipticE[I*ArcSinh[x/Sqrt[3]], -3] + 2*EllipticF[I*ArcSinh[x/Sqrt[3]
], -3])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4-2\,x^2+3}\,\left(x^2-3\right)}{x^4+2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 - 2*x^2 + 3)*(x^2 - 3)/(x^4 + 2*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

maple [B] time = 0.01, size = 95, normalized size = 3.52

$$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(-\operatorname{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)+\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{\sqrt{-x^4-2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x)
[Out] (-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1
/2))-EllipticE(x,1/3*I*3^(1/2)))+(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2
+3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(3 - x^4 - 2*x^2)^(1/2),x)
```

```
[Out] int(-(x^2 - 3)/(3 - x^4 - 2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{-x^4-2x^2+3}}\,dx-\int \left(-\frac{3}{\sqrt{-x^4-2x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 2*x*
*2 + 3), x)
```

3.118 $\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx$

Optimal. Leaf size=92

$$\sqrt{3+2\sqrt{21}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)-\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5+\sqrt{2\right.$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(-3+21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))*(
6+2*21^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(-3+21^(1/2))^(1/2),1/2*I*7^(1/2)-1
/2*I*3^(1/2))*(3+2*21^(1/2))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1180, 524, 424, 419}

$$\sqrt{3+2\sqrt{21}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)-\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5+\sqrt{2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

```
[Out] -(Sqrt[(3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(-3 + Sqrt[21])]*x], (-5 +
 Sqrt[21])/2]) + Sqrt[3 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(-3 + Sqrt[21
])]*x], (-5 + Sqrt[21])/2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = 2\int \frac{3-x^2}{\sqrt{-3+\sqrt{21}-2x^2}\sqrt{3+\sqrt{21}+2x^2}}\,dx$$
$$= \left(9+\sqrt{21}\right)\int \frac{1}{\sqrt{-3+\sqrt{21}-2x^2}\sqrt{3+\sqrt{21}+2x^2}}\,dx - \int \frac{\sqrt{3+\sqrt{21}+2x^2}}{\sqrt{-3+\sqrt{21}-2x^2}}\,dx$$
$$= -\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)\middle|\frac{1}{2}\left(-5+\sqrt{21}\right)\right) + \sqrt{3+2\sqrt{21}}\,F\left(\sin^{-1}\left(\sqrt{-\right.\right.$$

Mathematica [C] time = 0.17, size = 107, normalized size = 1.16

$$-\frac{i\left(\left(\sqrt{21}-3\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)\middle|-\frac{5}{2}-\frac{\sqrt{21}}{2}\right)-\left(\sqrt{21}-9\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)\middle|-\frac{5}{2}-\frac{\sqrt{21}}{2}\right)\right)}{\sqrt{2\left(\sqrt{21}-3\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

```
[Out] ((-I)*((-3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(3 + Sqrt[21])]*x], -5/2
- Sqrt[21]/2] - (-9 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(3 + Sqrt[21])]*
x], -5/2 - Sqrt[21]/2]))/Sqrt[2*(-3 + Sqrt[21])]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-x^4-3\,x^2+3}\,\left(x^2-3\right)}{x^4+3\,x^2-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - 3*x^2 + 3)*(x^2 - 3)/(x^4 + 3*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

maple [B] time = 0.09, size = 204, normalized size = 2.22

$$\frac{18\sqrt{-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2+1}\,\sqrt{-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{18+6\sqrt{21}}\,x}{6}, \frac{i\sqrt{7}}{2}-\frac{i\sqrt{3}}{2}\right)}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}} + \frac{36\sqrt{-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2+1}\,\sqrt{-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x)
```

```
[Out] 36/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*21^(1
/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)/(-3+21^(1/2))*(EllipticF(1/6*x*(18+6*2
1^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))-EllipticE(1/6*x*(18+6*21^(1/2))
^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2)))+18/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*2
1^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)*E
llipticF(1/6*x*(18+6*21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(3 - x^4 - 3*x^2)^(1/2),x)
[Out] int(-(x^2 - 3)/(3 - x^4 - 3*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{-x^4-3x^2+3}}\,dx-\int \left(-\frac{3}{\sqrt{-x^4-3x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4-3*x**2+3)**(1/2),x)
[Out] -Integral(x**2/sqrt(-x**4 - 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 3*x*
*2 + 3), x)
```

3.119 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=296

$$\frac{\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}$$

```
[Out] 2*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/(a^(1/2)+x^2*c^(1/2))-2*a^(1/4)*c^(1/4)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/2)-(-4*a*c+
b^2)^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)
/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1197, 1103, 1195}

$$\frac{\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) - (2*a^(1/4)*
c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[a]*Sqrt[c] - Sqrt[b^2 - 4*a*c])
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx = -\left(\left(2\sqrt{a}\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx\right)+\left(b+2\sqrt{a}\sqrt{c}-\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{a+bx^2}}$$

$$= \frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\right)}{\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.30, size = 187, normalized size = 0.63

$$-\frac{2i\sqrt{2}\,a\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^2 + c*x^4]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.05, size = 515, normalized size = 1.74

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)b}{ac}-4}}{2}\right)+\mathrm{Elliptic}\right.}{\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(b+\sqrt{-4ac+b^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/
a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-
4)^(1/2)))-1/4*(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+1/4*b*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2
)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(
1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2
))/a*b/c-4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,cx^{2}+b-\sqrt{b^{2}-4\,ac}}{\sqrt{cx^{4}+bx^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b+2\,c\,x^{2}-\sqrt{b^{2}-4\,a\,c}}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx^{2}-\sqrt{-4ac+b^{2}}}{\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/sqrt(a + b*x**2 + c*x**4), x)
```

3.120 $\int \left(d+ex^2\right)^4 \left(a+cx^4\right)\,dx$

Optimal. Leaf size=106

$$\frac{1}{9}e^2x^9\left(ae^2+6cd^2\right)+\frac{4}{7}dex^7\left(ae^2+cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+cd^2\right)+ad^4x+\frac{4}{3}ad^3ex^3+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

```
[Out] a*d^4*x+4/3*a*d^3*e*x^3+1/5*d^2*(6*a*e^2+c*d^2)*x^5+4/7*d*e*(a*e^2+c*d^2)*x
^7+1/9*e^2*(a*e^2+6*c*d^2)*x^9+4/11*c*d*e^3*x^11+1/13*c*e^4*x^13
```

Rubi [A] time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$\frac{1}{9}e^2x^9\left(ae^2+6cd^2\right)+\frac{4}{7}dex^7\left(ae^2+cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+cd^2\right)+\frac{4}{3}ad^3ex^3+ad^4x+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + c*x^4),x]
```

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2
 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c
*e^4*x^13)/13
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^4\left(a+cx^4\right)dx &= \int \left(ad^4+4ad^3ex^2+d^2\left(cd^2+6ae^2\right)x^4+4de\left(cd^2+ae^2\right)x^6+e^2\left(6cd^2+ae^2\right)x^8\right. \\ &= ad^4x+\frac{4}{3}ad^3ex^3+\frac{1}{5}d^2\left(cd^2+6ae^2\right)x^5+\frac{4}{7}de\left(cd^2+ae^2\right)x^7+\frac{1}{9}e^2\left(6cd^2+ae^2\right)x\end{aligned}$$

Mathematica [A] time = 0.02, size = 106, normalized size = 1.00

$$\frac{1}{9}e^2x^9\left(ae^2+6cd^2\right)+\frac{4}{7}dex^7\left(ae^2+cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+cd^2\right)+ad^4x+\frac{4}{3}ad^3ex^3+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + c*x^4),x]
```

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2
 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c
*e^4*x^13)/13
```

fricas [A] time = 0.35, size = 98, normalized size = 0.92

$$\frac{1}{13}x^{13}e^4c+\frac{4}{11}x^{11}e^3dc+\frac{2}{3}x^9e^2d^2c+\frac{1}{9}x^9e^4a+\frac{4}{7}x^7ed^3c+\frac{4}{7}x^7e^3da+\frac{1}{5}x^5d^4c+\frac{6}{5}x^5e^2d^2a+\frac{4}{3}x^3ed^3a+xd^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^4*c + 4/11*x^11*e^3*d*c + 2/3*x^9*e^2*d^2*c + 1/9*x^9*e^4*a + 4
/7*x^7*e*d^3*c + 4/7*x^7*e^3*d*a + 1/5*x^5*d^4*c + 6/5*x^5*e^2*d^2*a + 4/3*
x^3*e*d^3*a + x*d^4*a
```

giac [A] time = 0.15, size = 94, normalized size = 0.89

$$\frac{1}{13}cx^{13}e^4+\frac{4}{11}cdx^{11}e^3+\frac{2}{3}cd^2x^9e^2+\frac{4}{7}cd^3x^7e+\frac{1}{9}ax^9e^4+\frac{1}{5}cd^4x^5+\frac{4}{7}adx^7e^3+\frac{6}{5}ad^2x^5e^2+\frac{4}{3}ad^3x^3e+ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^4 + 4/11*c*d*x^11*e^3 + 2/3*c*d^2*x^9*e^2 + 4/7*c*d^3*x^7*e +
 1/9*a*x^9*e^4 + 1/5*c*d^4*x^5 + 4/7*a*d*x^7*e^3 + 6/5*a*d^2*x^5*e^2 + 4/3*
a*d^3*x^3*e + a*d^4*x
```

maple [A] time = 0.00, size = 97, normalized size = 0.92

$$\frac{c\,e^4x^{13}}{13}+\frac{4cd\,e^3x^{11}}{11}+\frac{\left(e^4a+6d^2e^2c\right)x^9}{9}+\frac{4a\,d^3e\,x^3}{3}+\frac{\left(4d\,e^3a+4d^3ec\right)x^7}{7}+a\,d^4x+\frac{\left(6d^2e^2a+d^4c\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(c*x^4+a),x)
```

```
[Out] 1/13*c*e^4*x^13+4/11*c*d*e^3*x^11+1/9*(a*e^4+6*c*d^2*e^2)*x^9+1/7*(4*a*d*e^
3+4*c*d^3*e)*x^7+1/5*(6*a*d^2*e^2+c*d^4)*x^5+4/3*a*d^3*e*x^3+a*d^4*x
```

maxima [A] time = 1.05, size = 94, normalized size = 0.89

$$\frac{1}{13}ce^4x^{13}+\frac{4}{11}cde^3x^{11}+\frac{1}{9}\left(6\,cd^2e^2+ae^4\right)x^9+\frac{4}{3}ad^3ex^3+\frac{4}{7}\left(cd^3e+ade^3\right)x^7+ad^4x+\frac{1}{5}\left(cd^4+6\,ad^2e^2\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/9*(6*c*d^2*e^2 + a*e^4)*x^9 + 4/3*a
*d^3*e*x^3 + 4/7*(c*d^3*e + a*d*e^3)*x^7 + a*d^4*x + 1/5*(c*d^4 + 6*a*d^2*e
^2)*x^5
```

mupad [B] time = 4.35, size = 95, normalized size = 0.90

$$x^5\left(\frac{c\,d^4}{5}+\frac{6\,a\,d^2\,e^2}{5}\right)+x^9\left(\frac{2\,c\,d^2\,e^2}{3}+\frac{a\,e^4}{9}\right)+x^7\left(\frac{4\,c\,d^3\,e}{7}+\frac{4\,a\,d\,e^3}{7}\right)+\frac{c\,e^4\,x^{13}}{13}+a\,d^4\,x+\frac{4\,a\,d^3\,e\,x^3}{3}+\frac{4\,c\,d\,e^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)*(d + e*x^2)^4,x)
```

```
[Out] x^5*((c*d^4)/5 + (6*a*d^2*e^2)/5) + x^9*((a*e^4)/9 + (2*c*d^2*e^2)/3) + x^7
*((4*a*d*e^3)/7 + (4*c*d^3*e)/7) + (c*e^4*x^13)/13 + a*d^4*x + (4*a*d^3*e*x
^3)/3 + (4*c*d*e^3*x^11)/11
```

sympy [A] time = 0.09, size = 110, normalized size = 1.04

$$ad^4x+\frac{4ad^3ex^3}{3}+\frac{4cde^3x^{11}}{11}+\frac{ce^4x^{13}}{13}+x^9\left(\frac{ae^4}{9}+\frac{2cd^2e^2}{3}\right)+x^7\left(\frac{4ade^3}{7}+\frac{4cd^3e}{7}\right)+x^5\left(\frac{6ad^2e^2}{5}+\frac{cd^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(c*x**4+a),x)
```

```
[Out] a*d**4*x + 4*a*d**3*e*x**3/3 + 4*c*d*e**3*x**11/11 + c*e**4*x**13/13 + x**9
*(a*e**4/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 4*c*d**3*e/7) + x**5*(
6*a*d**2*e**2/5 + c*d**4/5)
```

3.121 $\int \left(d+ex^2\right)^3 \left(a+cx^4\right) dx$

Optimal. Leaf size=79

$$\frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right)+ad^3x+ad^2ex^3+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

```
[Out] a*d^3*x+a*d^2*e*x^3+1/5*d*(3*a*e^2+c*d^2)*x^5+1/7*e*(a*e^2+3*c*d^2)*x^7+1/3
*c*d*e^2*x^9+1/11*c*e^3*x^11
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$\frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right)+ad^2ex^3+ad^3x+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + c*x^4),x]
```

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*
x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^3\left(a+cx^4\right)dx &= \int \left(ad^3+3ad^2ex^2+d\left(cd^2+3ae^2\right)x^4+e\left(3cd^2+ae^2\right)x^6+3cde^2x^8+ce^3x^{10}\right)dx\\ &= ad^3x+ad^2ex^3+\frac{1}{5}d\left(cd^2+3ae^2\right)x^5+\frac{1}{7}e\left(3cd^2+ae^2\right)x^7+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}\end{aligned}$$

Mathematica [A] time = 0.02, size = 79, normalized size = 1.00

$$\frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right)+ad^3x+ad^2ex^3+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + c*x^4),x]
```

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*
x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

fricas [A] time = 0.35, size = 73, normalized size = 0.92

$$\frac{1}{11}x^{11}e^3c+\frac{1}{3}x^9e^2dc+\frac{3}{7}x^7ed^2c+\frac{1}{7}x^7e^3a+\frac{1}{5}x^5d^3c+\frac{3}{5}x^5e^2da+x^3ed^2a+xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^3*c + 1/3*x^9*e^2*d*c + 3/7*x^7*e*d^2*c + 1/7*x^7*e^3*a + 1/5*x
^5*d^3*c + 3/5*x^5*e^2*d*a + x^3*e*d^2*a + x*d^3*a
```

giac [A] time = 0.15, size = 71, normalized size = 0.90

$$\frac{1}{11}\,cx^{11}e^{3} + \frac{1}{3}\,cdx^{9}e^{2} + \frac{3}{7}\,cd^{2}x^{7}e + \frac{1}{5}\,cd^{3}x^{5} + \frac{1}{7}\,ax^{7}e^{3} + \frac{3}{5}\,adx^{5}e^{2} + ad^{2}x^{3}e + ad^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^3 + 1/3*c*d*x^9*e^2 + 3/7*c*d^2*x^7*e + 1/5*c*d^3*x^5 + 1/7*a
*x^7*e^3 + 3/5*a*d*x^5*e^2 + a*d^2*x^3*e + a*d^3*x
```

maple [A] time = 0.00, size = 72, normalized size = 0.91

$$\frac{c\,e^{3}x^{11}}{11} + \frac{cd\,e^{2}x^{9}}{3} + a\,d^{2}e\,x^{3} + \frac{\left(a\,e^{3} + 3c\,d^{2}e\right)x^{7}}{7} + a\,d^{3}x + \frac{\left(3d\,e^{2}a + d^{3}c\right)x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(c*x^4+a),x)
```

```
[Out] 1/11*c*e^3*x^11+1/3*c*d*e^2*x^9+1/7*(a*e^3+3*c*d^2*e)*x^7+1/5*(3*a*d*e^2+c*
d^3)*x^5+a*d^2*e*x^3+a*d^3*x
```

maxima [A] time = 1.04, size = 71, normalized size = 0.90

$$\frac{1}{11}\,ce^{3}x^{11} + \frac{1}{3}\,cde^{2}x^{9} + \frac{1}{7}\left(3\,cd^{2}e + ae^{3}\right)x^{7} + ad^{2}ex^{3} + \frac{1}{5}\left(cd^{3} + 3\,ade^{2}\right)x^{5} + ad^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 1/7*(3*c*d^2*e + a*e^3)*x^7 + a*d^2*e*x
^3 + 1/5*(c*d^3 + 3*a*d*e^2)*x^5 + a*d^3*x
```

mupad [B] time = 0.03, size = 71, normalized size = 0.90

$$x^{5}\,\left(\frac{c\,d^{3}}{5} + \frac{3\,a\,d\,e^{2}}{5}\right) + x^{7}\,\left(\frac{3\,c\,d^{2}\,e}{7} + \frac{a\,e^{3}}{7}\right) + \frac{c\,e^{3}\,x^{11}}{11} + a\,d^{3}\,x + a\,d^{2}\,e\,x^{3} + \frac{c\,d\,e^{2}\,x^{9}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)*(d + e*x^2)^3,x)
```

```
[Out] x^5*((c*d^3)/5 + (3*a*d*e^2)/5) + x^7*((a*e^3)/7 + (3*c*d^2*e)/7) + (c*e^3*
x^11)/11 + a*d^3*x + a*d^2*e*x^3 + (c*d*e^2*x^9)/3
```

sympy [A] time = 0.09, size = 78, normalized size = 0.99

$$ad^{3}x + ad^{2}ex^{3} + \frac{cde^{2}x^{9}}{3} + \frac{ce^{3}x^{11}}{11} + x^{7}\left(\frac{ae^{3}}{7} + \frac{3cd^{2}e}{7}\right) + x^{5}\left(\frac{3ade^{2}}{5} + \frac{cd^{3}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+a),x)
```

```
[Out] a*d**3*x + a*d**2*e*x**3 + c*d*e**2*x**9/3 + c*e**3*x**11/11 + x**7*(a*e**3
/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + c*d**3/5)
```

3.122 $\int \left(d+ex^2\right)^2 \left(a+cx^4\right) dx$

Optimal. Leaf size=56

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + c*x^4),x]
```

```
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*
e^2*x^9)/9
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2\left(a+cx^4\right)dx &= \int \left(ad^2+2adex^2+\left(cd^2+ae^2\right)x^4+2cdex^6+ce^2x^8\right)dx\\ &= ad^2x+\frac{2}{3}adex^3+\frac{1}{5}\left(cd^2+ae^2\right)x^5+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + c*x^4),x]
```

```
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*
e^2*x^9)/9
```

fricas [A] time = 0.35, size = 50, normalized size = 0.89

$$\frac{1}{9}x^9e^2c+\frac{2}{7}x^7edc+\frac{1}{5}x^5d^2c+\frac{1}{5}x^5e^2a+\frac{2}{3}x^3eda+xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/5*x^5*d^2*c + 1/5*x^5*e^2*a + 2/3*x^3*e*d
*a + x*d^2*a
```

giac [A] time = 0.20, size = 50, normalized size = 0.89

$$\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{5}cd^2x^5 + \frac{1}{5}ax^5e^2 + \frac{2}{3}adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/5*c*d^2*x^5 + 1/5*a*x^5*e^2 + 2/3*a*d*x^3
*e + a*d^2*x
```

maple [A] time = 0.00, size = 49, normalized size = 0.88

$$\frac{c\,e^2x^9}{9} + \frac{2cde\,x^7}{7} + \frac{2ade\,x^3}{3} + \frac{\left(a\,e^2 + c\,d^2\right)x^5}{5} + a\,d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+a),x)
```

```
[Out] a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9
```

maxima [A] time = 0.97, size = 48, normalized size = 0.86

$$\frac{1}{9}ce^2x^9 + \frac{2}{7}cdex^7 + \frac{2}{3}adex^3 + \frac{1}{5}\left(cd^2 + ae^2\right)x^5 + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 2/3*a*d*e*x^3 + 1/5*(c*d^2 + a*e^2)*x^5 + a
*d^2*x
```

mupad [B] time = 0.02, size = 49, normalized size = 0.88

$$x^5\left(\frac{c\,d^2}{5} + \frac{a\,e^2}{5}\right) + \frac{c\,e^2\,x^9}{9} + a\,d^2\,x + \frac{2\,a\,d\,e\,x^3}{3} + \frac{2\,c\,d\,e\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)*(d + e*x^2)^2,x)
```

```
[Out] x^5*((a*e^2)/5 + (c*d^2)/5) + (c*e^2*x^9)/9 + a*d^2*x + (2*a*d*e*x^3)/3 + (
2*c*d*e*x^7)/7
```

sympy [A] time = 0.08, size = 56, normalized size = 1.00

$$ad^2x + \frac{2adex^3}{3} + \frac{2cdex^7}{7} + \frac{ce^2x^9}{9} + x^5\left(\frac{ae^2}{5} + \frac{cd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+a),x)
```

```
[Out] a*d**2*x + 2*a*d*e*x**3/3 + 2*c*d*e*x**7/7 + c*e**2*x**9/9 + x**5*(a*e**2/5
 + c*d**2/5)
```

3.123 $\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx$

Optimal. Leaf size=32

$$adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7$$

```
[Out] a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1154}

$$adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx &= \int \left(ad+aex^2+cdx^4+cex^6\right)\,dx\\ &= adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7\end{aligned}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$adx + \frac{1}{3}aex^3 + \frac{1}{5}cdx^5 + \frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

fricas [A] time = 0.35, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7ec + \frac{1}{5}x^5dc + \frac{1}{3}x^3ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e*c + 1/5*x^5*d*c + 1/3*x^3*e*a + x*d*a
```

giac [A] time = 0.18, size = 28, normalized size = 0.88

$$\frac{1}{7}cx^7e + \frac{1}{5}cdx^5 + \frac{1}{3}ax^3e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e + 1/5*c*d*x^5 + 1/3*a*x^3*e + a*d*x
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{1}{7} c e x^{7}+\frac{1}{5} c d x^{5}+\frac{1}{3} a e x^{3}+a d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a),x)
```

```
[Out] a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

maxima [A] time = 1.06, size = 26, normalized size = 0.81

$$\frac{1}{7} \, c e x^{7} + \frac{1}{5} \, c d x^{5} + \frac{1}{3} \, a e x^{3} + a d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

mupad [B] time = 0.04, size = 26, normalized size = 0.81

$$\frac{c\,e\,x^7}{7}+\frac{c\,d\,x^5}{5}+\frac{a\,e\,x^3}{3}+a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)*(d + e*x^2),x)
```

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

sympy [A] time = 0.08, size = 29, normalized size = 0.91

$$a d x + \frac{a e x^{3}}{3} + \frac{c d x^{5}}{5} + \frac{c e x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a),x)
```

```
[Out] a*d*x + a*e*x**3/3 + c*d*x**5/5 + c*e*x**7/7
```

3.124 $\int \frac{a+cx^4}{d+ex^2} \, dx$

Optimal. Leaf size=55

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

```
[Out] -c*d*x/e^2+1/3*c*x^3/e+(a*e^2+c*d^2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/d^(1
/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1154, 205}

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2),x]
```

```
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[d]*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^4}{d+ex^2}\,dx &= \int \left(-\frac{cd}{e^2}+\frac{cx^2}{e}+\frac{cd^2+ae^2}{e^2\left(d+ex^2\right)}\right)dx\\ &= -\frac{cdx}{e^2}+\frac{cx^3}{3e}+\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{d+ex^2}\,dx\\ &= -\frac{cdx}{e^2}+\frac{cx^3}{3e}+\frac{\left(cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 1.00

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2),x]
```

```
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[d]*e^(5/2))
```

fricas [A] time = 0.40, size = 131, normalized size = 2.38

$$\left[\frac{2\,cde^2x^3 - 6\,cd^2ex - 3\left(cd^2 + ae^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right)}{6\,de^3}, \frac{cde^2x^3 - 3\,cd^2ex + 3\left(cd^2 + ae^2\right)\sqrt{de}\,\arctan\left(\frac{\sqrt{de}\,x}{d}\right)}{3\,de^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c*d*e^2*x^3 - 6*c*d^2*e*x - 3*(c*d^2 + a*e^2)*sqrt(-d*e)*log((e*x^2
 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d*e^3), 1/3*(c*d*e^2*x^3 - 3*c*d^2*e*
x + 3*(c*d^2 + a*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d*e^3)]
```

giac [A] time = 0.17, size = 44, normalized size = 0.80

$$\frac{\left(cd^2 + ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}} + \frac{1}{3}\left(cx^3e^2 - 3\,cdxe\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c*d^2 + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + 1/3*(c*x^3*e^2
 - 3*c*d*x*e)*e^(-3)
```

maple [A] time = 0.01, size = 57, normalized size = 1.04

$$\frac{c\,x^3}{3e} + \frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + \frac{c\,d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\;e^2} - \frac{cdx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d),x)
```

```
[Out] 1/3*c*x^3/e-c*d*x/e^2+1/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/e^2/(d*e)^(
1/2)*arctan(x*e/(d*e)^(1/2))*c*d^2
```

maxima [A] time = 2.55, size = 47, normalized size = 0.85

$$\frac{\left(cd^2 + ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^2} + \frac{cex^3 - 3\,cdx}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] (c*d^2 + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) + 1/3*(c*e*x^3 - 3*c*
d*x)/e^2
```

mupad [B] time = 0.07, size = 45, normalized size = 0.82

$$\frac{c\,x^3}{3\,e} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(c\,d^2 + a\,e^2\right)}{\sqrt{d}\;e^{5/2}} - \frac{c\,d\,x}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/(d + e*x^2),x)
```

```
[Out] (c*x^3)/(3*e) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c*d^2))/(d^(1/2)*e^(5/2
)) - (c*d*x)/e^2
```

sympy [B] time = 0.32, size = 104, normalized size = 1.89

$$-\frac{cdx}{e^2}+\frac{cx^3}{3e}-\frac{\sqrt{-\frac{1}{de^5}}\left(ae^2+cd^2\right)\log\left(-de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{de^5}}\left(ae^2+cd^2\right)\log\left(de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d),x)
```

```
[Out] -c*d*x/e**2 + c*x**3/(3*e) - sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(-d*e**
2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(d*e**2
*sqrt(-1/(d*e**5)) + x)/2
```

3.125 $\int \frac{a+cx^4}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

```
[Out] c*x/e^2+1/2*(a+c*d^2/e^2)*x/d/(e*x^2+d)-1/2*(-a*e^2+3*c*d^2)*arctan(x*e^(1/
2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1158, 388, 205}

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\int \frac{-a+\frac{cd^2}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}$$
$$= \frac{cx}{e^2} + \frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\left(d+ex^2\right)} + \frac{\left(a-\frac{3cd^2}{e^2}\right)\int \frac{1}{d+ex^2}\,dx}{2d}$$
$$= \frac{cx}{e^2} + \frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\left(d+ex^2\right)} + \frac{\left(a-\frac{3cd^2}{e^2}\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 1.05

$$\frac{x\left(ae^2+cd^2\right)}{2de^2\left(d+ex^2\right)} - \frac{\left(3cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

fricas [A] time = 0.43, size = 222, normalized size = 3.00

$$\left[\frac{4\,cd^2e^2x^3 + \left(3\,cd^3 - ade^2 + \left(3\,cd^2e - ae^3\right)x^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right) + 2\left(3\,cd^3e + ade^3\right)x}{4\left(d^2e^4x^2 + d^3e^3\right)}, \frac{2\,cd^2e^2x^3 - \left(3\,cd\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - a*d*e^2 + (3*c*d^2*e - a*e^3)*x^2)*sqrt(
-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^3*e + a*d*e^
3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2*e^2*x^3 - (3*c*d^3 - a*d*e^2 +
(3*c*d^2*e - a*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e + a*d
*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3)]
```

giac [A] time = 0.16, size = 62, normalized size = 0.84

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x + axe^2\right)e^{(-2)}}{2\left(x^2e+d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(3/
2) + 1/2*(c*d^2*x + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

maple [A] time = 0.01, size = 82, normalized size = 1.11

$$\frac{ax}{2\left(e\,x^2+d\right)d} + \frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d} + \frac{cdx}{2\left(e\,x^2+d\right)e^2} - \frac{3cd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^2} + \frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a+1/2/e^2*d*x/(e*x^2+d)*c+1/2/d/(d*e)^(1/2)*arcta
n(1/(d*e)^(1/2)*e*x)*a-3/2/e^2*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.24, size = 74, normalized size = 1.00

$$\frac{\left(cd^2+ae^2\right)x}{2\left(de^3x^2+d^2e^2\right)}+\frac{cx}{e^2}-\frac{\left(3\,cd^2-ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*d^2 + a*e^2)*x/(d*e^3*x^2 + d^2*e^2) + c*x/e^2 - 1/2*(3*c*d^2 - a*e^
2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^2)
```

mupad [B] time = 4.44, size = 68, normalized size = 0.92

$$\frac{c\,x}{e^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(a\,e^2-3\,c\,d^2\right)}{2\,d^{3/2}\,e^{5/2}}+\frac{x\,\left(c\,d^2+a\,e^2\right)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2))/(2*d^(3/2)*e^(5/2
)) + (x*(a*e^2 + c*d^2))/(2*d*(d*e^2 + e^3*x^2))
```

sympy [B] time = 0.51, size = 138, normalized size = 1.86

$$\frac{cx}{e^2}+\frac{x\left(ae^2+cd^2\right)}{2d^2e^2+2de^3x^2}-\frac{\sqrt{-\frac{1}{d^3e^5}}\,\left(ae^2-3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^3e^5}}\,\left(ae^2-3cd^2\right)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt(-1/(d**
3*e**5))*(a*e**2 - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4 + s
qrt(-1/(d**3*e**5))*(a*e**2 - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5))
+ x)/4
```

3.126 $\int \frac{a+cx^4}{\left(d+ex^2\right)^3} \, dx$

Optimal. Leaf size=93

$$\frac{x\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)}{8\left(d+ex^2\right)}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{3\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

```
[Out] 1/4*(a+c*d^2/e^2)*x/d/(e*x^2+d)^2+1/8*(3*a/d^2-5*c/e^2)*x/(e*x^2+d)+3/8*(a*
e^2+c*d^2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1158, 385, 205}

$$\frac{x\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)}{8\left(d+ex^2\right)}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{3\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) + (((3*a)/d^2 - (5*c)/e^2)*x)/(8*
(d + e*x^2)) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e
^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2} - \frac{\int \frac{-3a+\frac{cd^2}{e^2}-\frac{4cdx^2}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}$$

$$= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2} + \frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8\left(d+ex^2\right)} + \frac{1}{8}\left(3\left(\frac{a}{d^2}+\frac{c}{e^2}\right)\right)\int \frac{1}{d+ex^2}\,dx$$

$$= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2} + \frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8\left(d+ex^2\right)} + \frac{3\left(cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Mathematica [A] time = 0.06, size = 92, normalized size = 0.99

$$\frac{ae^2x\left(5d+3ex^2\right)-cd^2x\left(3d+5ex^2\right)}{8d^2e^2\left(d+ex^2\right)^2} + \frac{3\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (a*e^2*x*(5*d + 3*e*x^2) - c*d^2*x*(3*d + 5*e*x^2))/(8*d^2*e^2*(d + e*x^2)^
2) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

fricas [A] time = 0.43, size = 306, normalized size = 3.29

$$\left[-\frac{2\left(5\,cd^3e^2-3\,ade^4\right)x^3+3\left(cd^4+ad^2e^2+\left(cd^2e^2+ae^4\right)x^4+2\left(cd^3e+ade^3\right)x^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right)+2\left(3\right.}{16\left(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*c*d^3*e^2 - 3*a*d*e^4)*x^3 + 3*(c*d^4 + a*d^2*e^2 + (c*d^2*e^2
 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-
d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^4*e - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*
d^4*e^4*x^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - 3*a*d*e^4)*x^3 - 3*(c*d^4 + a*
d^2*e^2 + (c*d^2*e^2 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(d*e)*ar
ctan(sqrt(d*e)*x/d) + (3*c*d^4*e - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4
*x^2 + d^5*e^3)]
```

giac [A] time = 0.16, size = 77, normalized size = 0.83

$$\frac{3\left(cd^2+ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}} - \frac{\left(5\,cd^2x^3e+3\,cd^3x-3\,ax^3e^3-5\,adxe^2\right)e^{(-2)}}{8\left(x^2e+d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 3/8*(c*d^2 + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2) - 1/8*(5*c*d
^2*x^3*e + 3*c*d^3*x - 3*a*x^3*e^3 - 5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2
)
```

maple [A] time = 0.01, size = 99, normalized size = 1.06

$$\frac{3a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^2}+\frac{3c\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^2}+\frac{\frac{\left(3a\,e^2-5c\,d^2\right)x^3}{8d^2e}+\frac{\left(5a\,e^2-3c\,d^2\right)x}{8d\,e^2}}{\left(e\,x^2+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d)^3,x)
```

```
[Out] (1/8*(3*a*e^2-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-3*c*d^2)/d/e^2*x)/(e*x^2+d)^2
+3/8/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+3/8/e^2/(d*e)^(1/2)*arctan
(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.56, size = 102, normalized size = 1.10

$$-\frac{\left(5\,cd^2e-3\,ae^3\right)x^3+\left(3\,cd^3-5\,ade^2\right)x}{8\left(d^2e^4x^4+2\,d^3e^3x^2+d^4e^2\right)}+\frac{3\left(cd^2+ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*c*d^2*e - 3*a*e^3)*x^3 + (3*c*d^3 - 5*a*d*e^2)*x)/(d^2*e^4*x^4 + 2
*d^3*e^3*x^2 + d^4*e^2) + 3/8*(c*d^2 + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d
*e)*d^2*e^2)
```

mupad [B] time = 4.48, size = 97, normalized size = 1.04

$$\frac{\frac{x^3\left(3\,a\,e^2-5\,c\,d^2\right)}{8\,d^2\,e}+\frac{x\left(5\,a\,e^2-3\,c\,d^2\right)}{8\,d\,e^2}}{d^2+2\,d\,e\,x^2+e^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(c\,d^2+a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/(d + e*x^2)^3,x)
```

```
[Out] ((x^3*(3*a*e^2 - 5*c*d^2))/(8*d^2*e) + (x*(5*a*e^2 - 3*c*d^2))/(8*d*e^2))/(
d^2 + e^2*x^4 + 2*d*e*x^2) + (3*atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c*d^2))/
(8*d^(5/2)*e^(5/2))
```

sympy [B] time = 0.75, size = 219, normalized size = 2.35

$$-\frac{3\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)\log\left(-\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)}{3ae^2+3cd^2}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)\log\left(\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)}{3ae^2+3cd^2}+x\right)}{16}+\frac{x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d)**3,x)
```

```
[Out] -3*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)*log(-3*d**3*e**2*sqrt(-1/(d**5*e*
*5))*(a*e**2 + c*d**2)/(3*a*e**2 + 3*c*d**2) + x)/16 + 3*sqrt(-1/(d**5*e**5
))*(a*e**2 + c*d**2)*log(3*d**3*e**2*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)
/(3*a*e**2 + 3*c*d**2) + x)/16 + (x**3*(3*a*e**3 - 5*c*d**2*e) + x*(5*a*d*e
**2 - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**4)
```

3.127 $\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx$

Optimal. Leaf size=123

$$\frac{x\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)}{16d\left(d+ex^2\right)}+\frac{x\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)}{24\left(d+ex^2\right)^2}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}+\frac{\left(5ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

```
[Out] 1/6*(a+c*d^2/e^2)*x/d/(e*x^2+d)^3+1/24*(5*a/d^2-7*c/e^2)*x/(e*x^2+d)^2+1/16
*(5*a/d^2+c/e^2)*x/d/(e*x^2+d)+1/16*(5*a*e^2+c*d^2)*arctan(x*e^(1/2)/d^(1/2
))/d^(7/2)/e^(5/2)
```

Rubi [A] time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1158, 385, 199, 205}

$$\frac{x\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)}{16d\left(d+ex^2\right)}+\frac{x\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)}{24\left(d+ex^2\right)^2}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}+\frac{\left(5ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] ((a + (c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) + (((5*a)/d^2 - (7*c)/e^2)*x)/(24
*(d + e*x^2)^2) + (((5*a)/d^2 + c/e^2)*x)/(16*d*(d + e*x^2)) + ((c*d^2 + 5*
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\left(d+ex^2\right)^3}-\frac{\int \frac{-5a+\frac{cd^2}{e^2}-\frac{6cdx^2}{e}}{\left(d+ex^2\right)^3}\,dx}{6d}\\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\left(d+ex^2\right)^3}+\frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\left(d+ex^2\right)^2}+\frac{1}{8}\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)\int \frac{1}{\left(d+ex^2\right)^2}\,dx\\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\left(d+ex^2\right)^3}+\frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\left(d+ex^2\right)^2}+\frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d\left(d+ex^2\right)}+\frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)\int \frac{1}{d+ex^2}\,dx}{16d}\\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\left(d+ex^2\right)^3}+\frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\left(d+ex^2\right)^2}+\frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d\left(d+ex^2\right)}+\frac{\left(cd^2+5ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 113, normalized size = 0.92

$$\frac{\left(5ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(ae^2\left(33d^2+40dex^2+15e^2x^4\right)+cd^2\left(-3d^2-8dex^2+3e^2x^4\right)\right)}{48d^3e^2\left(d+ex^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + a*e^2*(33*d^2 + 40*d*e*x^2 + 1
5*e^2*x^4)))/(48*d^3*e^2*(d + e*x^2)^3) + ((c*d^2 + 5*a*e^2)*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

fricas [A] time = 0.42, size = 424, normalized size = 3.45

$$\left[\frac{6\left(cd^3e^3+5\,ade^5\right)x^5-16\left(cd^4e^2-5\,ad^2e^4\right)x^3-3\left(\left(cd^2e^3+5\,ae^5\right)x^6+cd^5+5\,ad^3e^2+3\left(cd^3e^2+5\,ade^4\right)x^4+\right.}{96\left(d^4e^6x^6+3\,d^5e^5x^4+3\,d^6e^4x^2+d^7e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(c*d^3*e^3 + 5*a*d*e^5)*x^5 - 16*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 - 3
*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5 + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*e^
4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e
)*x - d)/(e*x^2 + d)) - 6*(c*d^5*e - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6 + 3*d^5*
e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3), 1/48*(3*(c*d^3*e^3 + 5*a*d*e^5)*x^5 - 8
*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 + 3*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5 + 5*a
*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)*x^2)*s
qrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*d^5*e - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6
 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]
```

giac [A] time = 0.15, size = 100, normalized size = 0.81

$$\frac{\left(cd^2+5\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{16\,d^{\frac{7}{2}}}+\frac{\left(3\,cd^2x^5e^2-8\,cd^3x^3e+15\,ax^5e^4-3\,cd^4x+40\,adx^3e^3+33\,ad^2xe^2\right)e^{(-2)}}{48\left(x^2e+d\right)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] 1/16*(c*d^2 + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(7/2) + 1/48*(3
*c*d^2*x^5*e^2 - 8*c*d^3*x^3*e + 15*a*x^5*e^4 - 3*c*d^4*x + 40*a*d*x^3*e^3
+ 33*a*d^2*x*e^2)*e^(-2)/((x^2*e + d)^3*d^3)
```

maple [A] time = 0.01, size = 122, normalized size = 0.99

$$\frac{5a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de}\,d^3}+\frac{c\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de}\,d\,e^2}+\frac{\frac{\left(5a\,e^2+c\,d^2\right)x^5}{16d^3}+\frac{\left(5a\,e^2-c\,d^2\right)x^3}{6d^2e}+\frac{\left(11a\,e^2-c\,d^2\right)x}{16d\,e^2}}{\left(e\,x^2+d\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d)^4,x)
```

```
[Out] (1/16*(5*a*e^2+c*d^2)/d^3*x^5+1/6*(5*a*e^2-c*d^2)/d^2/e*x^3+1/16*(11*a*e^2-
c*d^2)/d/e^2*x)/(e*x^2+d)^3+5/16/d^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*
a+1/16/d/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.36, size = 137, normalized size = 1.11

$$\frac{3\left(cd^2e^2+5\,ae^4\right)x^5-8\left(cd^3e-5\,ade^3\right)x^3-3\left(cd^4-11\,ad^2e^2\right)x}{48\left(d^3e^5x^6+3\,d^4e^4x^4+3\,d^5e^3x^2+d^6e^2\right)}+\frac{\left(cd^2+5\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}\,d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(c*d^2*e^2 + 5*a*e^4)*x^5 - 8*(c*d^3*e - 5*a*d*e^3)*x^3 - 3*(c*d^4
- 11*a*d^2*e^2)*x)/(d^3*e^5*x^6 + 3*d^4*e^4*x^4 + 3*d^5*e^3*x^2 + d^6*e^2)
+ 1/16*(c*d^2 + 5*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e^2)
```

mupad [B] time = 4.48, size = 129, normalized size = 1.05

$$\frac{\frac{x^5\left(c\,d^2+5\,a\,e^2\right)}{16\,d^3}+\frac{x^3\left(5\,a\,e^2-c\,d^2\right)}{6\,d^2\,e}+\frac{x\left(11\,a\,e^2-c\,d^2\right)}{16\,d\,e^2}}{d^3+3\,d^2\,e\,x^2+3\,d\,e^2\,x^4+e^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(c\,d^2+5\,a\,e^2\right)}{16\,d^{7/2}\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/(d + e*x^2)^4,x)
```

```
[Out] ((x^5*(5*a*e^2 + c*d^2))/(16*d^3) + (x^3*(5*a*e^2 - c*d^2))/(6*d^2*e) + (x*
(11*a*e^2 - c*d^2))/(16*d*e^2))/(d^3 + e^3*x^6 + 3*d^2*e*x^2 + 3*d*e^2*x^4)
 + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2))/(16*d^(7/2)*e^(5/2))
```

sympy [A] time = 0.95, size = 204, normalized size = 1.66

$$-\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+cd^2\right)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+cd^2\right)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{x^5\left(15ae^4+3cd^2e^2\right.}{48d^6e^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d)**4,x)
```

```
[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(-d**4*e**2*sqrt(-1/(d**7*e**5
)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(d**4*e**2*sqrt(-1
/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*c*d**2*e**2) + x**3*(40*a*d*e*
*3 - 8*c*d**3*e) + x*(33*a*d**2*e**2 - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*
e**3*x**2 + 144*d**4*e**4*x**4 + 48*d**3*e**5*x**6)
```

3.128 $\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx$

Optimal. Leaf size=133

$$a^2d^3x+a^2d^2ex^3+\frac{1}{11}cex^{11}\left(2ae^2+3cd^2\right)+\frac{1}{9}cdx^9\left(6ae^2+cd^2\right)+\frac{1}{7}aex^7\left(ae^2+6cd^2\right)+\frac{1}{5}adx^5\left(3ae^2+2cd^2\right)+\frac{3}{13}c^2$$

```
[Out] a^2*d^3*x+a^2*d^2*e*x^3+1/5*a*d*(3*a*e^2+2*c*d^2)*x^5+1/7*a*e*(a*e^2+6*c*d^
2)*x^7+1/9*c*d*(6*a*e^2+c*d^2)*x^9+1/11*c*e*(2*a*e^2+3*c*d^2)*x^11+3/13*c^2
*d*e^2*x^13+1/15*c^2*e^3*x^15
```

Rubi [A] time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1154}

$$a^2d^2ex^3+a^2d^3x+\frac{1}{11}cex^{11}\left(2ae^2+3cd^2\right)+\frac{1}{9}cdx^9\left(6ae^2+cd^2\right)+\frac{1}{7}aex^7\left(ae^2+6cd^2\right)+\frac{1}{5}adx^5\left(3ae^2+2cd^2\right)+\frac{3}{13}c^2$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2
 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)
*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(d+ex^2\right)^3\left(a+cx^4\right)^2 dx = \int \left(a^2d^3+3a^2d^2ex^2+ad\left(2cd^2+3ae^2\right)x^4+ae\left(6cd^2+ae^2\right)x^6+cd\left(cd^2+6ae^2\right.\right.$$
$$= a^2d^3x+a^2d^2ex^3+\frac{1}{5}ad\left(2cd^2+3ae^2\right)x^5+\frac{1}{7}ae\left(6cd^2+ae^2\right)x^7+\frac{1}{9}cd\left(cd^2+6ae\right.$$

Mathematica [A] time = 0.02, size = 133, normalized size = 1.00

$$a^2d^3x+a^2d^2ex^3+\frac{1}{11}cex^{11}\left(2ae^2+3cd^2\right)+\frac{1}{9}cdx^9\left(6ae^2+cd^2\right)+\frac{1}{7}aex^7\left(ae^2+6cd^2\right)+\frac{1}{5}adx^5\left(3ae^2+2cd^2\right)+\frac{3}{13}c^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2
 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)
*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

fricas [A] time = 0.35, size = 131, normalized size = 0.98

$$\frac{1}{15}x^{15}e^3c^2+\frac{3}{13}x^{13}e^2dc^2+\frac{3}{11}x^{11}ed^2c^2+\frac{2}{11}x^{11}e^3ca+\frac{1}{9}x^9d^3c^2+\frac{2}{3}x^9e^2dca+\frac{6}{7}x^7ed^2ca+\frac{1}{7}x^7e^3a^2+\frac{2}{5}x^5d^3ca+\frac{3}{5}x^5e^2da$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*e^3*c^2 + 3/13*x^13*e^2*d*c^2 + 3/11*x^11*e*d^2*c^2 + 2/11*x^11*e
^3*c*a + 1/9*x^9*d^3*c^2 + 2/3*x^9*e^2*d*c*a + 6/7*x^7*e*d^2*c*a + 1/7*x^7*
e^3*a^2 + 2/5*x^5*d^3*c*a + 3/5*x^5*e^2*d*a^2 + x^3*e*d^2*a^2 + x*d^3*a^2
```

giac [A] time = 0.16, size = 128, normalized size = 0.96

$$\frac{1}{15}c^2x^{15}e^3+\frac{3}{13}c^2dx^{13}e^2+\frac{3}{11}c^2d^2x^{11}e+\frac{1}{9}c^2d^3x^9+\frac{2}{11}acx^{11}e^3+\frac{2}{3}acdx^9e^2+\frac{6}{7}acd^2x^7e+\frac{2}{5}acd^3x^5+\frac{1}{7}a^2x^7e^3+\frac{3}{5}a^2dx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*c^2*x^15*e^3 + 3/13*c^2*d*x^13*e^2 + 3/11*c^2*d^2*x^11*e + 1/9*c^2*d^3
*x^9 + 2/11*a*c*x^11*e^3 + 2/3*a*c*d*x^9*e^2 + 6/7*a*c*d^2*x^7*e + 2/5*a*c*
d^3*x^5 + 1/7*a^2*x^7*e^3 + 3/5*a^2*d*x^5*e^2 + a^2*d^2*x^3*e + a^2*d^3*x
```

maple [A] time = 0.00, size = 130, normalized size = 0.98

$$\frac{c^2e^3x^{15}}{15}+\frac{3c^2d\,e^2x^{13}}{13}+\frac{\left(2e^3ac+3d^2e\,c^2\right)x^{11}}{11}+\frac{\left(6acd\,e^2+c^2d^3\right)x^9}{9}+a^2d^2e\,x^3+\frac{\left(e^3a^2+6d^2eac\right)x^7}{7}+a^2d^3x+\frac{\left(3d\,e^2a^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(c*x^4+a)^2,x)
```

```
[Out] 1/15*c^2*e^3*x^15+3/13*c^2*d*e^2*x^13+1/11*(2*a*c*e^3+3*c^2*d^2*e)*x^11+1/9
*(6*a*c*d*e^2+c^2*d^3)*x^9+1/7*(a^2*e^3+6*a*c*d^2*e)*x^7+1/5*(3*a^2*d*e^2+2
*a*c*d^3)*x^5+a^2*d^2*e*x^3+a^2*d^3*x
```

maxima [A] time = 1.07, size = 129, normalized size = 0.97

$$\frac{1}{15}c^2e^3x^{15}+\frac{3}{13}c^2de^2x^{13}+\frac{1}{11}\left(3\,c^2d^2e+2\,ace^3\right)x^{11}+\frac{1}{9}\left(c^2d^3+6\,acde^2\right)x^9+a^2d^2ex^3+\frac{1}{7}\left(6\,acd^2e+a^2e^3\right)x^7+a^2d^3x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 1/11*(3*c^2*d^2*e + 2*a*c*e^3)*x^
11 + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + a^2*d^2*e*x^3 + 1/7*(6*a*c*d^2*e + a
^2*e^3)*x^7 + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*a^2*d*e^2)*x^5
```

mupad [B] time = 0.06, size = 127, normalized size = 0.95

$$x^5\left(\frac{3\,a^2\,d\,e^2}{5}+\frac{2\,c\,a\,d^3}{5}\right)+x^7\left(\frac{a^2\,e^3}{7}+\frac{6\,c\,a\,d^2\,e}{7}\right)+x^9\left(\frac{c^2\,d^3}{9}+\frac{2\,a\,c\,d\,e^2}{3}\right)+x^{11}\left(\frac{3\,c^2\,d^2\,e}{11}+\frac{2\,a\,c\,e^3}{11}\right)+a^2\,d^3\,x+\frac{c^2\,e^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2*(d + e*x^2)^3,x)
```

```
[Out] x^5*((3*a^2*d*e^2)/5 + (2*a*c*d^3)/5) + x^7*((a^2*e^3)/7 + (6*a*c*d^2*e)/7)
 + x^9*((c^2*d^3)/9 + (2*a*c*d*e^2)/3) + x^11*((3*c^2*d^2*e)/11 + (2*a*c*e^
3)/11) + a^2*d^3*x + (c^2*e^3*x^15)/15 + a^2*d^2*e*x^3 + (3*c^2*d*e^2*x^13)
/13
```

sympy [A] time = 0.09, size = 144, normalized size = 1.08

$$a^2d^3x+a^2d^2ex^3+\frac{3c^2de^2x^{13}}{13}+\frac{c^2e^3x^{15}}{15}+x^{11}\left(\frac{2ace^3}{11}+\frac{3c^2d^2e}{11}\right)+x^9\left(\frac{2acde^2}{3}+\frac{c^2d^3}{9}\right)+x^7\left(\frac{a^2e^3}{7}+\frac{6acd^2e}{7}\right)+x^5\left(\frac{3a^2de}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**3*x + a**2*d**2*e*x**3 + 3*c**2*d*e**2*x**13/13 + c**2*e**3*x**15/1
5 + x**11*(2*a*c*e**3/11 + 3*c**2*d**2*e/11) + x**9*(2*a*c*d*e**2/3 + c**2*
d**3/9) + x**7*(a**2*e**3/7 + 6*a*c*d**2*e/7) + x**5*(3*a**2*d*e**2/5 + 2*a
*c*d**3/5)
```

3.129 $\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx$

Optimal. Leaf size=97

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

```
[Out] a^2*d^2*x+2/3*a^2*d*e*x^3+1/5*a*(a*e^2+2*c*d^2)*x^5+4/7*a*c*d*e*x^7+1/9*c*(
2*a*e^2+c*d^2)*x^9+2/11*c^2*d*e*x^11+1/13*c^2*e^2*x^13
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1154}

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x^
7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)/1
3
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx &= \int \left(a^2d^2+2a^2dex^2+a\left(2cd^2+ae^2\right)x^4+4acdex^6+c\left(cd^2+2ae^2\right)x^8+2c^2dex^{10}+\right.\\ &= a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{5}a\left(2cd^2+ae^2\right)x^5+\frac{4}{7}acdex^7+\frac{1}{9}c\left(cd^2+2ae^2\right)x^9+\frac{2}{11}c^2dex^{1}\end{aligned}$$

Mathematica [A] time = 0.02, size = 97, normalized size = 1.00

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x^
7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)/1
3
```

fricas [A] time = 0.35, size = 91, normalized size = 0.94

$$\frac{1}{13}x^{13}e^2c^2+\frac{2}{11}x^{11}edc^2+\frac{1}{9}x^9d^2c^2+\frac{2}{9}x^9e^2ca+\frac{4}{7}x^7edca+\frac{2}{5}x^5d^2ca+\frac{1}{5}x^5e^2a^2+\frac{2}{3}x^3eda^2+xd^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^2*c^2 + 2/11*x^11*e*d*c^2 + 1/9*x^9*d^2*c^2 + 2/9*x^9*e^2*c*a +
 4/7*x^7*e*d*c*a + 2/5*x^5*d^2*c*a + 1/5*x^5*e^2*a^2 + 2/3*x^3*e*d*a^2 + x*
d^2*a^2
```

giac [A] time = 0.15, size = 91, normalized size = 0.94

$$\frac{1}{13}c^2x^{13}e^2+\frac{2}{11}c^2dx^{11}e+\frac{1}{9}c^2d^2x^9+\frac{2}{9}acx^9e^2+\frac{4}{7}acdx^7e+\frac{2}{5}acd^2x^5+\frac{1}{5}a^2x^5e^2+\frac{2}{3}a^2dx^3e+a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13*e^2 + 2/11*c^2*d*x^11*e + 1/9*c^2*d^2*x^9 + 2/9*a*c*x^9*e^2 +
 4/7*a*c*d*x^7*e + 2/5*a*c*d^2*x^5 + 1/5*a^2*x^5*e^2 + 2/3*a^2*d*x^3*e + a^
2*d^2*x
```

maple [A] time = 0.00, size = 90, normalized size = 0.93

$$\frac{c^2e^2x^{13}}{13}+\frac{2c^2de\,x^{11}}{11}+\frac{4acde\,x^7}{7}+\frac{\left(2e^2ac+c^2d^2\right)x^9}{9}+\frac{2a^2de\,x^3}{3}+a^2d^2x+\frac{\left(e^2a^2+2d^2ac\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+a)^2,x)
```

```
[Out] 1/13*c^2*e^2*x^13+2/11*c^2*d*e*x^11+1/9*(2*a*c*e^2+c^2*d^2)*x^9+4/7*a*c*d*e
*x^7+1/5*(a^2*e^2+2*a*c*d^2)*x^5+2/3*a^2*d*e*x^3+a^2*d^2*x
```

maxima [A] time = 1.03, size = 89, normalized size = 0.92

$$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2dex^{11}+\frac{4}{7}acdex^7+\frac{1}{9}\left(c^2d^2+2\,ace^2\right)x^9+\frac{2}{3}a^2dex^3+\frac{1}{5}\left(2\,acd^2+a^2e^2\right)x^5+a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 4/7*a*c*d*e*x^7 + 1/9*(c^2*d^2 + 2*
a*c*e^2)*x^9 + 2/3*a^2*d*e*x^3 + 1/5*(2*a*c*d^2 + a^2*e^2)*x^5 + a^2*d^2*x
```

mupad [B] time = 0.05, size = 89, normalized size = 0.92

$$x^5\left(\frac{a^2\,e^2}{5}+\frac{2\,c\,a\,d^2}{5}\right)+x^9\left(\frac{c^2\,d^2}{9}+\frac{2\,a\,c\,e^2}{9}\right)+a^2\,d^2\,x+\frac{c^2\,e^2\,x^{13}}{13}+\frac{2\,a^2\,d\,e\,x^3}{3}+\frac{2\,c^2\,d\,e\,x^{11}}{11}+\frac{4\,a\,c\,d\,e\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2*(d + e*x^2)^2,x)
```

```
[Out] x^5*((a^2*e^2)/5 + (2*a*c*d^2)/5) + x^9*((c^2*d^2)/9 + (2*a*c*e^2)/9) + a^2
*d^2*x + (c^2*e^2*x^13)/13 + (2*a^2*d*e*x^3)/3 + (2*c^2*d*e*x^11)/11 + (4*a
*c*d*e*x^7)/7
```

sympy [A] time = 0.09, size = 104, normalized size = 1.07

$$a^2d^2x+\frac{2a^2dex^3}{3}+\frac{4acdex^7}{7}+\frac{2c^2dex^{11}}{11}+\frac{c^2e^2x^{13}}{13}+x^9\left(\frac{2ace^2}{9}+\frac{c^2d^2}{9}\right)+x^5\left(\frac{a^2e^2}{5}+\frac{2acd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**2*x + 2*a**2*d*e*x**3/3 + 4*a*c*d*e*x**7/7 + 2*c**2*d*e*x**11/11 +
c**2*e**2*x**13/13 + x**9*(2*a*c*e**2/9 + c**2*d**2/9) + x**5*(a**2*e**2/5
+ 2*a*c*d**2/5)
```

3.130 $\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx$

Optimal. Leaf size=60

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

```
[Out] a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1/11*c^2*e*
x^11
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9
 + (c^2*e*x^11)/11
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx &= \int \left(a^2d+a^2ex^2+2acdx^4+2acex^6+c^2dx^8+c^2ex^{10}\right)\,dx\\ &= a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 60, normalized size = 1.00

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9
 + (c^2*e*x^11)/11
```

fricas [A] time = 0.35, size = 50, normalized size = 0.83

$$\frac{1}{11}x^{11}ec^2+\frac{1}{9}x^9dc^2+\frac{2}{7}x^7eca+\frac{2}{5}x^5dca+\frac{1}{3}x^3ea^2+xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e*c^2 + 1/9*x^9*d*c^2 + 2/7*x^7*e*c*a + 2/5*x^5*d*c*a + 1/3*x^3*e
*a^2 + x*d*a^2
```

giac [A] time = 0.15, size = 53, normalized size = 0.88

$$\frac{1}{11}\,c^{2}x^{11}e+\frac{1}{9}\,c^{2}dx^{9}+\frac{2}{7}\,acx^{7}e+\frac{2}{5}\,acdx^{5}+\frac{1}{3}\,a^{2}x^{3}e+a^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11*e + 1/9*c^2*d*x^9 + 2/7*a*c*x^7*e + 2/5*a*c*d*x^5 + 1/3*a^2*x
^3*e + a^2*d*x
```

maple [A] time = 0.00, size = 51, normalized size = 0.85

$$\frac{1}{11}c^{2}e\,x^{11}+\frac{1}{9}c^{2}d\,x^{9}+\frac{2}{7}ace\,x^{7}+\frac{2}{5}acd\,x^{5}+\frac{1}{3}a^{2}e\,x^{3}+a^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^2,x)
```

```
[Out] a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1/11*c^2*e*
x^11
```

maxima [A] time = 1.04, size = 50, normalized size = 0.83

$$\frac{1}{11}\,c^{2}ex^{11}+\frac{1}{9}\,c^{2}dx^{9}+\frac{2}{7}\,acex^{7}+\frac{2}{5}\,acdx^{5}+\frac{1}{3}\,a^{2}ex^{3}+a^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*e
*x^3 + a^2*d*x
```

mupad [B] time = 0.03, size = 50, normalized size = 0.83

$$\frac{e\,a^2\,x^3}{3}+d\,a^2\,x+\frac{2\,e\,a\,c\,x^7}{7}+\frac{2\,d\,a\,c\,x^5}{5}+\frac{e\,c^2\,x^{11}}{11}+\frac{d\,c^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2*(d + e*x^2),x)
```

```
[Out] (a^2*e*x^3)/3 + (c^2*d*x^9)/9 + (c^2*e*x^11)/11 + a^2*d*x + (2*a*c*d*x^5)/5
 + (2*a*c*e*x^7)/7
```

sympy [A] time = 0.08, size = 60, normalized size = 1.00

$$a^{2}dx+\frac{a^{2}ex^{3}}{3}+\frac{2acdx^{5}}{5}+\frac{2acex^{7}}{7}+\frac{c^{2}dx^{9}}{9}+\frac{c^{2}ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**3/3 + 2*a*c*d*x**5/5 + 2*a*c*e*x**7/7 + c**2*d*x**9/9
+ c**2*e*x**11/11
```

3.131 $\int \left(a+cx^4\right)^2 dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}$$

```
[Out] a^2*x+2/5*a*c*x^5+1/9*c^2*x^9
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^4\right)^2 dx &= \int \left(a^2+2acx^4+c^2x^8\right) dx\\ &= a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

fricas [A] time = 0.34, size = 21, normalized size = 0.84

$$\frac{1}{9}x^9c^2+\frac{2}{5}x^5ca+xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/5*x^5*c*a + x*a^2
```

giac [A] time = 0.17, size = 21, normalized size = 0.84

$$\frac{1}{9}c^2x^9+\frac{2}{5}acx^5+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{9}c^2x^9 + \frac{2}{5}ac\,x^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2,x)
```

```
[Out] a^2*x+2/5*a*c*x^5+1/9*c^2*x^9
```

maxima [A] time = 1.00, size = 21, normalized size = 0.84

$$\frac{1}{9}\,c^2x^9 + \frac{2}{5}\,acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x + \frac{2\,a\,c\,x^5}{5} + \frac{c^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2,x)
```

```
[Out] a^2*x + (c^2*x^9)/9 + (2*a*c*x^5)/5
```

sympy [A] time = 0.07, size = 22, normalized size = 0.88

$$a^2x + \frac{2acx^5}{5} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2,x)
```

```
[Out] a**2*x + 2*a*c*x**5/5 + c**2*x**9/9
```

3.132 $\int \frac{(a+cx^4)^2}{d+ex^2} \, dx$

Optimal. Leaf size=108

$$\frac{(ae^2+cd^2)^2 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\, e^{9/2}} - \frac{cdx(2ae^2+cd^2)}{e^4} + \frac{cx^3(2ae^2+cd^2)}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e}$$

```
[Out] -c*d*(2*a*e^2+c*d^2)*x/e^4+1/3*c*(2*a*e^2+c*d^2)*x^3/e^3-1/5*c^2*d*x^5/e^2+
1/7*c^2*x^7/e+(a*e^2+c*d^2)^2*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1154, 205}

$$\frac{cx^3(2ae^2+cd^2)}{3e^3} - \frac{cdx(2ae^2+cd^2)}{e^4} + \frac{(ae^2+cd^2)^2 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\, e^{9/2}} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] -((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^3)/(3*e^3) - (c^2
*d*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(Sqrt[d]*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+cx^4)^2}{d+ex^2} \, dx &= \int \left(-\frac{cd(cd^2+2ae^2)}{e^4} + \frac{c(cd^2+2ae^2)x^2}{e^3} - \frac{c^2dx^4}{e^2} + \frac{c^2x^6}{e} + \frac{c^2d^4+2acd^2e^2+a^2e^4}{e^4(d+ex^2)}\right) dx \\
&= -\frac{cd(cd^2+2ae^2)x}{e^4} + \frac{c(cd^2+2ae^2)x^3}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + \frac{(cd^2+ae^2)^2 \int \frac{1}{d+ex^2} \, dx}{e^4} \\
&= -\frac{cd(cd^2+2ae^2)x}{e^4} + \frac{c(cd^2+2ae^2)x^3}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + \frac{(cd^2+ae^2)^2 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\, e^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 97, normalized size = 0.90

$$\frac{(ae^2+cd^2)^2 \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\, e^{9/2}} + \frac{cx\left(70ae^2(ex^2-3d)+c(-105d^3+35d^2ex^2-21de^2x^4+15e^3x^6)\right)}{105e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] (c*x*(70*a*e^2*(-3*d + e*x^2) + c*(-105*d^3 + 35*d^2*e*x^2 - 21*d*e^2*x^4 +
 15*e^3*x^6)))/(105*e^4) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(Sqrt[d]*e^(9/2))
```

fricas [A] time = 0.40, size = 268, normalized size = 2.48

$$\left[\frac{30\,c^2de^4x^7 - 42\,c^2d^2e^3x^5 + 70\left(c^2d^3e^2 + 2\,acde^4\right)x^3 - 105\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right) -}{210\,de^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/210*(30*c^2*d*e^4*x^7 - 42*c^2*d^2*e^3*x^5 + 70*(c^2*d^3*e^2 + 2*a*c*d*e
^4)*x^3 - 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-d*e)*log((e*x^2 - 2
*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 210*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5
), 1/105*(15*c^2*d*e^4*x^7 - 21*c^2*d^2*e^3*x^5 + 35*(c^2*d^3*e^2 + 2*a*c*d
*e^4)*x^3 + 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)*arctan(sqrt(d
*e)*x/d) - 105*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5)]
```

giac [A] time = 0.16, size = 105, normalized size = 0.97

$$\frac{\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} + \frac{1}{105}\left(15\,c^2x^7e^6 - 21\,c^2dx^5e^5 + 35\,c^2d^2x^3e^4 - 105\,c^2d^3xe^3 + 70\,acx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt
(d) + 1/105*(15*c^2*x^7*e^6 - 21*c^2*d*x^5*e^5 + 35*c^2*d^2*x^3*e^4 - 105*c
^2*d^3*x*e^3 + 70*a*c*x^3*e^6 - 210*a*c*d*x*e^5)*e^(-7)
```

maple [A] time = 0.00, size = 136, normalized size = 1.26

$$\frac{c^2x^7}{7e} - \frac{c^2d\,x^5}{5e^2} + \frac{2ac\,x^3}{3e} + \frac{c^2d^2x^3}{3e^3} + \frac{a^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + \frac{2ac\,d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\;e^2} + \frac{c^2d^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\;e^4} - \frac{2acdx}{e^2} - \frac{c^2d^3x}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d),x)
```

```
[Out] 1/7*c^2*x^7/e-1/5*c^2*d*x^5/e^2+2/3*c/e*x^3*a+1/3*c^2/e^3*x^3*d^2-2*c/e^2*d
*a*x-c^2/e^4*d^3*x+1/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a^2+2/e^2/(d*e)^
(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a*c*d^2+1/e^4/(d*e)^(1/2)*arctan(1/(d*e)^(1
/2)*e*x)*c^2*d^4
```

maxima [A] time = 2.45, size = 113, normalized size = 1.05

$$\frac{\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^4} + \frac{15\,c^2e^3x^7 - 21\,c^2de^2x^5 + 35\left(c^2d^2e + 2\,ace^3\right)x^3 - 105\left(c^2d^3 + 2\,acde^2\right)x}{105\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) +
 1/105*(15*c^2*e^3*x^7 - 21*c^2*d*e^2*x^5 + 35*(c^2*d^2*e + 2*a*c*e^3)*x^3
- 105*(c^2*d^3 + 2*a*c*d*e^2)*x)/e^4
```

mupad [B] time = 4.39, size = 141, normalized size = 1.31

$$x^3\left(\frac{c^2\,d^2}{3\,e^3}+\frac{2\,a\,c}{3\,e}\right)+\frac{c^2\,x^7}{7\,e}-\frac{c^2\,d\,x^5}{5\,e^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,{\left(c\,d^2+a\,e^2\right)}^2}{\sqrt{d}\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}\right)\,{\left(c\,d^2+a\,e^2\right)}^2}{\sqrt{d}\,e^{9/2}}-\frac{d\,x\,\left(\frac{c^2\,d^2}{e^3}+\frac{2\,a\,c}{e}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/(d + e*x^2),x)
```

```
[Out] x^3*((c^2*d^2)/(3*e^3) + (2*a*c)/(3*e)) + (c^2*x^7)/(7*e) - (c^2*d*x^5)/(5*
e^2) + (atan((e^(1/2)*x*(a*e^2 + c*d^2)^2)/(d^(1/2)*(a^2*e^4 + c^2*d^4 + 2*
a*c*d^2*e^2)))*(a*e^2 + c*d^2)^2)/(d^(1/2)*e^(9/2)) - (d*x*((c^2*d^2)/e^3 +
 (2*a*c)/e))/e
```

sympy [B] time = 0.50, size = 236, normalized size = 2.19

$$-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}+x^3\left(\frac{2ac}{3e}+\frac{c^2d^2}{3e^3}\right)+x\left(-\frac{2acd}{e^2}-\frac{c^2d^3}{e^4}\right)-\frac{\sqrt{-\frac{1}{de^9}}\,\left(ae^2+cd^2\right)^2\log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}\left(ae^2+cd^2\right)^2}{a^2e^4+2acd^2e^2+c^2d^4}+x\right)}{2}+\sqrt{-\frac{1}{de^9}}\,(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d),x)
```

```
[Out] -c**2*d*x**5/(5*e**2) + c**2*x**7/(7*e) + x**3*(2*a*c/(3*e) + c**2*d**2/(3*
e**3)) + x*(-2*a*c*d/e**2 - c**2*d**3/e**4) - sqrt(-1/(d*e**9))*(a*e**2 + c
*d**2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2
*a*c*d**2*e**2 + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2
*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2*a*c*d**2*
e**2 + c**2*d**4) + x)/2
```

3.133 $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=131

$$\frac{x\left(ae^2+cd^2\right)^2}{2de^4\left(d+ex^2\right)}+\frac{cx\left(2ae^2+3cd^2\right)}{e^4}-\frac{\left(7cd^2-ae^2\right)\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}$$

```
[Out] c*(2*a*e^2+3*c*d^2)*x/e^4-2/3*c^2*d*x^3/e^3+1/5*c^2*x^5/e^2+1/2*(a*e^2+c*d^
2)^2*x/d/e^4/(e*x^2+d)-1/2*(-a*e^2+7*c*d^2)*(a*e^2+c*d^2)*arctan(x*e^(1/2)/
d^(1/2))/d^(3/2)/e^(9/2)
```

Rubi [A] time = 0.19, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1158, 1810, 205}

$$\frac{x\left(ae^2+cd^2\right)^2}{2de^4\left(d+ex^2\right)}+\frac{cx\left(2ae^2+3cd^2\right)}{e^4}-\frac{\left(7cd^2-ae^2\right)\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) +
 ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c*d^2 - a*e^2)*(c*d^2 +
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{\int \frac{-a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{2cd(cd^2+2ae^2)x^2}{e^3}+\frac{2c^2d^2x^4}{e^2}-\frac{2c^2dx^6}{e}}{d+ex^2}\,dx}{2d}$$

$$= \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{\int \left(-\frac{2cd(3cd^2+2ae^2)}{e^4}+\frac{4c^2d^2x^2}{e^3}-\frac{2c^2dx^4}{e^2}+\frac{7c^2d^4+6acd^2e^2-a^2e^4}{e^4(d+ex^2)}\right)dx}{2d}$$

$$= \frac{c(3cd^2+2ae^2)x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{\left((7cd^2-ae^2)(cd^2+ae^2)\right)\int \frac{1}{d+ex^2}\,dx}{2de^4}$$

$$= \frac{c(3cd^2+2ae^2)x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{(7cd^2-ae^2)(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

Mathematica [A] time = 0.11, size = 134, normalized size = 1.02

$$-\frac{(-a^2e^4+6acd^2e^2+7c^2d^4)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}} + \frac{x(ae^2+cd^2)^2}{2de^4(d+ex^2)} + \frac{cx(2ae^2+3cd^2)}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) +
 ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c^2*d^4 + 6*a*c*d^2*e^2
- a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

fricas [A] time = 0.40, size = 394, normalized size = 3.01

$$\left[\frac{12\,c^2d^2e^4x^7 - 28\,c^2d^3e^3x^5 + 20\,(7\,c^2d^4e^2 + 6\,acd^2e^4)x^3 + 15\,(7\,c^2d^5 + 6\,acd^3e^2 - a^2de^4 + (7\,c^2d^4e + 6\,acd^2e^3 - a^2}{60\,(d^2e^6x^2 + d^3e^5)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^2*d^2*e^4*x^7 - 28*c^2*d^3*e^3*x^5 + 20*(7*c^2*d^4*e^2 + 6*a*c*
d^2*e^4)*x^3 + 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*d^4*e + 6
*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e
*x^2 + d)) + 30*(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x)/(d^2*e^6*x^2 +
 d^3*e^5), 1/30*(6*c^2*d^2*e^4*x^7 - 14*c^2*d^3*e^3*x^5 + 10*(7*c^2*d^4*e^2
 + 6*a*c*d^2*e^4)*x^3 - 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*
d^4*e + 6*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 15*
(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x)/(d^2*e^6*x^2 + d^3*e^5)]
```

giac [A] time = 0.17, size = 128, normalized size = 0.98

$$\frac{1}{15}\left(3\,c^2x^5e^8 - 10\,c^2dx^3e^7 + 45\,c^2d^2xe^6 + 30\,acxe^8\right)e^{(-10)} - \frac{(7\,c^2d^4 + 6\,acd^2e^2 - a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{(c^2d^4x +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*c^2*x^5*e^8 - 10*c^2*d*x^3*e^7 + 45*c^2*d^2*x*e^6 + 30*a*c*x*e^8)*e
^(-10) - 1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*arctan(x*e^(1/2)/sqrt(d)
)*e^(-9/2)/d^(3/2) + 1/2*(c^2*d^4*x + 2*a*c*d^2*x*e^2 + a^2*x*e^4)*e^(-4)/(
(x^2*e + d)*d)
```

maple [A] time = 0.01, size = 170, normalized size = 1.30

$$\frac{c^2x^5}{5e^2}-\frac{2c^2d\,x^3}{3e^3}+\frac{a^2x}{2\left(e\,x^2+d\right)d}+\frac{a^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d}+\frac{acdx}{\left(e\,x^2+d\right)e^2}-\frac{3acd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^2}+\frac{c^2d^3x}{2\left(e\,x^2+d\right)e^4}-\frac{7c^2d^3\arct}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^2,x)
```

```
[Out] 1/5*c^2*x^5/e^2-2/3*c^2*d*x^3/e^3+2*c/e^2*a*x+3*c^2/e^4*d^2*x+1/2/d*x/(e*x^
2+d)*a^2+1/e^2*d*x/(e*x^2+d)*a*c+1/2/e^4*d^3*x/(e*x^2+d)*c^2+1/2/d/(d*e)^(1
/2)*arctan(1/(d*e)^(1/2)*e*x)*a^2-3/e^2*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*
e*x)*a*c-7/2/e^4*d^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c^2
```

maxima [A] time = 2.28, size = 142, normalized size = 1.08

$$\frac{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)x}{2\left(de^5x^2+d^2e^4\right)}+\frac{3\,c^2e^2x^5-10\,c^2dex^3+15\left(3\,c^2d^2+2\,ace^2\right)x}{15\,e^4}-\frac{\left(7\,c^2d^4+6\,acd^2e^2-a^2e^4\right)\arctan\left(\frac{ex}{\sqrt{d}}\right.}{2\sqrt{de}\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*x/(d*e^5*x^2 + d^2*e^4) + 1/15*(3*c
^2*e^2*x^5 - 10*c^2*d*e*x^3 + 15*(3*c^2*d^2 + 2*a*c*e^2)*x)/e^4 - 1/2*(7*c^
2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^4)
```

mupad [B] time = 4.40, size = 183, normalized size = 1.40

$$x\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,a\,c}{e^2}\right)+\frac{c^2\,x^5}{5\,e^2}-\frac{2\,c^2\,d\,x^3}{3\,e^3}+\frac{x\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}{2\,d\,\left(e^5\,x^2+d\,e^4\right)}-\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,\left(c\,d^2+a\,e^2\right)\,\left(a\,e^2-7\,c\,d^2\right)}{\sqrt{d}\,\left(-a^2\,e^4+6\,a\,c\,d^2\,e^2+7\,c^2\,d^4\right)}\right)\,\left(c\,d^2+a\,e\right.}{2\,d^{3/2}\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/(d + e*x^2)^2,x)
```

```
[Out] x*((3*c^2*d^2)/e^4 + (2*a*c)/e^2) + (c^2*x^5)/(5*e^2) - (2*c^2*d*x^3)/(3*e^
3) + (x*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(2*d*(d*e^4 + e^5*x^2)) - (ata
n((e^(1/2)*x*(a*e^2 + c*d^2)*(a*e^2 - 7*c*d^2))/(d^(1/2)*(7*c^2*d^4 - a^2*e
^4 + 6*a*c*d^2*e^2)))*(a*e^2 + c*d^2)*(a*e^2 - 7*c*d^2))/(2*d^(3/2)*e^(9/2)
)
```

sympy [B] time = 0.93, size = 314, normalized size = 2.40

$$-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}+x\left(\frac{2ac}{e^2}+\frac{3c^2d^2}{e^4}\right)+\frac{x\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{2d^2e^4+2de^5x^2}-\frac{\sqrt{-\frac{1}{d^3e^9}}\,\left(ae^2-7cd^2\right)\left(ae^2+cd^2\right)\log\left(-\frac{d^2e^4\sqrt{-\frac{1}{d^3}}}{a^2e}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**2,x)
```

```
[Out] -2*c**2*d*x**3/(3*e**3) + c**2*x**5/(5*e**2) + x*(2*a*c/e**2 + 3*c**2*d**2/
e**4) + x*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)/(2*d**2*e**4 + 2*d*e**5
*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(-d*
*2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e*
*4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4 + sqrt(-1/(d**3*e**9))*(a*e**2 -
 7*c*d**2)*(a*e**2 + c*d**2)*log(d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7
*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)
/4
```

3.134 $\int \frac{(a+cx^4)^2}{(d+ex^2)^3} dx$

Optimal. Leaf size=155

$$\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{8d^2\left(d+ex^2\right)}+\frac{\left(3a^2e^4+6acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}$$

```
[Out] -3*c^2*d*x/e^4+1/3*c^2*x^3/e^3+1/4*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^2+1/8*
(3*a^2-13*c^2*d^4/e^4-10*a*c*d^2/e^2)*x/d^2/(e*x^2+d)+1/8*(3*a^2*e^4+6*a*c*
d^2*e^2+35*c^2*d^4)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(9/2)
```

Rubi [A] time = 0.25, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1158, 1814, 1153, 205}

$$\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{8d^2\left(d+ex^2\right)}+\frac{\left(3a^2e^4+6acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (-3*c^2*d*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + a*e^2)^2*x)/(4*d*e^4*(d +
e*x^2)^2) + ((3*a^2 - (13*c^2*d^4)/e^4 - (10*a*c*d^2)/e^2)*x)/(8*d^2*(d + e
*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(8*d^(5/2)*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx = \frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\int \frac{-3a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{4cd\left(cd^2+2ae^2\right)x^2}{e^3}+\frac{4c^2d^2x^4}{e^2}-\frac{4c^2dx^6}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}$$

$$= \frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} + \frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2\left(d+ex^2\right)} + \frac{\int \frac{3a^2+\frac{11c^2d^4}{e^4}+\frac{6acd^2}{e^2}-\frac{16c^2d^3x^2}{e^3}+\frac{8c^2d^2x^4}{e^2}}{d+ex^2}\,dx}{8d^2}$$

$$= \frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} + \frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2\left(d+ex^2\right)} + \frac{\int \left(-\frac{24c^2d^3}{e^4}+\frac{8c^2d^2x^2}{e^3}+\frac{35c^2d^4+6acd^2e^2+3a^2e^4}{e^4\left(d+ex^2\right)}\right)dx}{8d^2}$$

$$= -\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} + \frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} + \frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2\left(d+ex^2\right)} + \frac{\left(35c^2d^4+6acd^2e^2+3a^2e^4\right)\int}{8d^2e^4}$$

$$= -\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} + \frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} + \frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2\left(d+ex^2\right)} + \frac{\left(35c^2d^4+6acd^2e^2+3a^2e^4\right)\mathrm{ta}}{8d^{5/2}e^{9/2}}$$

Mathematica [A] time = 0.11, size = 154, normalized size = 0.99

$$\frac{x\left(3a^2e^4\left(5d+3ex^2\right)-6acd^2e^2\left(3d+5ex^2\right)-c^2d^2\left(105d^3+175d^2ex^2+56de^2x^4-8e^3x^6\right)\right)}{24d^2e^4\left(d+ex^2\right)^2}+\frac{\left(3a^2e^4+6acd^2e^2+3\right.}{8d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (x*(3*a^2*e^4*(5*d + 3*e*x^2) - 6*a*c*d^2*e^2*(3*d + 5*e*x^2) - c^2*d^2*(10
5*d^3 + 175*d^2*e*x^2 + 56*d*e^2*x^4 - 8*e^3*x^6)))/(24*d^2*e^4*(d + e*x^2)
^2) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(8*d^(5/2)*e^(9/2))
```

fricas [A] time = 0.41, size = 516, normalized size = 3.33

$$\left[\frac{16\,c^2d^3e^4x^7-112\,c^2d^4e^3x^5-2\left(175\,c^2d^5e^2+30\,acd^3e^4-9\,a^2de^6\right)x^3-3\left(35\,c^2d^6+6\,acd^4e^2+3\,a^2d^2e^4+\left(35\,c^2d^4\right.\right.}{48\left(d^3e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^2*d^3*e^4*x^7 - 112*c^2*d^4*e^3*x^5 - 2*(175*c^2*d^5*e^2 + 30*a
*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 - 3*(35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e
^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*e^6)*x^4 + 2*(35*c^2*d^5*e + 6
*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d
)/(e*x^2 + d)) - 6*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x)/(d^3*e
^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 56*c^2*d^4*e^3
*x^5 - (175*c^2*d^5*e^2 + 30*a*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 + 3*(35*c^2*d^6
```

```
 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*
e^6)*x^4 + 2*(35*c^2*d^5*e + 6*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*sqrt(d*e)*ar
ctan(sqrt(d*e)*x/d) - 3*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x)/(
d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]
```

giac [A] time = 0.17, size = 145, normalized size = 0.94

$$\frac{1}{3}\left(c^2x^3e^6 - 9\,c^2dxe^5\right)e^{(-9)} + \frac{\left(35\,c^2d^4 + 6\,acd^2e^2 + 3\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,d^{\frac{5}{2}}} - \frac{\left(13\,c^2d^4x^3e + 11\,c^2d^5x + 10\,acd^2x\right.}{8\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^6 - 9*c^2*d*x*e^5)*e^(-9) + 1/8*(35*c^2*d^4 + 6*a*c*d^2*e^2
+ 3*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(5/2) - 1/8*(13*c^2*d^4*x
^3*e + 11*c^2*d^5*x + 10*a*c*d^2*x^3*e^3 + 6*a*c*d^3*x*e^2 - 3*a^2*x^3*e^5
- 5*a^2*d*x*e^4)*e^(-4)/((x^2*e + d)^2*d^2)
```

maple [A] time = 0.01, size = 211, normalized size = 1.36

$$\frac{3a^2e\,x^3}{8\left(e\,x^2+d\right)^2d^2} - \frac{5ac\,x^3}{4\left(e\,x^2+d\right)^2e} - \frac{13c^2d^2x^3}{8\left(e\,x^2+d\right)^2e^3} + \frac{5a^2x}{8\left(e\,x^2+d\right)^2d} - \frac{3acdx}{4\left(e\,x^2+d\right)^2e^2} - \frac{11c^2d^3x}{8\left(e\,x^2+d\right)^2e^4} + \frac{c^2x^3}{3e^3} + \frac{3a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^3,x)
```

```
[Out] 1/3*c^2*x^3/e^3-3*c^2*d*x/e^4+3/8*e/(e*x^2+d)^2/d^2*x^3*a^2-5/4/e/(e*x^2+d)
^2*x^3*a*c-13/8/e^3/(e*x^2+d)^2*d^2*x^3*c^2+5/8/(e*x^2+d)^2/d*x*a^2-3/4/e^2
/(e*x^2+d)^2*d*x*a*c-11/8/e^4/(e*x^2+d)^2*d^3*x*c^2+3/8/d^2/(d*e)^(1/2)*arc
tan(1/(d*e)^(1/2)*e*x)*a^2+3/4/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a*
c+35/8/e^4*d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c^2
```

maxima [A] time = 2.31, size = 167, normalized size = 1.08

$$-\frac{\left(13\,c^2d^4e + 10\,acd^2e^3 - 3\,a^2e^5\right)x^3 + \left(11\,c^2d^5 + 6\,acd^3e^2 - 5\,a^2de^4\right)x}{8\left(d^2e^6x^4 + 2\,d^3e^5x^2 + d^4e^4\right)} + \frac{c^2ex^3 - 9\,c^2dx}{3\,e^4} + \frac{\left(35\,c^2d^4 + 6\,acd^2e^2 + 3\right.}{8\,\sqrt{de}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*c^2*d^4*e + 10*a*c*d^2*e^3 - 3*a^2*e^5)*x^3 + (11*c^2*d^5 + 6*a*c
*d^3*e^2 - 5*a^2*d*e^4)*x)/(d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4) + 1/3*(c
^2*e*x^3 - 9*c^2*d*x)/e^4 + 1/8*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ar
ctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^4)
```

mupad [B] time = 4.41, size = 164, normalized size = 1.06

$$\frac{c^2\,x^3}{3\,e^3} - \frac{\frac{x^3\,\left(-3\,a^2\,e^5+10\,a\,c\,d^2\,e^3+13\,c^2\,d^4\,e\right)}{8\,d^2} + \frac{x\,\left(-5\,a^2\,e^4+6\,a\,c\,d^2\,e^2+11\,c^2\,d^4\right)}{8\,d}}{d^2\,e^4 + 2\,d\,e^5\,x^2 + e^6\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3\,a^2\,e^4 + 6\,a\,c\,d^2\,e^2 + 35\,c^2\,d^4\right)}{8\,d^{5/2}\,e^{9/2}} - \frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/(d + e*x^2)^3,x)
```

```
[Out] (c^2*x^3)/(3*e^3) - ((x^3*(13*c^2*d^4*e - 3*a^2*e^5 + 10*a*c*d^2*e^3))/(8*d
^2) + (x*(11*c^2*d^4 - 5*a^2*e^4 + 6*a*c*d^2*e^2))/(8*d))/(d^2*e^4 + e^6*x^
4 + 2*d*e^5*x^2) + (atan((e^(1/2)*x)/d^(1/2))*(3*a^2*e^4 + 35*c^2*d^4 + 6*a
*c*d^2*e^2))/(8*d^(5/2)*e^(9/2)) - (3*c^2*d*x)/e^4
```

sympy [A] time = 1.71, size = 257, normalized size = 1.66

$$-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}-\frac{\sqrt{-\frac{1}{d^5e^9}}\left(3a^2e^4+6acd^2e^2+35c^2d^4\right)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^5e^9}}\left(3a^2e^4+6acd^2e^2+35c^2d^4\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**3,x)
```

```
[Out] -3*c**2*d*x/e**4 + c**2*x**3/(3*e**3) - sqrt(-1/(d**5*e**9))*(3*a**2*e**4 +
 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/1
6 + sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log
(d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + (x**3*(3*a**2*e**5 - 10*a*c*d**2*
e**3 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 6*a*c*d**3*e**2 - 11*c**2*d**5)
)/(8*d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)
```

3.135 $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^4} \, dx$

Optimal. Leaf size=184

$$\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{24d^2\left(d+ex^2\right)^2}-\frac{\left(-5a^2e^4-2acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}+\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{16d^3\left(d+ex^2\right)}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

```
[Out] c^2*x/e^4+1/6*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^3+1/24*(5*a^2-19*c^2*d^4/e^
4-14*a*c*d^2/e^2)*x/d^2/(e*x^2+d)^2+1/16*(5*a^2+29*c^2*d^4/e^4+2*a*c*d^2/e^
2)*x/d^3/(e*x^2+d)-1/16*(-5*a^2*e^4-2*a*c*d^2*e^2+35*c^2*d^4)*arctan(x*e^(1
/2)/d^(1/2))/d^(7/2)/e^(9/2)
```

Rubi [A] time = 0.30, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1158, 1814, 1157, 388, 205}

$$\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{16d^3\left(d+ex^2\right)}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{24d^2\left(d+ex^2\right)^2}-\frac{\left(-5a^2e^4-2acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) + ((5*a^2 - (19
*c^2*d^4)/e^4 - (14*a*c*d^2)/e^2)*x)/(24*d^2*(d + e*x^2)^2) + ((5*a^2 + (29
*c^2*d^4)/e^4 + (2*a*c*d^2)/e^2)*x)/(16*d^3*(d + e*x^2)) - ((35*c^2*d^4 - 2
*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
```

```
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} - \frac{\int \frac{-5a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{6cd(cd^2+2ae^2)x^2}{e^3}+\frac{6c^2d^2x^4}{e^2}-\frac{6c^2dx^6}{e}}{(d+ex^2)^3}\,dx}{6d}$$

$$= \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\int \frac{3\left(5a^2+\frac{5c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)-\frac{48c^2d^3x^2}{e^3}+\frac{24c^2d^2x^4}{e^2}}{(d+ex^2)^2}\,dx}{24d^2}$$

$$= \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)} - \frac{\int \frac{-3\left(5a^2-\frac{19c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)-}{d+ex^2}}{48d^3}$$

$$= \frac{c^2x}{e^4} + \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)} - \frac{(35c^2d^4-2aca}{}$$

$$= \frac{c^2x}{e^4} + \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)} - \frac{(35c^2d^4-2aca}{}$$

Mathematica [A] time = 0.14, size = 174, normalized size = 0.95

$$\frac{x\left(a^2e^4\left(33d^2+40dex^2+15e^2x^4\right)-2acd^2e^2\left(3d^2+8dex^2-3e^2x^4\right)+c^2d^3\left(105d^3+280d^2ex^2+231de^2x^4+48e^3x\right.\right.}{48d^3e^4(d+ex^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (x*(-2*a*c*d^2*e^2*(3*d^2 + 8*d*e*x^2 - 3*e^2*x^4) + a^2*e^4*(33*d^2 + 40*d
*e*x^2 + 15*e^2*x^4) + c^2*d^3*(105*d^3 + 280*d^2*e*x^2 + 231*d*e^2*x^4 + 4
8*e^3*x^6)))/(48*d^3*e^4*(d + e*x^2)^3) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*
a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

fricas [A] time = 0.45, size = 662, normalized size = 3.60

$$\left[\frac{96\,c^2d^4e^4x^7+6\left(77\,c^2d^5e^3+2\,acd^3e^5+5\,a^2de^7\right)x^5+16\left(35\,c^2d^6e^2-2\,acd^4e^4+5\,a^2d^2e^6\right)x^3+3\left(35\,c^2d^7-2\,acd^5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(96*c^2*d^4*e^4*x^7 + 6*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^7
)*x^5 + 16*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 + 3*(35*c^2
*d^7 - 2*a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 - 5*
a^2*e^7)*x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(35
*c^2*d^6*e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*
sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11*a^2*d
^3*e^5)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5), 1/48*(4
8*c^2*d^4*e^4*x^7 + 3*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^7)*x^5 +
8*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 - 3*(35*c^2*d^7 - 2*
a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 - 5*a^2*e^7)*
x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(35*c^2*d^6*
e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 3
*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11*a^2*d^3*e^5)*x)/(d^4*e^8*x^6 + 3*d^5*e^
7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5)]
```

giac [A] time = 0.16, size = 167, normalized size = 0.91

$$c^2xe^{(-4)} - \frac{\left(35\,c^2d^4 - 2\,acd^2e^2 - 5\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{16\,d^{\frac{7}{2}}} + \frac{\left(87\,c^2d^4x^5e^2 + 136\,c^2d^5x^3e + 6\,acd^2x^5e^4 + 57\,c^2d^6x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] c^2*x*e^(-4) - 1/16*(35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*arctan(x*e^(1/
2)/sqrt(d))*e^(-9/2)/d^(7/2) + 1/48*(87*c^2*d^4*x^5*e^2 + 136*c^2*d^5*x^3*e
 + 6*a*c*d^2*x^5*e^4 + 57*c^2*d^6*x - 16*a*c*d^3*x^3*e^3 + 15*a^2*x^5*e^6 -
 6*a*c*d^4*x*e^2 + 40*a^2*d*x^3*e^5 + 33*a^2*d^2*x*e^4)*e^(-4)/((x^2*e + d)
^3*d^3)
```

maple [A] time = 0.01, size = 262, normalized size = 1.42

$$\frac{5a^2e^2x^5}{16\left(e\,x^2+d\right)^3d^3} + \frac{ac\,x^5}{8\left(e\,x^2+d\right)^3d} + \frac{29c^2d\,x^5}{16\left(e\,x^2+d\right)^3e^2} + \frac{5a^2e\,x^3}{6\left(e\,x^2+d\right)^3d^2} - \frac{ac\,x^3}{3\left(e\,x^2+d\right)^3e} + \frac{17c^2d^2x^3}{6\left(e\,x^2+d\right)^3e^3} + \frac{11}{16\left(e\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^4,x)
```

```
[Out] c^2*x/e^4+5/16*e^2/(e*x^2+d)^3/d^3*x^5*a^2+1/8/(e*x^2+d)^3/d*x^5*a*c+29/16/
e^2/(e*x^2+d)^3*d*x^5*c^2+5/6*e/(e*x^2+d)^3/d^2*x^3*a^2-1/3/e/(e*x^2+d)^3*x
^3*a*c+17/6/e^3/(e*x^2+d)^3*d^2*x^3*c^2+11/16/(e*x^2+d)^3/d*x*a^2-1/8/e^2/(
e*x^2+d)^3*d*x*a*c+19/16/e^4/(e*x^2+d)^3*d^3*x*c^2+5/16/d^3/(d*e)^(1/2)*arc
tan(1/(d*e)^(1/2)*e*x)*a^2+1/8/e^2/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*
a*c-35/16/e^4*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c^2
```

maxima [A] time = 2.39, size = 205, normalized size = 1.11

$$\frac{3\left(29\,c^2d^4e^2 + 2\,acd^2e^4 + 5\,a^2e^6\right)x^5 + 8\left(17\,c^2d^5e - 2\,acd^3e^3 + 5\,a^2de^5\right)x^3 + 3\left(19\,c^2d^6 - 2\,acd^4e^2 + 11\,a^2d^2e^4\right)x}{48\left(d^3e^7x^6 + 3\,d^4e^6x^4 + 3\,d^5e^5x^2 + d^6e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(29*c^2*d^4*e^2 + 2*a*c*d^2*e^4 + 5*a^2*e^6)*x^5 + 8*(17*c^2*d^5*e
- 2*a*c*d^3*e^3 + 5*a^2*d*e^5)*x^3 + 3*(19*c^2*d^6 - 2*a*c*d^4*e^2 + 11*a^2
*d^2*e^4)*x)/(d^3*e^7*x^6 + 3*d^4*e^6*x^4 + 3*d^5*e^5*x^2 + d^6*e^4) + c^2*
x/e^4 - 1/16*(35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*arctan(e*x/sqrt(d*e))
/(sqrt(d*e)*d^3*e^4)
```

mupad [B] time = 4.49, size = 199, normalized size = 1.08

$$\frac{\frac{x^3\left(5\,a^2\,e^5-2\,a\,c\,d^2\,e^3+17\,c^2\,d^4\,e\right)}{6\,d^2}+\frac{x\left(11\,a^2\,e^4-2\,a\,c\,d^2\,e^2+19\,c^2\,d^4\right)}{16\,d}+\frac{x^5\left(5\,a^2\,e^6+2\,a\,c\,d^2\,e^4+29\,c^2\,d^4\,e^2\right)}{16\,d^3}}{d^3\,e^4+3\,d^2\,e^5\,x^2+3\,d\,e^6\,x^4+e^7\,x^6}+\frac{c^2\,x}{e^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(5\,a^2\,e^4+2\,a \cdots\right.}{16\,d^{7/2}\,e^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/(d + e*x^2)^4,x)
```

```
[Out] ((x^3*(5*a^2*e^5 + 17*c^2*d^4*e - 2*a*c*d^2*e^3))/(6*d^2) + (x*(11*a^2*e^4
+ 19*c^2*d^4 - 2*a*c*d^2*e^2))/(16*d) + (x^5*(5*a^2*e^6 + 29*c^2*d^4*e^2 +
2*a*c*d^2*e^4))/(16*d^3))/(d^3*e^4 + e^7*x^6 + 3*d*e^6*x^4 + 3*d^2*e^5*x^2)
 + (c^2*x)/e^4 + (atan((e^(1/2)*x)/d^(1/2))*(5*a^2*e^4 - 35*c^2*d^4 + 2*a*c
*d^2*e^2))/(16*d^(7/2)*e^(9/2))
```

sympy [A] time = 2.61, size = 292, normalized size = 1.59

$$\frac{c^2x}{e^4}-\frac{\sqrt{-\frac{1}{d^7e^9}}\left(5a^2e^4+2acd^2e^2-35c^2d^4\right)\log\left(-d^4e^4\sqrt{-\frac{1}{d^7e^9}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{d^7e^9}}\left(5a^2e^4+2acd^2e^2-35c^2d^4\right)\log\left(d^4e^4 \cdots\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**4,x)
```

```
[Out] c**2*x/e**4 - sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2
*d**4)*log(-d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + sqrt(-1/(d**7*e**9))*(
5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2*d**4)*log(d**4*e**4*sqrt(-1/(d**7*e
**9)) + x)/32 + (x**5*(15*a**2*e**6 + 6*a*c*d**2*e**4 + 87*c**2*d**4*e**2)
+ x**3*(40*a**2*d*e**5 - 16*a*c*d**3*e**3 + 136*c**2*d**5*e) + x*(33*a**2*d
**2*e**4 - 6*a*c*d**4*e**2 + 57*c**2*d**6))/(48*d**6*e**4 + 144*d**5*e**5*x
**2 + 144*d**4*e**6*x**4 + 48*d**3*e**7*x**6)
```

3.136 $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^5}\,dx$

Optimal. Leaf size=223

$$-\frac{x\left(-35a^2e^4-6acd^2e^2+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{48d^2\left(d+ex^2\right)^3}+\frac{\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}}+\frac{x\left(35\right.}{1}$$

```
[Out] 1/8*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^4+1/48*(7*a^2-25*c^2*d^4/e^4-18*a*c*d
^2/e^2)*x/d^2/(e*x^2+d)^3+1/192*(35*a^2+163*c^2*d^4/e^4+6*a*c*d^2/e^2)*x/d^
3/(e*x^2+d)^2-1/128*(-35*a^2*e^4-6*a*c*d^2*e^2+93*c^2*d^4)*x/d^4/e^4/(e*x^2
+d)+1/128*(35*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)*arctan(x*e^(1/2)/d^(1/2))/d
^(9/2)/e^(9/2)
```

Rubi [A] time = 0.34, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1158, 1814, 1157, 385, 205}

$$-\frac{x\left(-35a^2e^4-6acd^2e^2+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{192d^3\left(d+ex^2\right)^2}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{48d^2\left(d+ex^2\right)^3}+\frac{\left(35a^2e^4+6acd^2e^2+\right.}{128d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((c*d^2 + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) + ((7*a^2 - (25*c^2*d^4)/e^4
- (18*a*c*d^2)/e^2)*x)/(48*d^2*(d + e*x^2)^3) + ((35*a^2 + (163*c^2*d^4)/e^
4 + (6*a*c*d^2)/e^2)*x)/(192*d^3*(d + e*x^2)^2) - ((93*c^2*d^4 - 6*a*c*d^2*
e^2 - 35*a^2*e^4)*x)/(128*d^4*e^4*(d + e*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e
^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
```

```
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} - \frac{\int \frac{-7a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{8cd(cd^2+2ae^2)x^2}{e^3}+\frac{8c^2d^2x^4}{e^2}-\frac{8c^2dx^6}{e}}{(d+ex^2)^4}\,dx}{8d}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\int \frac{35a^2+\frac{19c^2d^4}{e^4}+\frac{6acd^2}{e^2}-\frac{96c^2d^3x^2}{e^3}+\frac{48c^2d^2x^4}{e^2}}{(d+ex^2)^3}\,dx}{48d^2}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3(d+ex^2)^2} - \frac{\int \frac{-3\left(35a^2-\frac{29c^2d^4}{e^4}+\frac{6acd}{e^2}\right.}{(d+ex^2)^2}}{192d^3}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3(d+ex^2)^2} - \frac{(93c^2d^4-6acd^2e^2}{128d^4e^4(d+}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3(d+ex^2)^2} - \frac{(93c^2d^4-6acd^2e^2}{128d^4e^4(d+}$$

Mathematica [A] time = 0.19, size = 200, normalized size = 0.90

$$\frac{3\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\frac{\sqrt{d}\sqrt{e}x\left(a^2e^4\left(279d^3+511d^2ex^2+385de^2x^4+105e^3x^6\right)-6acd^2e^2\left(3d^3+11d^2ex^2-11de^2x^4-3e\right.\right.}{(d+ex^2)^4}}{384d^{9/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((Sqrt[d]*Sqrt[e]*x*(-6*a*c*d^2*e^2*(3*d^3 + 11*d^2*e*x^2 - 11*d*e^2*x^4 -
3*e^3*x^6) + a^2*e^4*(279*d^3 + 511*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6
) - c^2*d^4*(105*d^3 + 385*d^2*e*x^2 + 511*d*e^2*x^4 + 279*e^3*x^6)))/(d +
e*x^2)^4 + 3*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(384*d^(9/2)*e^(9/2))
```

fricas [A] time = 0.45, size = 806, normalized size = 3.61

$$\left[-\frac{6\left(93\,c^2d^5e^4-6\,acd^3e^6-35\,a^2de^8\right)x^7+2\left(511\,c^2d^6e^3-66\,acd^4e^5-385\,a^2d^2e^7\right)x^5+2\left(385\,c^2d^7e^2+66\,acd^5e^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="fricas")
```

```
[Out] [-1/768*(6*(93*c^2*d^5*e^4 - 6*a*c*d^3*e^6 - 35*a^2*d*e^8)*x^7 + 2*(511*c^2
*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*d^2*e^7)*x^5 + 2*(385*c^2*d^7*e^2 + 66*
a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 + 3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2
*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^
5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e
^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5
)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2
*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 +
6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), -1/384*(3*(93*c^2*d^5*e^4 - 6*a*c
*d^3*e^6 - 35*a^2*d*e^8)*x^7 + (511*c^2*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*
d^2*e^7)*x^5 + (385*c^2*d^7*e^2 + 66*a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 - 3
*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2
*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*
x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d
^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d)
 + 3*(35*c^2*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^5*e^9*x^8 + 4*d^
6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5)]
```

giac [A] time = 0.25, size = 198, normalized size = 0.89

$$\frac{\left(35\,c^2d^4+6\,acd^2e^2+35\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{128\,d^{\frac{9}{2}}}-\frac{\left(279\,c^2d^4x^7e^3+511\,c^2d^5x^5e^2-18\,acd^2x^7e^5+385\,c^2d^6x^3e-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="giac")
```

```
[Out] 1/128*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e
^(-9/2)/d^(9/2) - 1/384*(279*c^2*d^4*x^7*e^3 + 511*c^2*d^5*x^5*e^2 - 18*a*c
*d^2*x^7*e^5 + 385*c^2*d^6*x^3*e - 66*a*c*d^3*x^5*e^4 + 105*c^2*d^7*x - 105
*a^2*x^7*e^7 + 66*a*c*d^4*x^3*e^3 - 385*a^2*d*x^5*e^6 + 18*a*c*d^5*x*e^2 -
511*a^2*d^2*x^3*e^5 - 279*a^2*d^3*x*e^4)*e^(-4)/((x^2*e + d)^4*d^4)
```

maple [A] time = 0.01, size = 231, normalized size = 1.04

$$\frac{35a^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\sqrt{de}\;d^4}+\frac{3ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{64\sqrt{de}\;d^2e^2}+\frac{35c^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\sqrt{de}\;e^4}+\frac{\frac{\left(35a^2e^4+6ac\,d^2e^2-93c^2d^4\right)x^7}{128d^4e}+\frac{\left(385a^2e^4+66ac\,d^2e^2-511c^2d^4\right)x^5}{384d^3e^2}\cdots}{\left(e\,x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^5,x)
```

```
[Out] (1/128*(35*a^2*e^4+6*a*c*d^2*e^2-93*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+6
6*a*c*d^2*e^2-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4-66*a*c*d^2*e^2-38
5*c^2*d^4)/d^2/e^3*x^3+1/128*(93*a^2*e^4-6*a*c*d^2*e^2-35*c^2*d^4)/d/e^4*x)
/(e*x^2+d)^4+35/128/d^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a^2+3/64/d^2/
e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a*c+35/128/e^4/(d*e)^(1/2)*arctan
(1/(d*e)^(1/2)*e*x)*c^2
```

maxima [A] time = 2.41, size = 244, normalized size = 1.09

$$-\frac{3\left(93\,c^2d^4e^3-6\,acd^2e^5-35\,a^2e^7\right)x^7+\left(511\,c^2d^5e^2-66\,acd^3e^4-385\,a^2de^6\right)x^5+\left(385\,c^2d^6e+66\,acd^4e^3-511\cdots\right.}{384\left(d^4e^8x^8+4\,d^5e^7x^6+6\,d^6e^6x^4+4\,d^7e^5x^2+d^8e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="maxima")
```

```
[Out] -1/384*(3*(93*c^2*d^4*e^3 - 6*a*c*d^2*e^5 - 35*a^2*e^7)*x^7 + (511*c^2*d^5*
e^2 - 66*a*c*d^3*e^4 - 385*a^2*d*e^6)*x^5 + (385*c^2*d^6*e + 66*a*c*d^4*e^3
 - 511*a^2*d^2*e^5)*x^3 + 3*(35*c^2*d^7 + 6*a*c*d^5*e^2 - 93*a^2*d^3*e^4)*x
)/(d^4*e^8*x^8 + 4*d^5*e^7*x^6 + 6*d^6*e^6*x^4 + 4*d^7*e^5*x^2 + d^8*e^4) +
 1/128*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqr
t(d*e)*d^4*e^4)
```

mupad [B] time = 4.49, size = 240, normalized size = 1.08

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(35\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4\right)}{128\,d^{9/2}\,e^{9/2}}-\frac{\frac{x\,\left(-93\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4\right)}{128\,d\,e^4}-\frac{x^7\,\left(35\,a^2\,e^4+6\,a\,c\,d^2\,e^2-93\,c^2\,d^4\right)}{128\,d^4\,e}+\frac{x^3\,\left(-511\,a^2\,e^4\cdots\right.}{\cdots}}{d^4+4\,d^3\,e\,x^2+6\,d^2\,e^2\,x^4+4\,d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/(d + e*x^2)^5,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(35*a^2*e^4 + 35*c^2*d^4 + 6*a*c*d^2*e^2))/(128*
d^(9/2)*e^(9/2)) - ((x*(35*c^2*d^4 - 93*a^2*e^4 + 6*a*c*d^2*e^2))/(128*d*e^
4) - (x^7*(35*a^2*e^4 - 93*c^2*d^4 + 6*a*c*d^2*e^2))/(128*d^4*e) + (x^3*(38
5*c^2*d^4 - 511*a^2*e^4 + 66*a*c*d^2*e^2))/(384*d^2*e^3) - (x^5*(385*a^2*e^
4 - 511*c^2*d^4 + 66*a*c*d^2*e^2))/(384*d^3*e^2))/(d^4 + e^4*x^8 + 4*d^3*e*
x^2 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4)
```

sympy [A] time = 4.11, size = 335, normalized size = 1.50

$$-\frac{\sqrt{-\frac{1}{d^9e^9}}\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\log\left(-d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256}+\frac{\sqrt{-\frac{1}{d^9e^9}}\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\log\left(d^5e^4\sqrt{\cdots}\right.}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**5,x)
```

```
[Out] -sqrt(-1/(d**9*e**9))*(35*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-
d**5*e**4*sqrt(-1/(d**9*e**9)) + x)/256 + sqrt(-1/(d**9*e**9))*(35*a**2*e**
4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(d**5*e**4*sqrt(-1/(d**9*e**9)) + x)
/256 + (x**7*(105*a**2*e**7 + 18*a*c*d**2*e**5 - 279*c**2*d**4*e**3) + x**5
*(385*a**2*d*e**6 + 66*a*c*d**3*e**4 - 511*c**2*d**5*e**2) + x**3*(511*a**2
*d**2*e**5 - 66*a*c*d**4*e**3 - 385*c**2*d**6*e) + x*(279*a**2*d**3*e**4 -
18*a*c*d**5*e**2 - 105*c**2*d**7))/(384*d**8*e**4 + 1536*d**7*e**5*x**2 + 2
304*d**6*e**6*x**4 + 1536*d**5*e**7*x**6 + 384*d**4*e**8*x**8)
```

3.137 $\int \frac{\left(d+ex^2\right)^4}{a+cx^4} \, dx$

Optimal. Leaf size=437

$$-\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\,\sqrt{c}\,de\left(cd^2-ae^2\right)+c^2d^4\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{9/4}}+\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\right.}{}$$

```
[Out] e^2*(-a*e^2+6*c*d^2)*x/c^2+4/3*d*e^3*x^3/c+1/5*e^4*x^5/c-1/8*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4-4*d*e*(
-a*e^2+c*d^2)*a^(1/2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4-4*d*e*(-a*
e^2+c*d^2)*a^(1/2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x
*2^(1/2)/a^(1/4))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4+4*d*e*(-a*e^2+c*d^2)*a^(1/
2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))
*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4+4*d*e*(-a*e^2+c*d^2)*a^(1/2)*c^(1/2))/a^(3/
4)/c^(9/4)*2^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\,\sqrt{c}\,de\left(cd^2-ae^2\right)+c^2d^4\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{9/4}}+\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(a + c*x^4),x]
```

```
[Out] (e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (4*d*e^3*x^3)/(3*c) + (e^4*x^5)/(5*c) - ((c
^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) + ((c^2
*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) - ((c^2*d
^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(9/
4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 -
a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*c^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^4}{a+cx^4}\,dx = \int \left(\frac{e^2\left(6cd^2-ae^2\right)}{c^2}+\frac{4de^3x^2}{c}+\frac{e^4x^4}{c}+\frac{c^2d^4-6acd^2e^2+a^2e^4+4cde\left(cd^2-ae^2\right)x^2}{c^2\left(a+cx^4\right)}\right)dx$$

$$=\frac{e^2\left(6cd^2-ae^2\right)x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}+\frac{\int\frac{c^2d^4-6acd^2e^2+a^2e^4+4cde\left(cd^2-ae^2\right)x^2}{a+cx^4}\,dx}{c^2}$$

$$=\frac{e^2\left(6cd^2-ae^2\right)x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}-\frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c^2}+\frac{(4cd \ldots}{}$$

$$=\frac{e^2\left(6cd^2-ae^2\right)x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}+\frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}\,c^{7/4}}+\ldots$$

$$=\frac{e^2\left(6cd^2-ae^2\right)x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}+\frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\ldots\right.}{4\sqrt{2}\sqrt[4]{a}\,c^{7/4}}$$

$$=\frac{e^2\left(6cd^2-ae^2\right)x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}-\frac{\left(4cd^3e-4ade^3+\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,c^{7/4}}+\ldots$$

Mathematica [A] time = 0.34, size = 444, normalized size = 1.02

$$160a^{3/4}c^{5/4}de^3x^3+24a^{3/4}c^{5/4}e^4x^5-120a^{3/4}\sqrt[4]{c}\,e^2x\left(ae^2-6cd^2\right)-15\sqrt{2}\left(4a^{3/2}\sqrt{c}\,de^3+a^2e^4-4\sqrt{a}\,c^{3/2}d^3e-6acd^2\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(a + c*x^4),x]
```

```
[Out] (-120*a^(3/4)*c^(1/4)*e^2*(-6*c*d^2 + a*e^2)*x + 160*a^(3/4)*c^(5/4)*d*e^3*
x^3 + 24*a^(3/4)*c^(5/4)*e^4*x^5 - 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2)*
d^3*e - 6*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)] + 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2)*d^3*e - 6
*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)] - 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*c*d^2*
e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2] + 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*
c*d^2*e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(120*a^(3/4)*c^(9/4))
```

fricas [B] time = 4.26, size = 2878, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*c*e^4*x^5 + 80*c*d*e^3*x^3 + 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a*c^2*
d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16 - 56*a*c^7
*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8
*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a
^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a^2*c^
6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d^6*e^1
0 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^8*d^12 - 3
4*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a^5*c^
4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 + 4*(a^3*c^8*d^3*e - a^4*c^7
*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a
^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^
2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c^3*d^7*e -
 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16
- 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a
^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^
2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) - 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a
*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16 - 56*
a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^
4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^1
4 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a
^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d^
6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x - (a*c^8*d^1
2 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a
^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 + 4*(a^3*c^8*d^3*e - a^
4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3
976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a
^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c^3*d^
7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*
d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6
470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7
*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) + 15*c^2*sqrt(-(8*c^3*d^7*e -
 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-(c^8*d^16
- 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a
^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^
2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 -
 36*a^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c
^3*d^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^
8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 +
```

```
239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 - 4*(a^3*c^8*d^3*e
 - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^
4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 +
924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c
^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-
(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^
6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 5
6*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) - 15*c^2*sqrt(-(8*c^3*d^
7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-(c^8*
d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6
470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7
*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*
e^2 - 36*a^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*
a^5*c^3*d^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x -
(a*c^8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e
^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 - 4*(a^3*c^8*
d^3*e - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^
12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^
10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(
-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*s
qrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^
10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^1
2 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) + 60*(6*c*d^2*e^2 -
 a*e^4)*x)/c^2
```

giac [A] time = 0.19, size = 498, normalized size = 1.14

$$\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^4-6\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e^2+4\left(ac^3\right)^{\frac{3}{4}}cd^3e+\left(ac^3\right)^{\frac{1}{4}}a^2ce^4-4\left(ac^3\right)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^4}+\frac{\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 4*(a*c
^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3/4)*a*d*e^3)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/4*sqrt(
2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 4*(a*c^3)^(3/4)
*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^
3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 4*(a*c^3)^(3/4)*c*d^3*e
+ (a*c^3)^(1/4)*a^2*c*e^4 + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 + sqrt(2)*x*(a
/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*
c^3)^(1/4)*a*c^2*d^2*e^2 - 4*(a*c^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^
4 + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^4) + 1/15*(3*c^4*x^5*e^4 + 20*c^4*d*x^3*e^3 + 90*c^4*d^2*x*e^2 - 15*a*c
^3*x*e^4)/c^5
```

maple [B] time = 0.01, size = 741, normalized size = 1.70

$$\frac{e^4x^5}{5c}+\frac{4d\,e^3x^3}{3c}-\frac{\sqrt{2}\,ad\,e^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}-\frac{\sqrt{2}\,ad\,e^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}-\frac{\sqrt{2}\,ad\,e^3\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}-\frac{a\,e^4x}{c^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*x^4+a),x)
```

```
[Out] 1/5*e^4*x^5/c+4/3*d*e^3*x^3/c-e^4/c^2*a*x+6*e^2/c*d^2*x+1/4/c^2*(a/c)^(1/4)
*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^4-3/2/c*(a/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e^2+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x-1)*d^4+1/8/c^2*(a/c)^(1/4)*a*2^(1/2)*ln((x^2+(a/c)^(1/4)*
x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*e^4-3/4/c*(
a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1
/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2*e^2+1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)
^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^4+
1/4/c^2*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^4-3/2/c*(a/
c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e^2+1/4*(a/c)^(1/4)/a*
2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^4-1/2/c^2/(a/c)^(1/4)*2^(1/2)*ln(
(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2)))*a*d*e^3+1/2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)
^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^3*e-1/c^2/(a/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*a*d*e^3+1/c/(a/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e-1/c^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/c)^(1/4)*x+1)*a*d*e^3+1/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^3*e
```

maxima [A] time = 2.45, size = 432, normalized size = 0.99

$$\frac{3\,ce^4x^5+20\,cde^3x^3+15\left(6\,cd^2e^2-ae^4\right)x}{15\,c^2}+\frac{\frac{2\sqrt{2}\left(c^{\frac{5}{2}}d^4+4\sqrt{a}\,c^2d^3e-6\,ac^{\frac{3}{2}}d^2e^2-4\,a^{\frac{3}{2}}cde^3+a^2\sqrt{c}\,e^4\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c*e^4*x^5 + 20*c*d*e^3*x^3 + 15*(6*c*d^2*e^2 - a*e^4)*x)/c^2 + 1/8*
(2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 - 4*a^(
3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a
^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(
c)) + 2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 -
4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*
sqrt(c)) + sqrt(2)*(c^(5/2)*d^4 - 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2
 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c
^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(5/2)*d^4 - 4*sqrt(a)*c^
2*d^3*e - 6*a*c^(3/2)*d^2*e^2 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*log(sq
rt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c^2
```

mupad [B] time = 5.08, size = 4022, normalized size = 9.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4/(a + c*x^4),x)
```

```
[Out] atan((((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a
^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*
((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8
*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2
*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*
(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^
4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5
```

```
*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^
2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))
^(1/2)*1i + ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6
 + 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2
*e^2)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7
*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d
^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^
4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2
) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c
^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^
3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3
*c^9))^(1/2)*1i)/(((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^
2*e^6 + 70*a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c
^5*d^2*e^2)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c
^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a
*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*
c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9
)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56
*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) -
 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(
16*a^3*c^9))^(1/2) - ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c
*d^2*e^6 + 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^
2*c^5*d^2*e^2)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^
2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 2
8*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a
^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*
c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 +
 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2
) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2)
)/(16*a^3*c^9))^(1/2) + (8*(a^5*d*e^11 - c^5*d^11*e - 3*a*c^4*d^9*e^3 + 3*a
^4*c*d^3*e^9 - 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7))/c^3))*((a^4*e^8*(-a^
3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7
 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1
/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/
2))/(16*a^3*c^9))^(1/2)*2i - x*((a*e^4)/c^2 - (6*d^2*e^2)/c) + atan((((4*x*
(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c^2*d^4*e
^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*(-(a^4*e^8*(
-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*
e^7 - 56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)
^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^
(1/2))/(16*a^3*c^9))^(1/2))/c^3)*(-(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^
3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 - 56*a^3*c^7*d^5*e^3 + 56*
a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^
3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2)*1i
+ ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c
^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*(-(a
^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^
5*c^5*d*e^7 - 56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-
a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a
^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*(-(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*
d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 - 56*a^3*c^7*d^5*e
^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*
e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(
1/2)*1i)/(((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 +
70*a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e
^2)*(-(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*
e - 8*a^5*c^5*d*e^7 - 56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^
6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4
*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*(-(a^4*e^8*(-a^3*c^9)^(1/2
```

```
) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 - 56*a^3*c
^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^
3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3
*c^9))^(1/2) - ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e
^6 + 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*
d^2*e^2)*(-(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8
*d^7*e - 8*a^5*c^5*d*e^7 - 56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c
^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^
2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*(-(a^4*e^8*(-a^3*c^9)
^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 - 56*
a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) -
28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(1
6*a^3*c^9))^(1/2) + (8*(a^5*d*e^11 - c^5*d^11*e - 3*a*c^4*d^9*e^3 + 3*a^4*c
*d^3*e^9 - 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7))/c^3))*(-(a^4*e^8*(-a^3*c
^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 -
56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2)
 - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))
/(16*a^3*c^9))^(1/2)*2i + (e^4*x^5)/(5*c) + (4*d*e^3*x^3)/(3*c)
```

sympy [A] time = 3.75, size = 500, normalized size = 1.14

$$x\left(-\frac{ae^4}{c^2}+\frac{6d^2e^2}{c}\right)+\text{RootSum}\left(256t^4a^3c^9+t^2\left(-256a^5c^5de^7+1792a^4c^6d^3e^5-1792a^3c^7d^5e^3+256a^2c^8d^7e\right)+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*x**4+a),x)
```

```
[Out] x*(-a*e**4/c**2 + 6*d**2*e**2/c) + RootSum(256*_t**4*a**3*c**9 + _t**2*(-25
6*a**5*c**5*d*e**7 + 1792*a**4*c**6*d**3*e**5 - 1792*a**3*c**7*d**5*e**3 +
256*a**2*c**8*d**7*e) + a**8*e**16 + 8*a**7*c*d**2*e**14 + 28*a**6*c**2*d**
4*e**12 + 56*a**5*c**3*d**6*e**10 + 70*a**4*c**4*d**8*e**8 + 56*a**3*c**5*d
**10*e**6 + 28*a**2*c**6*d**12*e**4 + 8*a*c**7*d**14*e**2 + c**8*d**16, Lam
bda(_t, _t*log(x + (256*_t**3*a**4*c**7*d*e**3 - 256*_t**3*a**3*c**8*d**3*e
 + 4*_t*a**7*c**2*e**12 - 264*_t*a**6*c**3*d**2*e**10 + 1980*_t*a**5*c**4*d
**4*e**8 - 3696*_t*a**4*c**5*d**6*e**6 + 1980*_t*a**3*c**6*d**8*e**4 - 264*
_t*a**2*c**7*d**10*e**2 + 4*_t*a*c**8*d**12)/(a**8*e**16 - 24*a**7*c*d**2*e
**14 - 36*a**6*c**2*d**4*e**12 + 88*a**5*c**3*d**6*e**10 + 198*a**4*c**4*d*
*8*e**8 + 88*a**3*c**5*d**10*e**6 - 36*a**2*c**6*d**12*e**4 - 24*a*c**7*d**
14*e**2 + c**8*d**16)))) + 4*d*e**3*x**3/(3*c) + e**4*x**5/(5*c)
```

3.138 $\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$

Optimal. Leaf size=370

$$-\frac{\left(\sqrt{c}\,d\left(cd^2-3ae^2\right)-\sqrt{a}\,e\left(3cd^2-ae^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}\,d\left(cd^2-3ae^2\right)-\sqrt{a}\,e\left(3cd^2-ae\right.\right.}{4\sqrt{2}\,a}$$

```
[Out] 3*d*e^2*x/c+1/3*e^3*x^3/c-1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(-e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^(7
/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*(-a*e
^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^(7/4)*2^(1/2)+1/4
*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e
^2+c*d^2)*c^(1/2))/a^(3/4)/c^(7/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a
^(1/4))*(e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^(
7/4)*2^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{c}\,d\left(cd^2-3ae^2\right)-\sqrt{a}\,e\left(3cd^2-ae^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}\,d\left(cd^2-3ae^2\right)-\sqrt{a}\,e\left(3cd^2-ae\right.\right.}{4\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + c*x^4),x]
```

```
[Out] (3*d*e^2*x)/c + (e^3*x^3)/(3*c) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e
*(3*c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e*(3*c*d^2 - a*e^2)
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(7/4)) - ((
Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqr
t[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{a+cx^4}\,dx = \int \left(\frac{3de^2}{c}+\frac{e^3x^2}{c}+\frac{cd^3-3ade^2+e\left(3cd^2-ae^2\right)x^2}{c\left(a+cx^4\right)}\right)dx$$

$$= \frac{3de^2x}{c}+\frac{e^3x^3}{3c}+\frac{\int \frac{cd^3-3ade^2+e\left(3cd^2-ae^2\right)x^2}{a+cx^4}\,dx}{c}$$

$$= \frac{3de^2x}{c}+\frac{e^3x^3}{3c}-\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}\,d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c^2}+\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}\,d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right.}{2c^2}$$

$$= \frac{3de^2x}{c}+\frac{e^3x^3}{3c}+\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}\,d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}\,c^{7/4}}+\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}\,d\left(cd^2\right.}{\sqrt{a}}\right.}{4\sqrt{2}}$$

$$= \frac{3de^2x}{c}+\frac{e^3x^3}{3c}+\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}\,d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,c^{7/4}}-\frac{\left(3cd^2e-\right.}{}$$

$$= \frac{3de^2x}{c}+\frac{e^3x^3}{3c}-\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}\,d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,c^{7/4}}+\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}\,d\left(cd^2-\right.}{\sqrt{a}}\right.}{2\sqrt{2}}$$

Mathematica [A] time = 0.28, size = 360, normalized size = 0.97

$$\frac{-3\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}\,cd^2e-3a\sqrt{c}\,de^2+c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+3\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}\,cd^2e-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + c*x^4),x]
```

```
[Out] (72*a^(3/4)*c^(3/4)*d*e^2*x + 8*a^(3/4)*c^(3/4)*e^3*x^3 + 6*Sqrt[2]*(-(c^(3
/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 -
(Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e -
 3*a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
 3*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e
^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*(c^(
3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^(3/4)*c^(7/4))
```

fricas [B] time = 1.27, size = 2133, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*e^3*x^3 + 36*d*e^2*x - 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*
a^2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4
 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12
)/(a^3*c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*
e^4 + 27*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x + (a*c^6*d^9 - 1
8*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e
^8 + (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 +
255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*
d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^
2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 -
 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/
(a^3*c^7)))/(a*c^3))) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e
^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*
a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*
c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*e^4 + 2
7*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x - (a*c^6*d^9 - 18*a^2*c
^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 + (3
*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2
*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^1
0 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5
 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^
3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^
7)))/(a*c^3))) - 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*
c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3
*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/
(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*e^4 + 27*a^4*c
^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x + (a*c^6*d^9 - 18*a^2*c^5*d^7*
e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 - (3*a^3*c^
6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^
8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6
*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*c^
3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d
^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a
*c^3))) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*c^3*sqr
t(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^
6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3)
)*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*e^4 + 27*a^4*c^2*d^4*
e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x - (a*c^6*d^9 - 18*a^2*c^5*d^7*e^2 + 6
0*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 - (3*a^3*c^6*d^2*e
 - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 -
 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/
(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*c^3*sqrt(
```

```
-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6
+ 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3)))
)/c
```

giac [A] time = 0.21, size = 405, normalized size = 1.09

$$\frac{c^2x^3e^3+9\,c^2dxe^2}{3\,c^3}+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3-3\,\left(ac^3\right)^{\frac{1}{4}}ac^2de^2+3\,\left(ac^3\right)^{\frac{3}{4}}cd^2e-\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^3 + 9*c^2*d*x*e^2)/c^3 + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3
- 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e
^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1
/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)
^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*
c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*log
(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1
/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^
3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4)
```

maple [A] time = 0.00, size = 572, normalized size = 1.55

$$\frac{e^3x^3}{3c}-\frac{\sqrt{2}\,a\,e^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}-\frac{\sqrt{2}\,a\,e^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}-\frac{\sqrt{2}\,a\,e^3\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\,\mathrm{arcta}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+a),x)
```

```
[Out] 1/3*e^3*x^3/c+3*d*e^2*x/c-3/4/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1
/4)*x-1)*d*e^2+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^
3-3/8/c*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2
-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^2+1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^
2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)
))*d^3-3/4/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2+1/4*
(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3-1/8/c^2/(a/c)^(1/
4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2)))*a*e^3+3/8/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(
1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e-1/4/c^2/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*a*e^3+3/4/c/(a/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e-1/4/c^2/(a/c)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/c)^(1/4)*x+1)*a*e^3+3/4/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x+1)*d^2*e
```

maxima [A] time = 2.48, size = 342, normalized size = 0.92

$$\frac{e^3x^3+9\,de^2x}{3\,c}+\frac{\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^3+3\sqrt{a}\,cd^2e-3\,a\sqrt{c}\,de^2-a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^3+3\sqrt{a}\,cd^2e-3\,a\sqrt{c}\,de^2-a^{\frac{3}{2}}e^3\right)\arctan}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/3*(e^3*x^3 + 9*d*e^2*x)/c + 1/8*(2*sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2
*e - 3*a*sqrt(c)*d*e^2 - a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqr
t(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))
*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2
- a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/s
qrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(c^
(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c
^(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*log(sqrt(
c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

mupad [B] time = 4.88, size = 2712, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + c*x^4),x)
```

```
[Out] (e^3*x^3)/(3*c) - atan((a^3*e^6*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (5*d^3
*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c^7)^(
1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^4*e^2
*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 +
 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^3*c^7)^
(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*(-a^3*c
^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-a^3*c^7
)^(1/2))/(a*c^2)) - (c^3*d^6*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (5*d^3*e^
3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c^7)^(1/2
))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^4*e^2*(-
a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 12
0*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^3*c^7)^(1/
2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*(-a^3*c^7)
^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-a^3*c^7)^(
1/2))/(a*c^2)) + (a*c^2*d^4*e^2*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (5*d^3
*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c^7)^(
1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^4*e^2
*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2
 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^3*c^7
)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*(-a^3
*c^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-a^3*c
^7)^(1/2))/(a*c^2)) - (a^2*c*d^2*e^4*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (
5*d^3*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c
^7)^(1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^
4*e^2*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9
)/c^2 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^
3*c^7)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*
(-a^3*c^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-
a^3*c^7)^(1/2))/(a*c^2)))*(-(c^3*d^6*(-a^3*c^7)^(1/2) - a^3*e^6*(-a^3*c^7)^
(1/2) + 6*a^2*c^6*d^5*e + 6*a^4*c^4*d*e^5 - 20*a^3*c^5*d^3*e^3 - 15*a*c^2*d
^4*e^2*(-a^3*c^7)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a^3*c^7))^
(1/2)*2i - atan((a^3*e^6*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2))/(1
6*c^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/2))/(
16*a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(-a^3*
c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 120*a^
2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^(1/2))/
(a^2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c^7)^(1/
2))/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7)^(1/2)
```

```
)/(a*c^2)) - (c^3*d^6*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2))/(16*c
^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/2))/(16*
a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(-a^3*c^7
)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 120*a^2*d
^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^(1/2))/(a^
2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c^7)^(1/2))
/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7)^(1/2))/(
a*c^2)) + (a*c^2*d^4*e^2*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2))/(1
6*c^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/2))/(
16*a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(-a^3*
c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 120*
a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^(1/2)
)/(a^2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c^7)^(
1/2))/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7)^(1/
2))/(a*c^2)) - (a^2*c*d^2*e^4*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2
))/(16*c^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/
2))/(16*a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(
-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 +
 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^
(1/2))/(a^2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c
^7)^(1/2))/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7
)^(1/2))/(a*c^2)))*(-(a^3*e^6*(-a^3*c^7)^(1/2) - c^3*d^6*(-a^3*c^7)^(1/2) +
 6*a^2*c^6*d^5*e + 6*a^4*c^4*d*e^5 - 20*a^3*c^5*d^3*e^3 + 15*a*c^2*d^4*e^2*
(-a^3*c^7)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a^3*c^7))^(1/2)*2
i + (3*d*e^2*x)/c
```

sympy [A] time = 2.27, size = 350, normalized size = 0.95

$$\mathrm{RootSum}\left(256t^4a^3c^7 + t^2\left(192a^4c^4de^5 - 640a^3c^5d^3e^3 + 192a^2c^6d^5e\right) + a^6e^{12} + 6a^5cd^2e^{10} + 15a^4c^2d^4e^8 + 20a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**7 + _t**2*(192*a**4*c**4*d*e**5 - 640*a**3*c**5*d
**3*e**3 + 192*a**2*c**6*d**5*e) + a**6*e**12 + 6*a**5*c*d**2*e**10 + 15*a*
*4*c**2*d**4*e**8 + 20*a**3*c**3*d**6*e**6 + 15*a**2*c**4*d**8*e**4 + 6*a*c
**5*d**10*e**2 + c**6*d**12, Lambda(_t, _t*log(x + (-64*_t**3*a**4*c**5*e**
3 + 192*_t**3*a**3*c**6*d**2*e - 36*_t*a**5*c**2*d*e**8 + 336*_t*a**4*c**3*
d**3*e**6 - 504*_t*a**3*c**4*d**5*e**4 + 144*_t*a**2*c**5*d**7*e**2 - 4*_t*
a*c**6*d**9)/(a**6*e**12 - 12*a**5*c*d**2*e**10 - 27*a**4*c**2*d**4*e**8 +
27*a**2*c**4*d**8*e**4 + 12*a*c**5*d**10*e**2 - c**6*d**12)))) + 3*d*e**2*x
/c + e**3*x**3/(3*c)
```

3.139 $\int \frac{(d+ex^2)^2}{a+cx^4} dx$

Optimal. Leaf size=297

$$-\frac{\left(-2\sqrt{a}\sqrt{c}\,de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}\,de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}\right.}{4\sqrt{2}\,a^{3/4}c^{5/4}}$$

```
[Out] e^2*x/c-1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c*d^2-a*e^2
-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(c*d^2-a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c^(5
/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*d^2-a*e^2+2*d*e*a^(
1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(c*d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(-2\sqrt{a}\sqrt{c}\,de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}\,de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}\right.}{4\sqrt{2}\,a^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + c*x^4),x]
```

```
[Out] (e^2*x)/c - ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 + 2*Sqrt[a]*Sqrt[c
]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
c^(5/4)) - ((c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 - 2*
Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{a+cx^4}\,dx=\int \left(\frac{e^2}{c}+\frac{cd^2-ae^2+2cdex^2}{c\left(a+cx^4\right)}\right)dx$$

$$=\frac{e^2x}{c}+\frac{\int \frac{cd^2-ae^2+2cdex^2}{a+cx^4}\,dx}{c}$$

$$=\frac{e^2x}{c}+\frac{\left(cd^2-2\sqrt{a}\sqrt{c}\,de-ae^2\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\sqrt{a}\,c^{3/2}}+\frac{\left(cd^2+2\sqrt{a}\sqrt{c}\,de-ae^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\sqrt{a}\,c^{3/2}}$$

$$=\frac{e^2x}{c}-\frac{\left(cd^2-2\sqrt{a}\sqrt{c}\,de-ae^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,a^{3/4}c^{5/4}}-\frac{\left(cd^2-2\sqrt{a}\sqrt{c}\,de-ae^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}}}{4\sqrt{2}\,a^{3/4}c^{5/4}}$$

$$=\frac{e^2x}{c}-\frac{\left(cd^2-2\sqrt{a}\sqrt{c}\,de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{\left(cd^2-2\sqrt{a}\sqrt{c}\,de-ae^2\right)}{4}$$

$$=\frac{e^2x}{c}-\frac{\left(cd^2+2\sqrt{a}\sqrt{c}\,de-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{\left(cd^2+2\sqrt{a}\sqrt{c}\,de-ae^2\right)\tan^{-1}\left(1+\right.}{2\sqrt{2}\,a^{3/4}c^{5/4}}$$

Mathematica [A] time = 0.26, size = 269, normalized size = 0.91

$$\frac{8a^{3/4}\sqrt[4]{c}\,e^2x+\sqrt{2}\left(2\sqrt{a}\sqrt{c}\,de+ae^2-cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+\sqrt{2}\left(-2\sqrt{a}\sqrt{c}\,de-ae^2+cd^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + c*x^4),x]
```

```
[Out] (8*a^(3/4)*c^(1/4)*e^2*x - 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*Sq
```

```
rt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(c*d
^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2] + Sqrt[2]*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*a^(3/4)*c^(5/4))
```

fricas [B] time = 0.61, size = 1480, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*e^2*x + c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a
*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))
/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2
*e^6 + a^4*e^8)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^
4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*
e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3
+ a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d
^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 +
a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2
*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^
2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e
^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^
3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sq
rt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a
^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) + c*sqrt
(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2
*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^
8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x + (
a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*a^3*c^4*d
*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^
6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^
8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^
3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8
- 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*
c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3
*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^
4 - a^4*c*e^6 - 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^
2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*
d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a
^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))))/c
```

giac [A] time = 0.18, size = 318, normalized size = 1.07

$$\frac{xe^2}{c}+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2+2\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2+2\left(\right.\right.}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] x*e^2/c + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a
*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*
(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/
4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 -
2*(a*c^3)^(3/4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) -
```

```
1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/
4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

maple [A] time = 0.00, size = 412, normalized size = 1.39

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8a}+\frac{\sqrt{2}\,de\,\mathrm{arcta}}{2\left(\frac{a}{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+a),x)
```

```
[Out] e^2*x/c-1/4/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^2+1/4*(
a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2-1/8/c*(a/c)^(1/4)*
2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)
*x+(a/c)^(1/2)))*e^2+1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2-1/4/c*(a/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^2+1/4*(a/c)^(1/4)/a*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2+1/4/c*d*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/
c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/
2/c*d*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/2/c*d*e/(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.36, size = 288, normalized size = 0.97

$$\frac{e^2x}{c}+\frac{\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^2+2\sqrt{a}\,cde-a\sqrt{c}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^2+2\sqrt{a}\,cde-a\sqrt{c}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(c^{\frac{3}{2}}d^2\right.}{}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] e^2*x/c + 1/8*(2*sqrt(2)*(c^(3/2)*d^2 + 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*ar
ctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(
c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^2 + 2*s
qrt(a)*c*d*e - a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(
1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)
) + sqrt(2)*(c^(3/2)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sqrt(c)*x^2
 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(3/2
)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*
c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

mupad [B] time = 4.79, size = 1479, normalized size = 4.98

$$\frac{e^2\,x}{c}-2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^4\,x\,\sqrt{\frac{d\,e^3}{4\,c^2}-\frac{d^3\,e}{4\,a\,c}+\frac{d^4\,\sqrt{-a^3\,c^5}}{16\,a^3\,c^3}+\frac{e^4\,\sqrt{-a^3\,c^5}}{16\,a\,c^5}-\frac{3\,d^2\,e^2\,\sqrt{-a^3\,c^5}}{8\,a^2\,c^4}}}{4\,a^2\,d\,e^5-\frac{2\,d^6\,\sqrt{-a^3\,c^5}}{a^2}+4\,c^2\,d^5\,e+\frac{2\,a\,e^6\,\sqrt{-a^3\,c^5}}{c^3}-24\,a\,c\,d^3\,e^3-\frac{14\,d^2\,e^4\,\sqrt{-a^3\,c^5}}{c^2}+\frac{14\,d^4\,e^2\,\sqrt{-a^3\,c^5}}{a\,c}}+\frac{}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + c*x^4),x)
```

```
[Out] (e^2*x)/c - 2*atanh((8*c^3*d^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*
(-a^3*c^5)^(1/2))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2
```

```
*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)
^(1/2))/a^2 + 4*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3
 - (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c))
 + (8*a^2*c*e^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*(-a^3*c^5)^(1/2
))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2*e^2*(-a^3*c^5)
^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)^(1/2))/a^2 + 4
*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 - (14*d^2*e^4*
(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)) - (48*a*c^2*d^
2*e^2*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*(-a^3*c^5)^(1/2))/(16*a^3
*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8
*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)^(1/2))/a^2 + 4*c^2*d^5*e
 + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 - (14*d^2*e^4*(-a^3*c^5)
^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)))*((a^2*e^4*(-a^3*c^5)^(1
/2) + c^2*d^4*(-a^3*c^5)^(1/2) - 4*a^2*c^4*d^3*e + 4*a^3*c^3*d*e^3 - 6*a*c*
d^2*e^2*(-a^3*c^5)^(1/2))/(16*a^3*c^5))^(1/2) - 2*atanh((8*c^3*d^4*x*((d*e^
3)/(4*c^2) - (d^3*e)/(4*a*c) - (d^4*(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (e^4*(
-a^3*c^5)^(1/2))/(16*a*c^5) + (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/
2))/((2*d^6*(-a^3*c^5)^(1/2))/a^2 + 4*a^2*d*e^5 + 4*c^2*d^5*e - (2*a*e^6*(-
a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 + (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 -
(14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)) + (8*a^2*c*e^4*x*((d*e^3)/(4*c^2) - (d
^3*e)/(4*a*c) - (d^4*(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (e^4*(-a^3*c^5)^(1/2)
)/(16*a*c^5) + (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/((2*d^6*(-a
^3*c^5)^(1/2))/a^2 + 4*a^2*d*e^5 + 4*c^2*d^5*e - (2*a*e^6*(-a^3*c^5)^(1/2))
/c^3 - 24*a*c*d^3*e^3 + (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 - (14*d^4*e^2*(-a
^3*c^5)^(1/2))/(a*c)) - (48*a*c^2*d^2*e^2*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a
*c) - (d^4*(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (e^4*(-a^3*c^5)^(1/2))/(16*a*c^
5) + (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/((2*d^6*(-a^3*c^5)^(1
/2))/a^2 + 4*a^2*d*e^5 + 4*c^2*d^5*e - (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*
a*c*d^3*e^3 + (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 - (14*d^4*e^2*(-a^3*c^5)^(1
/2))/(a*c)))*(-(a^2*e^4*(-a^3*c^5)^(1/2) + c^2*d^4*(-a^3*c^5)^(1/2) + 4*a^2
*c^4*d^3*e - 4*a^3*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^5)^(1/2))/(16*a^3*c^5)
)^(1/2)
```

sympy [A] time = 1.48, size = 238, normalized size = 0.80

$$\operatorname{RootSum}\left(256t^4a^3c^5 + t^2\left(-128a^3c^3de^3 + 128a^2c^4d^3e\right) + a^4e^8 + 4a^3cd^2e^6 + 6a^2c^2d^4e^4 + 4ac^3d^6e^2 + c^4d^8, \left(t \mapsto t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**5 + _t**2*(-128*a**3*c**3*d*e**3 + 128*a**2*c**4*
d**3*e) + a**4*e**8 + 4*a**3*c*d**2*e**6 + 6*a**2*c**2*d**4*e**4 + 4*a*c**3
*d**6*e**2 + c**4*d**8, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c**4*d*e - 4
*_t*a**4*c*e**6 + 60*_t*a**3*c**2*d**2*e**4 - 60*_t*a**2*c**3*d**4*e**2 + 4
*_t*a*c**4*d**6)/(a**4*e**8 - 4*a**3*c*d**2*e**6 - 10*a**2*c**2*d**4*e**4 -
 4*a*c**3*d**6*e**2 + c**4*d**8)))) + e**2*x/c
```

3.140 $\int \frac{d+ex^2}{a+cx^4}\,dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}\,e+\right.}{}$$

```
[Out] -1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/
2))/a^(3/4)/c^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^
(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)
*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arcta
n(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2
)
```

Rubi [A] time = 0.15, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}\,e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + c*x^4),x]
```

```
[Out] -((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)
) + ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rubi steps

$$\int \frac{d+ex^2}{a+cx^4}\,dx = \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c}+\frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c}$$

$$= \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c}+\frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,d}{4\sqrt{2}\,a^{3/4}c^{3/4}}$$

$$= -\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right.}{4\sqrt{2}\,a^{3/4}c^{3/4}}$$

$$= -\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{a}\right.}{4\sqrt{2}}$$

Mathematica [A] time = 0.05, size = 183, normalized size = 0.74

$$\frac{-\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)\right)-2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\right.}{4\sqrt{2}\,a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d -
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(4*Sqrt[2]*a^(3/4)*c^(3
/4))
```

fricas [B] time = 0.43, size = 767, normalized size = 3.11

$$-\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\log\left(-\left(c^2d^4-a^2e^4\right)x+\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+ac^2d^3-a^2cde^2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*
e)/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^
2*e^2 + a^2*e^4)/(a^3*c^3)) + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^
2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt(-(a
*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))*log
(-(c^2*d^4 - a^2*e^4)*x - (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e
^4)/(a^3*c^3)) + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c
*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt((a*c*sqrt(-(c^2*
d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-(c^2*d^4 - a
^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3))
 - a*c^2*d^3 + a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*
e^4)/(a^3*c^3)) - 2*d*e)/(a*c))) - 1/4*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x - (a^
3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - a*c^2*d^3 +
a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3))
 - 2*d*e)/(a*c)))
```

giac [A] time = 0.18, size = 245, normalized size = 0.99

$$\frac{\sqrt{2}\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d+\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^{3}}+\frac{\sqrt{2}\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d+\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^{3}}+\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c
^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(
x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/
4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c^3)
```

maple [A] time = 0.00, size = 260, normalized size = 1.05

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8a}+\frac{\sqrt{2}\,e\arctan\left(\cdots\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/8*d*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2
-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)+1/8*e/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2
))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*e/c/(a/c)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*e/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x-1)
```

maxima [A] time = 2.53, size = 221, normalized size = 0.89

$$\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{4\,\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{4\,\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}+\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*s
qrt(c)) + 1/4*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)
*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(c))*sqrt(c)) + 1/8*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 + s
qrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(sqrt(c
)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^
(3/4)*c^(3/4))
```

mupad [B] time = 4.68, size = 599, normalized size = 2.43

$$-2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}-\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}-\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}-\frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}-\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}-\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}\right)\sqrt{-\frac{c\,d}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + c*x^4),x)
```

```
[Out] - 2*atanh((8*c^3*d^2*x*((e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3) - (d^2*(-a^3*c^
3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 + (
2*c*d^3*(-a^3*c^3)^(1/2))/a^2 - (2*d*e^2*(-a^3*c^3)^(1/2))/a) - (8*a*c^2*e^
2*x*((e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3) - (d^2*(-a^3*c^3)^(1/2))/(16*a^3*c
^2) - (d*e)/(8*a*c))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 + (2*c*d^3*(-a^3*c^3)^
(1/2))/a^2 - (2*d*e^2*(-a^3*c^3)^(1/2))/a))*(-(c*d^2*(-a^3*c^3)^(1/2) - a*e
^2*(-a^3*c^3)^(1/2) + 2*a^2*c^2*d*e)/(16*a^3*c^3))^(1/2) - 2*atanh((8*c^3*d
^2*x*((d^2*(-a^3*c^3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c) - (e^2*(-a^3*c^3)
^(1/2))/(16*a^2*c^3))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 - (2*c*d^3*(-a^3*c^3)
^(1/2))/a^2 + (2*d*e^2*(-a^3*c^3)^(1/2))/a) - (8*a*c^2*e^2*x*((d^2*(-a^3*c^
3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c) - (e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3
))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 - (2*c*d^3*(-a^3*c^3)^(1/2))/a^2 + (2*d*
e^2*(-a^3*c^3)^(1/2))/a))*(-(a*e^2*(-a^3*c^3)^(1/2) - c*d^2*(-a^3*c^3)^(1/2
) + 2*a^2*c^2*d*e)/(16*a^3*c^3))^(1/2)
```

sympy [A] time = 0.68, size = 109, normalized size = 0.44

$$\mathrm{RootSum}\left(256t^4a^3c^3+64t^2a^2c^2de+a^2e^4+2acd^2e^2+c^2d^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^3c^2e+12ta^2cde^2-4tac^2d^3}{a^2e^4-c^2d^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**3 + 64*_t**2*a**2*c**2*d*e + a**2*e**4 + 2*a*c*d*
*2*e**2 + c**2*d**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*c**2*e + 12*_t*a*
*2*c*d*e**2 - 4*_t*a*c**2*d**3)/(a**2*e**4 - c**2*d**4))))
```

3.141 $\int \frac{1}{a+cx^4} \, dx$

Optimal. Leaf size=185

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

```
[Out] 1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*arctan
(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*ln(-a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*ln(a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+cx^4}\,dx = \frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 -\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\right.}{4\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 134, normalized size = 0.72

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))
```

fricas [A] time = 0.41, size = 121, normalized size = 0.65

$$\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}+\sqrt{a^2\sqrt{-\frac{1}{a^3c}}+x^2}\,a^2c\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right)+\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)-\frac{1}{4}\left(-\frac{1}{a^3c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*c))^(1/4)*arctan(-a^2*c*x*(-1/(a^3*c))^(3/4) + sqrt(a^2*sqrt(-1/(a
^3*c)) + x^2)*a^2*c*(-1/(a^3*c))^(3/4)) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/
(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) + x)
```

giac [A] time = 0.18, size = 179, normalized size = 0.97

$$\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac}+\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac}+\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\cdots\right.}{8\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

maple [A] time = 0.00, size = 128, normalized size = 0.69

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a),x)
```

```
[Out] 1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(
a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)
/(a/c)^(1/4)*x+1)+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.44, size = 169, normalized size = 0.91

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt
(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4
)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

mupad [B] time = 4.41, size = 33, normalized size = 0.18

$$-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)+\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4),x)
```

```
[Out] -(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3/
4)*c^(1/4))
```

sympy [A] time = 0.17, size = 20, normalized size = 0.11

$$\text{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

3.142 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}-$$

```
[Out] 1/4*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(
3/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*
(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*c^(1/4)*ln(-a^(1/4
)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e
^2+c*d^2)*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1
/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)+e^(3/2)*arctan(x*e
^(1/2)/d^(1/2))/(a*e^2+c*d^2)/d^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1171, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{e^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= \frac{c\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2}+\frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)}-\cdots$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d+\cdots\right.}{\cdots}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\cdots\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+a\cdots\right.}$$

Mathematica [A] time = 0.15, size = 234, normalized size = 0.70

$$\frac{8a^{3/4}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{2}\,\sqrt[4]{c}\,\sqrt{d}\,\left(-\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\right.\right.\right.}{8a^{3/4}\sqrt{d}\,\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((
-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d]*(c*
d^2 + a*e^2))
```

fricas [B] time = 1.05, size = 4084, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a
^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*
e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*
d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2
+ a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^
4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d
^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^
4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
 a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^
2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^
5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (
a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a
^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^
2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e
^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4
- 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2
*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*
e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4
)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c
^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*
a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 -
 a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
```

```
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - 2*e*sqrt(-e/d)*log((e*x^2 + 2*d
*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(c*d^2 + a*e^2), 1/4*(4*e*sqrt(e/d)*arctan
(x*sqrt(e/d)) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 -
 a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e +
 (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2
 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3
 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*
a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*
c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^
2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4
*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*
d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-
(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e
^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3
*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d
^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)
*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6
*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2
*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))
)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*
c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(
c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4
+ 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))))/(c*d^2 + a*e^2)]
```

giac [A] time = 0.21, size = 339, normalized size = 1.01

$$\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^{3}d^{2}+\sqrt{2}\,a^{2}c^{2}e^{2}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^{3}d^{2}+\sqrt{2}\,a^{2}c^{2}e^{2}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/
2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*
((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^
```

```
2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2
)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*
d^2 + a*e^2)*sqrt(d))
```

maple [A] time = 0.01, size = 363, normalized size = 1.08

$$\frac{e^{2}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(ae^{2}+cd^{2}\right)\sqrt{de}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(ae^{2}+cd^{2}\right)a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(ae^{2}+cd^{2}\right)a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,cd\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+}\right.}{8\left(ae^{2}+cd^{2}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] e^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)+1/8*c/(a*e^2+c*d^2)
*d*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a
/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*c/(a*e^2+c*d^2)*d*(a/c)^(1/4)/a*2^(1/
2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*c/(a*e^2+c*d^2)*d*(a/c)^(1/4)/a*2^(1
/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/8/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)
*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c
)^(1/2)))-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x+1)-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x
-1)
```

maxima [A] time = 2.38, size = 268, normalized size = 0.80

$$\frac{e^{2}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^{2}+ae^{2}\right)\sqrt{de}}+\frac{c\left(\frac{2\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}d+\sqrt{a}e\right)}{}\right.}{8\left(cd^{2}+ae^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] e^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/8*c*(2*sqrt(2)*(s
qrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(
1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sq
rt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(
1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)
) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqr
t(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c*d^2 +
 a*e^2)
```

mupad [B] time = 5.71, size = 4802, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^
5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-
```

a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
 c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d
^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^
5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3
*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*
((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
 a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*
(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)
))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) +
2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4
*c^4*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(
16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512
*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^
6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32
*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c
)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2
) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^
(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*
1i)/((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5
*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a
^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5
 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*
c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*(
(a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((a
e^2(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^
3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^
3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(
1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^
2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4
e^8 - x((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2
*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^
2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c
^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) +
20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2
) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*2i + atan(((((c*d^2*(-a^3*c)^(1/2) -
a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(2
56*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*
d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9
 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*
c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^
2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(
-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))
)^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^
(1/2)*1i - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(1
6*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d

```
^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*
c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1
/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d
^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 44
8*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e
^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*
e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) + 6*c^5*e^5*
x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^
4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i)/((((c*d^2*(-a^3*c)^(1/2) - a
*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2
*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/
2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(25
6*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d
*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9
- 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c
^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2
 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-
a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))
^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^
3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(
1/2) + (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a
^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(
-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*
d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1/2)
- a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*
d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e
^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^
3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))
*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4
+ a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(
(c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^
(1/2)*2i - (log(16*a^2*e^2*(-d*e^3)^(3/2) + c^2*d^5*e^3*x - c^2*d^5*e*(-d*e
^3)^(1/2) + 16*a^2*d*e^7*x + a*c*d^2*(-d*e^3)^(3/2) + a*c*d^3*e^5*x)*(-d*e^
3)^(1/2))/(2*(c*d^3 + a*d*e^2)) + (log(c^2*d^5*e^3*x - 16*a^2*e^2*(-d*e^3)^
(3/2) + c^2*d^5*e*(-d*e^3)^(1/2) + 16*a^2*d*e^7*x + 4*a*c*d^2*(-d*e^3)^(3/2
) + a*c*d^3*e^5*x + 5*a*c*d^3*e^3*(-d*e^3)^(1/2))*(-d*e^3)^(1/2))/(2*c*d^3
+ 2*a*d*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.143 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=453

$$-\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}\,de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}\,de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\right.}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}$$

```
[Out] 1/2*e^2*x/d/(a*e^2+c*d^2)/(e*x^2+d)+1/2*e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/d
^(3/2)/(a*e^2+c*d^2)+1/4*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*d^
2-a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*c^(3/4)*
arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*d^2-a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(
3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*c^(1/2))*(c*d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^
2*2^(1/2)+1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c*
d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+2*c*e^(3/2
)*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(a*e^2+c*d^2)^2
```

Rubi [A] time = 0.38, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1171, 199, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}\,de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}\,de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\right.}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

```
[Out] (e^2*x)/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (2*c*Sqrt[d]*e^(3/2)*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 + a*e^2)) - (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a
*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 +
a*e^2)^2) + (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)
*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(c*d
^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2(a+cx^4)}\,dx = \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)^2} + \frac{2cde^2}{(cd^2+ae^2)^2(d+ex^2)} + \frac{c(cd^2-ae^2-2cdex^2)}{(cd^2+ae^2)^2(a+cx^4)}\right)dx$$

$$= \frac{c\int \frac{cd^2-ae^2-2cdex^2}{a+cx^4}dx}{(cd^2+ae^2)^2} + \frac{(2cde^2)\int \frac{1}{d+ex^2}dx}{(cd^2+ae^2)^2} + \frac{e^2\int \frac{1}{(d+ex^2)^2}dx}{cd^2+ae^2}$$

$$= \frac{e^2x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\left(\sqrt{c}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int}{2\sqrt{a}\,(cd^2+ae^2)^2}$$

$$= \frac{e^2x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} + \frac{\left(\sqrt{c}\left(cd^2-\right.\right.}{}$$

$$= \frac{e^2x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} - \frac{c^{3/4}\left(cd^2+2\right.}{}$$

$$= \frac{e^2x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} - \frac{c^{3/4}\left(cd^2-2\right.}{}$$

Mathematica [A] time = 0.47, size = 362, normalized size = 0.80

$$\frac{\frac{\sqrt{2}c^{3/4}\left(-2\sqrt{a}\sqrt{c}de+ae^2-cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}} + \frac{\sqrt{2}c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}} + \frac{2\sqrt{2}c^{3/4}\left(2\sqrt{a}\sqrt{c}d\right.}{}}{8\left(ae^2+cd^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

```
[Out] ((4*e^2*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (4*e^(3/2)*(5*c*d^2 + a*e^2)*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sqrt
[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) -
 (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*c^(3/4)*(-(c*d^2) - 2*Sqrt[
a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/a^(3/4) + (Sqrt[2]*c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4))/(8*(c*d^2 + a
*e^2)^2)
```

fricas [B] time = 16.31, size = 8409, normalized size = 18.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 +
 a^2*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3
*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 -
12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(
```

```
a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e
^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^1
0*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*
e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*
e^4)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2
*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 +
 a^7*d*e^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3
*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6
*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10
+ 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e
- 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^
4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e
^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 +
 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^
3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*
d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)))
- (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2
*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6
*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a
*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*
c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 +
 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*
d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4
+ 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)
*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*(a^
3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7
*d*e^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4
*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^1
2*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28
*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*
a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*
d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 -
 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*
a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^
6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8
+ 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) + (c
^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e
^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2
 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6
*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*
d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*
a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*
e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*
a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x +
 (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^
4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e
^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^
4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9
*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^
2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*
e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*
a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*
c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^
10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*
a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) - (c^2*d
^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*
x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6
*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6
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*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16
 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*
c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14
 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*
c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x - (a*
c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^
9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*
sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6
 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 +
56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2
*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^2*d*
e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6
+ a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*
c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*
d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 +
 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*
c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) + (5*c*d^3*e
 + a*d*e^3 + (5*c*d^2*e^2 + a*e^4)*x^2)*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(
-e/d) - d)/(e*x^2 + d)) + 2*(c*d^2*e^2 + a*e^4)*x)/(c^2*d^6 + 2*a*c*d^4*e^2
 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x^2), 1/4*(2*(5*c*
d^3*e + a*d*e^3 + (5*c*d^2*e^2 + a*e^4)*x^2)*sqrt(e/d)*arctan(x*sqrt(e/d))
+ (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2
*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6
*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a
*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*
c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 +
 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*
d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4
+ 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)
*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*(a^
3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7
*d*e^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4
*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^1
2*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28
*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*
a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*
d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 -
 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*
a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^
6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8
+ 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) - (c
^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e
^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2
 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6
*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*
d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*
a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*
e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*
a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x -
 (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*(a^3*c^
4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e
^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^
4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9
*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^
2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*
e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*
a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*
c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^
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10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*
a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) + (c^2*d
^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*
x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6
*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6
*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16
 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*
c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14
 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*
c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x + (a*
c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^
9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*
sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6
 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 +
56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2
*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^2*d*
e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6
+ a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*
c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*
d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 +
 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*
c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) - (c^2*d^6 +
 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x^2)
*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3
*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2
 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8
*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*
d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a
^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^
2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x - (a*c^4*
d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^9*e
+ 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt
(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a
^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a
^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4
*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3
- (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^
5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*
d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12
*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*
a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*
d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) + 2*(c*d^2*e^2 +
 a*e^4)*x)/(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*
e^3 + a^2*d*e^5)*x^2)]
```

giac [A] time = 0.25, size = 517, normalized size = 1.14

$$\frac{\left(5\,cd^2e^2 + ae^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\left(c^2d^5 + 2\,acd^3e^2 + a^2de^4\right)\sqrt{d}} + \frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2 - \left(ac^3\right)^{\frac{1}{4}}ace^2 - 2\,\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^3d^4 + 2\,\sqrt{2}\,a^2c^2d^2e^2 + \sqrt{2}\,a^3ce^4\right)} + \frac{\left(\left(ac^3\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 + a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^2*d^5 + 2*
a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1
/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^
```

```
(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2
)*a^3*c*e^4) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^
3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(
sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) + 1/8*(s
qrt(2)*(a*c^3)^(1/4)*c^2*d^2 - sqrt(2)*(a*c^3)^(1/4)*a*c*e^2 + 2*sqrt(2)*(a
*c^3)^(3/4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3*d^4 +
2*a^2*c^2*d^2*e^2 + a^3*c*e^4) - 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^2*d^2 - sqrt(
2)*(a*c^3)^(1/4)*a*c*e^2 + 2*sqrt(2)*(a*c^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x
*(a/c)^(1/4) + sqrt(a/c))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4) + 1/2
*x*e^2/((c*d^3 + a*d*e^2)*(x^2*e + d))
```

maple [A] time = 0.01, size = 650, normalized size = 1.43

$$\frac{a e^{4} x}{2 \left(a e^{2}+c d^{2}\right)^{2} \left(e x^{2}+d\right) d}+\frac{a e^{4} \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2 \left(a e^{2}+c d^{2}\right)^{2} \sqrt{d e}\, d}+\frac{c d e^{2} x}{2 \left(a e^{2}+c d^{2}\right)^{2} \left(e x^{2}+d\right)}+\frac{5 c d e^{2} \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2 \left(a e^{2}+c d^{2}\right)^{2} \sqrt{d e}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, c^{2}\cdots}{4 \left(a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a),x)
```

```
[Out] 1/2*e^4/(a*e^2+c*d^2)^2/d*x/(e*x^2+d)*a+1/2*e^2/(a*e^2+c*d^2)^2*d*x/(e*x^2+
d)*c+1/2*e^4/(a*e^2+c*d^2)^2/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+5/2*
e^2/(a*e^2+c*d^2)^2*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c-1/8*c/(a*e^2+
c*d^2)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^
2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^2+1/8*c^2/(a*e^2+c*d^2)^2*(a/c)^(1/
4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^
(1/2)*x+(a/c)^(1/2)))*d^2-1/4*c/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x-1)*e^2+1/4*c^2/(a*e^2+c*d^2)^2*(a/c)^(1/4)/a*2^(1/2)*
arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2-1/4*c/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^2+1/4*c^2/(a*e^2+c*d^2)^2*(a/c)^(1/4)/
a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2-1/4*c/(a*e^2+c*d^2)^2*d*e/(a/
c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4
)*2^(1/2)*x+(a/c)^(1/2)))-1/2*c/(a*e^2+c*d^2)^2*d*e/(a/c)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/c)^(1/4)*x+1)-1/2*c/(a*e^2+c*d^2)^2*d*e/(a/c)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.45, size = 403, normalized size = 0.89

$$\frac{e^{2} x}{2 \left(c d^{4}+a d^{2} e^{2}+\left(c d^{3} e+a d e^{3}\right) x^{2}\right)}+\frac{c \left(\frac{2 \sqrt{2} \left(c^{\frac{3}{2}} d^{2}-2 \sqrt{a}\, c d e-a \sqrt{c}\, e^{2}\right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{c}\, x+\sqrt{2}\, a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}+\frac{2 \sqrt{2} \left(c^{\frac{3}{2}} d^{2}-2 \sqrt{a}\, c d e-a \sqrt{c}\, e^{2}\right) \operatorname{arct}\cdots}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}}\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*e^2*x/(c*d^4 + a*d^2*e^2 + (c*d^3*e + a*d*e^3)*x^2) + 1/8*c*(2*sqrt(2)*
(c^(3/2)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(
c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a
)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*
e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(c^(3/2)*d^2
+ 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4
```

```
)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(3/2)*d^2 + 2*sqrt(a)*c*d*e -
 a*sqrt(c)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(
3/4)*c^(3/4)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*(5*c*d^2*e^2 + a*e
^4)*arctan(e*x/sqrt(d*e))/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d*e))
```

mupad [B] time = 6.55, size = 16369, normalized size = 36.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x^2)^2),x)
```

```
[Out] (e^2*x)/(2*d*(d + e*x^2)*(a*e^2 + c*d^2)) - atan((((((256*a^8*c^4*d*e^16 -
 128*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960
*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^
7*c^5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*
e^6 + 6*a^2*c^2*d^6*e^4)) + (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c
^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1
/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a
^5*c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4
608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608
*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^1
0 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(
(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*
a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8
+ 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(32
*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*
e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))
/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^
6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d
^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3
*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)
 + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*
d^4*e^9 + 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2
 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*
d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-
a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11
*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*
e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^
2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*
(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*1i - (((((256*a^8*c^4*d*e^16 - 128*a*c
^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960*a^4*c^8
*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^7*c^5*d^
3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*
a^2*c^2*d^6*e^4)) - (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2
) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16
*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d
^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608*a^4*
c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^7*c^6
*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^10 + a^4*
d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4
*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*
d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*
c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(32*a^6*c^5
*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^6 + 22
08*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(c^4*d^
10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*
((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4
```

```
*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8
 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (480*
a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*d^4*e^9
+ 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3
*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^
3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)
^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a*c^8*d
^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*
a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-
a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^
3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2
+ 6*a^5*c^2*d^4*e^4)))^(1/2)*1i)/((((((256*a^8*c^4*d*e^16 - 128*a*c^11*d^15
*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960*a^4*c^8*d^9*e^8
 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^7*c^5*d^3*e^14)/
(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*
d^6*e^4)) + (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^
2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^
8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4))
)^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608*a^4*c^9*d^12
*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^7*c^6*d^6*e^1
3 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^10 + a^4*d^2*e^8
+ 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c
^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 -
6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^
6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(32*a^6*c^5*d*e^14
- 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c
^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(c^4*d^10 + a^4
*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^
4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2
*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6
*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (480*a^2*c^8*
d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*d^4*e^9 + 96*a^4
*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e
^6 + 6*a^2*c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(
1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/
(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^
2*d^4*e^4)))^(1/2) + (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 +
 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^
4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)
^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2)
)/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*
c^2*d^4*e^4)))^(1/2) + (((((256*a^8*c^4*d*e^16 - 128*a*c^11*d^15*e^2 + 256*
a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960*a^4*c^8*d^9*e^8 + 10880*a^
5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^7*c^5*d^3*e^14)/(2*(c^4*d^1
0 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) -
 (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e
 - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4
*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*(51
2*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560
*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^
8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d
^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) +
 c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e
^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c
^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(32*a^6*c^5*d*e^14 - 48*a*c^10
*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8
+ 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(c^4*d^10 + a^4*d^2*e^8 +
4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3
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)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*
a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6
+ 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (480*a^2*c^8*d^6*e^7 - 2
00*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^
11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*
c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^
2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^
8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4))
)^(1/2) - (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*
d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a
^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*
a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*
e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4
)))^(1/2) + (5*c^8*d^3*e^6 + a*c^7*d*e^8)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3
*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2
) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^
2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^
4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*2i - (atan(((((x*(a^3*c^6*e^11 -
27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^
2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - (((240*a^2
*c^8*d^6*e^7 - 100*a*c^9*d^8*e^5 - 4*a^5*c^5*e^13 + 392*a^3*c^7*d^4*e^9 + 4
8*a^4*c^6*d^2*e^11)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4
*e^6 + 6*a^2*c^2*d^6*e^4) - (((x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 -
16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c
^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*
e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - ((a*e^2 + 5*c*d^2)*((128*a^8*c
^4*d*e^16 - 64*a*c^11*d^15*e^2 + 128*a^2*c^10*d^13*e^4 + 1728*a^3*c^9*d^11*
e^6 + 4480*a^4*c^8*d^9*e^8 + 5440*a^5*c^7*d^7*e^10 + 3456*a^6*c^6*d^5*e^12
+ 1088*a^7*c^5*d^3*e^14)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*
c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - (x*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2)*(512
*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560*
a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^8
*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^
5*e^2)*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*
c^2*d^6*e^4)))*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)
))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^
5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*
a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2)*1i)/(4*(c^2*d^7 + a^2*d^3
*e^4 + 2*a*c*d^5*e^2)) + (((x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a*c^8*d^4
*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^
3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) + (((240*a^2*c^8*d^6*e^7 - 100*a*c^9*d^8*e
^5 - 4*a^5*c^5*e^13 + 392*a^3*c^7*d^4*e^9 + 48*a^4*c^6*d^2*e^11)/(c^4*d^10
+ a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) + ((
(x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^
9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*
e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*
c^2*d^6*e^4) + ((a*e^2 + 5*c*d^2)*((128*a^8*c^4*d*e^16 - 64*a*c^11*d^15*e^2
+ 128*a^2*c^10*d^13*e^4 + 1728*a^3*c^9*d^11*e^6 + 4480*a^4*c^8*d^9*e^8 + 5
440*a^5*c^7*d^7*e^10 + 3456*a^6*c^6*d^5*e^12 + 1088*a^7*c^5*d^3*e^14)/(c^4*
d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)
+ (x*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*
c^10*d^14*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^
7*d^8*e^11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^
2*e^17))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)*(c^4*d^10 + a^4*d^2*e^8
+ 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-d^3*e^3)^(1/2
))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3
)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d
^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^
2)*(-d^3*e^3)^(1/2)*1i)/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))/((5*c^

8*d^3*e^6 + a*c^7*d*e^8)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*
c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - (((x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a
*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^
2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - (((240*a^2*c^8*d^6*e^7 - 100*a*c
^9*d^8*e^5 - 4*a^5*c^5*e^13 + 392*a^3*c^7*d^4*e^9 + 48*a^4*c^6*d^2*e^11)/(c
^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e
^4) - (((x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 102
4*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*
c^6*d^3*e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6
+ 6*a^2*c^2*d^6*e^4) - ((a*e^2 + 5*c*d^2)*((128*a^8*c^4*d*e^16 - 64*a*c^11*
d^15*e^2 + 128*a^2*c^10*d^13*e^4 + 1728*a^3*c^9*d^11*e^6 + 4480*a^4*c^8*d^9
*e^8 + 5440*a^5*c^7*d^7*e^10 + 3456*a^6*c^6*d^5*e^12 + 1088*a^7*c^5*d^3*e^1
4)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*
d^6*e^4) - (x*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2)*(512*a^2*c^11*d^16*e^3 + 2
560*a^3*c^10*d^14*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 256
0*a^6*c^7*d^8*e^11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^
9*c^4*d^2*e^17))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)*(c^4*d^10 + a^4
*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-d^3*e
^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(
-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*
d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2
+ 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)) +
(((x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9)
)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d
^6*e^4) + (((240*a^2*c^8*d^6*e^7 - 100*a*c^9*d^8*e^5 - 4*a^5*c^5*e^13 + 392
*a^3*c^7*d^4*e^9 + 48*a^4*c^6*d^2*e^11)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d
^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) + (((x*(32*a^6*c^5*d*e^14 - 4
8*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*
d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(c^4*d^10 + a^4*d^
2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) + ((a*e^2 +
5*c*d^2)*((128*a^8*c^4*d*e^16 - 64*a*c^11*d^15*e^2 + 128*a^2*c^10*d^13*e^4
+ 1728*a^3*c^9*d^11*e^6 + 4480*a^4*c^8*d^9*e^8 + 5440*a^5*c^7*d^7*e^10 + 34
56*a^6*c^6*d^5*e^12 + 1088*a^7*c^5*d^3*e^14)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*
c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) + (x*(a*e^2 + 5*c*d^2)*(
-d^3*e^3)^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608*a^4*
c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^7*c^6
*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(4*(c^2*d^7 + a^
2*d^3*e^4 + 2*a*c*d^5*e^2)*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^
3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*
e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^
2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^
7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(
c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2))))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2
)*1i)/(2*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)) - atan((((((256*a^8*c^4*
d*e^16 - 128*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^
6 + 8960*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 +
2176*a^7*c^5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^
3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) + (x*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^
4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^
3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*
e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^1
4*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^
11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))
/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^
6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*
d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^
3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2
) + (x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^
2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*

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d^3*e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*
a^2*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) -
4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^
7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e
^4)))^(1/2) + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 7
84*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*
c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-(a^2*e^4*(-a^3*c^3)^
(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*
c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(a^3*c^6*e^11 - 27*c^9*
d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 +
4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^
3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6
*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*1i - (((((256*a^8*c^4*d*e
^16 - 128*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 +
 8960*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 21
76*a^7*c^5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c
*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) - (x*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(
-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c
^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2
 + 6*a^5*c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e
^5 + 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11
- 4608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c
^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e
^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3
*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c
^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) -
 (x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c
^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3
*e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2
*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a
^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e
^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)
))^(1/2) + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*
a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3
*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-(a^2*e^4*(-a^3*c^3)^(1/
2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d
^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(a^3*c^6*e^11 - 27*c^9*d^6
*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a
*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^
(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*
c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*1i)/((((((256*a^8*c^4*d*e^16
 - 128*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 89
60*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*
a^7*c^5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^
4*e^6 + 6*a^2*c^2*d^6*e^4)) + (x*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^
3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)
^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5
+ 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4
608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*
d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)
)*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e
+ 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*
d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x
*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*
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d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^
12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^
2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*
c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8
+ a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^
(1/2) + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3
*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^
8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-(a^2*e^4*(-a^3*c^3)^(1/2)
+ c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*
e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*
c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^
5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^
3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/
2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d
^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (((((256*a^8*c^4*d*e^16 - 128
*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960*a^4
*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^7*c^
5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6
+ 6*a^2*c^2*d^6*e^4)) - (x*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)
^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2)
)/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*
c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608
*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^
7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^10 +
 a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(-(a
^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^
3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 +
4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(32*a
^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^
6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(
c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*
e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^
3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*
c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)
+ (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*d
^4*e^9 + 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2
+ 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*
d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-
a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11
*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*
e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c
^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2
*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3
*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (5*c^8*d^3*e^6 + a*c^7*d*e^8)/(c^4*
d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)
))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e
 + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4
*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a),x)
[Out] Timed out
```

3.144 $\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=363

$$-\frac{3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{7/4}}+\frac{3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x\right.}{16\sqrt{2}\,a^{7/4}c^{7/4}}$$

```
[Out] -e^3*x^3/c/(c*x^4+a)+1/4*x*(d*(-3*a*e^2+c*d^2)+3*e*(a*e^2+c*d^2)*x^2)/a/c/(
c*x^4+a)-3/32*(a*e^2+c*d^2)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/
2))*(-e*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2)+3/32*(a*e^2+c*d^2)*ln(a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(7/4)
/c^(7/4)*2^(1/2)+3/16*(a*e^2+c*d^2)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e
*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2)+3/16*(a*e^2+c*d^2)*arctan(1+c^(
1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1207, 1858, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{7/4}}+\frac{3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x\right.}{16\sqrt{2}\,a^{7/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

```
[Out] -((e^3*x^3)/(c*(a + c*x^4))) + (x*(d*(c*d^2 - 3*a*e^2) + 3*e*(c*d^2 + a*e^2
)*x^2))/(4*a*c*(a + c*x^4)) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(7/4)) + (3*(Sq
rt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(8*Sqrt[2]*a^(7/4)*c^(7/4)) - (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4
)*c^(7/4)) + (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{\left(a+cx^4\right)^2}\,dx = -\frac{e^3x^3}{c\left(a+cx^4\right)} - \frac{\int \frac{-cd^3-3e\left(cd^2+ae^2\right)x^2-3cde^2x^4}{\left(a+cx^4\right)^2}\,dx}{c}$$

$$= -\frac{e^3x^3}{c\left(a+cx^4\right)} + \frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} + \frac{\int \frac{3cd\left(cd^2+ae^2\right)+3ce\left(cd^2+ae^2\right)x^2}{a+cx^4}\,dx}{4ac^2}$$

$$= -\frac{e^3x^3}{c\left(a+cx^4\right)} + \frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} + \frac{\left(3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(cd^2+ae^2\right)\right)\int \frac{\sqrt{a}\cdots}{\cdots}}{8a^{3/2}c^2}\cdots$$

$$= -\frac{e^3x^3}{c\left(a+cx^4\right)} + \frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} - \frac{\left(3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(cd^2+ae^2\right)\right)\int \frac{\cdots}{-\frac{\sqrt{\cdots}}{\sqrt{\cdots}}}}{16\sqrt{2}\,a^{7/4}c^{7/4}}\cdots$$

$$= -\frac{e^3x^3}{c\left(a+cx^4\right)} + \frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} - \frac{3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(cd^2+ae^2\right)\log\left(\sqrt{\cdots}\right.}{16\sqrt{2}\,a^{7/4}c^{7\cdots}}\cdots$$

$$= -\frac{e^3x^3}{c\left(a+cx^4\right)} + \frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} - \frac{3\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\left(cd^2+ae^2\right)\tan^{-1}\left(\cdots\right.}{8\sqrt{2}\,a^{7/4}c^{7/4}}\cdots$$

Mathematica [A] time = 0.26, size = 371, normalized size = 1.02

$$\frac{-\frac{8a^{3/4}c^{3/4}\left(ae^2x\left(3d+ex^2\right)-cd^2x\left(d+3ex^2\right)\right)}{a+cx^4} + 3\sqrt{2}\left(a^{3/2}e^3+\sqrt{a}\,cd^2e-a\sqrt{c}\,de^2-c^{3/2}d^3\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*c^(3/4)*(a*e^2*x*(3*d + e*x^2) - c*d^2*x*(d + 3*e*x^2)))/(a +
c*x^4) - 6*Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(3/
2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3 +
Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)] + 3*Sqrt[2]*(-(c^(3/2)*d^3) + Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e
^2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] +
3*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^(7/4)*c^(7/4))
```

fricas [B] time = 0.55, size = 2116, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(3*c*d^2*e - a*e^3)*x^3 - 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2*d^5*e
+ 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2
- a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10
+ a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^4*d^8*e^2 + 2*
a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x + 27*(a
^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 + a^6*c^5*e*
sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 -
```

```
a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sqrt(-(2*c^2*d^5
*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e
^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^
10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) + 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2
*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^
10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^
2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^4*d^8*e
^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x
- 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 + a^6
*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6
*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sqrt(-(2*
c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5
*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c
*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) - 3*(a*c^2*x^4 + a^2*c)*sqrt(
-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^12 + 2*a
*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a
^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^
4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e
^10)*x + 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^
6 - a^6*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*
c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sq
rt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^12 +
2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 +
2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) + 3*(a*c^2*x^4 + a^2*c
)*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^1
2 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^
8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 +
 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8
- a^5*e^10)*x - 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c
^2*d*e^6 - a^6*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 -
 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^
7)))*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*
d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4
*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) + 4*(c*d^3 - 3*
a*d*e^2)*x)/(a*c^2*x^4 + a^2*c)
```

giac [A] time = 0.19, size = 425, normalized size = 1.17

$$\frac{3\,cd^2x^3e + cd^3x - ax^3e^3 - 3\,adxe^2}{4\left(cx^4 + a\right)ac} + \frac{3\,\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3 + \left(ac^3\right)^{\frac{1}{4}}ac^2de^2 + \left(ac^3\right)^{\frac{3}{4}}cd^2e + \left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x\cdots\right.}{2\left(\cdots\right.}\right.}{16\,a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*x^3*e + c*d^3*x - a*x^3*e^3 - 3*a*d*x*e^2)/((c*x^4 + a)*a*c) +
 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^
(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*
c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/32*sqr
t(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*c*d
^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
^2*c^4) - 3/32*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 -
 (a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/
4) + sqrt(a/c))/(a^2*c^4)
```

maple [B] time = 0.01, size = 624, normalized size = 1.72

$$\frac{3\sqrt{2}\,d^{2}e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{3\sqrt{2}\,d^{2}e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{3\sqrt{2}\,d^{2}e\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\,e^{2}\arctan}{16ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+a)^2,x)
```

```
[Out] (-1/4*e*(a*e^2-3*c*d^2)/a/c*x^3-1/4*d*(3*a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+3/32
/a/c*d*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-
(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^2+3/32/a^2*d^3*(a/c)^(1/4)*2^(1/2)*ln
((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(
1/2)))+3/16/a/c*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^2+3
/16/a^2*d^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/a/c*d*
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^2+3/16/a^2*d^3*(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+3/32/c^2*e^3/(a/c)^(1/4)*2^(1
/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(
a/c)^(1/2)))+3/32/a/c*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(
a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2+3/16/c^2*e^3/(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/a/c*e/(a/c)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2+3/16/c^2*e^3/(a/c)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/c)^(1/4)*x-1)+3/16/a/c*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x-1)*d^2
```

maxima [A] time = 2.36, size = 292, normalized size = 0.80

$$\frac{\left(3\,cd^{2}e-ae^{3}\right)x^{3}+\left(cd^{3}-3\,ade^{2}\right)x}{4\left(ac^{2}x^{4}+a^{2}c\right)}+\frac{3\left(cd^{2}+ae^{2}\right)\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\arctan}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((3*c*d^2*e - a*e^3)*x^3 + (c*d^3 - 3*a*d*e^2)*x)/(a*c^2*x^4 + a^2*c) +
 3/32*(c*d^2 + a*e^2)*(2*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sq
rt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*log(
sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt
(2)*(sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(3/4)))/(a*c)
```

mupad [B] time = 4.94, size = 2560, normalized size = 7.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + c*x^4)^2,x)
```

```
[Out] - ((d*x*(3*a*e^2 - c*d^2))/(4*a*c) + (e*x^3*(a*e^2 - 3*c*d^2))/(4*a*c))/(a
+ c*x^4) - 2*atanh((9*c^3*d^6*x*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (
9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^7)^(1/2))/
(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a
^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*c*d^6*
e^3)/16 - (27*a^3*e^9)/(32*c^2) + (27*c^2*d^8*e)/(32*a) - (27*a^2*d^2*e^7)/
(16*c) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^5*c) - (27*d*e^8*(-a^7*c^7)^(1/2))
/(32*a*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^2*c^4) + (27*d^7*e^2*(-a^
7*c^7)^(1/2))/(16*a^4*c^2))) + (9*a*e^6*x*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^
4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^
7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/
2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*(
(27*a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a^2) - (27*c
^2*d^8*e)/(32*a^3) - (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c) + (27*d*e^8*(-a^7
*c^7)^(1/2))/(32*a^3*c^5) + (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^4*c^4) - (2
7*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) + (9*c*d^2*e^4*x*((9*e^6*(-a^7*c
^7)^(1/2))/(256*a^4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2)
 - (9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*
e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6
*c^5))^(1/2))/(2*((27*a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3
)/(16*a^2) - (27*c^2*d^8*e)/(32*a^3) - (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c)
 + (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^3*c^5) + (27*d^3*e^6*(-a^7*c^7)^(1/2))
/(16*a^4*c^4) - (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) - (9*c^2*d^4*e
^2*x*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d
^3*e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(
128*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*
c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*a^2*e^9)/(32*c^2) + (27*a*d^2*e^7
)/(16*c) - (27*c*d^6*e^3)/(16*a) - (27*c^2*d^8*e)/(32*a^2) - (27*d^9*(-a^7*
c^7)^(1/2))/(32*a^6*c) + (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^2*c^5) + (27*d^3
*e^6*(-a^7*c^7)^(1/2))/(16*a^3*c^4) - (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^5
*c^2))))*(-(9*(c^3*d^6*(-a^7*c^7)^(1/2) - a^3*e^6*(-a^7*c^7)^(1/2) + 2*a^4*
c^6*d^5*e + 2*a^6*c^4*d*e^5 + 4*a^5*c^5*d^3*e^3 + a*c^2*d^4*e^2*(-a^7*c^7)^
(1/2) - a^2*c*d^2*e^4*(-a^7*c^7)^(1/2)))/(256*a^7*c^7))^(1/2) - 2*atanh((9*
c^3*d^6*x*((9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d^5*e)/(128*a^3*c) -
 (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*c^7)^(1/2)
)/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) + (9*d^4*e^2*(
-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*c*d^6*e^3)/16 - (27*a^3*e^9)
/(32*c^2) + (27*c^2*d^8*e)/(32*a) - (27*a^2*d^2*e^7)/(16*c) - (27*d^9*(-a^7
*c^7)^(1/2))/(32*a^5*c) + (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a*c^5) + (27*d^3*
e^6*(-a^7*c^7)^(1/2))/(16*a^2*c^4) - (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^4*
c^2))) + (9*a*e^6*x*((9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d^5*e)/(12
8*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*
c^7)^(1/2))/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) + (9
*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*a*e^9)/(32*c^2) +
(27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a^2) - (27*c^2*d^8*e)/(32*a^3) + (
27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c) - (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^3*c
^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^4*c^4) + (27*d^7*e^2*(-a^7*c^7)^(
1/2))/(16*a^6*c^2))) + (9*c*d^2*e^4*x*((9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^
4) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^
3) - (9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^(1/2))/
(256*a^5*c^6) + (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*
a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a^2) - (27*c^2*d
^8*e)/(32*a^3) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c) - (27*d*e^8*(-a^7*c^7
)^(1/2))/(32*a^3*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^4*c^4) + (27*d^
7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) - (9*c^2*d^4*e^2*x*((9*d^6*(-a^7*c^7
)^(1/2))/(256*a^7*c^4) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) -
 (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^2*e^
4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) + (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c
^5))^(1/2))/(2*((27*a^2*e^9)/(32*c^2) + (27*a*d^2*e^7)/(16*c) - (27*c*d^6*e
```

```
^3)/(16*a) - (27*c^2*d^8*e)/(32*a^2) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^6*c)
 - (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^2*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))
/(16*a^3*c^4) + (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^5*c^2))))*(-(9*(a^3*e^6
*(-a^7*c^7)^(1/2) - c^3*d^6*(-a^7*c^7)^(1/2) + 2*a^4*c^6*d^5*e + 2*a^6*c^4*
d*e^5 + 4*a^5*c^5*d^3*e^3 - a*c^2*d^4*e^2*(-a^7*c^7)^(1/2) + a^2*c*d^2*e^4*
(-a^7*c^7)^(1/2)))/(256*a^7*c^7))^(1/2)
```

sympy [A] time = 3.37, size = 352, normalized size = 0.97

$$\text{RootSum}\Big(65536t^4a^7c^7 + t^2\left(9216a^6c^4de^5 + 18432a^5c^5d^3e^3 + 9216a^4c^6d^5e\right) + 81a^6e^{12} + 486a^5cd^2e^{10} + 1215a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**7 + _t**2*(9216*a**6*c**4*d*e**5 + 18432*a**5*c
**5*d**3*e**3 + 9216*a**4*c**6*d**5*e) + 81*a**6*e**12 + 486*a**5*c*d**2*e*
*10 + 1215*a**4*c**2*d**4*e**8 + 1620*a**3*c**3*d**6*e**6 + 1215*a**2*c**4*
d**8*e**4 + 486*a*c**5*d**10*e**2 + 81*c**6*d**12, Lambda(_t, _t*log(x + (4
096*_t**3*a**6*c**5*e + 432*_t*a**5*c**2*d*e**6 + 720*_t*a**4*c**3*d**3*e**
4 + 144*_t*a**3*c**4*d**5*e**2 - 144*_t*a**2*c**5*d**7)/(27*a**5*e**10 + 81
*a**4*c*d**2*e**8 + 54*a**3*c**2*d**4*e**6 - 54*a**2*c**3*d**6*e**4 - 81*a*
c**4*d**8*e**2 - 27*c**5*d**10)))) + (x**3*(-a*e**3 + 3*c*d**2*e) + x*(-3*a
*d*e**2 + c*d**3))/(4*a**2*c + 4*a*c**2*x**4)
```

3.145 $\int \frac{\left(d+ex^2\right)^2}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=349

$$-\frac{\left(-2\sqrt{a}\sqrt{c}\,de+ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}\,de+ae^2+3cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\right.}{16\sqrt{2}\,a^{7/4}c^{5/4}}$$

```
[Out] -1/3*e^2*x/c/(c*x^4+a)+1/12*x*(6*c*d*e*x^2+a*e^2+3*c*d^2)/a/c/(c*x^4+a)-1/3
2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2+a*e^2-2*d*e*a
^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)+1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(3*c*d^2+a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2
^(1/2)+1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2+a*e^2+2*d*e*a^(1/
2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4)
)*(3*c*d^2+a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1207, 1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(-2\sqrt{a}\sqrt{c}\,de+ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}\,de+ae^2+3cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\right.}{16\sqrt{2}\,a^{7/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

```
[Out] -(e^2*x)/(3*c*(a + c*x^4)) + (x*(3*c*d^2 + a*e^2 + 6*c*d*e*x^2))/(12*a*c*(a
 + c*x^4)) - ((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*c*d^2 + 2*Sqrt[a]*S
qrt[c]*d*e + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*c^(5/4)) - ((3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*
c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = -\frac{e^2x}{3c(a+cx^4)} - \frac{\int \frac{-3cd^2-ae^2-6cdex^2}{(a+cx^4)^2}\,dx}{3c}$$

$$= -\frac{e^2x}{3c(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac(a+cx^4)} + \frac{\int \frac{3(3cd^2+ae^2)+6cdex^2}{a+cx^4}\,dx}{12ac}$$

$$= -\frac{e^2x}{3c(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac(a+cx^4)} + \frac{(3cd^2-2\sqrt{a}\sqrt{c}\,de+ae^2)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a^{3/2}c^{3/2}} + \frac{(3cd^2}{}$$

$$= -\frac{e^2x}{3c(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac(a+cx^4)} - \frac{(3cd^2-2\sqrt{a}\sqrt{c}\,de+ae^2)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}\,a^{7/4}c^{5/4}} - \frac{(}{}$$

$$= -\frac{e^2x}{3c(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac(a+cx^4)} - \frac{(3cd^2-2\sqrt{a}\sqrt{c}\,de+ae^2)\log(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x}{16\sqrt{2}\,a^{7/4}c^{5/4}}$$

$$= -\frac{e^2x}{3c(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac(a+cx^4)} - \frac{(3cd^2+2\sqrt{a}\sqrt{c}\,de+ae^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{5/4}} + \frac{(}{}$$

Mathematica [**A**] time = 0.17, size = 295, normalized size = 0.85

$$\frac{-\frac{8a^{3/4}\sqrt[4]{c}(ae^2x-cdx(d+2ex^2))}{a+cx^4} - \sqrt{2}\left(-2\sqrt{a}\sqrt{c}\,de+ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right) + \sqrt{2}\left(-2\sqrt{a}\sqrt{c}\,de\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*c^(1/4)*(a*e^2*x - c*d*x*(d + 2*e*x^2)))/(a + c*x^4) - 2*Sqrt[
2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)] + 2*Sqrt[2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a
*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*(3*c
*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(32*a^(7/4)*c^(5/4))
```

fricas [**B**] time = 0.63, size = 1596, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*c*d*e*x^3 + (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 +
36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5
)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2
 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*sqrt
(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a
^4*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^
4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2
*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e
```

```
^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36
*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5))
 + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^4*d*e*sqrt(-
(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4
*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^4
+ a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c
^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3
)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*
c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) -
12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38
*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*sqrt(-(81
*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^
8)/(a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a
^5*c*e^6)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d
^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a
^3*c^2))) + (a*c^2*x^4 + a^2*c)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*
d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c
*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2
*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^4*d*e*sqrt(-(81*c^4
*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(
a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a^5*c
*e^6)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e
^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c
^2))) + 4*(c*d^2 - a*e^2)*x)/(a*c^2*x^4 + a^2*c)
```

giac [A] time = 0.19, size = 350, normalized size = 1.00

$$\frac{2\,cdx^3e+cd^2x-axe^2}{4\left(cx^4+a\right)ac}+\frac{\sqrt{2}\left(3\left(ac^3\right)^{\frac{1}{4}}c^2d^2+\left(ac^3\right)^{\frac{1}{4}}ace^2+2\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3}+\frac{\sqrt{2}\left(3\left(ac^3\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*c*d*x^3*e + c*d^2*x - a*x*e^2)/((c*x^4 + a)*a*c) + 1/16*sqrt(2)*(3*(
a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(
2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/3
2*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4
)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2
)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*l
og(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

maple [A] time = 0.01, size = 464, normalized size = 1.33

$$\frac{\sqrt{2}\,de\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{\sqrt{2}\,de\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{\sqrt{2}\,de\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,e^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+a)^2,x)
```

```
[Out] (1/2*d*e/a*x^3-1/4*(a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+1/16/a/c*(a/c)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^2+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x-1)*d^2+1/32/a/c*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1
/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^2+3/3
2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(
a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2+1/16/a/c*(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x+1)*e^2+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/c)^(1/4)*x+1)*d^2+1/16/a/c*d*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(
1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/8/a/c*d*e/(a
/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/8/a/c*d*e/(a/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.59, size = 324, normalized size = 0.93

$$\frac{2\,cdex^3+\left(cd^2-ae^2\right)x}{4\left(ac^2x^4+a^2c\right)}+\frac{\frac{2\sqrt{2}\left(3\,c^{\frac{3}{2}}d^2+2\sqrt{a}\,cde+a\sqrt{c}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(3\,c^{\frac{3}{2}}d^2+2\sqrt{a}\,cde+a\sqrt{c}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\right.}{2\sqrt{\sqrt{}}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*c*d*e*x^3 + (c*d^2 - a*e^2)*x)/(a*c^2*x^4 + a^2*c) + 1/32*(2*sqrt(2)
*(3*c^(3/2)*d^2 + 2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sq
rt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*c^(3/2)*d^2 + 2*sqrt(a)*c*d*e + a*sqr
t(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(s
qrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*c^(3/
2)*d^2 - 2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*c^(3/2)*d^2 - 2*sqrt(a
)*c*d*e + a*sqrt(c)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt
(a))/(a^(3/4)*c^(3/4)))/(a*c)
```

mupad [B] time = 4.79, size = 1565, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + c*x^4)^2,x)
```

```
[Out] 2*atanh((9*c^3*d^4*x*((9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (3*d^3*e)/(6
4*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) + (d
^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/(2*((27*d^6*(-a^7*c^5)^(1/2)
)/(32*a^5) - (c*d^3*e^3)/8 - (a*d*e^5)/16 - (9*c^2*d^5*e)/(16*a) + (e^6*(-a
^7*c^5)^(1/2))/(32*a^2*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^3*c^2) + (
15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^4*c))) + (c*e^4*x*((9*d^4*(-a^7*c^5)^(1/
2))/(256*a^7*c^3) - (3*d^3*e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^
7*c^5)^(1/2))/(256*a^5*c^5) + (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/
2))/(2*((27*d^6*(-a^7*c^5)^(1/2))/(32*a^7) - (d*e^5)/(16*a) - (c*d^3*e^3)/(
8*a^2) - (9*c^2*d^5*e)/(16*a^3) + (e^6*(-a^7*c^5)^(1/2))/(32*a^4*c^3) + (5*
d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^5*c^2) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*
a^6*c))) + (c^2*d^2*e^2*x*((9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (3*d^3*
e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5)
 + (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/((27*d^6*(-a^7*c^5)^(1/
2))/(32*a^6) - (d*e^5)/16 - (c*d^3*e^3)/(8*a) - (9*c^2*d^5*e)/(16*a^2) + (e
^6*(-a^7*c^5)^(1/2))/(32*a^3*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^4*c^
2) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^5*c))*((a^2*e^4*(-a^7*c^5)^(1/2)
```

```
+ 9*c^2*d^4*(-a^7*c^5)^(1/2) - 12*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 + 2*a*c*d
^2*e^2*(-a^7*c^5)^(1/2))/(256*a^7*c^5))^(1/2) - 2*atanh((9*c^3*d^4*x*(- (d*
e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c) - (9*d^4*(-a^7*c^5)^(1/2))/(256*a^
7*c^3) - (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) - (d^2*e^2*(-a^7*c^5)^(1/2))/
(128*a^6*c^4))^(1/2))/(2*((27*d^6*(-a^7*c^5)^(1/2))/(32*a^5) + (c*d^3*e^3)/
8 + (a*d*e^5)/16 + (9*c^2*d^5*e)/(16*a) + (e^6*(-a^7*c^5)^(1/2))/(32*a^2*c^
3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^3*c^2) + (15*d^4*e^2*(-a^7*c^5)^(1/
2))/(32*a^4*c))) + (c*e^4*x*(- (d*e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c)
- (9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (e^4*(-a^7*c^5)^(1/2))/(256*a^5*
c^5) - (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/(2*((27*d^6*(-a^7*c
^5)^(1/2))/(32*a^7) + (d*e^5)/(16*a) + (c*d^3*e^3)/(8*a^2) + (9*c^2*d^5*e)/
(16*a^3) + (e^6*(-a^7*c^5)^(1/2))/(32*a^4*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2
))/(32*a^5*c^2) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^6*c))) + (c^2*d^2*e^2
*x*(- (d*e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c) - (9*d^4*(-a^7*c^5)^(1/2)
)/(256*a^7*c^3) - (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) - (d^2*e^2*(-a^7*c^5
)^(1/2))/(128*a^6*c^4))^(1/2))/((d*e^5)/16 + (27*d^6*(-a^7*c^5)^(1/2))/(32*
a^6) + (c*d^3*e^3)/(8*a) + (9*c^2*d^5*e)/(16*a^2) + (e^6*(-a^7*c^5)^(1/2))/
(32*a^3*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^4*c^2) + (15*d^4*e^2*(-a^
7*c^5)^(1/2))/(32*a^5*c)))*(-(a^2*e^4*(-a^7*c^5)^(1/2) + 9*c^2*d^4*(-a^7*c^
5)^(1/2) + 12*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 + 2*a*c*d^2*e^2*(-a^7*c^5)^(1
/2))/(256*a^7*c^5))^(1/2) + ((d*e*x^3)/(2*a) - (x*(a*e^2 - c*d^2))/(4*a*c))
/(a + c*x^4)
```

sympy [A] time = 2.07, size = 275, normalized size = 0.79

$$\mathrm{RootSum}\left(65536t^4a^7c^5 + t^2\left(2048a^5c^3de^3 + 6144a^4c^4d^3e\right) + a^4e^8 + 20a^3cd^2e^6 + 118a^2c^2d^4e^4 + 180ac^3d^6e^2 + \dots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**5 + _t**2*(2048*a**5*c**3*d*e**3 + 6144*a**4*c*
*4*d**3*e) + a**4*e**8 + 20*a**3*c*d**2*e**6 + 118*a**2*c**2*d**4*e**4 + 18
0*a*c**3*d**6*e**2 + 81*c**4*d**8, Lambda(_t, _t*log(x + (-8192*_t**3*a**6*
c**4*d*e + 16*_t*a**5*c*e**6 - 48*_t*a**4*c**2*d**2*e**4 - 144*_t*a**3*c**3
*d**4*e**2 + 432*_t*a**2*c**4*d**6)/(a**4*e**8 + 12*a**3*c*d**2*e**6 + 38*a
**2*c**2*d**4*e**4 + 108*a*c**3*d**6*e**2 + 81*c**4*d**8)))) + (2*c*d*e*x**
3 + x*(-a*e**2 + c*d**2))/(4*a**2*c + 4*a*c**2*x**4)
```

3.146 $\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=275

$$-\frac{\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{\left(\sqrt{a}\,e+3\right.}{}$$

```
[Out] 1/4*x*(e*x^2+d)/a/(c*x^4+a)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*c
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/c^(3
/4)*2^(1/2)+1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*d*c^(1/2
))/a^(7/4)/c^(3/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1
/2)+3*d*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{\left(\sqrt{a}\,e+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x^2))/(4*a*(a + c*x^4)) - ((3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d +
Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3
/4)) - ((3*Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d - Sqrt[a]*e)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c
^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx = \frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d-ex^2}{a+cx^4}\,dx}{4a}$$

$$= \frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)} + \frac{\left(\frac{3\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{8ac} + \frac{\left(\frac{3\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{8ac}$$

$$= \frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)} + \frac{\left(\frac{3\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16ac} + \frac{\left(\frac{3\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16ac} - \frac{\left(3\sqrt{c}\,d-\right.}{}$$

$$= \frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)} - \frac{\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}} + \frac{\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{a}\right.}{16\sqrt{2}\,a}$$

$$= \frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)} - \frac{\left(3\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}} + \frac{\left(3\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}} -$$

Mathematica [A] time = 0.27, size = 267, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\left(a^{3/4}e-3\sqrt[4]{a}\,\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}} + \frac{\sqrt{2}\left(3\sqrt[4]{a}\,\sqrt{c}\,d-a^{3/4}e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}} - \frac{2\sqrt{2}\,\sqrt[4]{a}\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\tan^{-1}\left(1-\right.}{c^{3/4}}}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + c*x^4)^2,x]
```

```
[Out] ((8*a*x*(d + e*x^2))/(a + c*x^4) - (2*Sqrt[2]*a^(1/4)*(3*Sqrt[c]*d + Sqrt[a
]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (2*Sqrt[2]*a^(1/4)*
(3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4)
+ (Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d - a^(3/4
)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(32*a
^2)
```

fricas [B] time = 0.42, size = 873, normalized size = 3.17

$$\frac{4\,ex^3 - \left(acx^4 + a^2\right)\sqrt{-\frac{a^3c\sqrt{-\frac{81\,c^2d^4-18\,acd^2e^2+a^2e^4}{a^7c^3}}+6\,de}{a^3c}}\,\log\left(-\left(81\,c^2d^4 - a^2e^4\right)x + \left(a^6c^2e\sqrt{-\frac{81\,c^2d^4-18\,acd^2e^2+a^2e^4}{a^7c^3}} + 27\,a^2 \ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*e*x^3 - (a*c*x^4 + a^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2
*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^2*e^4)*x
+ (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 27*
a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^
2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (a*c*x^4 + a^2)*sqrt(-(a^3*c*s
qrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*l
og(-(81*c^2*d^4 - a^2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^
2 + a^2*e^4)/(a^7*c^3)) + 27*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^3*c*sqrt
(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (
a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a
^7*c^3)) - 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^2*e^4)*x + (a^6*c^2*e*sqrt(
-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 27*a^2*c^2*d^3 + 3*a^
3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c
^3)) - 6*d*e)/(a^3*c))) - (a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 1
8*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^
2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^
3)) - 27*a^2*c^2*d^3 + 3*a^3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*
c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))) + 4*d*x)/(a*c*x^4 + a^2)
```

giac [A] time = 0.44, size = 273, normalized size = 0.99

$$\frac{x^3e + dx}{4\left(cx^4 + a\right)a} + \frac{\sqrt{2}\left(3\left(ac^3\right)^{\frac{1}{4}}c^2d + \left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}\left(3\left(ac^3\right)^{\frac{1}{4}}c^2d + \left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\ldots\right)}{16\,a^2c^3} \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (
a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))
/(a^2*c^3) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/32*sqrt(
2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4
) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3
/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

maple [A] time = 0.01, size = 303, normalized size = 1.10

$$\frac{e\,x^{3}}{4\left(c\,x^{4}+a\right)a}+\frac{dx}{4\left(c\,x^{4}+a\right)a}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{\sqrt{2}\,e\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d*x/a/(c*x^4+a)+3/32*d/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d/a^2*(a/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d/a^2*(a/c)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/4*e*x^3/a/(c*x^4+a)+1/32*e/a/c/(a/c)^
(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2
^(1/2)*x+(a/c)^(1/2)))+1/16*e/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x+1)+1/16*e/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.27, size = 253, normalized size = 0.92

$$\frac{ex^{3}+dx}{4\left(acx^{4}+a^{2}\right)}+\frac{\frac{2\sqrt{2}\left(3\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(3\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(3\sqrt{c}d-\sqrt{a}e\right)\cdots}{\cdots}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e*x^3 + d*x)/(a*c*x^4 + a^2) + 1/32*(2*sqrt(2)*(3*sqrt(c)*d + sqrt(a)*
e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*sqrt(c)*d
+ sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*sq
rt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))
/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/a
```

mupad [B] time = 0.40, size = 637, normalized size = 2.32

$$\frac{\frac{e\,x^{3}}{4\,a}+\frac{d\,x}{4\,a}}{c\,x^{4}+a}-2\,\mathrm{atanh}\left(\frac{c^{2}\,e^{2}\,x\,\sqrt{\frac{e^{2}\,\sqrt{-a^{7}\,c^{3}}}{256\,a^{6}\,c^{3}}-\frac{9\,d^{2}\,\sqrt{-a^{7}\,c^{3}}}{256\,a^{7}\,c^{2}}-\frac{3\,d\,e}{128\,a^{3}\,c}}}{2\left(\frac{c\,e^{3}}{32\,a}-\frac{9\,c^{2}\,d^{2}\,e}{32\,a^{2}}-\frac{27\,c\,d^{3}\,\sqrt{-a^{7}\,c^{3}}}{32\,a^{6}}+\frac{3\,d\,e^{2}\,\sqrt{-a^{7}\,c^{3}}}{32\,a^{5}}\right)}-\frac{9\,c^{3}\,d^{2}\,x\,\sqrt{\frac{e^{2}\,\sqrt{-a^{7}\,c^{3}}}{256\,a^{6}\,c^{3}}-\frac{9\,d^{2}\,\sqrt{-a^{7}\,c^{3}}}{256\,a^{7}\,c^{2}}-\cdots}}{2\left(\frac{c\,e^{3}}{32}-\frac{9\,c^{2}\,d^{2}\,e}{32\,a}-\frac{27\,c\,d^{3}\,\sqrt{-a^{7}\,c^{3}}}{32\,a^{5}}+\frac{3\,d\,e^{2}\,\sqrt{-a^{7}\,c^{3}}}{32\,a^{4}}\cdots\right)}\cdots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + c*x^4)^2,x)
```

```
[Out] ((e*x^3)/(4*a) + (d*x)/(4*a))/(a + c*x^4) - 2*atanh((c^2*e^2*x*((e^2*(-a^7*
c^3)^(1/2))/(256*a^6*c^3) - (9*d^2*(-a^7*c^3)^(1/2))/(256*a^7*c^2) - (3*d*e
)/(128*a^3*c))^(1/2))/(2*((c*e^3)/(32*a) - (9*c^2*d^2*e)/(32*a^2) - (27*c*d
^3*(-a^7*c^3)^(1/2))/(32*a^6) + (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^5))) - (9*
c^3*d^2*x*((e^2*(-a^7*c^3)^(1/2))/(256*a^6*c^3) - (9*d^2*(-a^7*c^3)^(1/2))/
(256*a^7*c^2) - (3*d*e)/(128*a^3*c))^(1/2))/(2*((c*e^3)/32 - (9*c^2*d^2*e)/
(32*a) - (27*c*d^3*(-a^7*c^3)^(1/2))/(32*a^5) + (3*d*e^2*(-a^7*c^3)^(1/2))/
```

```
(32*a^4))))*(-(9*c*d^2*(-a^7*c^3)^(1/2) - a*e^2*(-a^7*c^3)^(1/2) + 6*a^4*c^
2*d*e)/(256*a^7*c^3))^(1/2) - 2*atanh((c^2*e^2*x*((9*d^2*(-a^7*c^3)^(1/2))/
(256*a^7*c^2) - (3*d*e)/(128*a^3*c) - (e^2*(-a^7*c^3)^(1/2))/(256*a^6*c^3))
^(1/2))/(2*((c*e^3)/(32*a) - (9*c^2*d^2*e)/(32*a^2) + (27*c*d^3*(-a^7*c^3)^
(1/2))/(32*a^6) - (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^5))) - (9*c^3*d^2*x*((9*
d^2*(-a^7*c^3)^(1/2))/(256*a^7*c^2) - (3*d*e)/(128*a^3*c) - (e^2*(-a^7*c^3)
^(1/2))/(256*a^6*c^3))^(1/2))/(2*((c*e^3)/32 - (9*c^2*d^2*e)/(32*a) + (27*c
*d^3*(-a^7*c^3)^(1/2))/(32*a^5) - (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^4))))*(-
(a*e^2*(-a^7*c^3)^(1/2) - 9*c*d^2*(-a^7*c^3)^(1/2) + 6*a^4*c^2*d*e)/(256*a^
7*c^3))^(1/2)
```

sympy [A] time = 1.03, size = 136, normalized size = 0.49

$$\operatorname{RootSum}\left(65536t^4a^7c^3 + 3072t^2a^4c^2de + a^2e^4 + 18acd^2e^2 + 81c^2d^4, \left(t \mapsto t\log\left(x + \frac{4096t^3a^6c^2e + 144ta^3cde^2 -}{a^2e^4 - 81c^2d^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 3072*_t**2*a**4*c**2*d*e + a**2*e**4 + 18*a
*c*d**2*e**2 + 81*c**2*d**4, Lambda(_t, _t*log(x + (4096*_t**3*a**6*c**2*e
+ 144*_t*a**3*c*d*e**2 - 432*_t*a**2*c**2*d**3)/(a**2*e**4 - 81*c**2*d**4))
)) + (d*x + e*x**3)/(4*a**2 + 4*a*c*x**4)
```

3.147 $\int \frac{1}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\ldots}\right.}{8\sqrt{2}\,a^{7/\ldots}}$$

```
[Out] 1/4*x/a/(c*x^4+a)+3/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)
*2^(1/2)+3/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)*2^(1/2)-3
/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*2^(1
/2)+3/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*
2^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\ldots}\right.}{8\sqrt{2}\,a^{7/\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx = \frac{x}{4a\left(a+cx^4\right)}+\frac{3\int \frac{1}{a+cx^4}\,dx}{4a}$$

$$=\frac{x}{4a\left(a+cx^4\right)}+\frac{3\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}}+\frac{3\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}}$$

$$=\frac{x}{4a\left(a+cx^4\right)}+\frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}}+\frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}}-\frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\int \cdots}{16\cdots}$$

$$=\frac{x}{4a\left(a+cx^4\right)}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\,\mathrm{Su}\cdots}{\cdots}$$

$$=\frac{x}{4a\left(a+cx^4\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\cdots\right.}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.12, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4)
 - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/
```

```
4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

fricas [A] time = 0.41, size = 173, normalized size = 0.86

$$\frac{12\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\arctan\left(-a^5cx\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}+\sqrt{a^4\sqrt{-\frac{1}{a^7c}}+x^2}\,a^5c\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}\right)+3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\right.}{16\left(acx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*arctan(-a^5*c*x*(-1/(a^7*c))^(3
/4) + sqrt(a^4*sqrt(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^(3/4)) + 3*(a*c*x
^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x) - 3*(a*c*x^4 +
 a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) + x) + 4*x)/(a*c*x^4 +
 a^2)
```

giac [A] time = 0.18, size = 194, normalized size = 0.96

$$\frac{x}{4\left(cx^4+a\right)a}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) -
3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
2*c)
```

maple [A] time = 0.00, size = 143, normalized size = 0.71

$$\frac{x}{4\left(c\,x^4+a\right)a}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16/a^2*(a/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.43, size = 189, normalized size = 0.94

$$\frac{x}{4\left(acx^4+a^2\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{\right.}}{}\right.}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + s
qrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c
))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*lo
g(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a
```

mupad [B] time = 0.08, size = 58, normalized size = 0.29

$$\frac{x}{4\,a\,\left(c\,x^4+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)}{8\,{\left(-a\right)}^{7/4}\,c^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)}{8\,{\left(-a\right)}^{7/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^2,x)
```

```
[Out] x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4
)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

sympy [A] time = 0.35, size = 39, normalized size = 0.19

$$\frac{x}{4a^2+4acx^4}+\mathrm{RootSum}\left(65536t^4a^7c+81,\left(t\mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*lo
g(16*_t*a**2/3 + x)))
```

3.148 $\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=689

$$-\frac{\sqrt[4]{c}\,e^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}\,e^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}$$

```
[Out] 1/4*c*x*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/4*c^(1/4)*e^2*arctan(-1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/
2)+1/4*c^(1/4)*e^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2
))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*e^2*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^
(1/2)+1/8*c^(1/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*
a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/16*c^(1/4)*arctan(-1+c
^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^
(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1
/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/
2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2
)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+e^(7/2)*arctan(x*e^(1/2)/d^(1/
2))/(a*e^2+c*d^2)^2/d^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1239, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\,e^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}\,e^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1239

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \int \left(\frac{e^4}{\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)^2} - \frac{ce^2\left(-d+ex^2\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}\right) dx$$

$$= -\frac{\left(ce^2\right)\int \frac{-d+ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{e^4\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{c\int \frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} + \frac{\left(\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)e^2\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2} + \frac{(}{}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} + \frac{\left(\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)^2}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\right.}{4\sqrt{2}\,a^{3/4}\left(cd^2+ae\right.}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.30, size = 429, normalized size = 0.62

$$-\frac{\sqrt{2}\,\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}\,cd^2e+7a\sqrt{c}\,de^2+3c^{3/2}d^3\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{7/4}} + \frac{\sqrt{2}\,\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}\,cd^2e+7a\sqrt{c}\,de^2+3c^{3/2}d^3\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x\right.}{a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]
*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e
- 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c
]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqr
t[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

fricas [B] time = 19.11, size = 9892, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*
e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e +
44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^
5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d
^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8
 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e
^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*
a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))
/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a
^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*
a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2
+ 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^
10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^1
0*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a
^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*
d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*
d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^1
0 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5
*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*
c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^
4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d
^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8
+ 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^
16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^
6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*
a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70
*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6
*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^
5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e
^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*
e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 2
8*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4
*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5
*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625
*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4
 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*
e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*
e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748
*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*
e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^
5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e
^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3
 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a
^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*
d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*
d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^1
0 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2
*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3
*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c
^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)
*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*
c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)
/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^1
0*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*
```

```
c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*
e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a
^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^
6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*
e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d
^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 52
9*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 +
 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*
c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^
14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (
a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7
*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748
*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*
e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^
5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e
^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*
c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 +
 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4
*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^
8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d
^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 +
a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c
*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^
4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*
c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 -
 (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*
e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*
e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1
950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 +
 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12
*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqr
t((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c
^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d
^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 52
9*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 +
 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*
c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^
14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*
a^6*c*d^2*e^6 + a^7*e^8))) - 8*(a*c*e^3*x^4 + a^2*e^3)*sqrt(-e/d)*log((e*x^
2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^
2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*
e^4)*x^4), -1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 - 16*(a*c*e^3*x^4 + a^2*e^3)*
sqrt(e/d)*arctan(x*sqrt(e/d)) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 +
(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c
^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^
2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 195
0*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 2
8*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c
^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*
c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^
2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*
a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e +
 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^
2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
```

```
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 4
4*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5
*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^
10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8
- 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^
2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a
^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^
2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c
*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2
*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*
e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 +
625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 +
56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13
*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 +
 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c
*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198
*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 +
26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*
a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*
c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*
c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2*c^2*d
^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4
)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(
-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^
6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*
c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6
+ 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^1
4*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^
4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 +
1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5
*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 17
5*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 +
34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 +
738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*
c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8
*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^
8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a
^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
```

```
8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c
^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
+ a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^
5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2
+ 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a
*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d
^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*
d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8
 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e
^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e
^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4
+ 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^
7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*
c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 +
21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 +
2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^
5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^
9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d
^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c
^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6
*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 +
738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*
c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8
*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^
8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a
^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8))) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]
```

giac [A] time = 0.21, size = 603, normalized size = 0.88

$$\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-\left(ac^3\right)^{\frac{3}{4}}cd^2e-5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}+\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*
c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2
)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2
 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3
*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)
^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*
c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c
^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(
x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a
^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/(
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d)) - 1/4*(c*x^3*e - c*d*x)/((c*x^
4 + a)*(a*c*d^2 + a^2*e^2))
```

maple [A] time = 0.02, size = 873, normalized size = 1.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] e^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)-1/4/(a*e^2+c*d^2)
^2*c/(c*x^4+a)*e^3*x^3-1/4/(a*e^2+c*d^2)^2*c^2/(c*x^4+a)*e/a*x^3*d^2+1/4/(a
*e^2+c*d^2)^2*c/(c*x^4+a)*d*x*e^2+1/4/(a*e^2+c*d^2)^2*c^2/(c*x^4+a)*d^3/a*x
+7/16/(a*e^2+c*d^2)^2*c/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-
1)*d*e^2+3/16/(a*e^2+c*d^2)^2*c^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/c)^(1/4)*x-1)*d^3+7/32/(a*e^2+c*d^2)^2*c/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/
c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*
e^2+3/32/(a*e^2+c*d^2)^2*c^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^
(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3+7/16/(a*e
^2+c*d^2)^2*c/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2+3
/16/(a*e^2+c*d^2)^2*c^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^3-5/32/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/
2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^3-1/32/(a*e^2+
c*d^2)^2*c/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))
/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e-5/16/(a*e^2+c*d^2)^2/(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^3-1/16/(a*e^2+c*d^2)^2*c/a/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e-5/16/(a*e^2+c*d^2
)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^3-1/16/(a*e^2+c*d
^2)^2*c/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e
```

maxima [A] time = 2.45, size = 506, normalized size = 0.73

$$\frac{e^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{de}}+\frac{c\left(\frac{2\sqrt{2}\left(3c^{\frac{3}{2}}d^3-\sqrt{a}\,cd^2e+7\,a\sqrt{c}\,de^2-5\,a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(3c^{\frac{3}{2}}d^3-\sqrt{a}\,cd^2e+7\,a\sqrt{c}\cdots\right.}{\sqrt{\cdots}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)) +
 1/32*c*(2*sqrt(2)*(3*c^(3/2)*d^3 - sqrt(a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 - 5
*a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sq
rt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3
*c^(3/2)*d^3 - sqrt(a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 - 5*a^(3/2)*e^3)*arctan(
1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*c^(3/2)*d^3 + sqrt(a)*
c*d^2*e + 7*a*sqrt(c)*d*e^2 + 5*a^(3/2)*e^3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*c^(3/2)*d^3 + sqrt(
a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 + 5*a^(3/2)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a
^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4) - 1/4*((c^2*d^2*e + a*c*e^3)*x^3 - (c^2*d^3 + a*c*d*e^2)*x)/(a^
2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^
3*c*e^4)*x^4)
```

mupad [B] time = 6.78, size = 17945, normalized size = 26.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^
4) - atan((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c
^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8
*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a
^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*
e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c
^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^
(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a
^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*
c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*
c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a
^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 +
 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a
^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*
e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(
-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a
^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c
^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e
^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)
))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) +
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e
^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 +
21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8
*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(
128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^
2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4
*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c
)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4
*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i - (((((
65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 3
6864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 +
663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4
*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c
^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^
5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c
*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536
*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 3
27680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13
 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25
*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d
*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(
256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*
c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 +
7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 666
88*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
```

```
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c
^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^
5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e
^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) -
25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c
*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))
/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^
9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8
*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9
*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*
a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6
+ 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i)/((125*a^2*c^5*e^12 + 8
1*c^7*d^4*e^8 + 270*a*c^6*d^2*e^10)/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d
^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^11*c^4*e^16
 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e
^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*
e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*
e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/
2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*
a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(
1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 +
 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3
- 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e
^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*
d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^
2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(
1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4
*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7
*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2
) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*
e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10
 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
+ 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*
a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*
e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(2
56*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c
^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^
2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c
^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*
e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)
^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d
^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a
^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1
/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2
*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*
c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(
1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 7
0*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)
^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d
^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8
*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*
c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6
*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a
^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*
c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^
8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)
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))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d
^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^
7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^
8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e
^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^
5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2)
 + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c
*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^
11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9
*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d
^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1
/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*
e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*
c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)
 - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 123
20*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(
a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4
*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*
d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/
2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10
*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*
c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532
*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(
-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*
a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*
e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^
4)))^(1/2)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4
*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c
)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4
*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*2i - atan(
((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e
^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e
^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x
*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e +
 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39
*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*
e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 -
 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e
^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^
4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d
^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/
2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a
^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1
/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 +
6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e
^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8
 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c
^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^
3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*
c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d
^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*
a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432
*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*
c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c
*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(
1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70
```

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*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^
(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612
*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e
^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)
))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) -
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i - (((((65536*a^11
*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c
^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9
*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a
^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-
a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*
e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(
-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*
d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*
c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*
a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(
-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*
a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*
e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^
4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c
^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7
*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c
*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(
1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70
*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^
(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13
+ 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33
296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5
*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6
*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 4
1*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^1
1*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*
e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 +
 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*
(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i)/((125*a^2*c^5*e^12 + 81*c^7*d^4*
e^8 + 270*a*c^6*d^2*e^10)/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4
*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^11*c^4*e^16 - 12288*a
^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 24576
0*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331
776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^
5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^
3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^
5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256
*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2
*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a
^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 32768
0*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))
/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*
c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a
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^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7
*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 +
 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(11
52*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 2035
2*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*
a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^
3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-
a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a
*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e
^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4
)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*
e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^1
1)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^
6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6
*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a
^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8
 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(
1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e
^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*
c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*
e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(2
56*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c
^2*d^4*e^4)))^(1/2) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 -
57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 +
 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^
14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a
^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2
) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^
2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^
4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(
65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 -
589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11
 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^
4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25
*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a
^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*
c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 +
 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^
2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 +
 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/
(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c
^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^
4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*
c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 +
4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*
c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8
*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 +
a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((
25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44
*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^
2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6
 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 +
 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d
^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1
/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*
e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*
c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)
```

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))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) -
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*2i + (atan(-(((((((4
5*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*e^5)/16
 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6*d^3*e^
11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6
*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 - 224*a^
5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*a^8*c^7
*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x
*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7
*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*
a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(
512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*
d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*
d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*
c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*
d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a^8*e^8
+ a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*
(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/
(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*e^13 + 81*c^9*
d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11)
)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6
*c^2*d^4*e^4)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2) -
((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*
e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6
*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*
e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 -
 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*
a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*
e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4
)) + (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327
680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 +
327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e
^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4
*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(
2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1152*a^2*c^11*d^13*e^2 - 490
24*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a
^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a
^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*
e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^
(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1425*a^4*c^5*e^13 +
81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^
2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
+ 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*
e^2))/(((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c
^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*
a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^1
4*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8
+ 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(
2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*
d^4*e^4)) - (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^
3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8
*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^
5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*
d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(
1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^
```

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2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 +
 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/
(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c
^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-
d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*
e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3
*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d
^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(c^2*d^5 + a^2*d*e^4 + 2*a*c*
d^3*e^2) - ((125*a^2*c^5*e^12)/128 + (81*c^7*d^4*e^8)/128 + (135*a*c^6*d^2*
e^10)/64)/(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*
a^6*c^2*d^4*e^4) + ((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16
+ (305*a^2*c^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9
)/8 + (2081*a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e
^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*
a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*
c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5
*d^2*e^14)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
+ 6*a^6*c^2*d^4*e^4)) + (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*
c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680
*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 32
7680*a^12*c^5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^
8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))
)*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1152*a^2*
c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c
^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6
*d^3*e^12))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e
^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d
^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1
425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^
9 + 2532*a^3*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 +
 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(c^2*d^5 + a^2*d*
e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*
e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.149 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=864

$$\frac{xe^4}{2d\left(cd^2+ae^2\right)^2\left(ex^2+d\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{4c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\left(cd^2+ae^2\right)^3}-\frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\tan^{-1}\left(1-\right.}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$

```
[Out] 1/2*e^4*x/d/(a*e^2+c*d^2)^2/(e*x^2+d)+1/4*c*x*(-2*c*d*e*x^2-a*e^2+c*d^2)/a/
(a*e^2+c*d^2)^2/(c*x^4+a)+1/2*e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*
e^2+c*d^2)^2+1/4*c^(3/4)*e^2*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2-
a*e^2-4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^3*2^(1/2)+1/4*c^(3/4)*e^
2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2-a*e^2-4*d*e*a^(1/2)*c^(1/2))
/a^(3/4)/(a*e^2+c*d^2)^3*2^(1/2)+1/16*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a
^(1/4))*(3*c*d^2-3*a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)^2*2^(
1/2)+1/16*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2-3*a*e^2-2*d*
e*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/32*c^(3/4)*ln(-a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2-3*a*e^2+2*d*e*a^(1/2)*c^(1
/2))/a^(7/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/32*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1
/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2-3*a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/(a
*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*e^2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(3*c*d^2-a*e^2+4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^3
*2^(1/2)+1/8*c^(3/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*
(3*c*d^2-a*e^2+4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^3*2^(1/2)+4*c*e
^(7/2)*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(a*e^2+c*d^2)^3
```

Rubi [A] time = 0.91, antiderivative size = 864, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1239, 199, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{xe^4}{2d\left(cd^2+ae^2\right)^2\left(ex^2+d\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{4c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\left(cd^2+ae^2\right)^3}-\frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\tan^{-1}\left(1-\right.}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (c*x*(c*d^2 - a*e^2 - 2*c*d*e
*x^2))/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^4)) + (4*c*Sqrt[d]*e^(7/2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^3 + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*d^(3/2)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c
]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^
2) + (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*
(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 + 4
*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 + 2*Sq
rt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 +
4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 + 2*S
qrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
```

```
*p]
```

Rule 1239

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx = \int\left(\frac{e^4}{\left(cd^2+ae^2\right)^2\left(d+ex^2\right)^2} + \frac{4cde^4}{\left(cd^2+ae^2\right)^3\left(d+ex^2\right)} + \frac{c\left(cd^2-ae^2-2cdex^2\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^4\right)^2} - \frac{c}{}\right.$$

$$= -\frac{\left(ce^2\right)\int\frac{-3cd^2+ae^2+4cdex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^3} + \frac{\left(4cde^4\right)\int\frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^3} + \frac{c\int\frac{cd^2-ae^2-2cdex^2}{\left(a+cx^4\right)^2}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{e^4\int\frac{1}{(d+e}}{\left(cd^2+\right.}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} +$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} + \frac{}{2}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} + \frac{}{2}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} + \frac{}{2}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} + \frac{}{2}$$

Mathematica [A] time = 0.58, size = 540, normalized size = 0.62

$$-\frac{\sqrt{2}c^{3/4}\left(18a^{3/2}\sqrt{c}de^3-7a^2e^4+2\sqrt{a}c^{3/2}d^3e+12acd^2e^2+3c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}} + \frac{\sqrt{2}c^{3/4}\left(18a^{3/2}\sqrt{c}de^3-7a^2e^4+2\sqrt{a}c^{3/2}d^3e+12acd^2e^2\right.}{a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

```
[Out] ((16*e^4*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (8*c*(c*d^2 + a*e^2)*x*(-(a*e
^2) + c*d*(d - 2*e*x^2)))/(a*(a + c*x^4)) + (16*e^(7/2)*(9*c*d^2 + a*e^2)*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-3*c^2*d^4 + 2*Sq
rt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 + 7*a^2*e^4
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(3/4)*(-3
*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*
```

```
e^3 + 7*a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (Sqrt[2
]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*e^2 + 18*a^(3/2
)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e +
 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.25, size = 855, normalized size = 0.99

$$\frac{\left(9\,cd^2e^4+ae^6\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\left(c^3d^7+3\,ac^2d^5e^2+3\,a^2cd^3e^4+a^3de^6\right)\sqrt{d}}+\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^4+12\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e^2-2\left(ac^3\right)^{\frac{3}{4}}cd^3e-7\left(ac^3\right)^{\frac{1}{4}}a\right.}{8\left(\sqrt{2}\,a^2c^4d^6+3\sqrt{2}\,a^3c^3d^4e^2+3\sqrt{2}\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*c*d^2*e^4 + a*e^6)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^3*d^7 + 3*
a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*sqrt(d)) + 1/8*(3*(a*c^3)^(1/4
)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 2*(a*c^3)^(3/4)*c*d^3*e - 7*(a
*c^3)^(1/4)*a^2*c*e^4 - 18*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sqrt(2)*(2*x +
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^6 + 3*sqrt(2)*a^3*c^3
*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6) + 1/8*(3*(a*c^3)^
(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 2*(a*c^3)^(3/4)*c*d^3*e -
7*(a*c^3)^(1/4)*a^2*c*e^4 - 18*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^6 + 3*sqrt(2)*a^3
*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6) + 1/32*(3*sqr
t(2)*(a*c^3)^(1/4)*c^3*d^4 + 12*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 2*sqr
t(2)*(a*c^3)^(3/4)*c*d^3*e - 7*sqrt(2)*(a*c^3)^(1/4)*a^2*c*e^4 + 18*sqrt(2)
*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
^4*d^6 + 3*a^3*c^3*d^4*e^2 + 3*a^4*c^2*d^2*e^4 + a^5*c*e^6) - 1/32*(3*sqrt(
2)*(a*c^3)^(1/4)*c^3*d^4 + 12*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 2*sqrt(
2)*(a*c^3)^(3/4)*c*d^3*e - 7*sqrt(2)*(a*c^3)^(1/4)*a^2*c*e^4 + 18*sqrt(2)*(
a*c^3)^(3/4)*a*d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4
*d^6 + 3*a^3*c^3*d^4*e^2 + 3*a^4*c^2*d^2*e^4 + a^5*c*e^6) - 1/4*(2*c^2*d^2*
x^5*e^2 + c^2*d^3*x^3*e - 2*a*c*x^5*e^4 - c^2*d^4*x + a*c*d*x^3*e^3 + a*c*d
^2*x*e^2 - 2*a^2*x*e^4)/((a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4)*(c*x^6*e
 + c*d*x^4 + a*x^2*e + a*d))
```

maple [A] time = 0.02, size = 1169, normalized size = 1.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a)^2,x)
```

```
[Out] 1/2*e^6/(a*e^2+c*d^2)^3/d*x/(e*x^2+d)*a+1/2*e^4/(a*e^2+c*d^2)^3*d*x/(e*x^2+
d)*c+1/2*e^6/(a*e^2+c*d^2)^3/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+9/2*
e^4/(a*e^2+c*d^2)^3*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c-1/2*c^2/(a*e^
```

```
2+c*d^2)^3/(c*x^4+a)*d*e^3*x^3-1/2*c^3/(a*e^2+c*d^2)^3/(c*x^4+a)*d^3*e/a*x^
3-1/4*c/(a*e^2+c*d^2)^3/(c*x^4+a)*a*x*e^4+1/4*c^3/(a*e^2+c*d^2)^3/(c*x^4+a)
/a*x*d^4-7/16*c/(a*e^2+c*d^2)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1
/4)*x+1)*e^4+3/4*c^2/(a*e^2+c*d^2)^3/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/c)^(1/4)*x+1)*d^2*e^2+3/16*c^3/(a*e^2+c*d^2)^3/a^2*(a/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^4-7/16*c/(a*e^2+c*d^2)^3*(a/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^4+3/4*c^2/(a*e^2+c*d^2)^3/a*(a/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e^2+3/16*c^3/(a*e^2+c*d^2)^3/a
^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^4-7/32*c/(a*e^2+c*
d^2)^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-
(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^4+3/8*c^2/(a*e^2+c*d^2)^3/a*(a/c)^(1/
4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2)))*d^2*e^2+3/32*c^3/(a*e^2+c*d^2)^3/a^2*(a/c)^(1/4)*2^(1/2
)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/
c)^(1/2)))*d^4-9/16*c/(a*e^2+c*d^2)^3/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/
4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^3-1/
16*c^2/(a*e^2+c*d^2)^3/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+
(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3*e-9/8*c/(a*e^2+c*
d^2)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^3-1/8*c^2/(a
*e^2+c*d^2)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e-9
/8*c/(a*e^2+c*d^2)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*
e^3-1/8*c^2/(a*e^2+c*d^2)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)*d^3*e
```

maxima [A] time = 2.61, size = 732, normalized size = 0.85

$$c\left(\frac{2\,\sqrt{2}\left(3\,c^{\frac{5}{2}}d^4-2\,\sqrt{a}\,c^2d^3e+12\,ac^{\frac{3}{2}}d^2e^2-18\,a^{\frac{3}{2}}cde^3-7\,a^2\,\sqrt{c}\,e^4\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\left(3\,c^{\frac{5}{2}}d^4-2\,\sqrt{a}\,c^2d^3e+12\,ac^{\frac{3}{2}}d^2e^2-18\,a^{\frac{3}{2}}cde^3-7\right.}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/32*c*(2*sqrt(2)*(3*c^(5/2)*d^4 - 2*sqrt(a)*c^2*d^3*e + 12*a*c^(3/2)*d^2*e
^2 - 18*a^(3/2)*c*d*e^3 - 7*a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*
x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*s
qrt(c))*sqrt(c)) + 2*sqrt(2)*(3*c^(5/2)*d^4 - 2*sqrt(a)*c^2*d^3*e + 12*a*c^
(3/2)*d^2*e^2 - 18*a^(3/2)*c*d*e^3 - 7*a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*
(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*c^(5/2)*d^4 + 2*sqrt(a)*c^2*d^3*e
+ 12*a*c^(3/2)*d^2*e^2 + 18*a^(3/2)*c*d*e^3 - 7*a^2*sqrt(c)*e^4)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3
*c^(5/2)*d^4 + 2*sqrt(a)*c^2*d^3*e + 12*a*c^(3/2)*d^2*e^2 + 18*a^(3/2)*c*d*
e^3 - 7*a^2*sqrt(c)*e^4)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt
(a))/(a^(3/4)*c^(3/4)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 +
a^4*e^6) + 1/2*(9*c*d^2*e^4 + a*e^6)*arctan(e*x/sqrt(d*e))/((c^3*d^7 + 3*a*
c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*sqrt(d*e)) - 1/4*(2*(c^2*d^2*e^2
 - a*c*e^4)*x^5 + (c^2*d^3*e + a*c*d*e^3)*x^3 - (c^2*d^4 - a*c*d^2*e^2 + 2*
a^2*e^4)*x)/(a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*e^4 + (a*c^3*d^5*e + 2
*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^6 + (a*c^3*d^6 + 2*a^2*c^2*d^4*e^2 + a^3*
c*d^2*e^4)*x^4 + (a^2*c^2*d^5*e + 2*a^3*c*d^3*e^3 + a^4*d*e^5)*x^2)
```

mupad [B] time = 8.33, size = 28923, normalized size = 33.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x^2)^2),x)
```

```
[Out] ((x*(2*a^2*e^4 + c^2*d^4 - a*c*d^2*e^2))/(4*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c*
d^2*e^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)) + (c*e^2*x^5*(a*e^2 - c*d^2))/(
2*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a*d + a*e*x^2 + c*d*x^4 + c*e*
x^6) + atan(((((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*
d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5
*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254
784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^
2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*
a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*
d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 - 212992*a^
5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10
960896*a^8*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9
*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5554
176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12
*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 +
56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c
^2*d^6*e^12)) - (x*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1
/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*
c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c
^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*
d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^1
0*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589
824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e
^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^
12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 -
 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*
e^25))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d
^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 +
 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/
2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 15
6*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2)
 - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2)
)/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 +
 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) -
(x*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4
 - 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^1
3*e^10 + 2219776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^
9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*
(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28
*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*
d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d
^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^
5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c
*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13
*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*
d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2))*(-(49*a^4*e^8
*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7
*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(
-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(
-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8
*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*
e^8)))^(1/2) - (x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e
^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6
*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18
+ a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*
e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*
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a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^
(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^
6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7
*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^
6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a
^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*1i - (((3584*a^10*c^5*e^21 +
 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 +
 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8
*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6
*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c
^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e
^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^2
4 - 24576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^
19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736
*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e
^16 + 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*
c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^
5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^1
0*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) + (x*(-(49*a^4*e^8*(-a
^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2
*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7
*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7
*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5
*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)
))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*
c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2
752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*
d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*
a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^2
*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a
^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d
^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12
*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*
e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2
) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6
*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^
6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*
c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*
a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12
+ 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5
*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^
11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*
e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(
-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7
*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c
^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c
^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*
e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a
^11*c^2*d^4*e^8)))^(1/2))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*
c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 4
04*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*
(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a
^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 +
 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x*(81*c^13*d^14*e^5
- 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636
*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575
*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14
+ 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c
^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(
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-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*1i)/((((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (x*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^

3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404
*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-
a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7
*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 2
0*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (((3584*a^10*c^5*e^21 +
 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 +
 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8
*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6
*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c
^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e
^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^2
4 - 24576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^
19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736
*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e
^16 + 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*
c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^
5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^1
0*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) + (x*(-(49*a^4*e^8*(-a
^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2
*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7
*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7
c^3)^(1/2))/(256(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5
*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)
))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*
c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2
752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*
d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*
a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^2
*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a
^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d
^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12
*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*
e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2
) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6
*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^
6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*
c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*
a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12
+ 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5
*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^
11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*
e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(
-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7
*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c
^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c
^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*
e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a
^11*c^2*d^4*e^8)))^(1/2))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*
c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 4
04*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*
(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a
^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 +
 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x*(81*c^13*d^14*e^5
- 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636
*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575
*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14
+ 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c
^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(
-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c

^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a
^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a
^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c
^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^
8)))^(1/2) - (729*c^11*d^9*e^8 + 2916*a*c^10*d^7*e^10 + 2009*a^4*c^7*d*e^16
 - 2538*a^2*c^9*d^5*e^12 + 17764*a^3*c^8*d^3*e^14)/(256*(a^4*c^8*d^18 + a^1
2*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 +
 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*
c^2*d^6*e^12))))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2
) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^
3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3
)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^
12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*
c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*2i + atan(((((3584*a^10*c^5*e^21
 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7
 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d
^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c
^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5
*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10
*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e
^24 - 24576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*
d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 254607
36*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9
*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^1
4*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*
a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d
^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (x*((49*a^4*e^8*(-
a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^
2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^
7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^
7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^
5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8
)))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8
*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 +
2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8
*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824
*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^
2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*
a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*
d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12
*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*
e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2
) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6
*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^
6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*
c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*
a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12
+ 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5
*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^
11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*
e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(
(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*
e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^
3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^
2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e
^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^
11*c^2*d^4*e^8)))^(1/2))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^
3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404
*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-

a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*1i - (((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) + (x*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7

c^3)^(1/2))/(256(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5
*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)
))^(1/2) + (x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 +
 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^1
3 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^
12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4
+ 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10
*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2)
 + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3
*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)
^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^1
2 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c
^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*1i)/((((3584*a^10*c^5*e^21 + 1152
*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 2968
32*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13
 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*
e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^
16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 +
56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 2
4576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^
6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736*a^9*
c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 +
 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d
^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7
*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8
 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (x*((49*a^4*e^8*(-a^7*c^3
)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7
 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^
(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^
(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*
e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/
2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d
^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512
*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e
^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c
^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16
+ 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5
*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^1
2)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^
5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 6
8*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*
a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c
*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 +
 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^
21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^1
2*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12 + 31554
56*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 -
 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^
4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 7
0*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4
e^8(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252
*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e
^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e
^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6
*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*
d^4*e^8)))^(1/2))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2
) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^
3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3
)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^

12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*
c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(81*c^13*d^14*e^5 - 392*a^7
*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10
*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*
d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c
^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e
^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^
(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 +
 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1
/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1
/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^
2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)
 + (((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 +
 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10
*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^
7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8
*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^
12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12))
 - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^2
1*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8
*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14
 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c
^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16
+ 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5
*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^1
2)) + (x*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^
4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5
 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) +
 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^
12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e
^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^1
4*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 589824
0*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12
*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^
15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128
*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 2
8*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3
*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d
^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^
5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c
*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13
*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*
d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x*(4096*a^1
2*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4
*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 221
9776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^
16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^1
8 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^1
4*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 2
8*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)
^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a
^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^
7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c
^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*
a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2))*((49*a^4*e^8*(-a^7*c^3)^(1
/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 1
56*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2
) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2
))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2

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+ 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) +
 (x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2
*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a
^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^1
6 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c
^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e
^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*
c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 +
 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 3
0*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12
*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6
 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (729*c^11*d^9*e^8 + 2916*a*c^10*d^7*e^10
+ 2009*a^4*c^7*d*e^16 - 2538*a^2*c^9*d^5*e^12 + 17764*a^3*c^8*d^3*e^14)/(25
6*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 +
28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^
3*d^8*e^10 + 28*a^10*c^2*d^6*e^12))))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4
*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*
d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3
*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^
13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^
4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*2i + (atan((
(((x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^
2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*
a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^
16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*
c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*
e^12)) - (((7*a^10*c^5*e^21 + (9*a*c^14*d^18*e^3)/4 + (103*a^2*c^13*d^16*e^
5)/4 + (429*a^3*c^12*d^14*e^7)/4 + (2319*a^4*c^11*d^12*e^9)/4 + (10019*a^5*
c^10*d^10*e^11)/4 + (6009*a^6*c^9*d^8*e^13)/4 - (11105*a^7*c^8*d^6*e^15)/4
- (9803*a^8*c^7*d^4*e^17)/4 - 174*a^9*c^6*d^2*e^19)/(a^4*c^8*d^18 + a^12*d^
2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*
a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*
d^6*e^12) + ((a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*((x*(4096*a^12*c^5*d*e^22 -
 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6
 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*
d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^
10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e
^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7
*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6
*e^12)) - (((128*a^15*c^4*d*e^24 - 48*a^4*c^15*d^23*e^2 - 416*a^5*c^14*d^21
*e^4 - 688*a^6*c^13*d^19*e^6 + 3840*a^7*c^12*d^17*e^8 + 21408*a^8*c^11*d^15
*e^10 + 49728*a^9*c^10*d^13*e^12 + 67872*a^10*c^9*d^11*e^14 + 58752*a^11*c^
8*d^9*e^16 + 32400*a^12*c^7*d^7*e^18 + 10848*a^13*c^6*d^5*e^20 + 1936*a^14*
c^5*d^3*e^22)/(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7
*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8
 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12) - (x*(a*e^2 + 9*c*d^2)*(-d^3
*e^7)^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a
^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11
+ 2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c
^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 5898
24*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(512*(c^3*d^9 + a^3*d^3*e^
6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^
11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*
e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(
a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*
e^2 + 3*a^2*c*d^5*e^4))))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a
^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e
^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2
)*1i)/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)) + (((
```

x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c
^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5
*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16
+ 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5
*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^1
2)) + (((7*a^10*c^5*e^21 + (9*a*c^14*d^18*e^3)/4 + (103*a^2*c^13*d^16*e^5)/
4 + (429*a^3*c^12*d^14*e^7)/4 + (2319*a^4*c^11*d^12*e^9)/4 + (10019*a^5*c^1
0*d^10*e^11)/4 + (6009*a^6*c^9*d^8*e^13)/4 - (11105*a^7*c^8*d^6*e^15)/4 - (
9803*a^8*c^7*d^4*e^17)/4 - 174*a^9*c^6*d^2*e^19)/(a^4*c^8*d^18 + a^12*d^2*e
^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7
*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6
*e^12) - ((a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*((x*(4096*a^12*c^5*d*e^22 - 11
52*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 -
140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^1
1*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*
c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16
 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^
5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^
12)) + (((128*a^15*c^4*d*e^24 - 48*a^4*c^15*d^23*e^2 - 416*a^5*c^14*d^21*e^
4 - 688*a^6*c^13*d^19*e^6 + 3840*a^7*c^12*d^17*e^8 + 21408*a^8*c^11*d^15*e^
10 + 49728*a^9*c^10*d^13*e^12 + 67872*a^10*c^9*d^11*e^14 + 58752*a^11*c^8*d
^9*e^16 + 32400*a^12*c^7*d^7*e^18 + 10848*a^13*c^6*d^5*e^20 + 1936*a^14*c^5
*d^3*e^22)/(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^
16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 +
56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12) + (x*(a*e^2 + 9*c*d^2)*(-d^3*e^
7)^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*
c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2
752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*
d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*
a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(512*(c^3*d^9 + a^3*d^3*e^6 +
 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*
c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6
 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(a*e
^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2
 + 3*a^2*c*d^5*e^4))))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*
c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6
+ 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*1
i)/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))/(((729*
c^11*d^9*e^8)/256 + (729*a*c^10*d^7*e^10)/64 + (2009*a^4*c^7*d*e^16)/256 -
(1269*a^2*c^9*d^5*e^12)/128 + (4441*a^3*c^8*d^3*e^14)/64)/(a^4*c^8*d^18 + a
^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4
 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^1
0*c^2*d^6*e^12) + (((x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d
^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^
9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*
d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*
d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10
+ 28*a^10*c^2*d^6*e^12)) - (((7*a^10*c^5*e^21 + (9*a*c^14*d^18*e^3)/4 + (10
3*a^2*c^13*d^16*e^5)/4 + (429*a^3*c^12*d^14*e^7)/4 + (2319*a^4*c^11*d^12*e^
9)/4 + (10019*a^5*c^10*d^10*e^11)/4 + (6009*a^6*c^9*d^8*e^13)/4 - (11105*a^
7*c^8*d^6*e^15)/4 - (9803*a^8*c^7*d^4*e^17)/4 - 174*a^9*c^6*d^2*e^19)/(a^4*
c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*
c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e
^10 + 28*a^10*c^2*d^6*e^12) + ((a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*((x*(4096
*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336
*a^4*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 +
 2219776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^
7*e^16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8
*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6

*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((128*a^15*c^4*d*e^24 - 48*a^4*c^15*d^23*e^2 - 416*a^5*c^14*d^21*e^4 - 688*a^6*c^13*d^19*e^6 + 3840*a^7*c^12*d^17*e^8 + 21408*a^8*c^11*d^15*e^10 + 49728*a^9*c^10*d^13*e^12 + 67872*a^10*c^9*d^11*e^14 + 58752*a^11*c^8*d^9*e^16 + 32400*a^12*c^7*d^7*e^18 + 10848*a^13*c^6*d^5*e^20 + 1936*a^14*c^5*d^3*e^22)/(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12) - (x*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(512*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4))))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)) - (((x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) + (((7*a^10*c^5*e^21 + (9*a*c^14*d^18*e^3)/4 + (103*a^2*c^13*d^16*e^5)/4 + (429*a^3*c^12*d^14*e^7)/4 + (2319*a^4*c^11*d^12*e^9)/4 + (10019*a^5*c^10*d^10*e^11)/4 + (6009*a^6*c^9*d^8*e^13)/4 - (11105*a^7*c^8*d^6*e^15)/4 - (9803*a^8*c^7*d^4*e^17)/4 - 174*a^9*c^6*d^2*e^19)/(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12) - ((a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*((x*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) + (((128*a^15*c^4*d*e^24 - 48*a^4*c^15*d^23*e^2 - 416*a^5*c^14*d^21*e^4 - 688*a^6*c^13*d^19*e^6 + 3840*a^7*c^12*d^17*e^8 + 21408*a^8*c^11*d^15*e^10 + 49728*a^9*c^10*d^13*e^12 + 67872*a^10*c^9*d^11*e^14 + 58752*a^11*c^8*d^9*e^16 + 32400*a^12*c^7*d^7*e^18 + 10848*a^13*c^6*d^5*e^20 + 1936*a^14*c^5*d^3*e^22)/(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12) + (x*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(512*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4))))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^

```
4))))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*1i)/(2*(c^3*d^9 + a^3*d^3*e^6 + 3*
a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.150 $\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=388

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(-252a^{3/2}\sqrt{c}\,de^3+25a^2e^4+420\sqrt{a}\,c^{3/2}d^3e-210acd^2e^2+105c^2d^4\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{210\sqrt[4]{a}\,c^{9/4}\sqrt{a+cx^4}}$$

```
[Out] 1/21*e^2*(-5*a*e^2+42*c*d^2)*x*(c*x^4+a)^(1/2)/c^2+4/5*d*e^3*x^3*(c*x^4+a)^
(1/2)/c+1/7*e^4*x^5*(c*x^4+a)^(1/2)/c+4/5*d*e*(-3*a*e^2+5*c*d^2)*x*(c*x^4+a
)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-4/5*a^(1/4)*d*e*(-3*a*e^2+5*c*d^2)*(c
os(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*E
llipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2)
)*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)+1/210*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(105*c^2*d^4-210*a*
c*d^2*e^2+25*a^2*e^4+420*c^(3/2)*d^3*e*a^(1/2)-252*a^(3/2)*d*e^3*c^(1/2))*(
a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(9
/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 386, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1207, 1888, 1198, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(5\left(5a^2e^4-42acd^2e^2+21c^2d^4\right)+84\sqrt{a}\,\sqrt{c}\,de\left(5cd^2-3ae^2\right)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{210\sqrt[4]{a}\,c^{9/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*(42*c*d^2 - 5*a*e^2)*x*Sqrt[a + c*x^4])/(21*c^2) + (4*d*e^3*x^3*Sqrt[a
 + c*x^4])/(5*c) + (e^4*x^5*Sqrt[a + c*x^4])/(7*c) + (4*d*e*(5*c*d^2 - 3*a*
e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (4*a^(1/4)*d*
e*(5*c*d^2 - 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sq
rt[a + c*x^4]) + ((84*Sqrt[a]*Sqrt[c]*d*e*(5*c*d^2 - 3*a*e^2) + 5*(21*c^2*d
^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
210*a^(1/4)*c^(9/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^4}{\sqrt{a+cx^4}}\,dx = \frac{e^4x^5\sqrt{a+cx^4}}{7c}+\frac{\int \frac{7cd^4+28cd^3ex^2+e^2\left(42cd^2-5ae^2\right)x^4+28cde^3x^6}{\sqrt{a+cx^4}}\,dx}{7c}$$

$$= \frac{4de^3x^3\sqrt{a+cx^4}}{5c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}+\frac{\int \frac{35c^2d^4+28cde\left(5cd^2-3ae^2\right)x^2+5ce^2\left(42cd^2-5ae^2\right)x^4}{\sqrt{a+cx^4}}\,dx}{35c^2}$$

$$= \frac{e^2\left(42cd^2-5ae^2\right)x\sqrt{a+cx^4}}{21c^2}+\frac{4de^3x^3\sqrt{a+cx^4}}{5c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}+\frac{\int \frac{5c\left(21c^2d^4-42acd^2e^2+5a^2e^4\right)+}{\sqrt{a+cx^4}}}{105c^3}$$

$$= \frac{e^2\left(42cd^2-5ae^2\right)x\sqrt{a+cx^4}}{21c^2}+\frac{4de^3x^3\sqrt{a+cx^4}}{5c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}-\frac{\left(4\sqrt{a}\,de\left(5cd^2-3ae^2\right)\right)\int}{5c^{3/2}}$$

$$= \frac{e^2\left(42cd^2-5ae^2\right)x\sqrt{a+cx^4}}{21c^2}+\frac{4de^3x^3\sqrt{a+cx^4}}{5c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}+\frac{4de\left(5cd^2-3ae^2\right)x\sqrt{a+}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Mathematica [C] time = 0.21, size = 203, normalized size = 0.52

$$\frac{ex\left(-25a^2e^3+28cdx^2\sqrt{\frac{cx^4}{a}+1}\left(5cd^2-3ae^2\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)+2ace\left(105d^2+42dex^2-5e^2x^4\right)+3c^2ex^4\left(70d^2\right.\right.}{105c^2\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

```
[Out] (5*(21*c^2*d^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + e*x*(-25*a^2*e^3 + 2*a*c*e*(105*d^
2 + 42*d*e*x^2 - 5*e^2*x^4) + 3*c^2*e*x^4*(70*d^2 + 28*d*e*x^2 + 5*e^2*x^4)
 + 28*c*d*(5*c*d^2 - 3*a*e^2)*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2
, 3/4, 7/4, -((c*x^4)/a)]))/(105*c^2*Sqrt[a + c*x^4])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^4x^8 + 4\,de^3x^6 + 6\,d^2e^2x^4 + 4\,d^3ex^2 + d^4}{\sqrt{cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^4*x^8 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4 + 4*d^3*e*x^2 + d^4)/sqrt(c
*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^4}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)
```

maple [C] time = 0.02, size = 506, normalized size = 1.30

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,d^4\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x, i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{4i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x, i\right)+\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*x^4+a)^(1/2),x)
```

```
[Out] e^4*(1/7/c*x^5*(c*x^4+a)^(1/2)-5/21*a/c^2*x*(c*x^4+a)^(1/2)+5/21*a^2/c^2/(I
/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)
*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I))+4*d
*e^3*(1/5/c*x^3*(c*x^4+a)^(1/2)-3/5*I*a^(3/2)/c^(3/2)/(I/a^(1/2)*c^(1/2))^(
1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^
4+a)^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c
^(1/2))^(1/2)*x,I)))+6*d^2*e^2*(1/3*(c*x^4+a)^(1/2)/c*x-1/3*a/c/(I/a^(1/2)*
c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(
1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I))+4*I*d^3*e*a^
(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)
*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2)
)^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))+d^4/(I/a^(1/2)*c^(1/
2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/
(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^4}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^4}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^4/(a + c*x^4)^(1/2), x)
```

sympy [C] time = 6.17, size = 214, normalized size = 0.55

$$\frac{d^4x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{d^3ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{3d^2e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{de^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*x**4+a)**(1/2),x)
```

```
[Out] d**4*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + d**3*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4
*exp_polar(I*pi)/a)/(sqrt(a)*gamma(7/4)) + 3*d**2*e**2*x**5*gamma(5/4)*hype
r((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + d*
e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(
sqrt(a)*gamma(11/4)) + e**4*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), c*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4))
```

3.151 $\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=326

$$\frac{3ex\sqrt{a+cx^4}\left(5cd^2-ae^2\right)}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{3\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(5cd^2-ae^2\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\cdots\right.}{}$$

```
[Out] d*e^2*x*(c*x^4+a)^(1/2)/c+1/5*e^3*x^3*(c*x^4+a)^(1/2)/c+3/5*e*(-a*e^2+5*c*d
^2)*x*(c*x^4+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-3/5*a^(1/4)*e*(-a*e^2+5
*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a
^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x
^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/
2)+1/10*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*c^(1/2))*(15*c*d^2*e-3*a*e^3+5*d*(-a*e^2+c*d^2)*c^(1/2)/a^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1207, 1888, 1198, 220, 1196}

$$\frac{3ex\sqrt{a+cx^4}\left(5cd^2-ae^2\right)}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{5\sqrt{c}\,d\left(cd^2-ae^2\right)}{\sqrt{a}}-3ae^3+15cd^2e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\cdots\right.}{10c^{7/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (d*e^2*x*Sqrt[a + c*x^4])/c + (e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (3*e*(5*c*d
^2 - a*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (3*a^(
1/4)*e*(5*c*d^2 - a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)
*Sqrt[a + c*x^4]) + (a^(1/4)*(15*c*d^2*e - 3*a*e^3 + (5*Sqrt[c]*d*(c*d^2 -
a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a
 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+cx^4}}\,dx = \frac{e^3x^3\sqrt{a+cx^4}}{5c} + \frac{\int \frac{5cd^3+3e\left(5cd^2-ae^2\right)x^2+15cde^2x^4}{\sqrt{a+cx^4}}\,dx}{5c}$$

$$= \frac{de^2x\sqrt{a+cx^4}}{c} + \frac{e^3x^3\sqrt{a+cx^4}}{5c} + \frac{\int \frac{15cd\left(cd^2-ae^2\right)+9ce\left(5cd^2-ae^2\right)x^2}{\sqrt{a+cx^4}}\,dx}{15c^2}$$

$$= \frac{de^2x\sqrt{a+cx^4}}{c} + \frac{e^3x^3\sqrt{a+cx^4}}{5c} - \frac{\left(3\sqrt{a}\,e\left(5cd^2-ae^2\right)\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{5c^{3/2}} + \frac{\left(5\sqrt{c}\,d\left(cd^2-ae^2\right)+3\right.}{}$$

$$= \frac{de^2x\sqrt{a+cx^4}}{c} + \frac{e^3x^3\sqrt{a+cx^4}}{5c} + \frac{3e\left(5cd^2-ae^2\right)x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{3\sqrt[4]{a}\,e\left(5cd^2-ae^2\right)\left(\sqrt{a}+\sqrt{c}\right.}{5c^{7}}$$

Mathematica [C] time = 0.14, size = 140, normalized size = 0.43

$$\frac{5dx\sqrt{\frac{cx^4}{a}+1}\left(cd^2-ae^2\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex\left(x^2\sqrt{\frac{cx^4}{a}+1}\left(5cd^2-ae^2\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)+e\left(a+cx^4\right)\left(5d+\right.\right.}{5c\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (5*d*(c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
 -((c*x^4)/a)] + e*x*(e*(5*d + e*x^2)*(a + c*x^4) + (5*c*d^2 - a*e^2)*x^2*S
qrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(5*c*Sq
rt[a + c*x^4])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

maple [C] time = 0.01, size = 388, normalized size = 1.19

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,d^3\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{3i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\Big(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)+}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] e^3*(1/5*(c*x^4+a)^(1/2)/c*x^3-3/5*I*a^(3/2)/c^(3/2)/(I/a^(1/2)*c^(1/2))^(1
/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4
+a)^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^
(1/2))^(1/2)*x,I)))+3*d*e^2*(1/3*(c*x^4+a)^(1/2)/c*x-1/3*a/c/(I/a^(1/2)*c^(
1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2
)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I))+3*I*d^2*e*a^(1/
2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^
(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(
1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))+d^3/(I/a^(1/2)*c^(1/2))
^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*
x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^3}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^3/(a + c*x^4)^(1/2), x)
```

sympy [C] time = 4.73, size = 173, normalized size = 0.53

$$\frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)*hyp
er((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e
**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(4
*sqrt(a)*gamma(11/4))
```

3.152 $\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=264

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(6\sqrt{a}\,\sqrt{c}\,de-ae^2+3cd^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}\,c^{5/4}\sqrt{a+cx^4}}-\frac{2\sqrt[4]{a}\,de\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{c^{3/4}\sqrt{a+cx}}$$

```
[Out] 1/3*e^2*x*(c*x^4+a)^(1/2)/c+2*d*e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^
(1/2))-2*a^(1/4)*d*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/
(c*x^4+a)^(1/2)+1/6*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*(3*c*d^2-a*e^2+6*d*e*a^(1/2)*c^(1/2))*((c*x^4+a)/(a
^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(5/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1207, 1198, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(6\sqrt{a}\,\sqrt{c}\,de-ae^2+3cd^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}\,c^{5/4}\sqrt{a+cx^4}}-\frac{2\sqrt[4]{a}\,de\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{c^{3/4}\sqrt{a+cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(3*c) + (2*d*e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a]
 + Sqrt[c]*x^2)) - (2*a^(1/4)*d*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
c^(3/4)*Sqrt[a + c*x^4]) + ((3*c*d^2 + 6*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*c^(5/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx &= \frac{e^2x\sqrt{a+cx^4}}{3c}+\frac{\int \frac{3cd^2-ae^2+6cdex^2}{\sqrt{a+cx^4}}\,dx}{3c}\\ &= \frac{e^2x\sqrt{a+cx^4}}{3c}-\frac{\left(2\sqrt{a}\,de\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}+\frac{\left(3cd^2+6\sqrt{a}\,\sqrt{c}\,de-ae^2\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3c}\\ &= \frac{e^2x\sqrt{a+cx^4}}{3c}+\frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2\sqrt[4]{a}\,de\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.09, size = 120, normalized size = 0.45

$$\frac{x\sqrt{\frac{cx^4}{a}+1}\left(3cd^2-ae^2\right)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex\left(2cdx^2\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)+e\left(a+cx^4\right)\right)}{3c\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

```
[Out] ((3*c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -
((c*x^4)/a)] + e*x*(e*(a + c*x^4) + 2*c*d*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*c*Sqrt[a + c*x^4])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^2x^4+2\,dex^2+d^2}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

maple [C] time = 0.01, size = 266, normalized size = 1.01

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,d^2\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{2i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] e^2*(1/3*(c*x^4+a)^(1/2)/c*x-1/3*a/c/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*
c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icF((I/a^(1/2)*c^(1/2))^(1/2)*x,I))+2*I*d*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/
2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+
a)^(1/2)/c^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(
1/2)*c^(1/2))^(1/2)*x,I))+d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)
*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/
a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(a + c*x^4)^(1/2), x)
```

sympy [C] time = 3.57, size = 124, normalized size = 0.47

$$\frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*ex
p_polar(I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1/2,
5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

3.153 $\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*a
rctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipti
cF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d
*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+
a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1198, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sq
rt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sq
rt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = -\frac{\left(\sqrt{a}\,e\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{ex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)}{}$$

Mathematica [C] time = 0.03, size = 77, normalized size = 0.34

$$\frac{\sqrt{\frac{cx^4}{a}+1}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
+ e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[a + c*x^4]
)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ex^2+d}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

maple [C] time = 0.00, size = 169, normalized size = 0.75

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,d\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}} + \frac{i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\mathrm{E}\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] I*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a
^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF((I/a^(1/2)*c
^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))+d/(I/a^(1/2)*c
^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1
/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{\sqrt{c\,x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(a + c*x^4)^(1/2), x)
```

sympy [C] time = 2.06, size = 78, normalized size = 0.35

$$\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_pol
ar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

3.154 $\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=334

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,de};2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}\,d\sqrt{a+cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\,\tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,\sqrt{ae^2+cd^2}}+$$

```
[Out] 1/2*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))*e^(1/2)/d
^(1/2)/(a*e^2+c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/a^(1/4)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)-1/4*a^(3/4)*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellip
ticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^
(1/2)/c^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((
c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(c*x^4+a)^
(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1217, 220, 1707}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,de};2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}\,d\sqrt{a+cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\,\tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,\sqrt{ae^2+cd^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])
/(2*Sqrt[d]*Sqrt[c*d^2 + a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4]) - (a^(3/4)*((S
qrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*
d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*Sqr
t[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
```

```
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx = \frac{\sqrt{c}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e} - \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e}$$

$$= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2+ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+cx^4}} - \ldots$$

Mathematica [C] time = 0.15, size = 95, normalized size = 0.28

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}\,\Pi\left(-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSin
h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[a +
c*x^4])
```

fricas [F] time = 11.32, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+a}}{cex^6+cdx^4+aex^2+ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

maple [C] time = 0.04, size = 107, normalized size = 0.32

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}+1}\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/d/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c
^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)*c^(1/2))^(1/2)*x,
I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{4}+a}\left(ex^{2}+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^{4}+a}\,\left(e\,x^{2}+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)),x)
```

```
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^{4}}\left(d+ex^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)), x)
```

3.155 $\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=581

$$\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}-\frac{\sqrt{c}\,ex\sqrt{a+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{a}\,\sqrt[4]{c}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\sqrt{a+cx^4}\left(ae^2+cd^2\right)}$$

```
[Out] 1/4*(a*e^2+3*c*d^2)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^
(1/2))*e^(1/2)/d^(3/2)/(a*e^2+c*d^2)^(3/2)+1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e
^2+c*d^2)/(e*x^2+d)-1/2*e*x*c^(1/2)*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(a^(1/2
)+x^2*c^(1/2))+1/2*a^(1/4)*c^(1/4)*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2
)^(1/2)/d/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x
/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan
(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)
+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)-1/8
*(a*e^2+3*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1
/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^
(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4
)/d^2/(a*e^2+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1224, 1715, 1196, 1709, 220, 1707}

$$\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}-\frac{\sqrt{c}\,ex\sqrt{a+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{e}\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\left(ae^2+cd^2\right)^{3/2}}+\frac{\sqrt[4]{a}\,\sqrt[4]{c}\,e\left(\sqrt{a}+\sqrt{\right.}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)
) + (e^2*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (Sqrt[e]*(3
*c*d^2 + a*e^2)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*
x^4])])/(4*d^(3/2)*(c*d^2 + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (c^(1/4
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]
*e)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqr
t[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqr
t[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+cx^4}}\,dx = \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{-2cd^2-ae^2+2cdex^2+ce^2x^4}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{2d\left(cd^2+ae^2\right)}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{\sqrt{a}\,c^{3/2}de^2+ce\left(-2cd^2-ae^2\right)+\left(2c^2de^2-ce^2\left(cd-\sqrt{a}\,\sqrt{c}\,e\right)\right)x^2}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{2cde\left(cd^2+ae^2\right)} + \frac{\left(\sqrt{a}\ \ldots}{\ldots}$$

$$= -\frac{\sqrt{c}\,ex\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{\sqrt[4]{a}\,\sqrt[4]{c}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\ldots}{2d\left(cd^2\ \ldots\right.}$$

$$= -\frac{\sqrt{c}\,ex\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{\sqrt{e}\left(3cd^2+ae^2\right)\tan^{-1}\ldots}{4d^{3/2}\left(cd^2+a\ldots\right.}$$

Mathematica [C] time = 0.76, size = 522, normalized size = 0.90

$$\frac{-3icd^3\sqrt{\frac{cx^4}{a}+1}\,\Pi\left(-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right) - 3icd^2ex^2\sqrt{\frac{cx^4}{a}+1}\,\Pi\left(-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right) - iae^3\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] (a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e^2*x + Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d*e^2*x^
5 - Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh
[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*d*(I*Sqrt[c]*d + Sqrt[a]*e)*(d
 + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]
*x], -1] - (3*I)*c*d^3*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqr
t[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a*d*e^2*Sqrt[1 + (
c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c
])/Sqrt[a]]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I
)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a
*e^3*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*Arc
Sinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d^2*(c
*d^2 + a*e^2)*(d + e*x^2)*Sqrt[a + c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

maple [C] time = 0.03, size = 556, normalized size = 0.96

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,a\,e^2\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{2\left(a\,e^2+c\,d^2\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,d^2}+\frac{i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{a}\,\sqrt{c}\,e\,\mathrm{Ellip}}{2\left(a\,e^2+c\,d^2\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)-1/2*c/(a*e^2+c*d^2)/(I/
a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*
x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-1/2*I
*c^(1/2)*e/(a*e^2+c*d^2)/d*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^
(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*Elliptic
F((I/a^(1/2)*c^(1/2))^(1/2)*x,I)+1/2*I*c^(1/2)*e/(a*e^2+c*d^2)/d*a^(1/2)/(I
/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)
*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I)+1/2/
(a*e^2+c*d^2)/d^2*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^
(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)
*c^(1/2))^(1/2)*x,I*a^(1/2)/c^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/
2)*c^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2
)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*Elli
pticPi((I/a^(1/2)*c^(1/2))^(1/2)*x,I*a^(1/2)/c^(1/2)/d*e,(-I/a^(1/2)*c^(1/2
))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^4+a}\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^2),x)
```

```
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^4}\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**2), x)
```

3.156 $\int \frac{1}{\left(d+ex^2\right)^3 \sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=729

$$\frac{3\sqrt{e}\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}\left(ae^2+cd^2\right)^{5/2}}-\frac{3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\left(a^2e^4+2acd^2e^2\right.}{32\sqrt[4]{a}\sqrt[4]{c}d^3\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{\right.}}$$

```
[Out] 3/16*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)
/e^(1/2)/(c*x^4+a)^(1/2))*e^(1/2)/d^(5/2)/(a*e^2+c*d^2)^(5/2)+1/4*e^2*x*(c*
x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^2)*x*(c*x^4+a
)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)-3/8*e*(a*e^2+3*c*d^2)*x*c^(1/2)*(c*x^
4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(a^(1/2)+x^2*c^(1/2))+3/8*a^(1/4)*c^(1/4)*e*
(a*e^2+3*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/d^2/(a*e^2+c*
d^2)^2/(c*x^4+a)^(1/2)-3/32*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(s
in(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c
^(1/2),1/2*2^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^3/(a*e^2+c*d^2)^2/(-e*a^(1
/2)+d*c^(1/2))/(c*x^4+a)^(1/2)+1/8*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(4*c*d^2+3*a*e^2-d*e*a^(1/2)
*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d^2/(a*e^2+c*d^
2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A] time = 1.25, antiderivative size = 729, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1224, 1697, 1715, 1196, 1709, 220, 1707}

$$\frac{3\sqrt{e}\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}\left(ae^2+cd^2\right)^{5/2}}-\frac{3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\left(a^2e^4+2acd^2e^2\right.}{32\sqrt[4]{a}\sqrt[4]{c}d^3\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (-3*Sqrt[c]*e*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2
*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d
 + e*x^2)^2) + (3*e^2*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 +
a*e^2)^2*(d + e*x^2)) + (3*Sqrt[e]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*Ar
cTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(16*d^(5/2
)*(c*d^2 + a*e^2)^(5/2)) + (3*a^(1/4)*c^(1/4)*e*(3*c*d^2 + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) +
(c^(1/4)*(4*c*d^2 - Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt
[a + c*x^4]) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*
e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(32*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d
^2 + a*e^2)^2*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1697

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] &&
 PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx = \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} - \frac{\int \frac{-4cd^2-3ae^2+4cdex^2-ce^2x^4}{\left(d+ex^2\right)^2\sqrt{a+cx^4}}\,dx}{4d\left(cd^2+ae^2\right)}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} + \frac{3e^2\left(3cd^2+ae^2\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{\int \frac{8c^2d^4+5acd^2e^2+3a^2e^4-4cde(\cdots}{\left(d+ex^2\cdots\right.}}{8d^2\left(cd\cdots\right.}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} + \frac{3e^2\left(3cd^2+ae^2\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{\int \frac{-3\sqrt{a}\,c^{3/2}de^2\left(3cd^2+ae^2\right)+ce(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{3\sqrt{c}\,e\left(3cd^2+ae^2\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} + \frac{3e^2\left(3cd^2+ae^2\right)x\sqrt{\cdots}}{8d^2\left(cd^2+ae^2\right)^2\left(d\cdots\right.}$$

$$= -\frac{3\sqrt{c}\,e\left(3cd^2+ae^2\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} + \frac{3e^2\left(3cd^2+ae^2\right)x\sqrt{\cdots}}{8d^2\left(cd^2+ae^2\right)^2\left(d\cdots\right.}$$

Mathematica [C] time = 1.10, size = 332, normalized size = 0.46

$$\frac{\frac{de^2x\left(a+cx^4\right)\left(ae^2\left(5d+3ex^2\right)+cd^2\left(11d+9ex^2\right)\right)}{\left(d+ex^2\right)^2} + \frac{\sqrt{\frac{cx^4}{a}+1}\left(i\left(\sqrt{c}\,d\left(-3ia^{3/2}e^3-9i\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+7c^{3/2}d^3\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)-3\left(a^2e^4+2acd^2e^2\cdots\right.\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\cdots}}{8d^3\sqrt{a+cx^4}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] ((d*e^2*x*(a + c*x^4)*(a*e^2*(5*d + 3*e*x^2) + c*d^2*(11*d + 9*e*x^2)))/(d
+ e*x^2)^2 + (Sqrt[1 + (c*x^4)/a]*(-3*Sqrt[a]*Sqrt[c]*d*e*(3*c*d^2 + a*e^2)
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*(Sqrt[c]*d*(7*c^
(3/2)*d^3 - (9*I)*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - (3*I)*a^(3/2)*e^3)*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - 3*(5*c^2*d^4 + 2*a*c*
d^2*e^2 + a^2*e^4)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[
(I*Sqrt[c])/Sqrt[a]]*x], -1])))/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(8*d^3*(c*d^2 +
a*e^2)^2*Sqrt[a + c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}\left(ex^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

maple [C] time = 0.03, size = 1018, normalized size = 1.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/4*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^
2)*x*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)-1/8*c/d/(a*e^2+c*d^2)^2/
(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/
2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)*a*
e^2-7/8*c^2*d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)
*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/
a^(1/2)*c^(1/2))^(1/2)*x,I)-9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a^(1
/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+
1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-3/8*I*c^(
1/2)*e^3/(a*e^2+c*d^2)^2/d^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*
c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)+9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/
(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/
2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I)+3/
8*I*c^(1/2)*e^3/(a*e^2+c*d^2)^2/d^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a
^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)
*EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I)+3/8/(a*e^2+c*d^2)^2/d^3*e^4/(I/a^
(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^
2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)*c^(1/2))^(1/2)*x,I*a^(1/2)
/c^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a^2+3/4/
(a*e^2+c*d^2)^2/d*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^
(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)
*c^(1/2))^(1/2)*x,I*a^(1/2)/c^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/
2)*c^(1/2))^(1/2))*a*c+15/8/(a*e^2+c*d^2)^2*d/(I/a^(1/2)*c^(1/2))^(1/2)*(-I
/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/
2)*EllipticPi((I/a^(1/2)*c^(1/2))^(1/2)*x,I*a^(1/2)/c^(1/2)/d*e,(-I/a^(1/2)
*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + a}\left(ex^2 + d\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^4 + a}\,\left(e\,x^2 + d\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^3),x)
```

```
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^4}\left(d+ex^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**3), x)
```

3.157 $\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=213

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{5\sqrt{c}\,d(ae^2+cd^2)}{\sqrt{a}}-3e\left(ae^2+5cd^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right)E\left(\sin^{-1}\left(\cdots\right.\right.}{5c^{7/4}\sqrt{a-cx^4}}$$

```
[Out] -d*e^2*x*(-c*x^4+a)^(1/2)/c-1/5*e^3*x^3*(-c*x^4+a)^(1/2)/c+3/5*a^(3/4)*e*(a
*e^2+5*c*d^2)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(7/4)/(-c*
x^4+a)^(1/2)+1/5*a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-3*e*(a*e^2+5*c*d^
2)+5*d*(a*e^2+c*d^2)*c^(1/2)/a^(1/2))*(1-c*x^4/a)^(1/2)/c^(7/4)/(-c*x^4+a)^
(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1207, 1888, 1201, 224, 221, 1200, 1199, 424}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{5\sqrt{c}\,d(ae^2+cd^2)}{\sqrt{a}}-3e\left(ae^2+5cd^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right)E\left(\sin^{-1}\left(\cdots\right.\right.}{5c^{7/4}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

```
[Out] -((d*e^2*x*Sqrt[a - c*x^4])/c) - (e^3*x^3*Sqrt[a - c*x^4])/(5*c) + (3*a^(3/
4)*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(
1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((5*Sqrt[c]*d*(c*d^2 + a
*e^2))/Sqrt[a] - 3*e*(5*c*d^2 + a*e^2))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a-cx^4}}\,dx = -\frac{e^3x^3\sqrt{a-cx^4}}{5c} - \frac{\int \frac{-5cd^3-3e\left(5cd^2+ae^2\right)x^2-15cde^2x^4}{\sqrt{a-cx^4}}\,dx}{5c}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\int \frac{15cd\left(cd^2+ae^2\right)+9ce\left(5cd^2+ae^2\right)x^2}{\sqrt{a-cx^4}}\,dx}{15c^2}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\left(3\sqrt{a}\,e\left(5cd^2+ae^2\right)\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{5c^{3/2}} + \frac{\left(5\sqrt{c}\,d\left(cd^2+ae^2\right)-\right.}{}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\left(3\sqrt{a}\,e\left(5cd^2+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{5c^{3/2}\sqrt{a-cx^4}} + \frac{\left(\left(5\sqrt{c}\,d\left(c\right.\right.\right.}{}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\sqrt[4]{a}\left(5\sqrt{c}\,d\left(cd^2+ae^2\right)-3\sqrt{a}\,e\left(5cd^2+ae^2\right)\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\right.}{5c^{7/4}\sqrt{a-cx^4}}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{3a^{3/4}e\left(5cd^2+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(\right.}{}$$

Mathematica [C] time = 0.16, size = 141, normalized size = 0.66

$$\frac{5dx\sqrt{1-\frac{cx^4}{a}}\left(ae^2+cd^2\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex\left(x^2\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)+e\left(cx^4-a\right)\left(5d+\right.\right.}{5c\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

```
[Out] (5*d*(c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
 (c*x^4)/a] + e*x*(e*(5*d + e*x^2)*(-a + c*x^4) + (5*c*d^2 + a*e^2)*x^2*Sqr
t[1 - (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(5*c*Sqrt[a
- c*x^4])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3\right)\sqrt{-cx^4+a}}{cx^4-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(-c*x^4 + a)/(c*x
^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)^3}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

maple [B] time = 0.03, size = 360, normalized size = 1.69

$$\frac{\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,d^3\,\mathrm{EllipticF}\left(\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{3\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\mathrm{El}\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(-c*x^4+a)^(1/2),x)
```

```
[Out] e^3*(-1/5/c*x^3*(-c*x^4+a)^(1/2)-3/5*a^(3/2)/c^(3/2)/(1/a^(1/2)*c^(1/2))^(1
/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4
+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*
c^(1/2))^(1/2),I)))+3*d*e^2*(-1/3/c*x*(-c*x^4+a)^(1/2)+1/3*a/c/(1/a^(1/2)*c
^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I))-3*d^2*e*a^(1/
2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))
^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))+d^3/(1/a^(1/2)*c^(1/2))
^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*
x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^3}{\sqrt{a-c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^3/(a - c*x^4)^(1/2), x)
```

sympy [A] time = 4.89, size = 180, normalized size = 0.85

$$\frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

```
[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*
x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)
*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4
)) + e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*p
i)/a)/(4*sqrt(a)*gamma(11/4))
```

3.158 $\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=162

$$\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}\,de+ae^2+3cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{5/4}\sqrt{a-cx^4}}-\frac{e^2x\sqrt{a}}{3c}$$

```
[Out] -1/3*e^2*x*(-c*x^4+a)^(1/2)/c+2*a^(3/4)*d*e*EllipticE(c^(1/4)*x/a^(1/4),I)*
(1-c*x^4/a)^(1/2)/c^(3/4)/(-c*x^4+a)^(1/2)+1/3*a^(1/4)*EllipticF(c^(1/4)*x/
a^(1/4),I)*(3*c*d^2+a*e^2-6*d*e*a^(1/2)*c^(1/2))*(1-c*x^4/a)^(1/2)/c^(5/4)/
(-c*x^4+a)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1207, 1201, 224, 221, 1200, 1199, 424}

$$\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}\,de+ae^2+3cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{5/4}\sqrt{a-cx^4}}-\frac{e^2x\sqrt{a}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(3*c) + (2*a^(3/4)*d*e*Sqrt[1 - (c*x^4)/a]*Ellipti
cE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(
3*c*d^2 - 6*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcS
in[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(5/4)*Sqrt[a - c*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
```

```
a, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{3c}-\frac{\int \frac{-3cd^2-ae^2-6cdex^2}{\sqrt{a-cx^4}}\,dx}{3c}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c}+\frac{\left(2\sqrt{a}\,de\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{\sqrt{c}}-\frac{\left(-3cd^2+6\sqrt{a}\sqrt{c}\,de-ae^2\right)\int \frac{1}{\sqrt{a-cx^4}}\,dx}{3c}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c}+\frac{\left(2\sqrt{a}\,de\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{c}\sqrt{a-cx^4}}-\frac{\left(\left(-3cd^2+6\sqrt{a}\sqrt{c}\,de-ae^2\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \cdots}{3c\sqrt{a-cx^4}\cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c}+\frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\sqrt{c}\,de+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{5/4}\sqrt{a-cx^4}}+\frac{\left(2\sqrt{a}\,de\sqrt{1-\cdots}\right.}{\sqrt{c}\cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c}+\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\sqrt{c}\,de+ae^2\right)\sqrt{1-\cdots}}{3c^{5/4}\sqrt{a-cx\cdots}}$$

Mathematica [C] time = 0.10, size = 121, normalized size = 0.75

$$\frac{x\sqrt{1-\frac{cx^4}{a}}\left(ae^2+3cd^2\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex\left(2cdx^2\sqrt{1-\frac{cx^4}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)-ae+cex^4\right)}{3c\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

```
[Out] ((3*c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (
c*x^4)/a] + e*x*(-(a*e) + c*e*x^4 + 2*c*d*x^2*Sqrt[1 - (c*x^4)/a]*Hypergeom
etric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*c*Sqrt[a - c*x^4])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(e^2x^4+2\,dex^2+d^2\right)\sqrt{-cx^4+a}}{cx^4-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-c*x^4 + a)/(c*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

maple [A] time = 0.01, size = 246, normalized size = 1.52

$$\frac{\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,d^2\,\mathrm{EllipticF}\left(\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{2\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\mathrm{Ell}\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(-c*x^4+a)^(1/2),x)
```

```
[Out] e^2*(-1/3*(-c*x^4+a)^(1/2)/c*x+1/3*a/c/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2
)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*Ell
ipticF((1/a^(1/2)*c^(1/2))^(1/2)*x,I))-2*d*e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1
/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^
4+a)^(1/2)/c^(1/2)*(EllipticF((1/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((1/a
^(1/2)*c^(1/2))^(1/2)*x,I))+d^2/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/
2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(
(1/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{a-c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(a - c*x^4)^(1/2), x)
```

sympy [A] time = 3.77, size = 129, normalized size = 0.80

$$\frac{d^{2}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^{4}e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{dex^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^{4}e^{2i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{e^{2}x^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^{4}e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

```
[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*
exp_polar(2*I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1
/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

3.159 $\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=124

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$

```
[Out] a^(3/4)*e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(3/4)/(-c*x^4+
a)^(1/2)+a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-e+d*c^(1/2)/a^(1/2))*(1-c
*x^4/a)^(1/2)/c^(3/4)/(-c*x^4+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1201, 224, 221, 1200, 1199, 424}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(c^(3/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c*
x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4
])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx &= \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{\sqrt{c}} + \left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}\,dx\\
&= \frac{\left(\sqrt{a}\,e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{c}\,\sqrt{a-cx^4}} + \frac{\left(\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{a-cx^4}}\\
&= \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,\sqrt{a-cx^4}} + \frac{\left(\sqrt{a}\,e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}\,dx}{\sqrt{c}\,\sqrt{a-cx^4}}\\
&= \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,\sqrt{a-cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 77, normalized size = 0.62

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[a - c*x^4])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-cx^4+a}\left(ex^2+d\right)}{cx^4-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + a)*(e*x^2 + d)/(c*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

maple [A] time = 0.00, size = 154, normalized size = 1.24

$$\frac{\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,d\,\mathrm{EllipticF}\left(\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\mathrm{Ellip}\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4+a)^(1/2),x)
[Out] -e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^
(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF((1/a^(1/2)*c
^(1/2))^(1/2)*x,I)-EllipticE((1/a^(1/2)*c^(1/2))^(1/2)*x,I))+d/(1/a^(1/2)*c
^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1
/2)/(-c*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{\sqrt{a-c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)/(a - c*x^4)^(1/2), x)
```

sympy [A] time = 2.24, size = 82, normalized size = 0.66

$$\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4+a)**(1/2),x)
[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqr
t(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_p
olar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

3.160 $\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{a-cx^4}}$$

```
[Out] a^(1/4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1
/2)/c^(1/4)/d/(-c*x^4+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1219, 1218}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x^4])
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx &= \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{(d+ex^2)\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{a-cx^4}}\\ &= \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{a-cx^4}}\end{aligned}$$

Mathematica [C] time = 0.15, size = 91, normalized size = 1.26

$$-\frac{i\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[a - c*x^
4])
```

fricas [F] time = 10.03, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-cx^4+a}}{cex^6+cdx^4-aex^2-ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

maple [A] time = 0.03, size = 97, normalized size = 1.35

$$\frac{\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,x, -\frac{\sqrt{a}\,e}{\sqrt{c}\,d}, \frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

```
[Out] 1/d/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c
^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*c^(1/2))^(1/2)*x
,-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

```
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-cx^4}\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)), x)
```

3.161 $\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a-cx^4}} \, dx$

Optimal. Leaf size=299

$$-\frac{a^{3/4}\sqrt[4]{c}\,e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\,\left(cd^2-ae^2\right)}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{cx^4}{a}}\,\left(3cd^2-ae^2\right)\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{c}\,d^2\sqrt{a-cx^4}\,\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a}}{2d\left(d+ex^2\right)}$$

```
[Out] -1/2*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)-1/2*a^(3/4)*c^(1/4)*
e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d/(-a*e^2+c*d^2)/(-c*x^4
+a)^(1/2)+1/2*a^(1/4)*(-a*e^2+3*c*d^2)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1
/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/d^2/(-a*e^2+c*d^2)/(-c*x^4+a)^(1
/2)-1/2*a^(1/4)*c^(1/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d/
(e*a^(1/2)+d*c^(1/2))/(-c*x^4+a)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1224, 1717, 1201, 224, 221, 1200, 1199, 424, 1219, 1218}

$$-\frac{a^{3/4}\sqrt[4]{c}\,e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\,\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{cx^4}{a}}\,\left(3cd^2-ae^2\right)\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\right.}{2\sqrt[4]{c}\,d^2\sqrt{a-cx^4}\,\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(2*d*(c*d^2 - a*e^2)*(d + e*x^2)) - (a^(3/4)*c^(1/
4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*(
c*d^2 - a*e^2)*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*Sqrt[1 - (c*x^4)/a]*Elli
pticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt[a
 - c*x^4]) + (a^(1/4)*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((S
qrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(1/4)*d^2*(c
*d^2 - a*e^2)*Sqrt[a - c*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
```

```
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1717

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -D
ist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} + \frac{\int \frac{2cd^2-ae^2-2cdex^2-ce^2x^4}{(d+ex^2)\sqrt{a-cx^4}}dx}{2d(cd^2-ae^2)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} - \frac{\int \frac{cde^2+ce^3x^2}{\sqrt{a-cx^4}}dx}{2de^2(cd^2-ae^2)} + \frac{(3cd^2-ae^2)\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}dx}{2d(cd^2-ae^2)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} - \frac{\sqrt{c}\int \frac{1}{\sqrt{a-cx^4}}dx}{2d(\sqrt{c}\,d+\sqrt{a}\,e)} - \frac{(\sqrt{a}\sqrt{c}\,e)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}dx}{2d(cd^2-ae^2)} + \frac{((3cd^2}{}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} + \frac{\sqrt[4]{a}\,(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-\right.}{2\sqrt[4]{c}\,d^2(cd^2-ae^2)\sqrt{a-cx^4}}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} - \frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d(\sqrt{c}\,d+\sqrt{a}\,e)\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\,(3cd^2-}{}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} - \frac{a^{3/4}\sqrt[4]{c}\,e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d(cd^2-ae^2)\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1}}{2d(\sqrt{}}$$

Mathematica [C] time = 0.96, size = 508, normalized size = 1.70

$$\frac{-3icd^3\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-3icd^2ex^2\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

```
[Out] (-(a*Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e^2*x) + Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d*e^2*x
^5 + I*Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcS
inh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*Sqrt[c]*d*(-(Sqrt[c]*d) + Sqrt[a]*
e)*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[
a])]*x], -1] - (3*I)*c*d^3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sq
rt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*d*e^2*Sqrt[1 -
(c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/
Sqrt[a])]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqr
t[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*e^3*
x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sq
rt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(2*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*(c*d^2 - a*
e^2)*(d + e*x^2)*Sqrt[a - c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4 + a}\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

maple [B] time = 0.03, size = 523, normalized size = 1.75

$$\frac{\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,a\,e^2\,\mathrm{EllipticPi}\left(\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,x,-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{2\left(a\,e^2-c\,d^2\right)\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,d^2}+\frac{\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{a}\,\sqrt{c}\,e\,\mathrm{EllipticE}\Big(}{2\left(a\,e^2-c\,d^2\right)\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/2*c/(a*e^2-c*d^2)/(1
/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)
*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*c^(1/2))^(1/2)*x,I)-1/2
*c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^
(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*Ellipti
cF((1/a^(1/2)*c^(1/2))^(1/2)*x,I)+1/2*c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)/(1/
a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*
x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE((1/a^(1/2)*c^(1/2))^(1/2)*x,I)+1/2/
(a*e^2-c*d^2)/d^2*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^
(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi((1/a^(1/2
)*c^(1/2))^(1/2)*x,-a^(1/2)/c^(1/2)/d*e,(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/
2)*c^(1/2))^(1/2))*a-3/2/(a*e^2-c*d^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2
)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*Ell
ipticPi((1/a^(1/2)*c^(1/2))^(1/2)*x,-a^(1/2)/c^(1/2)/d*e,(-1/a^(1/2)*c^(1/2
))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4 + a}\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a - c\,x^4}\,\left(e\,x^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^2),x)
```

```
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - cx^4}\left(d + ex^2\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**2), x)
```

3.162 $\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=425

$$-\frac{3a^{3/4}\sqrt[4]{c}\,e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}+\frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\right.\right.}{8\sqrt[4]{c}\,d^3\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}$$

```
[Out] -1/4*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^2-3/8*e^2*(-a*e^2+3*
c*d^2)*x*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)-3/8*a^(3/4)*c^(1/4
)*e*(-a*e^2+3*c*d^2)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d^2/(
-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)+3/8*a^(1/4)*(a^2*e^4-2*a*c*d^2*e^2+5*c^2*d
^4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1/2)/
c^(1/4)/d^3/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)-1/8*a^(1/4)*c^(1/4)*EllipticF
(c^(1/4)*x/a^(1/4),I)*(7*c*d^2-3*a*e^2-2*d*e*a^(1/2)*c^(1/2))*(1-c*x^4/a)^(
1/2)/d^2/(-a*e^2+c*d^2)/(e*a^(1/2)+d*c^(1/2))/(-c*x^4+a)^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1224, 1697, 1717, 1201, 224, 221, 1200, 1199, 424, 1219, 1218}

$$\frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{c}\,d^3\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}-\frac{3a^{3/4}\sqrt[4]{c}\,e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)E\left(\sin^{-1}\left(\right.\right.}{8d^2\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(4*d*(c*d^2 - a*e^2)*(d + e*x^2)^2) - (3*e^2*(3*c*
d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(8*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)) - (3*
a^(3/4)*c^(1/4)*e*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c
^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) - (a^(1/
4)*c^(1/4)*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Sqrt[1 - (c*x^4)/a]*
EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(Sqrt[c]*d + Sqrt[a]*e)*
(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) + (3*a^(1/4)*(5*c^2*d^4 - 2*a*c*d^2*e^2 +
a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(8*c^(1/4)*d^3*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]
)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1697

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] &&
 PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]
```

Rule 1717

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -D
ist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
```

```
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}+\frac{\int \frac{4cd^2-3ae^2-4cdex^2+ce^2x^4}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx}{4d\left(cd^2-ae^2\right)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}-\frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)}+\frac{\int \frac{8c^2d^4-5acd^2e^2+3a^2e^4-4cde(\ldots}{(d+ex\ldots}}{8d^2\left(c\ldots\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}-\frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)}-\frac{\int \frac{-3cde^2\left(3cd^2-ae^2\right)+4cde^2(4c\ldots}{\sqrt{a-\ldots}}}{8d^2e^2\left(cd\ldots\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}-\frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)}-\frac{\left(\sqrt{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(7cd^{\ldots}\right.\right.}{8d\ldots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}-\frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)}+\frac{3\sqrt[4]{a}\left(5c^2d^4-2acd^2e^2+\ldots\right.}{8\sqrt[4]{c}\ldots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}-\frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)}-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(7\ldots\right.}{\ldots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}-\frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)}-\frac{3a^{3/4}\sqrt[4]{c}\,e\left(3cd^2-ae^2\right)\sqrt{\ldots}}{8d^2\left(cd^2-\ldots\right.}$$

Mathematica [C] time = 1.24, size = 321, normalized size = 0.76

$$\frac{\frac{de^2x\left(a-cx^4\right)\left(ae^2\left(5d+3ex^2\right)-cd^2\left(11d+9ex^2\right)\right)}{\left(d+ex^2\right)^2}-\frac{i\sqrt{1-\frac{cx^4}{a}}\left(\left(-3a^{3/2}\sqrt{c}\,de^3+9\sqrt{a}\,c^{3/2}d^3e+acd^2e^2-7c^2d^4\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+3\left(a^2e^4-2acd^2e^2+5\ldots\right.\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\ldots}}{8d^3\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

```
[Out] ((d*e^2*x*(a - c*x^4)*(a*e^2*(5*d + 3*e*x^2) - c*d^2*(11*d + 9*e*x^2)))/(d
+ e*x^2)^2 - (I*Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*Sqrt[c]*d*e*(-3*c*d^2 + a*e^
2)*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (-7*c^2*d^4 + 9*S
qrt[a]*c^(3/2)*d^3*e + a*c*d^2*e^2 - 3*a^(3/2)*Sqrt[c]*d*e^3)*EllipticF[I*A
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + 3*(5*c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt
```

```
[a])]*x], -1]))/Sqrt[-(Sqrt[c]/Sqrt[a])])/(8*d^3*(c*d^2 - a*e^2)^2*Sqrt[a -
 c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4 + a}\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

maple [B] time = 0.03, size = 961, normalized size = 2.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x)
```

```
[Out] 1/4*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+3/8*e^2*(a*e^2-3*c*d
^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/8*c/d/(a*e^2-c*d^2)^
2/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(
1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*c^(1/2))^(1/2)*x,I)
*a*e^2-7/8*c^2*d/(a*e^2-c*d^2)^2/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1
/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF
((1/a^(1/2)*c^(1/2))^(1/2)*x,I)-3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^(3/2)
/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1
/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*c^(1/2))^(1/2)*x,I)+
9/8*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)
*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*Elli
pticF((1/a^(1/2)*c^(1/2))^(1/2)*x,I)+3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^
(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)
*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE((1/a^(1/2)*c^(1/2))^(1/2)*
x,I)-9/8*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^
(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)
*EllipticE((1/a^(1/2)*c^(1/2))^(1/2)*x,I)+3/8/(a*e^2-c*d^2)^2/d^3*e^4/(1/a^
(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^
2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*c^(1/2))^(1/2)*x,-a^(1/2)
/c^(1/2)/d*e,(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a^2-3/4/
(a*e^2-c*d^2)^2/d*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^
(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi((1/a^(1/2
)*c^(1/2))^(1/2)*x,-a^(1/2)/c^(1/2)/d*e,(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/
2)*c^(1/2))^(1/2))*a*c+15/8/(a*e^2-c*d^2)^2*d/(1/a^(1/2)*c^(1/2))^(1/2)*(-1
/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1
/2)*EllipticPi((1/a^(1/2)*c^(1/2))^(1/2)*x,-a^(1/2)/c^(1/2)/d*e,(-1/a^(1/2)
*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4 + a}\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^3),x)
```

```
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-cx^4}\,\left(d+ex^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**3), x)
```

3.163 $\int \frac{1}{\left(d+ex^2\right)^4\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=563

$$-\frac{e^2x\sqrt{a-cx^4}\,\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{16d^3\left(d+ex^2\right)\left(cd^2-ae^2\right)^3}-\frac{a^{3/4}\sqrt[4]{c}\,e\sqrt{1-\frac{cx^4}{a}}\,\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\Big|-\right.}{16d^3\sqrt{a-cx^4}\,\left(cd^2-ae^2\right)^3}$$

```
[Out] -1/6*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^3-5/24*e^2*(-a*e^2+3
*c*d^2)*x*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)^2-1/16*e^2*(5*a^2
*e^4-14*a*c*d^2*e^2+29*c^2*d^4)*x*(-c*x^4+a)^(1/2)/d^3/(-a*e^2+c*d^2)^3/(e*
x^2+d)-1/16*a^(3/4)*c^(1/4)*e*(5*a^2*e^4-14*a*c*d^2*e^2+29*c^2*d^4)*Ellipti
cE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d^3/(-a*e^2+c*d^2)^3/(-c*x^4+a)^(
1/2)+1/16*a^(1/4)*(-5*a^3*e^6+17*a^2*c*d^2*e^4-7*a*c^2*d^4*e^2+35*c^3*d^6)*
EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1/2)/c^(1
/4)/d^4/(-a*e^2+c*d^2)^3/(-c*x^4+a)^(1/2)-1/48*a^(1/4)*c^(1/4)*EllipticF(c^
(1/4)*x/a^(1/4),I)*(57*c^2*d^4-32*a*c*d^2*e^2+15*a^2*e^4-30*c^(3/2)*d^3*e*a
^(1/2)+10*a^(3/2)*d*e^3*c^(1/2))*(1-c*x^4/a)^(1/2)/d^3/(-e*a^(1/2)+d*c^(1/2
))^2/(e*a^(1/2)+d*c^(1/2))^3/(-c*x^4+a)^(1/2)
```

Rubi [A] time = 1.21, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1224, 1697, 1717, 1201, 224, 221, 1200, 1199, 424, 1219, 1218}

$$-\frac{e^2x\sqrt{a-cx^4}\,\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{16d^3\left(d+ex^2\right)\left(cd^2-ae^2\right)^3}-\frac{\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{1-\frac{cx^4}{a}}\,\left(10a^{3/2}\sqrt{c}\,de^3+15a^2e^4-30\sqrt{a}\,c^{3/2}d^3e-32acd^2\right.}{48d^3\sqrt{a-cx^4}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2\left(\sqrt{a}\,e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(6*d*(c*d^2 - a*e^2)*(d + e*x^2)^3) - (5*e^2*(3*c*
d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(24*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)^2) -
(e^2*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[a - c*x^4])/(16*d^3*(
c*d^2 - a*e^2)^3*(d + e*x^2)) - (a^(3/4)*c^(1/4)*e*(29*c^2*d^4 - 14*a*c*d^2
*e^2 + 5*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)]
, -1])/(16*d^3*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*(57*c^
2*d^4 - 30*Sqrt[a]*c^(3/2)*d^3*e - 32*a*c*d^2*e^2 + 10*a^(3/2)*Sqrt[c]*d*e^
3 + 15*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)],
-1])/(48*d^3*(Sqrt[c]*d - Sqrt[a]*e)^2*(Sqrt[c]*d + Sqrt[a]*e)^3*Sqrt[a - c
*x^4]) + (a^(1/4)*(35*c^3*d^6 - 7*a*c^2*d^4*e^2 + 17*a^2*c*d^2*e^4 - 5*a^3*
e^6)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(
1/4)*x)/a^(1/4)], -1])/(16*c^(1/4)*d^4*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1697

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] &&
 PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]
```

Rule 1717

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -D
ist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^4\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}+\frac{\int \frac{6cd^2-5ae^2-6cdex^2+3ce^2x^4}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx}{6d\left(cd^2-ae^2\right)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}+\frac{\int \frac{24c^2d^4-29acd^2e^2+15a^2\cdots}{\cdots}}{2\cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2\cdots\right.}{16d^3\left(cd^2-\cdots\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2\cdots\right.}{16d^3\left(cd^2-\cdots\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2\cdots\right.}{16d^3\left(cd^2-\cdots\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2\cdots\right.}{16d^3\left(cd^2-\cdots\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2\cdots\right.}{16d^3\left(cd^2-\cdots\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2\cdots\right.}{16d^3\left(cd^2-\cdots\right.}$$

Mathematica [C] time = 1.93, size = 458, normalized size = 0.81

$$-\frac{\frac{de^2x\left(a-cx^4\right)\left(3\left(d+ex^2\right)^2\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)+8\left(cd^3-ade^2\right)^2+10d\left(d+ex^2\right)\left(cd^2-ae^2\right)\left(3cd^2-ae^2\right)\right)}{\left(d+ex^2\right)^3\left(cd^2-ae^2\right)^3}-\frac{i\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}\,de\left(5a^2e^4-14acd^2e^2+\cdots\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```

```
[Out] (-((d*e^2*x*(a - c*x^4)*(8*(c*d^3 - a*d*e^2)^2 + 10*d*(c*d^2 - a*e^2)*(3*c*
d^2 - a*e^2)*(d + e*x^2) + 3*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*(d +
 e*x^2)^2))/((c*d^2 - a*e^2)^3*(d + e*x^2)^3)) - (I*Sqrt[1 - (c*x^4)/a]*(3*
Sqrt[a]*Sqrt[c]*d*e*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*EllipticE[I*A
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*d*(57*c^(5/2)*d^5 - 87*Sq
rt[a]*c^2*d^4*e - 2*a*c^(3/2)*d^3*e^2 + 42*a^(3/2)*c*d^2*e^3 + 5*a^2*Sqrt[c
]*d*e^4 - 15*a^(5/2)*e^5)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x],
-1] + 3*(-35*c^3*d^6 + 7*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 + 5*a^3*e^6)*Elli
pticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -
1]))/(Sqrt[-(Sqrt[c]/Sqrt[a])]*(-(c*d^2) + a*e^2)^3))/(48*d^4*Sqrt[a - c*x^
4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4 + a}\left(ex^2 + d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)
```

maple [B] time = 0.04, size = 1420, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x)
```

```
[Out] 1/6*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^3+5/24*e^2*(a*e^2-3*c*
d^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+1/16*e^2*(5*a^2*e^4
-14*a*c*d^2*e^2+29*c^2*d^4)/(a*e^2-c*d^2)^3/d^3*x*(-c*x^4+a)^(1/2)/(e*x^2+d
)-35/16/(a*e^2-c*d^2)^3*d^2/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x
^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi((1/
a^(1/2)*c^(1/2))^(1/2)*x,-a^(1/2)/c^(1/2)/d*e,(-1/a^(1/2)*c^(1/2))^(1/2)/(1
/a^(1/2)*c^(1/2))^(1/2))*c^3+5/16/(a*e^2-c*d^2)^3/d^4*e^6/(1/a^(1/2)*c^(1/2
))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(
-c*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*c^(1/2))^(1/2)*x,-a^(1/2)/c^(1/2)/d*e
,(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a^3+7/16/(a*e^2-c*d^
2)^3*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1
/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*c^(1/2))^(1
/2)*x,-a^(1/2)/c^(1/2)/d*e,(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(
1/2))*a*c^2+19/16*c^3*d^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(
1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*
EllipticF((1/a^(1/2)*c^(1/2))^(1/2)*x,I)+5/48*c/d^2/(a*e^2-c*d^2)^3/(1/a^(1
/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+
1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*c^(1/2))^(1/2)*x,I)*a^2*e^4-
1/24*c^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+
1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF((1/a^(1
/2)*c^(1/2))^(1/2)*x,I)*a*e^2-5/16*c^(1/2)*e^5/(a*e^2-c*d^2)^3/d^3*a^(5/2)/
```

```
(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/
2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*c^(1/2))^(1/2)*x,I)+7
/8*c^(3/2)*e^3/(a*e^2-c*d^2)^3/d*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1
/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*E
llipticF((1/a^(1/2)*c^(1/2))^(1/2)*x,I)-29/16*c^(5/2)*e/(a*e^2-c*d^2)^3*d*a
^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2
)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*c^(1/2))^(1/2)
*x,I)+5/16*c^(1/2)*e^5/(a*e^2-c*d^2)^3/d^3*a^(5/2)/(1/a^(1/2)*c^(1/2))^(1/2
)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+
a)^(1/2)*EllipticE((1/a^(1/2)*c^(1/2))^(1/2)*x,I)-7/8*c^(3/2)*e^3/(a*e^2-c*
d^2)^3/d*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)
*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE((1/a^(1/2)*c^(1
/2))^(1/2)*x,I)+29/16*c^(5/2)*e/(a*e^2-c*d^2)^3*d*a^(1/2)/(1/a^(1/2)*c^(1/2
))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(
-c*x^4+a)^(1/2)*EllipticE((1/a^(1/2)*c^(1/2))^(1/2)*x,I)-17/16/(a*e^2-c*d^2
)^3/d^2*e^4/(1/a^(1/2)*c^(1/2))^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(1/a
^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*c^(1/2))
^(1/2)*x,-a^(1/2)/c^(1/2)/d*e,(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2)
)^(1/2))*a^2*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4+a}\left(ex^2+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^4),x)
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-cx^4}\left(d+ex^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**4/(-c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**4), x)
```

3.164 $\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=126

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}$$

```
[Out] a^(3/4)*e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(3/4)/(c*x^4-a
)^(1/2)+a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-e+d*c^(1/2)/a^(1/2))*(1-c*
x^4/a)^(1/2)/c^(3/4)/(c*x^4-a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1201, 224, 221, 1200, 1199, 424}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(c^(3/4)*Sqrt[-a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c
*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[-a + c*x
^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{-a+cx^4}}\,dx}{\sqrt{c}} + \left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-a+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}\,e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{c}\,\sqrt{-a+cx^4}} + \frac{\left(\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,\sqrt{-a+cx^4}} + \frac{\left(\sqrt{a}\,e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}\,dx}{\sqrt{c}\,\sqrt{-a+cx^4}}$$

$$= \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,\sqrt{-a+cx^4}}$$

Mathematica [C] time = 0.03, size = 78, normalized size = 0.62

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4])
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ex^2+d}{\sqrt{cx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

maple [A] time = 0.01, size = 160, normalized size = 1.27

$$\frac{\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,d\,\mathrm{EllipticF}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}+\frac{\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\mathrm{Ellip}\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4-a)^(1/2),x)
```

```
[Out] e*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(-1/a^
(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-1/a^(1/2)
*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))+d/(-1/a^(1/2)
*c^(1/2))^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^
(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{\sqrt{c\,x^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(c*x^4 - a)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(c*x^4 - a)^(1/2), x)
```

sympy [A] time = 2.18, size = 73, normalized size = 0.58

$$-\frac{idx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}-\frac{iex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a)*gamma(5/4)
) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamm
a(7/4))
```

3.165 $\int \frac{1}{\left(d+ex^2\right)\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{cx^4-a}}$$

```
[Out] a^(1/4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1
/2)/c^(1/4)/d/(c*x^4-a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1219, 1218}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[-a + c*x^4])
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(d+ex^2\right)\sqrt{-a+cx^4}}\,dx &= \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{\left(d+ex^2\right)\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}\\
&= \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{-a+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.15, size = 92, normalized size = 1.26

$$-\frac{i\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[-a + c*x
^4])
```

fricas [F] time = 13.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4 - a}}{cex^6 + cdx^4 - aex^2 - ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 - a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 - a}\left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

maple [A] time = 0.02, size = 99, normalized size = 1.36

$$\frac{\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x, \frac{\sqrt{a}\,e}{\sqrt{c}\,d}, \frac{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4-a)^(1/2),x)
```

```
[Out] 1/d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(-1/a^(1/2)*
c^(1/2)*x^2+1)^(1/2)/(c*x^4-a)^(1/2)*EllipticPi((-1/a^(1/2)*c^(1/2))^(1/2)*
x,a^(1/2)/c^(1/2)/d*e,(1/a^(1/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 - a}\left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x^4 - a}\,\left(e\,x^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^4 - a)^(1/2)*(d + e*x^2)),x)
[Out] int(1/((c*x^4 - a)^(1/2)*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a + cx^4}\left(d + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4-a)**(1/2),x)
[Out] Integral(1/(sqrt(-a + c*x**4)*(d + e*x**2)), x)
```

3.166 $\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=54

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,\sqrt{cx^4-a}}$$

```
[Out] a^(3/4)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/(c*x^4-a)^
(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1200, 1199, 424}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (a^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c
^(1/4)*Sqrt[-a + c*x^4])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{-a+cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{\left(\sqrt{a}\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}}$$

Mathematica [C] time = 0.04, size = 86, normalized size = 1.59

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+\sqrt{c}\,x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/
a] + Sqrt[c]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a +
 c*x^4])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c}\,x^2+\sqrt{a}}{\sqrt{cx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c}\,x^2+\sqrt{a}}{\sqrt{cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

maple [B] time = 0.05, size = 158, normalized size = 2.93

$$\frac{\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{a}\,\operatorname{EllipticF}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}+\frac{\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\operatorname{EllipticE}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/2)+c^(1/2)*x^2)/(c*x^4-a)^(1/2),x)
```

```
[Out] a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(-1/a^(1
/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4-a)^(1/2)*(EllipticF((-1/a^(1/2)*c^(1/2))^(1
/2)*x,I)-EllipticE((-1/a^(1/2)*c^(1/2))^(1/2)*x,I))+a^(1/2)/(-1/a^(1/2)*c^(
1/2))^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2
)/(c*x^4-a)^(1/2)*EllipticF((-1/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c}\,x^2+\sqrt{a}}{\sqrt{cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{c\,x^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/2) + c^(1/2)*x^2)/(c*x^4 - a)^(1/2),x)
```

```
[Out] int((a^(1/2) + c^(1/2)*x^2)/(c*x^4 - a)^(1/2), x)
```

sympy [A] time = 2.41, size = 70, normalized size = 1.30

$$-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}-\frac{i\sqrt{c}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/2)+x**2*c**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*gamma(5/4)) - I*sqrt
(c)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamma(7/
4))
```

3.167 $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{cx^4-a}}$$

```
[Out] EllipticE((c/a)^(1/4)*x,I)*(1-c*x^4/a)^(1/2)/(c/a)^(1/4)/(c*x^4-a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1200, 1199, 424}

$$\frac{\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c/a)^(1/4)*x], -1])/((c/a)^(1/4)*Sqr
t[-a + c*x^4])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{\frac{c}{a}}\,x^2}{\sqrt{-a+cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1+\sqrt{\frac{c}{a}}\,x^2}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}$$
$$= \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{1+\sqrt{\frac{c}{a}}\,x^2}}{\sqrt{1-\sqrt{\frac{c}{a}}\,x^2}}\,dx}{\sqrt{-a+cx^4}}$$
$$= \frac{\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}\,x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\,\sqrt{-a+cx^4}}$$

Mathematica [C] time = 0.03, size = 85, normalized size = 1.63

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+x^3\sqrt{\frac{c}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + Sqr
t[c/a]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4
])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

maple [B] time = 0.04, size = 165, normalized size = 3.17

$$\frac{\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\,\mathrm{EllipticF}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}+\frac{\sqrt{\frac{c}{a}}\,\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\mathrm{E}\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x)
```

```
[Out] 1/(-1/a^(1/2)*c^(1/2))^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(-1/a^(1/2)*c^
(1/2)*x^2+1)^(1/2)/(c*x^4-a)^(1/2)*EllipticF((-1/a^(1/2)*c^(1/2))^(1/2)*x,I
)+(c/a)^(1/2)*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^
(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF((
-1/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((-1/a^(1/2)*c^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,\sqrt{\frac{c}{a}}+1}{\sqrt{c\,x^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c/a)^(1/2) + 1)/(c*x^4 - a)^(1/2),x)
```

```
[Out] int((x^2*(c/a)^(1/2) + 1)/(c*x^4 - a)^(1/2), x)
```

sympy [B] time = 2.30, size = 76, normalized size = 1.46

$$-\frac{ix^3\sqrt{\frac{c}{a}}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**2*(c/a)**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*x**3*sqrt(c/a)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)
*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a
)*gamma(5/4))
```

3.168 $\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{-a-cx^4}}$$

```
[Out] -e*x*(-c*x^4-a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a
)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-c*x^4-a)^(1/2)+1/2*a^(1/4)*(cos(
2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elli
pticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(
e+d*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-c*
x^4-a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1198, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{-a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```

```
[Out] -((e*x*Sqrt[-a - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^(1/4)*e*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[-a - c*x^4]) + (a^(1/4)
*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/
4)*Sqrt[-a - c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = -\frac{\left(\sqrt{a}\,e\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{-a-cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-a-cx^4}}\,dx$$

$$= -\frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}} + \frac{\left(\sqrt{c}\,d+\sqrt{\right.}}{}$$

Mathematica [C] time = 0.04, size = 80, normalized size = 0.34

$$\frac{\sqrt{\frac{cx^4}{a}+1}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)}{3\sqrt{-a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
+ e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[-a - c*x^4
])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-cx^4-a}\left(ex^2+d\right)}{cx^4+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 - a)*(e*x^2 + d)/(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{-cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

maple [C] time = 0.01, size = 175, normalized size = 0.74

$$\frac{\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,d\,\mathrm{EllipticF}\left(\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,x, i\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4-a}} - \frac{i\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,x, i\right)\right.}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4-a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4-a)^(1/2),x)
```

```
[Out] -I*e*a^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(-I
/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(-c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-I/a^(
1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-I/a^(1/2)*c^(1/2))^(1/2),I))+d/(-I/a^(
1/2)*c^(1/2))^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2
+1)^(1/2)/(-c*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*c^(1/2))^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{\sqrt{-c\,x^4 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(- a - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(- a - c*x^4)^(1/2), x)
```

sympy [C] time = 2.10, size = 83, normalized size = 0.35

$$-\frac{idx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} - \frac{iex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*ex
p_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

3.169 $\int \frac{1}{\left(d+ex^2\right)\sqrt{-a-cx^4}}\,dx$

Optimal. Leaf size=347

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,de};2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}\,d\sqrt{-a-cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\tan^{-1}\left(\frac{x\sqrt{-ae^2-cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\,\sqrt{-ae^2-cd^2}}+\cdots$$

```
[Out] 1/2*arctan(x*(-a*e^2-c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(-c*x^4-a)^(1/2))*e^(1/2)
/d^(1/2)/(-a*e^2-c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/a^(1/4)/(-e*a^(1/2)+d*c^(1/2))/(-c*x^4-a)^(1/2)-1/4*a^(3/4)*(c
os(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*E
llipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/
e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^
2*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(-c*x^
4-a)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1217, 220, 1707}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,de};2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}\,d\sqrt{-a-cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\tan^{-1}\left(\frac{x\sqrt{-ae^2-cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\,\sqrt{-ae^2-cd^2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a - c*x^4
])])/(2*Sqrt[d]*Sqrt[-(c*d^2) - a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a - c*x^4]) - (a^(
3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*
Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 - a*
e^2)*Sqrt[-a - c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
```

```
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{-a-cx^4}}\,dx = \frac{\sqrt{c}\int \frac{1}{\sqrt{-a-cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e} - \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{-a-cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e}$$

$$= \frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{-cd^2-ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\,\sqrt{-cd^2-ae^2}} + \frac{\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{-a-cx^4}} -$$

Mathematica [C] time = 0.15, size = 98, normalized size = 0.28

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}\,\Pi\left(-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSin
h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[-a -
 c*x^4])
```

fricas [F] time = 10.93, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-cx^4-a}}{cex^6+cdx^4+aex^2+ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 - a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4-a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

maple [C] time = 0.02, size = 110, normalized size = 0.32

$$\frac{\sqrt{\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}+1}\,\sqrt{-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}+1}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,x,-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x)
```

```
[Out] 1/d/(-I/a^(1/2)*c^(1/2))^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(-I/a^(1/2)*
c^(1/2)*x^2+1)^(1/2)/(-c*x^4-a)^(1/2)*EllipticPi((-I/a^(1/2)*c^(1/2))^(1/2)
*x,-I*a^(1/2)/c^(1/2)*e/d,(I/a^(1/2)*c^(1/2))^(1/2)/(-I/a^(1/2)*c^(1/2))^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^{4}-a}\,\left(ex^{2}+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{-c\,x^{4}-a}\,\left(e\,x^{2}+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

```
[Out] int(1/((- a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a-cx^{4}}\,\left(d+ex^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a - c*x**4)*(d + e*x**2)), x)
```

3.170 $\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx$

Optimal. Leaf size=40

$$\frac{\Pi\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

```
[Out] 1/10*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b/a*5^(1/2),I)*5^(3/4)/a*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1213, 537}

$$\frac{\Pi\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5^
(1/4)*a)
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx &= \sqrt{5}\int \frac{1}{\sqrt{2\sqrt{5}-5x^2}\sqrt{2\sqrt{5}+5x^2}\,(a+bx^2)}\,dx\\ &= \frac{\Pi\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}\end{aligned}$$

Mathematica [A] time = 0.13, size = 40, normalized size = 1.00

$$\frac{\Pi\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5^
(1/4)*a)
```

fricas [F] time = 7.83, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-5\,x^4+4}}{5\,bx^6+5\,ax^4-4\,bx^2-4\,a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 - 4*b*x^2 - 4*a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-5\,x^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

maple [B] time = 0.06, size = 79, normalized size = 1.98

$$\frac{\sqrt{2}\,5^{\frac{3}{4}}\sqrt{-\frac{\sqrt{5}\,x^2}{2}+1}\,\sqrt{\frac{\sqrt{5}\,x^2}{2}+1}\ \text{EllipticPi}\left(\frac{5^{\frac{1}{4}}\sqrt{2}\,x}{2}, -\frac{2\sqrt{5}\,b}{5a}, \frac{\sqrt{-\frac{\sqrt{5}}{2}}\,\sqrt{2}\,5^{\frac{3}{4}}}{5}\right)}{5\sqrt{-5x^4+4}\ a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x)
```

```
[Out] 1/5/a*2^(1/2)*5^(3/4)*(1-1/2*x^2*5^(1/2))^(1/2)*(1+1/2*x^2*5^(1/2))^(1/2)/(
-5*x^4+4)^(1/2)*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b/a*5^(1/2),1/5*(-1/2
*5^(1/2))^(1/2)*2^(1/2)*5^(3/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-5\,x^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^2+a\right)\,\sqrt{4-5\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(4 - 5*x^4)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(4 - 5*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4-5x^4}\,\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(-5*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(4 - 5*x**4)*(a + b*x**2)), x)
```

3.171 $\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx$

Optimal. Leaf size=310

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{5a^2+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{5x^4+4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}}+\frac{\sqrt[4]{5}\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(\sqrt{5}a+2b\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{5x^4+4}\left(5a^2-4b^2\right)}-\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4}{\left(\sqrt{5}x\right.}}}{}$$

```
[Out] 1/2*arctan(x*(5*a^2+4*b^2)^(1/2)/a^(1/2)/b^(1/2)/(5*x^4+4)^(1/2))*b^(1/2)/a
^(1/2)/(5*a^2+4*b^2)^(1/2)+1/4*5^(1/4)*(cos(2*arctan(1/2*5^(1/4)*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*5^(1/4)*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*
5^(1/4)*x*2^(1/2))),1/2*2^(1/2))*(2*b+a*5^(1/2))*(2+x^2*5^(1/2))*((5*x^4+4)
/(2+x^2*5^(1/2))^2)^(1/2)/(5*a^2-4*b^2)*2^(1/2)/(5*x^4+4)^(1/2)-1/40*(cos(2
*arctan(1/2*5^(1/4)*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*5^(1/4)*x*2^(1/2)
))*EllipticPi(sin(2*arctan(1/2*5^(1/4)*x*2^(1/2))),-1/40*(-2*b+a*5^(1/2))^2
/a/b*5^(1/2),1/2*2^(1/2))*(2*b+a*5^(1/2))^2*(2+x^2*5^(1/2))*((5*x^4+4)/(2+x
^2*5^(1/2))^2)^(1/2)*5^(3/4)/a/(5*a^2-4*b^2)*2^(1/2)/(5*x^4+4)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1217, 220, 1707}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{5a^2+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{5x^4+4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}}+\frac{\sqrt[4]{5}\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(\sqrt{5}a+2b\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{5x^4+4}\left(5a^2-4b^2\right)}-\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4}{\left(\sqrt{5}x\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[5*a^2 + 4*b^2]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + 5*x^4])])
/(2*Sqrt[a]*Sqrt[5*a^2 + 4*b^2]) + (5^(1/4)*(Sqrt[5]*a + 2*b)*(2 + Sqrt[5]*
x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticF[2*ArcTan[(5^(1/4)*x)/S
qrt[2]], 1/2])/(2*Sqrt[2]*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4]) - ((Sqrt[5]*a +
2*b)^2*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticPi[-
(Sqrt[5]*a - 2*b)^2/(8*Sqrt[5]*a*b), 2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2])/(
4*Sqrt[2]*5^(1/4)*a*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
```

```
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = -\frac{\left(2b\left(\sqrt{5}\,a+2b\right)\right)\int \frac{1+\frac{\sqrt{5}x^2}{2}}{(a+bx^2)\sqrt{4+5x^4}}\,dx}{5a^2-4b^2} + \frac{\left(5a+2\sqrt{5}\,b\right)\int \frac{1}{\sqrt{4+5x^4}}\,dx}{5a^2-4b^2}$$

$$= \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{5a^2+4b^2}\,x}{\sqrt{a}\,\sqrt{b}\,\sqrt{4+5x^4}}\right)}{2\sqrt{a}\,\sqrt{5a^2+4b^2}} + \frac{\sqrt[4]{5}\,\left(\sqrt{5}\,a+2b\right)\left(2+\sqrt{5}\,x^2\right)\sqrt{\frac{4+5x^4}{\left(2+\sqrt{5}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}\ldots}{\sqrt{2}}\right.\right.}{2\sqrt{2}\left(5a^2-4b^2\right)\sqrt{4+5x^4}}$$

Mathematica [C] time = 0.10, size = 50, normalized size = 0.16

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\Pi\left(-\frac{2ib}{\sqrt{5}\,a};i\sinh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{5}\,x\right)\middle|-1\right)}{\sqrt[4]{5}\,a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

```
[Out] ((-1/2 - I/2)*EllipticPi[((-2*I)*b)/(Sqrt[5]*a), I*ArcSinh[(1/2 + I/2)*5^(1
/4)*x], -1])/(5^(1/4)*a)
```

fricas [F] time = 7.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\,x^4+4}}{5\,bx^6+5\,ax^4+4\,bx^2+4\,a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 + 4*b*x^2 + 4*a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5\,x^4+4}\,\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

maple [C] time = 0.08, size = 86, normalized size = 0.28

$$\frac{\sqrt{2}\,\sqrt{-\frac{i\sqrt{5}\,x^2}{2}+1}\,\sqrt{\frac{i\sqrt{5}\,x^2}{2}+1}\;\mathrm{EllipticPi}\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{5}}\,x}{2},\frac{2i\sqrt{5}\,b}{5a},\frac{\sqrt{-\frac{i\sqrt{5}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{5}}}\right)}{\sqrt{i\sqrt{5}}\,\sqrt{5x^4+4}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(5*x^4+4)^(1/2),x)
```

```
[Out] 1/a/(1/2*I*5^(1/2))^(1/2)*(1-1/2*I*x^2*5^(1/2))^(1/2)*(1+1/2*I*x^2*5^(1/2))
^(1/2)/(5*x^4+4)^(1/2)*EllipticPi((1/2*I*5^(1/2))^(1/2)*x,2/5*I*5^(1/2)*b/a
,(-1/2*I*5^(1/2))^(1/2)/(1/2*I*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5 x^4 + 4} \left(b x^2 + a\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)\,\sqrt{5\,x^4+4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(5*x^4 + 4)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(5*x^4 + 4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b x^{2}\right) \sqrt{5 x^{4} + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(5*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(5*x**4 + 4)), x)
```

3.172 $\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}} dx$

Optimal. Leaf size=40

$$\frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

```
[Out] 1/2*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d^(1/2),I)/a/d^(1/4)*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1218}

$$\frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(a*Sqrt[d]), ArcSin[(d^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*a*
d^(1/4))
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{4-dx^4}}\,dx = \frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

Mathematica [C] time = 0.12, size = 59, normalized size = 1.48

$$-\frac{i\Pi\left(-\frac{2b}{a\sqrt{d}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{d}}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt{-\sqrt{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

```
[Out] ((-I)*EllipticPi[(-2*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[-Sqrt[d]]*x)/Sqrt[2]],
 -1])/(Sqrt[2]*a*Sqrt[-Sqrt[d]])
```

fricas [F] time = 177.33, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-dx^4+4}}{bdx^6+adx^4-4bx^2-4a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-d*x^4 + 4)/(b*d*x^6 + a*d*x^4 - 4*b*x^2 - 4*a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-dx^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

maple [B] time = 0.03, size = 78, normalized size = 1.95

$$\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{d}\,x^2}{2}+1}\,\sqrt{\frac{\sqrt{d}\,x^2}{2}+1}\,\operatorname{EllipticPi}\left(\frac{\sqrt{2}\,d^{\frac{1}{4}}x}{2},-\frac{2b}{a\sqrt{d}},\frac{\sqrt{-\frac{\sqrt{d}}{2}}\,\sqrt{2}}{d^{\frac{1}{4}}}\right)}{\sqrt{-d\,x^4+4}\;a\,d^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x)
[Out] 1/a*2^(1/2)/d^(1/4)*(1-1/2*x^2*d^(1/2))^(1/2)*(1+1/2*x^2*d^(1/2))^(1/2)/(-d
*x^4+4)^(1/2)*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d^(1/2),(-1/2*d^(1/2)
)^(1/2)*2^(1/2)/d^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-dx^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^2+a\right)\,\sqrt{4-d\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(4 - d*x^4)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(4 - d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{-dx^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(-d*x**4+4)**(1/2),x)
[Out] Integral(1/((a + b*x**2)*sqrt(-d*x**4 + 4)), x)
```

3.173 $\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx$

Optimal. Leaf size=300

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{a^2d+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{dx^4+4}}\right)}{2\sqrt{a}\sqrt{a^2d+4b^2}}-\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}+\frac{\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\left(a\sqrt{d}+2b\right.}{4\sqrt{2}a\sqrt[4]{d}\sqrt{dx^4}}$$

```
[Out] 1/2*arctan(x*(a^2*d+4*b^2)^(1/2)/a^(1/2)/b^(1/2)/(d*x^4+4)^(1/2))*b^(1/2)/a
^(1/2)/(a^2*d+4*b^2)^(1/2)-1/4*d^(1/4)*(cos(2*arctan(1/2*d^(1/4)*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*d^(1/4)*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*
d^(1/4)*x*2^(1/2))),1/2*2^(1/2))*(2+x^2*d^(1/2))*((d*x^4+4)/(2+x^2*d^(1/2))
^2)^(1/2)*2^(1/2)/(2*b-a*d^(1/2))/(d*x^4+4)^(1/2)+1/8*(cos(2*arctan(1/2*d^(
1/4)*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*d^(1/4)*x*2^(1/2)))*EllipticPi(s
in(2*arctan(1/2*d^(1/4)*x*2^(1/2))),-1/8*(2*b-a*d^(1/2))^2/a/b/d^(1/2),1/2*
2^(1/2))*(2*b+a*d^(1/2))*(2+x^2*d^(1/2))*((d*x^4+4)/(2+x^2*d^(1/2))^2)^(1/2
)/a/d^(1/4)*2^(1/2)/(2*b-a*d^(1/2))/(d*x^4+4)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1217, 220, 1707}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{a^2d+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{dx^4+4}}\right)}{2\sqrt{a}\sqrt{a^2d+4b^2}}-\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}+\frac{\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\left(a\sqrt{d}+2b\right.}{4\sqrt{2}a\sqrt[4]{d}\sqrt{dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[4*b^2 + a^2*d]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + d*x^4])])
/(2*Sqrt[a]*Sqrt[4*b^2 + a^2*d]) - (d^(1/4)*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x
^4)/(2 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(2*
Sqrt[2]*(2*b - a*Sqrt[d])*Sqrt[4 + d*x^4]) + ((2*b + a*Sqrt[d])*(2 + Sqrt[d
]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*EllipticPi[-(2*b - a*Sqrt[d])^
2/(8*a*b*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(4*Sqrt[2]*a*(2*b -
 a*Sqrt[d])*d^(1/4)*Sqrt[4 + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
```

```
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \frac{(2b)\int \frac{1+\frac{\sqrt{d}x^2}{2}}{(a+bx^2)\sqrt{4+dx^4}}\,dx}{2b-a\sqrt{d}} - \frac{\sqrt{d}\int \frac{1}{\sqrt{4+dx^4}}\,dx}{2b-a\sqrt{d}}$$

$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{4b^2+a^2d}\,x}{\sqrt{a}\sqrt{b}\sqrt{4+dx^4}}\right)}{2\sqrt{a}\sqrt{4b^2+a^2d}} - \frac{\sqrt[4]{d}\left(2+\sqrt{d}x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\left(2b-a\sqrt{d}\right)\sqrt{4+dx^4}} + \cdots$$

Mathematica [C] time = 0.11, size = 65, normalized size = 0.22

$$-\frac{i\Pi\left(-\frac{2ib}{a\sqrt{d}};i\sinh^{-1}\left(\frac{\sqrt{i\sqrt{d}}\,x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt{i\sqrt{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
[Out] ((-I)*EllipticPi[((-2*I)*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[I*Sqrt[d]]*x)/Sqrt
[2]], -1])/(Sqrt[2]*a*Sqrt[I*Sqrt[d]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

maple [C] time = 0.03, size = 86, normalized size = 0.29

$$\frac{\sqrt{2}\sqrt{-\frac{i\sqrt{d}\,x^2}{2}+1}\sqrt{\frac{i\sqrt{d}\,x^2}{2}+1}\ \mathrm{EllipticPi}\left(\frac{\sqrt{2}\sqrt{i\sqrt{d}}\,x}{2},\frac{2ib}{a\sqrt{d}},\frac{\sqrt{-\frac{i\sqrt{d}}{2}}\sqrt{2}}{\sqrt{i\sqrt{d}}}\right)}{\sqrt{i\sqrt{d}}\ \sqrt{d\,x^4+4}\ a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^4+4)^(1/2),x)
```

```
[Out] 1/a/(1/2*I*d^(1/2))^(1/2)*(1-1/2*I*d^(1/2)*x^2)^(1/2)*(1+1/2*I*d^(1/2)*x^2)
^(1/2)/(d*x^4+4)^(1/2)*EllipticPi((1/2*I*d^(1/2))^(1/2)*x,2*I/d^(1/2)*b/a,(
-1/2*I*d^(1/2))^(1/2)/(1/2*I*d^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)\,\sqrt{d\,x^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(d*x^4 + 4)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(d*x^4 + 4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{dx^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(d*x**4 + 4)), x)
```

3.174 $\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=112

$$\frac{a\sqrt{1-x^2}\sqrt{\frac{a(x^2+1)}{a+bx^2}}\,\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}\,x}{\sqrt{bx^2+a}}\right)\middle|-\frac{a-b}{a+b}\right)}{\sqrt{x^2+1}\sqrt{a+b}\sqrt{\frac{a(1-x^2)}{a+bx^2}}}$$

```
[Out] a*EllipticPi(x*(a+b)^(1/2)/(b*x^2+a)^(1/2),b/(a+b),((-a+b)/(a+b))^(1/2))*(-
x^2+1)^(1/2)*(a*(x^2+1)/(b*x^2+a))^(1/2)/(a+b)^(1/2)/(x^2+1)^(1/2)/(a*(-x^2
+1)/(b*x^2+a))^(1/2)
```

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
[Out] Defer[Int][Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
[Out] Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}\sqrt{bx^2+a}}{x^4-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 1)*sqrt(b*x^2 + a)/(x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
[Out] int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(1 - x^4)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)/(1 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(-x**4+1)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.175 $\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=22

$$\text{Int}\left(\left(a+bx^4\right)^p \left(c+ex^2\right)^q ,x\right)$$

```
[Out] Unintegrable((e*x^2+c)^q*(b*x^4+a)^p,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + e*x^2)^q*(a + b*x^4)^p,x]
[Out] Defer[Int][(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Rubi steps

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$$

Mathematica [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + e*x^2)^q*(a + b*x^4)^p,x]
[Out] Integrate[(c + e*x^2)^q*(a + b*x^4)^p, x]
```

fricas [A] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^p \left(ex^2+c\right)^q ,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^p \left(ex^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^p \left(e\,x^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^p \left(ex^2 + c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \left(b\,x^4 + a\right)^p \,\left(e\,x^2 + c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p*(c + e*x^2)^q,x)
```

```
[Out] int((a + b*x^4)^p*(c + e*x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**q*(b*x**4+a)**p,x)
```

```
[Out] Timed out
```

3.176 $\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=204

$$c^3x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)-\frac{ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(ae^2-bc^2(4p+7)\right){}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};\right.}{b(4p+7)}$$

```
[Out] e^3*x^3*(b*x^4+a)^(1+p)/b/(7+4*p)+c^3*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/
4],-b*x^4/a)/((1+b*x^4/a)^p)-e*(a*e^2-b*c^2*(7+4*p))*x^3*(b*x^4+a)^p*hyperg
eom([3/4, -p],[7/4],-b*x^4/a)/b/(7+4*p)/((1+b*x^4/a)^p)+3/5*c*e^2*x^5*(b*x^
4+a)^p*hypergeom([5/4, -p],[9/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A] time = 0.23, antiderivative size = 196, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1207, 1893, 246, 245, 365, 364}

$$ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c^2-\frac{ae^2}{4bp+7b}\right){}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+c^3x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

```
[Out] (e^3*x^3*(a + b*x^4)^(1 + p))/(b*(7 + 4*p)) + (c^3*x*(a + b*x^4)^p*Hypergeo
metric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (e*(c^2 - (a*e^2
)/(7*b + 4*b*p))*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4
)/a)])/(1 + (b*x^4)/a)^p + (3*c*e^2*x^5*(a + b*x^4)^p*Hypergeometric2F1[5/4
, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = \frac{e^3x^3\left(a+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\int \left(a+bx^4\right)^p \left(bc^3(7+4p)-3e\left(ae^2-bc^2(7+4p)\right)x^2+3bce^2(7\right.}{b(7+4p)}$$

$$= \frac{e^3x^3\left(a+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\int \left(bc^3(7+4p)\left(a+bx^4\right)^p+3e\left(-ae^2+bc^2(7+4p)\right)x^2\left(a+bx\right.\right.}{b(7+4p)}$$

$$= \frac{e^3x^3\left(a+bx^4\right)^{1+p}}{b(7+4p)} + c^3\int \left(a+bx^4\right)^p \, dx + \left(3ce^2\right)\int x^4\left(a+bx^4\right)^p \, dx + \left(3e\left(c^2-\frac{}{7l}\right.\right.$$

$$= \frac{e^3x^3\left(a+bx^4\right)^{1+p}}{b(7+4p)} + \left(c^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \left(1+\frac{bx^4}{a}\right)^p dx + \left(3ce^2\left(a+bx^4\right.\right.$$

$$= \frac{e^3x^3\left(a+bx^4\right)^{1+p}}{b(7+4p)} + c^3x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right) + e\left(c^2-\frac{a}{7b+}\right.$$

Mathematica [A] time = 0.07, size = 136, normalized size = 0.67

$$\frac{1}{35}x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(35c^3\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^2\left(35c^2\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+ex^2\left(21c\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4}\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(35*c^3*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*
x^2*(35*c^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + e*x^2*(21*c*Hyp
ergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 5*e*x^2*Hypergeometric2F1[7/4,
 -p, 11/4, -((b*x^4)/a)]))))/(35*(1 + (b*x^4)/a)^p)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^6+3\,ce^2x^4+3\,c^2ex^2+c^3\right)\left(bx^4+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+c\right)^3\left(bx^4+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="giac")
[Out] integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+c\right)^{3} \left(b\,x^{4}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^3*(b*x^4+a)^p,x)
[Out] int((e*x^2+c)^3*(b*x^4+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(ex^{2} + c\right)}^{3} {\left(bx^{4} + a\right)}^{p}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="maxima")
[Out] integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^4+a\right)}^p\,{\left(e\,x^2+c\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p*(c + e*x^2)^3,x)
[Out] int((a + b*x^4)^p*(c + e*x^2)^3, x)
```

sympy [C] time = 139.10, size = 167, normalized size = 0.82

$$\frac{a^{p} c^{3} x \Gamma\left(\frac{1}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| {\frac{b x^{4} e^{i \pi}}{a}} \right)}}{4 \Gamma\left(\frac{5}{4}\right)} + \frac{3 a^{p} c^{2} e x^{3} \Gamma\left(\frac{3}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix}\middle| {\frac{b x^{4} e^{i \pi}}{a}} \right)}}{4 \Gamma\left(\frac{7}{4}\right)} + \frac{3 a^{p} c e^{2} x^{5} \Gamma\left(\frac{5}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix}\middle| {\frac{b x^{4} e^{i \pi}}{a}} \right)}}{4 \Gamma\left(\frac{9}{4}\right)} + \frac{a^{p} e^{3} x^{7} \Gamma\left(\frac{7}{4}\right) \cdots}{4 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**3*(b*x**4+a)**p,x)
[Out] a**p*c**3*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4)) + 3*a**p*c**2*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + 3*a**p*c*e**2*x**5*gamma(5/4)*hyper(
(5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + a**p*e**3*x**
7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(1
1/4))
```

3.177 $\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=150

$$-\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(ae^2-bc^2(4p+5)\right)\,_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)}{b(4p+5)}+\frac{2}{3}cex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\,_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\right.$$

```
[Out] e^2*x*(b*x^4+a)^(1+p)/b/(5+4*p)-(a*e^2-b*c^2*(5+4*p))*x*(b*x^4+a)^p*hyperge
om([1/4, -p],[5/4],-b*x^4/a)/b/(5+4*p)/((1+b*x^4/a)^p)+2/3*c*e*x^3*(b*x^4+a
)^p*hypergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A] time = 0.13, antiderivative size = 142, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1207, 1204, 246, 245, 365, 364}

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c^2-\frac{ae^2}{4bp+5b}\right)\,_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{2}{3}cex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\,_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx}{a}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

```
[Out] (e^2*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c^2 - (a*e^2)/(5*b + 4*b*p))*
x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)
/a)^p + (2*c*e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/
a)])/(3*(1 + (b*x^4)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
```

```
NeQ[c*d^2 + a*e^2, 0]
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx = \frac{e^2x\left(a+bx^4\right)^{1+p}}{b(5+4p)} + \frac{\int \left(-ae^2+bc^2(5+4p)+2bce(5+4p)x^2\right)\left(a+bx^4\right)^p \, dx}{b(5+4p)}$$

$$= \frac{e^2x\left(a+bx^4\right)^{1+p}}{b(5+4p)} + \frac{\int \left(-ae^2\left(1-\frac{bc^2(5+4p)}{ae^2}\right)\left(a+bx^4\right)^p + 2bce(5+4p)x^2\left(a+bx^4\right)\right.}{b(5+4p)}$$

$$= \frac{e^2x\left(a+bx^4\right)^{1+p}}{b(5+4p)} + (2ce)\int x^2\left(a+bx^4\right)^p \, dx - \left(-c^2+\frac{ae^2}{5b+4bp}\right)\int \left(a+bx^4\right)^p \, d$$

$$= \frac{e^2x\left(a+bx^4\right)^{1+p}}{b(5+4p)} + \left(2ce\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^4}{a}\right)^p \, dx - \left(\left(-c^2+\right.\right.$$

$$= \frac{e^2x\left(a+bx^4\right)^{1+p}}{b(5+4p)} + \left(c^2-\frac{ae^2}{5b+4bp}\right)x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx}{a}\right.$$

Mathematica [A] time = 0.04, size = 106, normalized size = 0.71

$$\frac{1}{15}x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(15c^2\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^2\left(10c\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+3ex^2\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(15*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*
x^2*(10*c*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + 3*e*x^2*Hypergeom
etric2F1[5/4, -p, 9/4, -((b*x^4)/a)])))/(15*(1 + (b*x^4)/a)^p)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^4+2\,cex^2+c^2\right)\left(bx^4+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+c\right)^2\left(bx^4+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)^2*(b*x^4 + a)^p, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+c\right)^{2} \left(b\,x^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^{2}+c\right)^{2}\left(bx^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)^2*(b*x^4 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^{4}+a\right)^{p}\,\left(e\,x^{2}+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p*(c + e*x^2)^2,x)
```

```
[Out] int((a + b*x^4)^p*(c + e*x^2)^2, x)
```

sympy [C] time = 79.16, size = 119, normalized size = 0.79

$$\frac{a^{p}c^{2}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{p}cex^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},-p\\\frac{7}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}+\frac{a^{p}e^{2}x^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{5}{4},-p\\\frac{9}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**2*(b*x**4+a)**p,x)
```

```
[Out] a**p*c**2*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4)) + a**p*c*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(2*gamma(7/4)) + a**p*e**2*x**5*gamma(5/4)*hyper((5/4, -p)
, (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

3.178 $\int \left(c+ex^2\right)\left(a+bx^4\right)^p\,dx$

Optimal. Leaf size=96

$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)$$

```
[Out] c*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)+1/3*e*x
^3*(b*x^4+a)^p*hypergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A] time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1204, 246, 245, 365, 364}

$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)*(a + b*x^4)^p,x]
```

```
[Out] (c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x
^4)/a)^p + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a
)])/(3*(1 + (b*x^4)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\int \left(c+ex^2\right)\left(a+bx^4\right)^p\,dx = \int \left(c\left(a+bx^4\right)^p + ex^2\left(a+bx^4\right)^p\right)dx$$
$$= c\int \left(a+bx^4\right)^p\,dx + e\int x^2\left(a+bx^4\right)^p\,dx$$
$$= \left(c\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \left(1+\frac{bx^4}{a}\right)^p dx + \left(e\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^2\left(1+\right.$$
$$= cx\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right) + \frac{1}{3}ex^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\right.$$

Mathematica [A] time = 0.02, size = 75, normalized size = 0.78

$$\frac{1}{3}x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(3c\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^2\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(3*c*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*x^2
*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)]))/(3*(1 + (b*x^4)/a)^p)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^2+c\right)\left(bx^4+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + c)*(b*x^4 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+c\right)\left(bx^4+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)*(b*x^4 + a)^p, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+c\right)\left(b\,x^4+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)*(b*x^4+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+c\right)\left(bx^4+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)*(b*x^4 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^4+a\right)^p\,\left(e\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p*(c + e*x^2),x)
```

```
[Out] int((a + b*x^4)^p*(c + e*x^2), x)
```

sympy [C] time = 42.84, size = 75, normalized size = 0.78

$$\frac{a^{p}cx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{p}ex^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},-p\\\frac{7}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)*(b*x**4+a)**p,x)
```

```
[Out] a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(5/4)) + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*gamma(7/4))
```

3.179 $\int \left(a+bx^4\right)^p\,dx$

Optimal. Leaf size=44

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

```
[Out] x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4
)/a)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)^p\,dx &= \left(\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \left(1+\frac{bx^4}{a}\right)^p\,dx\\ &= x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4
)/a)^p
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^p, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p,x)
[Out] int((b*x^4+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^p, x)
```

mupad [B] time = 4.36, size = 41, normalized size = 0.93

$$\frac{x\left(b\,x^4+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(\frac{b\,x^4}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p,x)
[Out] (x*(a + b*x^4)^p*hypergeom([1/4, -p], 5/4, -(b*x^4)/a))/((b*x^4)/a + 1)^p
```

sympy [C] time = 9.18, size = 34, normalized size = 0.77

$$\frac{a^p x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p,x)
```

```
[Out] a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gam
ma(5/4))
```

3.180 $\int \frac{(a+bx^4)^p}{c+ex^2}\,dx$

Optimal. Leaf size=123

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,1;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

```
[Out] x*(b*x^4+a)^p*AppellF1(1/4,1,-p,5/4,e^2*x^4/c^2,-b*x^4/a)/c/((1+b*x^4/a)^p)
-1/3*e*x^3*(b*x^4+a)^p*AppellF1(3/4,1,-p,7/4,e^2*x^4/c^2,-b*x^4/a)/c^2/((1+
b*x^4/a)^p)
```

Rubi [A] time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1240, 430, 429, 511, 510}

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,1;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p/(c + e*x^2),x]
```

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c
*(1 + (b*x^4)/a)^p) - (e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 1, 7/4, -((b*x
^4)/a), (e^2*x^4)/c^2])/(3*c^2*(1 + (b*x^4)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^p}{c+ex^2}\,dx &= \int \left(\frac{c\left(a+bx^4\right)^p}{c^2-e^2x^4}+\frac{ex^2\left(a+bx^4\right)^p}{-c^2+e^2x^4}\right)dx\\
&= c\int \frac{\left(a+bx^4\right)^p}{c^2-e^2x^4}\,dx+e\int \frac{x^2\left(a+bx^4\right)^p}{-c^2+e^2x^4}\,dx\\
&= \left(c\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{c^2-e^2x^4}\,dx+\left(e\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^4}{a}\right)^p}{-c^2+e^2x^4}\,dx\\
&= \frac{x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{3}{4};-p,1;\frac{7}{4};\right.}{3c^2}
\end{aligned}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^4\right)^p}{c+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^4)^p/(c + e*x^2),x]
[Out] Integrate[(a + b*x^4)^p/(c + e*x^2), x]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^4+a\right)^p}{ex^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p/(e*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^p}{ex^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^p}{e\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p/(e*x^2+c),x)
```

```
[Out] int((b*x^4+a)^p/(e*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^p}{ex^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^4 + a)^p}{e\,x^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p/(c + e*x^2),x)
```

```
[Out] int((a + b*x^4)^p/(c + e*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p/(e*x**2+c),x)
```

```
[Out] Timed out
```

3.181 $\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx$

Optimal. Leaf size=189

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2}+\frac{e^2x^5\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{5}{4};-p,2;\frac{9}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{5c^4}-\frac{2ex^3\left(a\right.}{}$$

```
[Out] x*(b*x^4+a)^p*AppellF1(1/4,2,-p,5/4,e^2*x^4/c^2,-b*x^4/a)/c^2/((1+b*x^4/a)^
p)-2/3*e*x^3*(b*x^4+a)^p*AppellF1(3/4,2,-p,7/4,e^2*x^4/c^2,-b*x^4/a)/c^3/((
1+b*x^4/a)^p)+1/5*e^2*x^5*(b*x^4+a)^p*AppellF1(5/4,2,-p,9/4,e^2*x^4/c^2,-b*
x^4/a)/c^4/((1+b*x^4/a)^p)
```

Rubi [A] time = 0.19, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1240, 430, 429, 511, 510}

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2}-\frac{2ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,2;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3}+\frac{e^2x^5\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p/(c + e*x^2)^2,x]
```

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c
^2*(1 + (b*x^4)/a)^p) - (2*e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 2, 7/4, -(
(b*x^4)/a), (e^2*x^4)/c^2])/(3*c^3*(1 + (b*x^4)/a)^p) + (e^2*x^5*(a + b*x^4
)^p*AppellF1[5/4, -p, 2, 9/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(5*c^4*(1 + (b*
x^4)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^p}{\left(c+ex^2\right)^2}\,dx = \int \left(\frac{c^2\left(a+bx^4\right)^p}{\left(c^2-e^2x^4\right)^2} - \frac{2cex^2\left(a+bx^4\right)^p}{\left(c^2-e^2x^4\right)^2} + \frac{e^2x^4\left(a+bx^4\right)^p}{\left(-c^2+e^2x^4\right)^2}\right)dx$$

$$= c^2\int \frac{\left(a+bx^4\right)^p}{\left(c^2-e^2x^4\right)^2}\,dx - (2ce)\int \frac{x^2\left(a+bx^4\right)^p}{\left(c^2-e^2x^4\right)^2}\,dx + e^2\int \frac{x^4\left(a+bx^4\right)^p}{\left(-c^2+e^2x^4\right)^2}\,dx$$

$$= \left(c^2\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{\left(c^2-e^2x^4\right)^2}\,dx - \left(2ce\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{b}{}\right.}{\left(c^2-e^2x\right.}$$

$$= \frac{x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2} - \frac{2ex^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{3}{4};-p,\right.}{3c^3}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^4\right)^p}{\left(c+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^4)^p/(c + e*x^2)^2,x]
[Out] Integrate[(a + b*x^4)^p/(c + e*x^2)^2, x]
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^p}{e^2x^4+2\,cex^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^p}{\left(ex^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+a\right)^{p}}{\left(e\,x^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p/(e*x^2+c)^2,x)
```

```
[Out] int((b*x^4+a)^p/(e*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{4}+a\right)^{p}}{\left(ex^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^{4}+a\right)^{p}}{\left(e\,x^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p/(c + e*x^2)^2,x)
```

```
[Out] int((a + b*x^4)^p/(c + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p/(e*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.182 $\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx$

Optimal. Leaf size=108

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)+\frac{3}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)+\frac{x^3(1-b(4p+7))\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)}{b(4p+7)}-\frac{x^3\left(bx^4+1\right)^{p+1}}{b(4p+7)}$$

```
[Out] -x^3*(b*x^4+1)^(1+p)/b/(7+4*p)+x*hypergeom([1/4, -p],[5/4],-b*x^4)+(1-b*(7+
4*p))*x^3*hypergeom([3/4, -p],[7/4],-b*x^4)/b/(7+4*p)+3/5*x^5*hypergeom([5/
4, -p],[9/4],-b*x^4)
```

Rubi [A] time = 0.11, antiderivative size = 103, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1207, 1893, 245, 364}

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-x^3\left(1-\frac{1}{4bp+7b}\right)\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)+\frac{3}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)-\frac{x^3\left(bx^4+1\right)^{p+1}}{b(4p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

```
[Out] -((x^3*(1 + b*x^4)^(1 + p))/(b*(7 + 4*p))) + x*Hypergeometric2F1[1/4, -p, 5
/4, -(b*x^4)] - (1 - (7*b + 4*b*p)^(-1))*x^3*Hypergeometric2F1[3/4, -p, 7/4
, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = -\frac{x^3\left(1+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\int \left(1+bx^4\right)^p \left(b(7+4p)+3(1-b(7+4p))x^2+3b(7+4p)x^4\right) dx}{b(7+4p)}$$

$$= -\frac{x^3\left(1+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\int \left(b(7+4p)\left(1+bx^4\right)^p+3(1-b(7+4p))x^2\left(1+bx^4\right)^p+3b(7+\right.}{b(7+4p)}$$

$$= -\frac{x^3\left(1+bx^4\right)^{1+p}}{b(7+4p)} + 3\int x^4\left(1+bx^4\right)^p dx - \left(3\left(1-\frac{1}{7b+4bp}\right)\right)\int x^2\left(1+bx^4\right)^p dx +$$

$$= -\frac{x^3\left(1+bx^4\right)^{1+p}}{b(7+4p)} + x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \left(1-\frac{1}{7b+4bp}\right)x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Mathematica [A] time = 0.02, size = 86, normalized size = 0.80

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{7}x^7\,{}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-bx^4\right)+\frac{3}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)-x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - x^3*Hypergeometric2F1[3/4, -p
, 7/4, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5 - (x
^7*Hypergeometric2F1[7/4, -p, 11/4, -(b*x^4)])/7
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(x^6-3\,x^4+3\,x^2-1\right)\left(bx^4+1\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral(-(x^6 - 3*x^4 + 3*x^2 - 1)*(b*x^4 + 1)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(x^2-1\right)^3\left(bx^4+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

maple [A] time = 0.15, size = 75, normalized size = 0.69

$$-\frac{x^7\,\text{hypergeom}\left(\left[\frac{7}{4},-p\right],\left[\frac{11}{4}\right],-b\,x^4\right)}{7}+\frac{3x^5\,\text{hypergeom}\left(\left[\frac{5}{4},-p\right],\left[\frac{9}{4}\right],-b\,x^4\right)}{5}-x^3\,\text{hypergeom}\left(\left[\frac{3}{4},-p\right],\left[\frac{7}{4}\right],-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^3*(b*x^4+1)^p,x)
```

```
[Out] -1/7*x^7*hypergeom([7/4,-p],[11/4],-b*x^4)+3/5*x^5*hypergeom([5/4,-p],[9/4]
,-b*x^4)-x^3*hypergeom([3/4,-p],[7/4],-b*x^4)+x*hypergeom([1/4,-p],[5/4],-b
*x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (x^2-1)^3 (bx^4+1)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int (x^2-1)^3 (b\,x^4+1)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)^3*(b*x^4 + 1)^p,x)
```

```
[Out] -int((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

sympy [C] time = 120.37, size = 129, normalized size = 1.19

$$-\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},-p\\ \frac{11}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{3x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{3x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle| bx^4\right.}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**3*(b*x**4+1)**p,x)
```

```
[Out] -x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi))/(4*gamma
(11/4)) + 3*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)
)/(4*gamma(9/4)) - 3*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_po
lar(I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*ex
p_polar(I*pi))/(4*gamma(5/4))
```

3.183 $\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p\, dx$

Optimal. Leaf size=86

$$-\frac{x(1-b(4p+5))\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)}{b(4p+5)}-\frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)+\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}$$

```
[Out] x*(b*x^4+1)^(1+p)/b/(5+4*p)-(1-b*(5+4*p))*x*hypergeom([1/4, -p],[5/4],-b*x^
4)/b/(5+4*p)-2/3*x^3*hypergeom([3/4, -p],[7/4],-b*x^4)
```

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1207, 1204, 245, 364}

$$x\left(1-\frac{1}{4bp+5b}\right)\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)+\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

```
[Out] (x*(1 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + (1 - (5*b + 4*b*p)^(-1))*x*Hypergeo
metric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4, -p, 7/4,
-(b*x^4)])/3
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p dx = \frac{x\left(1+bx^4\right)^{1+p}}{b(5+4p)} + \frac{\int \left(-1+b(5+4p)-2b(5+4p)x^2\right)\left(1+bx^4\right)^p dx}{b(5+4p)}$$
$$= \frac{x\left(1+bx^4\right)^{1+p}}{b(5+4p)} + \frac{\int \left((-1+b(5+4p))\left(1+bx^4\right)^p - 2b(5+4p)x^2\left(1+bx^4\right)^p\right) dx}{b(5+4p)}$$
$$= \frac{x\left(1+bx^4\right)^{1+p}}{b(5+4p)} - 2\int x^2\left(1+bx^4\right)^p dx + \left(1-\frac{1}{5b+4bp}\right)\int \left(1+bx^4\right)^p dx$$
$$= \frac{x\left(1+bx^4\right)^{1+p}}{b(5+4p)} + \left(1-\frac{1}{5b+4bp}\right)x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right.$$

Mathematica [A] time = 0.01, size = 65, normalized size = 0.76

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) + \frac{1}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right) - \frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4,
 -p, 7/4, -(b*x^4)])/3 + (x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(x^4-2\,x^2+1\right)\left(bx^4+1\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral((x^4 - 2*x^2 + 1)*(b*x^4 + 1)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2-1\right)^2\left(bx^4+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

maple [A] time = 0.09, size = 56, normalized size = 0.65

$$\frac{x^5\,\mathrm{hypergeom}\left(\left[\frac{5}{4},-p\right],\left[\frac{9}{4}\right],-b\,x^4\right)}{5} - \frac{2x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{4},-p\right],\left[\frac{7}{4}\right],-b\,x^4\right)}{3} + x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},-p\right],\left[\frac{5}{4}\right],-l\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^2*(b*x^4+1)^p,x)
```

```
[Out] 1/5*x^5*hypergeom([5/4,-p],[9/4],-b*x^4)-2/3*x^3*hypergeom([3/4,-p],[7/4],-
b*x^4)+x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2-1\right)^2 \left(bx^4+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2-1\right)^2 \left(b\,x^4+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^2*(b*x^4 + 1)^p,x)
```

```
[Out] int((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

sympy [C] time = 69.12, size = 94, normalized size = 1.09

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**2*(b*x**4+1)**p,x)
```

```
[Out] x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi))/(4*gamma(9
/4)) - x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(2*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))
/(4*gamma(5/4))
```

3.184 $\int (1-x^2)(1+bx^4)^p \, dx$

Optimal. Leaf size=42

$$x\, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{3}x^3\, {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

```
[Out] x*hypergeom([1/4, -p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4, -p],[7/4],-b*x^
4)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1204, 245, 364}

$$x\, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{3}x^3\, {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)])/3
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}\int (1-x^2)(1+bx^4)^p \, dx &= \int \left((1+bx^4)^p - x^2(1+bx^4)^p\right) dx\\ &= \int (1+bx^4)^p \, dx - \int x^2(1+bx^4)^p \, dx\\ &= x\, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{3}x^3\, {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$x\, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{3}x^3\, {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)])/3
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(x^2-1\right)\left(bx^4+1\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(x^2-1\right)\left(bx^4+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

maple [A] time = 0.08, size = 37, normalized size = 0.88

$$-\frac{x^3 \,\text{hypergeom}\left(\left[\frac{3}{4}, -p\right], \left[\frac{7}{4}\right], -b\,x^4\right)}{3} + x\,\text{hypergeom}\left(\left[\frac{1}{4}, -p\right], \left[\frac{5}{4}\right], -b\,x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)*(b*x^4+1)^p,x)
```

```
[Out] x*hypergeom([1/4,-p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4,-p],[7/4],-b*x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(x^2-1\right)\left(bx^4+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)*(b*x^4 + 1)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \left(x^2-1\right)\,\left(b\,x^4+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)*(b*x^4 + 1)^p,x)
```

```
[Out] -int((x^2 - 1)*(b*x^4 + 1)^p, x)
```

sympy [C] time = 35.98, size = 61, normalized size = 1.45

$$-\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)*(b*x**4+1)**p,x)
```

```
[Out] -x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(4*gamma(
7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*ga
mma(5/4))
```

3.185 $\int (1+bx^4)^p \, dx$

Optimal. Leaf size=18

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

[Out] x*hypergeom([1/4, -p],[5/4],-b*x^4)

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {245}

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + b*x^4)^p,x]

[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (1+bx^4)^p \, dx = x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + b*x^4)^p,x]

[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^4+1)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+1)^p,x, algorithm="fricas")

[Out] integral((b*x^4 + 1)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^4+1)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p,x, algorithm="giac")
[Out] integrate((b*x^4 + 1)^p, x)
```

maple [A] time = 0.08, size = 17, normalized size = 0.94

$$x\,\text{hypergeom}\left(\left[\frac{1}{4},-p\right],\left[\frac{5}{4}\right],-b\,x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p,x)
[Out] x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{4}+1\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p,x, algorithm="maxima")
[Out] integrate((b*x^4 + 1)^p, x)
```

mupad [B] time = 0.07, size = 15, normalized size = 0.83

$$x\,{}_2\text{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -b\,x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4 + 1)^p,x)
[Out] x*hypergeom([1/4, -p], 5/4, -b*x^4)
```

sympy [C] time = 7.71, size = 29, normalized size = 1.61

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\right|bx^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p,x)
[Out] x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*gamma(5/4)
)
```

3.186 $\int \frac{(1+bx^4)^p}{1-x^2}\,dx$

Optimal. Leaf size=50

$$xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)$$

```
[Out] x*AppellF1(1/4,1,-p,5/4,x^4,-b*x^4)+1/3*x^3*AppellF1(3/4,1,-p,7/4,x^4,-b*x^
4)
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1240, 429, 510}

$$\frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)+xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^4)^p/(1 - x^2),x]
```

```
[Out] x*AppellF1[1/4, 1, -p, 5/4, x^4, -(b*x^4)] + (x^3*AppellF1[3/4, 1, -p, 7/4,
 x^4, -(b*x^4)])/3
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+bx^4\right)^p}{1-x^2}\,dx&=\int \left(\frac{\left(1+bx^4\right)^p}{1-x^4}-\frac{x^2\left(1+bx^4\right)^p}{-1+x^4}\right)dx\\&=\int \frac{\left(1+bx^4\right)^p}{1-x^4}\,dx-\int \frac{x^2\left(1+bx^4\right)^p}{-1+x^4}\,dx\\&=xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)\end{aligned}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(1+bx^4\right)^p}{1-x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2),x]
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2), x]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(bx^4+1\right)^p}{x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="fricas")
[Out] integral(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^4+1\right)^p}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="giac")
[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+1\right)^p}{-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p/(-x^2+1),x)
[Out] int((b*x^4+1)^p/(-x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^4+1\right)^p}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="maxima")
[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(b\,x^4+1\right)^p}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^4 + 1)^p/(x^2 - 1),x)
```

```
[Out] -int((b*x^4 + 1)^p/(x^2 - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p/(-x**2+1),x)
```

```
[Out] Timed out
```

3.187 $\int \frac{(1+bx^4)^p}{(1-x^2)^2} \, dx$

Optimal. Leaf size=77

$$xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)$$

```
[Out] x*AppellF1(1/4,2,-p,5/4,x^4,-b*x^4)+2/3*x^3*AppellF1(3/4,2,-p,7/4,x^4,-b*x^
4)+1/5*x^5*AppellF1(5/4,2,-p,9/4,x^4,-b*x^4)
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1240, 429, 510}

$$\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)+xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^4)^p/(1 - x^2)^2,x]
```

```
[Out] x*AppellF1[1/4, 2, -p, 5/4, x^4, -(b*x^4)] + (2*x^3*AppellF1[3/4, 2, -p, 7/
4, x^4, -(b*x^4)])/3 + (x^5*AppellF1[5/4, 2, -p, 9/4, x^4, -(b*x^4)])/5
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+bx^4\right)^p}{\left(1-x^2\right)^2} \, dx &= \int \left(\frac{\left(1+bx^4\right)^p}{\left(-1+x^4\right)^2}+\frac{2x^2\left(1+bx^4\right)^p}{\left(-1+x^4\right)^2}+\frac{x^4\left(1+bx^4\right)^p}{\left(-1+x^4\right)^2}\right) dx\\
&= 2\int \frac{x^2\left(1+bx^4\right)^p}{\left(-1+x^4\right)^2} \, dx+\int \frac{\left(1+bx^4\right)^p}{\left(-1+x^4\right)^2} \, dx+\int \frac{x^4\left(1+bx^4\right)^p}{\left(-1+x^4\right)^2} \, dx\\
&= xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)+\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-bx^4\right)
\end{aligned}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2)^2,x]
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^2, x]
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^4+1)^p}{x^4-2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="fricas")
[Out] integral((b*x^4 + 1)^p/(x^4 - 2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4+1)^p}{(-x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p/(-x^2+1)^2,x)
[Out] int((b*x^4+1)^p/(-x^2+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^4+1)^p}{(x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4 + 1)^p/(x^2 - 1)^2,x)
```

```
[Out] int((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p/(-x**2+1)**2,x)
```

```
[Out] Timed out
```

3.188 $\int \frac{(1+bx^4)^p}{(1-x^2)^3} dx$

Optimal. Leaf size=101

$$xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{7}x^7F_1\left(\frac{7}{4};3,-p;\frac{11}{4};x^4,-bx^4\right)+\frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right)+x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-\right.$$

```
[Out] x*AppellF1(1/4,3,-p,5/4,x^4,-b*x^4)+x^3*AppellF1(3/4,3,-p,7/4,x^4,-b*x^4)+3
/5*x^5*AppellF1(5/4,3,-p,9/4,x^4,-b*x^4)+1/7*x^7*AppellF1(7/4,3,-p,11/4,x^4
,-b*x^4)
```

Rubi [A] time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1240, 429, 510}

$$\frac{1}{7}x^7F_1\left(\frac{7}{4};3,-p;\frac{11}{4};x^4,-bx^4\right)+\frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right)+x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right)+xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^4)^p/(1 - x^2)^3,x]
```

```
[Out] x*AppellF1[1/4, 3, -p, 5/4, x^4, -(b*x^4)] + x^3*AppellF1[3/4, 3, -p, 7/4,
x^4, -(b*x^4)] + (3*x^5*AppellF1[5/4, 3, -p, 9/4, x^4, -(b*x^4)])/5 + (x^7*
AppellF1[7/4, 3, -p, 11/4, x^4, -(b*x^4)])/7
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int \left(-\frac{(1+bx^4)^p}{(-1+x^4)^3} - \frac{3x^2(1+bx^4)^p}{(-1+x^4)^3} - \frac{3x^4(1+bx^4)^p}{(-1+x^4)^3} - \frac{x^6(1+bx^4)^p}{(-1+x^4)^3}\right) dx$$
$$= -\left(3\int \frac{x^2(1+bx^4)^p}{(-1+x^4)^3}\,dx\right) - 3\int \frac{x^4(1+bx^4)^p}{(-1+x^4)^3}\,dx - \int \frac{(1+bx^4)^p}{(-1+x^4)^3}\,dx - \int \frac{x^6(1+bx^4)^p}{(-1+x^4)^3}\,dx$$
$$= xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right) + x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right) + \frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right) +$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2)^3,x]
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^3, x]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(bx^4+1)^p}{x^6-3x^4+3x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="fricas")
[Out] integral(-(b*x^4 + 1)^p/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="giac")
[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx^4+1)^p}{(-x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p/(-x^2+1)^3,x)
[Out] int((b*x^4+1)^p/(-x^2+1)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(b\,x^4+1\right)^p}{\left(x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^4 + 1)^p/(x^2 - 1)^3,x)
```

```
[Out] int(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p/(-x**2+1)**3,x)
```

```
[Out] Timed out
```

3.189 $\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=51

$$\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4}{3}dex^3-\frac{1}{5}e^2x^5$$

```
[Out] -7*d^2*x-4/3*d*e*x^3-1/5*e^2*x^5+8*d^(5/2)*arctanh(x*e^(1/2)/d^(1/2))/e^(1/
2)
```

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1150, 390, 208}

$$\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4}{3}dex^3-\frac{1}{5}e^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

```
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqr
t[d]])/Sqrt[e]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^4}{d^2-e^2x^4}\,dx&=\int \frac{\left(d+ex^2\right)^3}{d-ex^2}\,dx\\&=\int \left(-7d^2-4dex^2-e^2x^4+\frac{8d^3}{d-ex^2}\right)dx\\&=-7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\left(8d^3\right)\int \frac{1}{d-ex^2}\,dx\\&=-7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4}{3}dex^3-\frac{1}{5}e^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

```
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqr
t[d]])/Sqrt[e]
```

fricas [A] time = 0.82, size = 116, normalized size = 2.27

$$\left[-\frac{1}{5}e^2x^5-\frac{4}{3}dex^3+4d^2\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-7d^2x,-\frac{1}{5}e^2x^5-\frac{4}{3}dex^3-8d^2\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [-1/5*e^2*x^5 - 4/3*d*e*x^3 + 4*d^2*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e)
+ d)/(e*x^2 - d)) - 7*d^2*x, -1/5*e^2*x^5 - 4/3*d*e*x^3 - 8*d^2*sqrt(-d/e)*
arctan(e*x*sqrt(-d/e)/d) - 7*d^2*x]
```

giac [B] time = 0.21, size = 144, normalized size = 2.82

$$4\left(\left(d^2\right)^{\frac{1}{4}}d^2e^{\frac{11}{2}}-\left(d^2\right)^{\frac{1}{4}}d|d|e^{\frac{11}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-6)}+2\left(\left(d^2\right)^{\frac{1}{4}}d^2e^{\frac{15}{2}}+\left(d^2\right)^{\frac{3}{4}}de^{\frac{15}{2}}\right)e^{(-8)}\log\left(\left|\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+x\right|\right)-2\left(\left(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 4*((d^2)^(1/4)*d^2*e^(11/2) - (d^2)^(1/4)*d*abs(d)*e^(11/2))*arctan(x*e^(1/
2)/(d^2)^(1/4))*e^(-6) + 2*((d^2)^(1/4)*d^2*e^(15/2) + (d^2)^(3/4)*d*e^(15/
2))*e^(-8)*log(abs((d^2)^(1/4)*e^(-1/2) + x)) - 2*((d^2)^(1/4)*d^2*e^(11/2)
 + (d^2)^(1/4)*d*abs(d)*e^(11/2))*e^(-6)*log(abs(-(d^2)^(1/4)*e^(-1/2) + x)
) - 1/15*(3*x^5*e^12 + 20*d*x^3*e^11 + 105*d^2*x*e^10)*e^(-10)
```

maple [A] time = 0.00, size = 42, normalized size = 0.82

$$-\frac{e^2x^5}{5}-\frac{4dex^3}{3}+\frac{8d^3\,\mathrm{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}-7d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(-e^2*x^4+d^2),x)
```

```
[Out] -1/5*e^2*x^5-4/3*d*e*x^3-7*d^2*x+8*d^3/(d*e)^(1/2)*arctanh(1/(d*e)^(1/2)*e*
x)
```

maxima [A] time = 2.25, size = 56, normalized size = 1.10

$$-\frac{1}{5}e^2x^5-\frac{4}{3}dex^3-\frac{4d^3\log\left(\frac{ex-\sqrt{de}}{ex+\sqrt{de}}\right)}{\sqrt{de}}-7d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -1/5*e^2*x^5 - 4/3*d*e*x^3 - 4*d^3*log((e*x - sqrt(d*e))/(e*x + sqrt(d*e)))
/sqrt(d*e) - 7*d^2*x
```

mupad [B] time = 0.09, size = 42, normalized size = 0.82

$$-7\,d^2\,x-\frac{e^2\,x^5}{5}-\frac{4\,d\,e\,x^3}{3}-\frac{d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x\,1\mathrm{i}}{\sqrt{d}}\right)\,8\mathrm{i}}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4/(d^2 - e^2*x^4),x)
```

```
[Out] - 7*d^2*x - (e^2*x^5)/5 - (d^(5/2)*atan((e^(1/2)*x*1i)/d^(1/2))*8i)/e^(1/2)
 - (4*d*e*x^3)/3
```

sympy [A] time = 0.24, size = 75, normalized size = 1.47

$$-7d^2x-\frac{4dex^3}{3}-\frac{e^2x^5}{5}-4\sqrt{\frac{d^5}{e}}\log\left(x-\frac{\sqrt{\frac{d^5}{e}}}{d^2}\right)+4\sqrt{\frac{d^5}{e}}\log\left(x+\frac{\sqrt{\frac{d^5}{e}}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(-e**2*x**4+d**2),x)
```

```
[Out] -7*d**2*x - 4*d*e*x**3/3 - e**2*x**5/5 - 4*sqrt(d**5/e)*log(x - sqrt(d**5/e
)/d**2) + 4*sqrt(d**5/e)*log(x + sqrt(d**5/e)/d**2)
```

3.190 $\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=38

$$\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

```
[Out] -3*d*x-1/3*e*x^3+4*d^(3/2)*arctanh(x*e^(1/2)/d^(1/2))/e^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1150, 390, 208}

$$\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
```

```
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx &= \int \frac{(d+ex^2)^2}{d-ex^2}\,dx \\
&= \int \left(-3d-ex^2+\frac{4d^2}{d-ex^2}\right)dx \\
&= -3dx-\frac{ex^3}{3}+\left(4d^2\right)\int \frac{1}{d-ex^2}\,dx \\
&= -3dx-\frac{ex^3}{3}+\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

fricas [A] time = 0.65, size = 90, normalized size = 2.37

$$\left[-\frac{1}{3}ex^3 + 2d\sqrt{\frac{d}{e}}\log\left(\frac{ex^2 + 2ex\sqrt{\frac{d}{e}} + d}{ex^2 - d}\right) - 3dx, -\frac{1}{3}ex^3 - 4d\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right) - 3dx\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="fricas")
[Out] [-1/3*e*x^3 + 2*d*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d))
- 3*d*x, -1/3*e*x^3 - 4*d*sqrt(-d/e)*arctan(e*x*sqrt(-d/e)/d) - 3*d*x]
```

giac [B] time = 0.23, size = 123, normalized size = 3.24

$$2\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}} - \left(d^2\right)^{\frac{1}{4}}|d|e^{\frac{11}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-6)}+\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{15}{2}} + \left(d^2\right)^{\frac{3}{4}}e^{\frac{15}{2}}\right)e^{(-8)}\log\left(\left|\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)} + x\right|\right)-\left(\left(d^2\right)^{\frac{1}{4}}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="giac")
[Out] 2*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(1/4)*abs(d)*e^(11/2))*arctan(x*e^(1/2)/(
d^2)^(1/4))*e^(-6) + ((d^2)^(1/4)*d*e^(15/2) + (d^2)^(3/4)*e^(15/2))*e^(-8)
*log(abs((d^2)^(1/4)*e^(-1/2) + x)) - ((d^2)^(1/4)*d*e^(11/2) + (d^2)^(1/4)
*abs(d)*e^(11/2))*e^(-6)*log(abs(-(d^2)^(1/4)*e^(-1/2) + x)) - 1/3*(x^3*e^7
 + 9*d*x*e^6)*e^(-6)
```

maple [A] time = 0.00, size = 31, normalized size = 0.82

$$-\frac{e\,x^3}{3} + \frac{4d^2\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - 3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(-e^2*x^4+d^2),x)
[Out] -1/3*e*x^3-3*d*x+4*d^2/(d*e)^(1/2)*arctanh(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.45, size = 45, normalized size = 1.18

$$-\frac{1}{3}ex^3 - \frac{2d^2\log\left(\frac{ex-\sqrt{de}}{ex+\sqrt{de}}\right)}{\sqrt{de}} - 3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="maxima")
[Out] -1/3*e*x^3 - 2*d^2*log((e*x - sqrt(d*e))/(e*x + sqrt(d*e)))/sqrt(d*e) - 3*d
*x
```

mupad [B] time = 0.05, size = 28, normalized size = 0.74

$$\frac{4\,d^{3/2}\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{e\,x^3}{3} - 3\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(d^2 - e^2*x^4),x)
```

```
[Out] (4*d^(3/2)*atanh((e^(1/2)*x)/d^(1/2)))/e^(1/2) - (e*x^3)/3 - 3*d*x
```

sympy [A] time = 0.20, size = 58, normalized size = 1.53

$$-3dx-\frac{ex^3}{3}-2\sqrt{\frac{d^3}{e}}\log\left(x-\frac{\sqrt{\frac{d^3}{e}}}{d}\right)+2\sqrt{\frac{d^3}{e}}\log\left(x+\frac{\sqrt{\frac{d^3}{e}}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(-e**2*x**4+d**2),x)
```

```
[Out] -3*d*x - e*x**3/3 - 2*sqrt(d**3/e)*log(x - sqrt(d**3/e)/d) + 2*sqrt(d**3/e)
*log(x + sqrt(d**3/e)/d)
```

3.191 $\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=29

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

```
[Out] -x+2*arctanh(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1150, 388, 208}

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(d^2 - e^2*x^4),x]
```

```
[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)^2}{d^2-e^2x^4}\,dx &= \int \frac{d+ex^2}{d-ex^2}\,dx\\
&= -x+(2d)\int \frac{1}{d-ex^2}\,dx\\
&= -x+\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(d^2 - e^2*x^4),x]
```

```
[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

fricas [A] time = 0.80, size = 73, normalized size = 2.52

$$\left[\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-x,-2\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right)-x\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d)) - x, -2*sqrt(-d/e
)*arctan(e*x*sqrt(-d/e)/d) - x]
```

giac [B] time = 0.21, size = 118, normalized size = 4.07

$$\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{7}{2}}-\left(d^2\right)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-4)}}{d}+\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)e^{(-6)}\log\left(\left|\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+x\right|\right)}{2\,d}-\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{7}{2}}+\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] ((d^2)^(1/4)*d*e^(7/2) - (d^2)^(1/4)*abs(d)*e^(7/2))*arctan(x*e^(1/2)/(d^2)
^(1/4))*e^(-4)/d + 1/2*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-
6)*log(abs((d^2)^(1/4)*e^(-1/2) + x))/d - 1/2*((d^2)^(1/4)*d*e^(7/2) + (d^2
)^(1/4)*abs(d)*e^(7/2))*e^(-4)*log(abs(-(d^2)^(1/4)*e^(-1/2) + x))/d - x
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$\frac{2d\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(-e^2*x^4+d^2),x)
```

```
[Out] -x+2*d/(d*e)^(1/2)*arctanh(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.45, size = 36, normalized size = 1.24

$$-\frac{d\log\left(\frac{ex-\sqrt{de}}{ex+\sqrt{de}}\right)}{\sqrt{de}}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -d*log((e*x - sqrt(d*e))/(e*x + sqrt(d*e)))/sqrt(d*e) - x
```

mupad [B] time = 4.43, size = 21, normalized size = 0.72

$$\frac{2\sqrt{d}\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(d^2 - e^2*x^4),x)
```

```
[Out] (2*d^(1/2)*atanh((e^(1/2)*x)/d^(1/2)))/e^(1/2) - x
```

sympy [A] time = 0.18, size = 34, normalized size = 1.17

$$-x - \sqrt{\frac{d}{e}} \log\left(x - \sqrt{\frac{d}{e}}\right) + \sqrt{\frac{d}{e}} \log\left(x + \sqrt{\frac{d}{e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

```
[Out] -x - sqrt(d/e)*log(x - sqrt(d/e)) + sqrt(d/e)*log(x + sqrt(d/e))
```

3.192 $\int \frac{d+ex^2}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}$$

[Out] arctanh(x*e^(1/2)/d^(1/2))/d^(1/2)/e^(1/2)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{1150, 208\}$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(d^2 - e^2*x^4),x]

[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^2}{d^2-e^2x^4}\,dx &= \int \frac{1}{d-ex^2}\,dx\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(d^2 - e^2*x^4),x]

[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])

fricas [A] time = 0.54, size = 68, normalized size = 2.83

$$\left[\frac{\sqrt{de}\,\log\left(\frac{ex^2+2\,\sqrt{de}\,x+d}{ex^2-d}\right)}{2\,de},\,-\frac{\sqrt{-de}\,\arctan\left(\frac{\sqrt{-de}\,x}{d}\right)}{de}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d))/(d*e), -sqrt(-d
*e)*arctan(sqrt(-d*e)*x/d)/(d*e)]
```

giac [B] time = 0.29, size = 116, normalized size = 4.83

$$\frac{\left(\left(d^2\right)^{\frac{1}{4}} d e^{\frac{7}{2}} - \left(d^2\right)^{\frac{1}{4}} |d| e^{\frac{7}{2}}\right) \arctan\left(\frac{x e^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right) e^{(-4)}}{2 d^2} + \frac{\left(\left(d^2\right)^{\frac{1}{4}} d e^{\frac{11}{2}} + \left(d^2\right)^{\frac{3}{4}} e^{\frac{11}{2}}\right) e^{(-6)} \log\left(\left|\left(d^2\right)^{\frac{1}{4}} e^{\left(-\frac{1}{2}\right)} + x\right|\right)}{4 d^2} - \frac{\left(\left(d^2\right)^{\frac{1}{4}} d e^{\frac{7}{2}} \cdot\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/2*((d^2)^(1/4)*d*e^(7/2) - (d^2)^(1/4)*abs(d)*e^(7/2))*arctan(x*e^(1/2)/(
d^2)^(1/4))*e^(-4)/d^2 + 1/4*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2)
)*e^(-6)*log(abs((d^2)^(1/4)*e^(-1/2) + x))/d^2 - 1/4*((d^2)^(1/4)*d*e^(7/2
) + (d^2)^(1/4)*abs(d)*e^(7/2))*e^(-4)*log(abs(-(d^2)^(1/4)*e^(-1/2) + x))/
d^2
```

maple [A] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\operatorname{arctanh}\left(\frac{e x}{\sqrt{d e}}\right)}{\sqrt{d e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-e^2*x^4+d^2),x)
```

```
[Out] 1/(d*e)^(1/2)*arctanh(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.35, size = 31, normalized size = 1.29

$$-\frac{\log\left(\frac{e x - \sqrt{d e}}{e x + \sqrt{d e}}\right)}{2 \sqrt{d e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -1/2*log((e*x - sqrt(d*e))/(e*x + sqrt(d*e)))/sqrt(d*e)
```

mupad [B] time = 0.06, size = 16, normalized size = 0.67

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)}{\sqrt{d}\, \sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(d^2 - e^2*x^4),x)
```

```
[Out] atanh((e^(1/2)*x)/d^(1/2))/(d^(1/2)*e^(1/2))
```

sympy [B] time = 0.15, size = 46, normalized size = 1.92

$$-\frac{\sqrt{\frac{1}{d e}} \log\left(-d \sqrt{\frac{1}{d e}} + x\right)}{2} + \frac{\sqrt{\frac{1}{d e}} \log\left(d \sqrt{\frac{1}{d e}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-e**2*x**4+d**2),x)
```

```
[Out] -sqrt(1/(d*e))*log(-d*sqrt(1/(d*e)) + x)/2 + sqrt(1/(d*e))*log(d*sqrt(1/(d*
e)) + x)/2
```

3.193 $\int \frac{1}{\left(d+ex^2\right)\left(d^2-e^2x^4\right)} dx$

Optimal. Leaf size=72

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}+\frac{x}{4d^2\left(d+ex^2\right)}$$

```
[Out] 1/4*x/d^2/(e*x^2+d)+1/2*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)+1/4*arcta
nh(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1150, 414, 522, 208, 205}

$$\frac{x}{4d^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(4*d^2*(d + e*x^2)) + ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(2*d^(5/2)*Sqrt[e]) + A
rcTanh[(Sqrt[e]*x)/Sqrt[d]]/(4*d^(5/2)*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \int \frac{1}{(d-ex^2)(d+ex^2)^2}\,dx$$
$$= \frac{x}{4d^2(d+ex^2)} - \frac{\int \frac{-3de+e^2x^2}{(d-ex^2)(d+ex^2)}\,dx}{4d^2e}$$
$$= \frac{x}{4d^2(d+ex^2)} + \frac{\int \frac{1}{d-ex^2}\,dx}{4d^2} + \frac{\int \frac{1}{d+ex^2}\,dx}{2d^2}$$
$$= \frac{x}{4d^2(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.90

$$\frac{\frac{\sqrt{d}x}{d+ex^2} + \frac{2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{4d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

```
[Out] ((Sqrt[d]*x)/(d + e*x^2) + (2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ArcTan
h[(Sqrt[e]*x)/Sqrt[d]]/Sqrt[e])/(4*d^(5/2))
```

fricas [A] time = 0.69, size = 189, normalized size = 2.62

$$\left[\frac{2dex + 4(ex^2+d)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right) + (ex^2+d)\sqrt{de}\log\left(\frac{ex^2+2\sqrt{de}x+d}{ex^2-d}\right)}{8(d^3e^2x^2+d^4e)}, \frac{dex - (ex^2+d)\sqrt{-de}\arctan\left(\frac{\sqrt{-de}x}{d}\right)}{4(d^3e^2x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*d*e*x + 4*(e*x^2 + d)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (e*x^2 + d)
*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d)))/(d^3*e^2*x^2 + d^4
*e), 1/4*(d*e*x - (e*x^2 + d)*sqrt(-d*e)*arctan(sqrt(-d*e)*x/d) - (e*x^2 +
d)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^2*x^2 +
 d^4*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -((d^
2*exp(2)^3)^(1/4)*abs(d)*exp(1)^2-d*exp(2)*(d^2*exp(2)^3)^(1/4))/(4*d^4*exp
(2)*exp(1)^2-4*d^4*exp(2)^2)*ln(abs(x-(d^2/exp(2))^(1/4)))+((d^2*exp(2)^3)^
(1/4))^3/(4*d^4*exp(2)^2*exp(1)-4*d^4*exp(1)*exp(2)^2)*ln(abs(x+(d^2/exp(2)
)^(1/4)))-((d^2*exp(2)^3)^(1/4)*abs(d)*exp(1)^2+d*exp(2)*(d^2*exp(2)^3)^(1/
4))/(2*d^4*exp(2)*exp(1)^2-2*d^4*exp(2)^2)*atan(x/(d^2/exp(2))^(1/4))-2*exp
```

```
(1)^2*1/2/(exp(2)*d^2-d^2*exp(1)^2)/sqrt(d*exp(1))*atan(x*exp(1)/sqrt(d*exp
(1)))
```

maple [A] time = 0.01, size = 55, normalized size = 0.76

$$\frac{x}{4\left(e\,x^{2}+d\right)d^{2}}+\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{4\sqrt{de}\;d^{2}}+\frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-e^2*x^4+d^2),x)
```

```
[Out] 1/4*x/d^2/(e*x^2+d)+1/2/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)+1/4/d^2/(
d*e)^(1/2)*arctanh(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.44, size = 71, normalized size = 0.99

$$\frac{x}{4\left(d^{2}ex^{2}+d^{3}\right)}+\frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d^{2}}-\frac{\log\left(\frac{ex-\sqrt{de}}{ex+\sqrt{de}}\right)}{8\sqrt{de}\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] 1/4*x/(d^2*e*x^2 + d^3) + 1/2*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2) - 1/8*l
og((e*x - sqrt(d*e))/(e*x + sqrt(d*e)))/(sqrt(d*e)*d^2)
```

mupad [B] time = 0.16, size = 74, normalized size = 1.03

$$\frac{x}{4\,d^{2}\,\left(e\,x^{2}+d\right)}+\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{d^{5}\,e}}{d^{3}}\right)\,\sqrt{d^{5}\,e}}{4\,d^{5}\,e}-\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{-d^{5}\,e}}{d^{3}}\right)\,\sqrt{-d^{5}\,e}}{2\,d^{5}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)),x)
```

```
[Out] x/(4*d^2*(d + e*x^2)) + (atanh((x*(d^5*e)^(1/2))/d^3)*(d^5*e)^(1/2))/(4*d^5
*e) - (atanh((x*(-d^5*e)^(1/2))/d^3)*(-d^5*e)^(1/2))/(2*d^5*e)
```

sympy [B] time = 0.45, size = 226, normalized size = 3.14

$$\frac{x}{4d^{3}+4d^{2}ex^{2}}-\frac{\sqrt{\frac{1}{d^{5}e}}\log\left(-\frac{d^{8}e\left(\frac{1}{d^{5}e}\right)^{\frac{3}{2}}}{10}-\frac{9d^{3}\sqrt{\frac{1}{d^{5}e}}}{10}+x\right)}{8}+\frac{\sqrt{\frac{1}{d^{5}e}}\log\left(\frac{d^{8}e\left(\frac{1}{d^{5}e}\right)^{\frac{3}{2}}}{10}+\frac{9d^{3}\sqrt{\frac{1}{d^{5}e}}}{10}+x\right)}{8}-\sqrt{-\frac{1}{d^{5}e}}\log\left(-\frac{4d^{8}e(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-e**2*x**4+d**2),x)
```

```
[Out] x/(4*d**3 + 4*d**2*e*x**2) - sqrt(1/(d**5*e))*log(-d**8*e*(1/(d**5*e))**(3/
2)/10 - 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 + sqrt(1/(d**5*e))*log(d**8*e*(1/
(d**5*e))**(3/2)/10 + 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 - sqrt(-1/(d**5*e))
*log(-4*d**8*e*(-1/(d**5*e))**(3/2)/5 - 9*d**3*sqrt(-1/(d**5*e))/5 + x)/4 +
 sqrt(-1/(d**5*e))*log(4*d**8*e*(-1/(d**5*e))**(3/2)/5 + 9*d**3*sqrt(-1/(d*
*5*e))/5 + x)/4
```

3.194 $\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx$

Optimal. Leaf size=89

$$\frac{7\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}+\frac{5x}{16d^3\left(d+ex^2\right)}+\frac{x}{8d^2\left(d+ex^2\right)^2}$$

```
[Out] 1/8*x/d^2/(e*x^2+d)^2+5/16*x/d^3/(e*x^2+d)+7/16*arctan(x*e^(1/2)/d^(1/2))/d
^(7/2)/e^(1/2)+1/8*arctanh(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1150, 414, 527, 522, 208, 205}

$$\frac{5x}{16d^3\left(d+ex^2\right)}+\frac{x}{8d^2\left(d+ex^2\right)^2}+\frac{7\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(8*d^2*(d + e*x^2)^2) + (5*x)/(16*d^3*(d + e*x^2)) + (7*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(16*d^(7/2)*Sqrt[e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(8*d^(7/2)*
Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(d+ex^2\right)^2\left(d^2-e^2x^4\right)}\,dx &= \int \frac{1}{\left(d-ex^2\right)\left(d+ex^2\right)^3}\,dx\\
&= \frac{x}{8d^2\left(d+ex^2\right)^2}-\frac{\int \frac{-7de+3e^2x^2}{\left(d-ex^2\right)\left(d+ex^2\right)^2}\,dx}{8d^2e}\\
&= \frac{x}{8d^2\left(d+ex^2\right)^2}+\frac{5x}{16d^3\left(d+ex^2\right)}+\frac{\int \frac{18d^2e^2-10de^3x^2}{\left(d-ex^2\right)\left(d+ex^2\right)}\,dx}{32d^4e^2}\\
&= \frac{x}{8d^2\left(d+ex^2\right)^2}+\frac{5x}{16d^3\left(d+ex^2\right)}+\frac{\int \frac{1}{d-ex^2}\,dx}{8d^3}+\frac{7\int \frac{1}{d+ex^2}\,dx}{16d^3}\\
&= \frac{x}{8d^2\left(d+ex^2\right)^2}+\frac{5x}{16d^3\left(d+ex^2\right)}+\frac{7\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.85

$$\frac{\frac{\sqrt{d}x\left(7d+5ex^2\right)}{\left(d+ex^2\right)^2}+\frac{7\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{16d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

```
[Out] ((Sqrt[d]*x*(7*d + 5*e*x^2))/(d + e*x^2)^2 + (7*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/Sqrt[e] + (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e])/(16*d^(7/2))
```

fricas [B] time = 0.86, size = 278, normalized size = 3.12

$$\left[\frac{5\,de^2x^3+7\,d^2ex+7\left(e^2x^4+2\,dex^2+d^2\right)\sqrt{de}\,\arctan\left(\frac{\sqrt{de}\,x}{d}\right)+\left(e^2x^4+2\,dex^2+d^2\right)\sqrt{de}\,\log\left(\frac{ex^2+2\,\sqrt{de}\,x+d}{ex^2-d}\right)}{16\left(d^4e^3x^4+2\,d^5e^2x^2+d^6e\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/16*(5*d*e^2*x^3 + 7*d^2*e*x + 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(d*e)*ar
ctan(sqrt(d*e)*x/d) + (e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(d*e)*log((e*x^2 + 2*
sqrt(d*e)*x + d)/(e*x^2 - d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e), 1/32*
```

```
(10*d*e^2*x^3 + 14*d^2*e*x - 4*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-d*e)*arcta
n(sqrt(-d*e)*x/d) - 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-d*e)*log((e*x^2 - 2
*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -(-2*
(d^2*exp(2)^3)^(1/4)*abs(d)*exp(1)^2+d*(d^2*exp(2)^3)^(1/4)*exp(1)^2+d*exp(
2)*(d^2*exp(2)^3)^(1/4))/(4*d^5*exp(1)^4-8*d^5*exp(2)*exp(1)^2+4*d^5*exp(2)
^2)*ln(abs(x-(d^2/exp(2))^(1/4)))+exp(2)*(d^2*exp(2)^3)^(1/4)/(4*d^4*exp(2)
*exp(1)^2-8*d^4*exp(1)*exp(2)*exp(1)+4*d^4*exp(2)^2)*ln(abs(x+(d^2/exp(2))^
(1/4)))-(-2*(d^2*exp(2)^3)^(1/4)*abs(d)*exp(1)^2-d*(d^2*exp(2)^3)^(1/4)*exp
(1)^2-d*exp(2)*(d^2*exp(2)^3)^(1/4))/(2*d^5*exp(1)^4-4*d^5*exp(2)*exp(1)^2+
2*d^5*exp(2)^2)*atan(x/(d^2/exp(2))^(1/4))-(-5*exp(2)*exp(1)^2+exp(1)^4)*1/
2/(-exp(2)^2*d^3+2*exp(2)*d^3*exp(1)^2-d^3*exp(1)^4)/sqrt(d*exp(1))*atan(x*
exp(1)/sqrt(d*exp(1)))+x*exp(1)^2/(-2*exp(2)*d^3+2*d^3*exp(1)^2)/(x^2*exp(1
)+d)
```

maple [A] time = 0.01, size = 73, normalized size = 0.82

$$\frac{5e\,x^{3}}{16\left(e\,x^{2}+d\right)^{2}d^{3}}+\frac{7x}{16\left(e\,x^{2}+d\right)^{2}d^{2}}+\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^{3}}+\frac{7\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de}\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x)
```

```
[Out] 5/16/d^3/(e*x^2+d)^2*x^3*e+7/16*x/d^2/(e*x^2+d)^2+7/16/d^3/(d*e)^(1/2)*arct
an(1/(d*e)^(1/2)*e*x)+1/8/d^3/(d*e)^(1/2)*arctanh(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.49, size = 92, normalized size = 1.03

$$\frac{5\,ex^{3}+7\,dx}{16\left(d^{3}e^{2}x^{4}+2\,d^{4}ex^{2}+d^{5}\right)}+\frac{7\,\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}\,d^{3}}-\frac{\log\left(\frac{ex-\sqrt{de}}{ex+\sqrt{de}}\right)}{16\,\sqrt{de}\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] 1/16*(5*e*x^3 + 7*d*x)/(d^3*e^2*x^4 + 2*d^4*e*x^2 + d^5) + 7/16*arctan(e*x/
sqrt(d*e))/(sqrt(d*e)*d^3) - 1/16*log((e*x - sqrt(d*e))/(e*x + sqrt(d*e)))/
(sqrt(d*e)*d^3)
```

mupad [B] time = 0.16, size = 96, normalized size = 1.08

$$\frac{\frac{7\,x}{16\,d^{2}}+\frac{5\,e\,x^{3}}{16\,d^{3}}}{d^{2}+2\,d\,e\,x^{2}+e^{2}\,x^{4}}+\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{d^{7}\,e}}{d^{4}}\right)\,\sqrt{d^{7}\,e}}{8\,d^{7}\,e}-\frac{7\,\operatorname{atanh}\left(\frac{x\,\sqrt{-d^{7}\,e}}{d^{4}}\right)\,\sqrt{-d^{7}\,e}}{16\,d^{7}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^2),x)
```

```
[Out] ((7*x)/(16*d^2) + (5*e*x^3)/(16*d^3))/(d^2 + e^2*x^4 + 2*d*e*x^2) + (atanh(
(x*(d^7*e)^(1/2))/d^4)*(d^7*e)^(1/2))/(8*d^7*e) - (7*atanh((x*(-d^7*e)^(1/2
))/d^4)*(-d^7*e)^(1/2))/(16*d^7*e)
```

sympy [B] time = 0.72, size = 257, normalized size = 2.89

$$-\frac{\sqrt{\frac{1}{d^7e}}\log\left(-\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371}-\frac{351d^4\sqrt{\frac{1}{d^7e}}}{371}+x\right)}{16}+\frac{\sqrt{\frac{1}{d^7e}}\log\left(\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371}+\frac{351d^4\sqrt{\frac{1}{d^7e}}}{371}+x\right)}{16}-\frac{7\sqrt{-\frac{1}{d^7e}}\log\left(-\frac{245d^{11}e\left(-\right.}{106}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

```
[Out] -sqrt(1/(d**7*e))*log(-20*d**11*e*(1/(d**7*e))**(3/2)/371 - 351*d**4*sqrt(1
/(d**7*e))/371 + x)/16 + sqrt(1/(d**7*e))*log(20*d**11*e*(1/(d**7*e))**(3/2
)/371 + 351*d**4*sqrt(1/(d**7*e))/371 + x)/16 - 7*sqrt(-1/(d**7*e))*log(-24
5*d**11*e*(-1/(d**7*e))**(3/2)/106 - 351*d**4*sqrt(-1/(d**7*e))/106 + x)/32
 + 7*sqrt(-1/(d**7*e))*log(245*d**11*e*(-1/(d**7*e))**(3/2)/106 + 351*d**4*
sqrt(-1/(d**7*e))/106 + x)/32 - (-7*d*x - 5*e*x**3)/(16*d**5 + 32*d**4*e*x*
*2 + 16*d**3*e**2*x**4)
```

3.195 $\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

```
[Out] -arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(1/2)+arctanh(x*2^(1/2)*e^(1/2)/(e*x^
2+d)^(1/2))*2^(1/2)/e^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1150, 402, 217, 206, 377, 208}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[e]) + (Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
```

```
&& EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx &= \int \frac{\sqrt{d+ex^2}}{d-ex^2}\,dx\\ &= (2d)\int \frac{1}{(d-ex^2)\sqrt{d+ex^2}}\,dx - \int \frac{1}{\sqrt{d+ex^2}}\,dx\\ &= (2d)\,\text{Subst}\left(\int \frac{1}{d-2dex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right) - \text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.98

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]] - Log[e*x + Sqrt[e]*S
qrt[d + e*x^2]])/Sqrt[e]
```

fricas [A] time = 0.74, size = 199, normalized size = 3.21

$$\left[\frac{\sqrt{2}\sqrt{e}\log\left(\frac{17e^2x^4+14dex^2+d^2+\frac{4\sqrt{2}(3e^2x^3+dex)\sqrt{ex^2+d}}{\sqrt{e}}}{e^2x^4-2dex^2+d^2}\right) + 2\sqrt{e}\log\left(-2ex^2+2\sqrt{ex^2+d}\sqrt{e}x-d\right)}{4e}, -\frac{\sqrt{2}e\sqrt{-\frac{1}{e}}\ \text{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + d^2 + 4*sqrt(2)*(3*e^2
*x^3 + d*e*x)*sqrt(e*x^2 + d)/sqrt(e))/(e^2*x^4 - 2*d*e*x^2 + d^2)) + 2*sqr
t(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/e, -1/2*(sqrt(2)*e*sq
rt(-1/e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-1/e)/(e*x^3
 + d*x)) - 2*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e]
```

giac [A] time = 0.25, size = 24, normalized size = 0.39

$$\frac{1}{2}e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/2*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)
```

maple [B] time = 0.06, size = 1442, normalized size = 23.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x)
```

```
[Out] 1/6*e/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))/(-d*e)^(1/2)*(
(x-1/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(3/2)+1/4*e/(
(-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*((x-1/e*(-d*e)^(1/2))
^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2)*x+1/4*e^(1/2)/((-d*e)^(1/2)
+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d*ln(((x-1/e*(-d*e)^(1/2))*e+(-d*
e)^(1/2))/e^(1/2)+((x-1/e*(-d*e)^(1/2))^2*e+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1
/2)))^(1/2))-1/6*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d
*e)^(1/2))*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(3/2
)-1/4*e/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*((x-(d*e)^(1
/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2)*x-5/4*e^(1/2)/((-d*e)
^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d*ln(((x-(d*e)^(1/2)/e)*e+(
d*e)^(1/2))/e^(1/2)+((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+
2*d)^(1/2))-e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(
1/2))*d*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2)+e
/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d^(3/2)
*2^(1/2)*ln((4*d+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x-(d*e
)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2))/(x-(d*e)^(1/2)/e
))-1/6*e/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))/(-d*e)^(1/2
)*((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(3/2)+1/4*
e/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*((x+1/e*(-d*e)^(1/
2))^2*e-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2)*x+1/4*e^(1/2)/((-d*e)^(1
/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d*ln(((x+1/e*(-d*e)^(1/2))*e-(
-d*e)^(1/2))/e^(1/2)+((x+1/e*(-d*e)^(1/2))^2*e-2*(-d*e)^(1/2)*(x+1/e*(-d*e)
^(1/2)))^(1/2))+1/6*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)
+(d*e)^(1/2))*((x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(
3/2)-1/4*e/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*((x+(d*e)
^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2)*x-5/4*e^(1/2)/((-d
*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d*ln(((x+(d*e)^(1/2)/e)*
e-(d*e)^(1/2))/e^(1/2)+((x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/
e)+2*d)^(1/2))+e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e
)^(1/2))*d*((x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2
)-e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d^(3
/2)*2^(1/2)*ln((4*d-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x+(
d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2))/(x+(d*e)^(1/2
)/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{e^2x^4-d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 + d)^(3/2)/(e^2*x^4 - d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(e\,x^2+d\right)^{3/2}}{d^2-e^2\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x)
```

```
[Out] int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{d+ex^2}}{-d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(sqrt(d + e*x**2)/(-d + e*x**2), x)
```

3.196 $\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}\,d\sqrt{e}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d*2^(1/2)/e^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1150, 377, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}\,d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx &= \int \frac{1}{\left(d-ex^2\right)\sqrt{d+ex^2}}\,dx\\ &= \text{Subst}\left(\int \frac{1}{d-2dex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}\,d\sqrt{e}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 38, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}\,d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

fricas [A] time = 0.62, size = 138, normalized size = 3.63

$$\left[\frac{\sqrt{2}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}\left(3\,ex^3+dx\right)\sqrt{ex^2+d}\,\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right)}{8\,d\sqrt{e}},\ -\frac{\sqrt{2}\,\sqrt{-e}\,\arctan\left(\frac{\sqrt{2}\left(3\,ex^2+d\right)\sqrt{ex^2+d}\,\sqrt{-e}}{4\left(e^2x^3+dex\right)}\right)}{4\,de}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(2)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt(
e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2))/(d*sqrt(e)), -1/4*sq
rt(2)*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e
^2*x^3 + d*e*x))/(d*e)]
```

giac [B] time = 0.53, size = 131, normalized size = 3.45

$$-\frac{\left(\sqrt{2}\,i\arctan\left(\frac{e^{\frac{1}{2}}}{\sqrt{-\frac{de+\sqrt{d^2}\,e}{d}}}\right)e^{\frac{1}{2}}-\sqrt{2}\,i\arctan\left(\frac{e^{\frac{1}{2}}}{\sqrt{-\frac{de-\sqrt{d^2}\,e}{d}}}\right)e^{\frac{1}{2}}\right)e^{(-1)}\mathrm{sgn}(x)}{4\,|d|}+\frac{\sqrt{2}\,i\arctan\left(\frac{\sqrt{\frac{d}{x^2}+e}}{\sqrt{-\frac{d\mathrm{sgn}(x)+\sqrt{d^2}\,e}{d\mathrm{sgn}(x)}}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,|d|\left|\mathrm{sgn}(x)\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*i*arctan(e^(1/2)/sqrt(-(d*e + sqrt(d^2)*e)/d))*e^(1/2) - sqrt
(2)*i*arctan(e^(1/2)/sqrt(-(d*e - sqrt(d^2)*e)/d))*e^(1/2))*e^(-1)*sgn(x)/a
bs(d) + 1/2*sqrt(2)*i*arctan(sqrt(d/x^2 + e)/sqrt(-(d*e*sgn(x) + sqrt(d^2)*
e)/(d*sgn(x))))*e^(-1/2)/(abs(d)*abs(sgn(x)))
```

maple [B] time = 0.02, size = 986, normalized size = 25.95

$$-\frac{\sqrt{2}\,\sqrt{d}\,e\ln\left(\frac{4d+2\sqrt{2}\,\sqrt{2d+\left(x-\frac{\sqrt{de}}{e}\right)^2e+2\sqrt{de}\,\left(x-\frac{\sqrt{de}}{e}\right)}\,\sqrt{d}+2\sqrt{de}\,\left(x-\frac{\sqrt{de}}{e}\right)}{x-\frac{\sqrt{de}}{e}}\right)}{2\sqrt{de}\,\left(\sqrt{-de}+\sqrt{de}\right)\left(\sqrt{-de}-\sqrt{de}\right)}+\frac{\sqrt{2}\,\sqrt{d}\,e\ln\left(\frac{4d+2\sqrt{2}\,\sqrt{2d+\left(x+\frac{\sqrt{de}}{e}\right)^2e-2\sqrt{de}\,\left(x\cdots\right.}\cdots}{x+\frac{\sqrt{de}}{e}}\cdots\right)}{2\sqrt{de}\,\left(\sqrt{-de}+\sqrt{de}\right)\left(\sqrt{-de}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x)
```

```
[Out] -1/2*e/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/(-d*e)^(1/2)*(
(x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)-1/2*e^(1/2)
/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*ln(((x-(-d*e)^(1/2)/
e)*e+(-d*e)^(1/2))/e^(1/2)+((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)
^(1/2)/e))^(1/2))+1/2*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2
)-(d*e)^(1/2))*(2*d+(x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e))^
(1/2)+1/2*e^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*ln(
((x-(d*e)^(1/2)/e)*e+(d*e)^(1/2))/e^(1/2)+(2*d+(x-(d*e)^(1/2)/e)^2*e+2*(d*e
)^(1/2)*(x-(d*e)^(1/2)/e))^(1/2))-1/2*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/
2))/((-d*e)^(1/2)-(d*e)^(1/2))*d^(1/2)*2^(1/2)*ln((4*d+2*2^(1/2)*(2*d+(x-(d
*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e))^(1/2)*d^(1/2)+2*(d*e)^(1/
```

```
2)*(x-(d*e)^(1/2)/e))/(x-(d*e)^(1/2)/e))+1/2*e/((-d*e)^(1/2)+(d*e)^(1/2))/(
(-d*e)^(1/2)-(d*e)^(1/2))/(-d*e)^(1/2)*((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/
2)*(x+(-d*e)^(1/2)/e))^(1/2)-1/2*e^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)
^(1/2)-(d*e)^(1/2))*ln(((x+(-d*e)^(1/2)/e)*e-(-d*e)^(1/2))/e^(1/2)+((x+(-d*
e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2))-1/2*e/(d*e)^(1/2)
/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*(2*d+(x+(d*e)^(1/2)/
e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e))^(1/2)+1/2*e^(1/2)/((-d*e)^(1/2)+(d*
e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*ln(((x+(d*e)^(1/2)/e)*e-(d*e)^(1/2))/e
^(1/2)+(2*d+(x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e))^(1/2))+1
/2*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*d^(1
/2)*2^(1/2)*ln((4*d+2*2^(1/2)*(2*d+(x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(
d*e)^(1/2)/e))^(1/2)*d^(1/2)-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e))/(x+(d*e)^(1/2
)/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ex^2+d}}{e^2x^4-d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(e*x^2 + d)/(e^2*x^4 - d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{e\,x^2+d}}{d^2-e^2\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4),x)
```

```
[Out] int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-d\sqrt{d+ex^2}+ex^2\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d*sqrt(d + e*x**2) + e*x**2*sqrt(d + e*x**2)), x)
```

3.197 $\int \frac{1}{\sqrt{d+ex^2}\left(d^2-e^2x^4\right)}\,dx$

Optimal. Leaf size=61

$$\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}\,d^2\sqrt{e}}$$

```
[Out] 1/4*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d^2*2^(1/2)/e^(1/2)+1/2*x/d^
2/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1150, 382, 377, 208}

$$\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}\,d^2\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(2
*Sqrt[2]*d^2*Sqrt[e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(d^2-e^2x^4\right)}\,dx=\int \frac{1}{\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\,dx$$
$$=\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\int \frac{1}{\left(d-ex^2\right)\sqrt{d+ex^2}}\,dx}{2d}$$
$$=\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\text{Subst}\left(\int \frac{1}{d-2dex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2d}$$
$$=\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

Mathematica [**A**] time = 0.13, size = 108, normalized size = 1.77

$$\frac{\frac{4x}{\sqrt{d+ex^2}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{2}\sqrt{d+ex^2}}\right)}{\sqrt{e}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{e}x}{\sqrt{2}\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

```
[Out] ((4*x)/Sqrt[d + e*x^2] - (Sqrt[2]*ArcTanh[(Sqrt[d] - Sqrt[e]*x)/(Sqrt[2]*Sq
rt[d + e*x^2])])/Sqrt[e] + (Sqrt[2]*ArcTanh[(Sqrt[d] + Sqrt[e]*x)/(Sqrt[2]*
Sqrt[d + e*x^2])])/Sqrt[e])/(8*d^2)
```

fricas [**B**] time = 0.61, size = 209, normalized size = 3.43

$$\left[\frac{\sqrt{2}\left(ex^2+d\right)\sqrt{e}\log\left(\frac{17e^2x^4+14dex^2+4\sqrt{2}\left(3ex^3+dx\right)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2dex^2+d^2}\right)+8\sqrt{ex^2+d}ex}{16\left(d^2e^2x^2+d^3e\right)},-\frac{\sqrt{2}\left(ex^2+d\right)\sqrt{-e}\arctan\left(\frac{\sqrt{2}\left(3e\right.}{}\right.}{8\left(d^2e^2x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(2)*(e*x^2 + d)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2)
*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)
) + 8*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e), -1/8*(sqrt(2)*(e*x^2 + d)
*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^2*x^
3 + d*e*x)) - 4*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e)]
```

giac [**A**] time = 0.33, size = 1, normalized size = 0.02

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [**B**] time = 0.02, size = 441, normalized size = 7.23

$$-\frac{\sqrt{2}\,e\ln\left(\frac{4d+2\sqrt{2}\sqrt{2d+\left(x-\frac{\sqrt{de}}{e}\right)^2e+2\sqrt{de}\left(x-\frac{\sqrt{de}}{e}\right)}\sqrt{d}+2\sqrt{de}\left(x-\frac{\sqrt{de}}{e}\right)}{x-\frac{\sqrt{de}}{e}}\right)}{4\sqrt{de}\left(\sqrt{-de}+\sqrt{de}\right)\left(\sqrt{-de}-\sqrt{de}\right)\sqrt{d}}+\frac{\sqrt{2}\,e\ln\left(\frac{4d+2\sqrt{2}\sqrt{2d+\left(x+\frac{\sqrt{de}}{e}\right)^2e-2\sqrt{de}\left(x+\frac{\sqrt{de}}{e}\right)}\sqrt{d}-2\sqrt{}}{x+\frac{\sqrt{de}}{e}}\right)}{4\sqrt{de}\left(\sqrt{-de}+\sqrt{de}\right)\left(\sqrt{-de}-\sqrt{de}\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x)
```

```
[Out] -1/2/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d/(x-(-d*e)^(1/2
)/e)*((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)-1/4*e
/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*2^(1/2)/
d^(1/2)*ln((4*d+2*2^(1/2)*(2*d+(x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)
^(1/2)/e))^(1/2)*d^(1/2)+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e))/(x-(d*e)^(1/2)/e)
)-1/2/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d/(x+(-d*e)^(1/
2)/e)*((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)+1/4*
e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*2^(1/2)
/d^(1/2)*ln((4*d+2*2^(1/2)*(2*d+(x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e
)^(1/2)/e))^(1/2)*d^(1/2)-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e))/(x+(d*e)^(1/2)/e
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(e^2x^4-d^2\right)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((e^2*x^4 - d^2)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(d^2-e^2\,x^4\right)\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(1/2)),x)
```

```
[Out] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-d^2\sqrt{d+ex^2}+e^2x^4\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d**2*sqrt(d + e*x**2) + e**2*x**4*sqrt(d + e*x**2)), x)
```

3.198 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(d^2-e^2x^4\right)}\,dx$

Optimal. Leaf size=80

$$\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}\,d^3\sqrt{e}}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}$$

```
[Out] 1/6*x/d^2/(e*x^2+d)^(3/2)+1/8*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d^
3*2^(1/2)/e^(1/2)+7/12*x/d^3/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1150, 414, 527, 12, 377, 208}

$$\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}\,d^3\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(6*d^2*(d + e*x^2)^(3/2)) + (7*x)/(12*d^3*Sqrt[d + e*x^2]) + ArcTanh[(Sqr
t[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*Sqrt[2]*d^3*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(d^2-e^2x^4\right)}\,dx &= \int \frac{1}{\left(d-ex^2\right)\left(d+ex^2\right)^{5/2}}\,dx \\
&= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}} - \frac{\int \frac{-5de+2e^2x^2}{\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\,dx}{6d^2e} \\
&= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\int \frac{3d^2e^2}{\left(d-ex^2\right)\sqrt{d+ex^2}}\,dx}{12d^4e^2} \\
&= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\int \frac{1}{\left(d-ex^2\right)\sqrt{d+ex^2}}\,dx}{4d^2} \\
&= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{d-2dex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{4d^2} \\
&= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}\,d^3\sqrt{e}}
\end{aligned}
$$

Mathematica [C] time = 3.34, size = 345, normalized size = 4.31

$$
\frac{\frac{384e^4x^8\left(d+ex^2\right)^2 {}_3F_2\left(2,2,2;1,\frac{9}{2};-\frac{2ex^2}{d-ex^2}\right)}{ex^2-d} + \frac{384e^4x^8\left(4d^2+7dex^2+3e^2x^4\right) {}_2F_1\left(2,2;\frac{9}{2};-\frac{2ex^2}{d-ex^2}\right)}{ex^2-d} + \frac{35\sqrt{2}\sqrt{\frac{ex^2}{ex^2-d}}\left(-15d^3-5d^2ex^2+12de^2x^4+8e^3x^6\right)\left(\ldots\right.}{\ldots}}{2520d^5e^3x^5\sqrt{d+ex^2}\left(1-\frac{e^2x^4}{d^2}\right)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

```
[Out] ((35*Sqrt[2]*Sqrt[(e*x^2)/(-d + e*x^2)]*(-15*d^3 - 5*d^2*e*x^2 + 12*d*e^2*x
^4 + 8*e^3*x^6)*(Sqrt[2]*Sqrt[(e*x^2)/(-d + e*x^2)]*Sqrt[(d + e*x^2)/(d - e
*x^2)]*(-3*d^2 - 2*d*e*x^2 + 5*e^2*x^4) + 3*(d + e*x^2)^2*ArcSin[Sqrt[2]*Sq
rt[(e*x^2)/(-d + e*x^2)]]))/Sqrt[(d + e*x^2)/(d - e*x^2)] + (384*e^4*x^8*(4
*d^2 + 7*d*e*x^2 + 3*e^2*x^4)*Hypergeometric2F1[2, 2, 9/2, (-2*e*x^2)/(d -
e*x^2)])/(-d + e*x^2) + (384*e^4*x^8*(d + e*x^2)^2*HypergeometricPFQ[{2, 2,
 2}, {1, 9/2}, (-2*e*x^2)/(d - e*x^2)])/(-d + e*x^2))/(2520*d^5*e^3*x^5*Sqr
t[d + e*x^2]*(1 - (e^2*x^4)/d^2))
```

fricas [B] time = 0.77, size = 279, normalized size = 3.49

$$\left[\frac{3\sqrt{2}\left(e^2x^4+2\,dex^2+d^2\right)\sqrt{e}\,\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}\left(3\,ex^3+dx\right)\sqrt{ex^2+d}\,\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right)+8\left(7\,e^2x^3+9\,dex\right)\sqrt{ex^2+d}}{96\left(d^3e^3x^4+2\,d^4e^2x^2+d^5e\right)},-\frac{3\sqrt{2}(\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e)*log((17*e^2*x^4 + 14*d
*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4
- 2*d*e*x^2 + d^2)) + 8*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*x^4
 + 2*d^4*e^2*x^2 + d^5*e), -1/48*(3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqr
t(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^2*x^3 +
d*e*x)) - 4*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*x^4 + 2*d^4*e^2
*x^2 + d^5*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.03, size = 911, normalized size = 11.39

$$-\frac{\sqrt{2}\,e\ln\left(\frac{4d+2\sqrt{2}\,\sqrt{2d+\left(x-\frac{\sqrt{de}}{e}\right)^2e+2\sqrt{de}\left(x-\frac{\sqrt{de}}{e}\right)}\,\sqrt{d}+2\sqrt{de}\left(x-\frac{\sqrt{de}}{e}\right)}{x-\frac{\sqrt{de}}{e}}\right)}{8\sqrt{de}\,\left(\sqrt{-de}+\sqrt{de}\right)\left(\sqrt{-de}-\sqrt{de}\right)d^{\frac{3}{2}}}+\frac{\sqrt{2}\,e\ln\left(\frac{4d+2\sqrt{2}\,\sqrt{2d+\left(x+\frac{\sqrt{de}}{e}\right)^2e-2\sqrt{de}\left(x+\frac{\sqrt{de}}{e}\right)}\,\sqrt{d}-2\cdots}{x+\frac{\sqrt{de}}{e}}\right)}{8\sqrt{de}\,\left(\sqrt{-de}+\sqrt{de}\right)\left(\sqrt{-de}-\sqrt{de}\right)d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x)
```

```
[Out] -1/6/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d/(x-(-d*e)^(1/2
)/e)/((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)-1/3*e
/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d^2/((x-(-d*e)^(1/2)
/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)*x+1/4*e/(d*e)^(1/2)/((-d*e
)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d/(2*d+(x-(d*e)^(1/2)/e)^2*
e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e))^(1/2)-1/4*e/((-d*e)^(1/2)+(d*e)^(1/2))/(
(-d*e)^(1/2)-(d*e)^(1/2))/d^2/(2*d+(x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(
d*e)^(1/2)/e))^(1/2)*x-1/8*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)
^(1/2)-(d*e)^(1/2))/d^(3/2)*2^(1/2)*ln((4*d+2*2^(1/2)*(2*d+(x-(d*e)^(1/2)/e
)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e))^(1/2)*d^(1/2)+2*(d*e)^(1/2)*(x-(d*e)
^(1/2)/e))/(x-(d*e)^(1/2)/e))-1/6/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-
(d*e)^(1/2))/d/(x+(-d*e)^(1/2)/e)/((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x
+(-d*e)^(1/2)/e))^(1/2)-1/3*e/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e
)^(1/2))/d^2/((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/
2)*x-1/4*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2)
)/d/(2*d+(x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e))^(1/2)-1/4*e
/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d^2/(2*d+(x+(d*e)^(1
```

```
/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e))^(1/2)*x+1/8*e/(d*e)^(1/2)/((-d*
e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d^(3/2)*2^(1/2)*ln((4*d+2*
2^(1/2)*(2*d+(x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e))^(1/2)*d
^(1/2)-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e))/(x+(d*e)^(1/2)/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(e^2x^4-d^2\right)\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((e^2*x^4 - d^2)*(e*x^2 + d)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d^2-e^2\,x^4\right)\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(3/2)),x)
```

```
[Out] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-d^3\sqrt{d+ex^2}-d^2ex^2\sqrt{d+ex^2}+de^2x^4\sqrt{d+ex^2}+e^3x^6\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d**3*sqrt(d + e*x**2) - d**2*e*x**2*sqrt(d + e*x**2) + d*e**2
*x**4*sqrt(d + e*x**2) + e**3*x**6*sqrt(d + e*x**2)), x)
```

3.199 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=153

$$-\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] -1/4*x*(-b*x^2+a)*(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)-9/8*a*x*(-b*x^2+a)*(
b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/8*a^2*arctan(x*b^(1/2)/(-b*x^2+a)^(1
/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1152, 416, 388, 217, 203}

$$-\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-9*a*x*(a - b*x^2)*Sqrt[a + b*x^2])/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^
2)*(a + b*x^2)^(3/2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqr
t[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^2
*x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{\left(a+bx^2\right)^2}{\sqrt{a-bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$
$$= -\frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}} - \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{-5a^2b-9ab^2x^2}{\sqrt{a-bx^2}}\,dx}{4b\sqrt{a^2-b^2x^4}}$$
$$= -\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx}}}{8\sqrt{a^2-b^2x^4}}$$
$$= -\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\Big(}{8\sqrt{a^2-b^2x^4}}$$
$$= -\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\Big(\frac{\sqrt{}}{\sqrt{a}}}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [C] time = 0.17, size = 98, normalized size = 0.64

$$-\frac{\left(11ax+2bx^3\right)\sqrt{a^2-b^2x^4}}{8\sqrt{a+bx^2}} + \frac{19ia^2\log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}} - 2i\sqrt{b}\,x\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -1/8*((11*a*x + 2*b*x^3)*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2] + (((19*I)/8)
*a^2*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[
b]
```

fricas [A] time = 0.73, size = 251, normalized size = 1.64

$$\left[-\frac{19\left(a^2bx^2+a^3\right)\sqrt{-b}\,\log\left(-\frac{2\,b^2x^4+abx^2-2\,\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{-b}\,x-a^2}{bx^2+a}\right)+2\,\sqrt{-b^2x^4+a^2}\,\left(2\,b^2x^3+11\,abx\right)\sqrt{bx^2+a}}{16\left(b^2x^2+ab\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(19*(a^2*b*x^2 + a^3)*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b
^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a)) + 2*sqrt(-b^2*
x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*sqrt(b*x^2 + a))/(b^2*x^2 + a*b), -1/8*(1
9*(a^2*b*x^2 + a^3)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqr
t(b)/(b^2*x^3 + a*b*x)) + sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*sqrt(
b*x^2 + a))/(b^2*x^2 + a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

maple [A] time = 0.07, size = 132, normalized size = 0.86

$$-\frac{\sqrt{-b^2x^4+a^2}\,\left(2\sqrt{-b\,x^2+a}\,b^{\frac{3}{2}}x^3-32a^2\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{\left(-bx+\sqrt{ab}\right)\left(bx+\sqrt{ab}\right)}{b}}}\right)+13a^2\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+a}}\right)+11\sqrt{-b\,x^2+a}\,a}{8\sqrt{b\,x^2+a}\,\sqrt{-b\,x^2+a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/8*(-b^2*x^4+a^2)^(1/2)*(2*x^3*b^(3/2)*(-b*x^2+a)^(1/2)+11*(-b*x^2+a)^(1/
2)*b^(1/2)*x*a+13*arctan(1/(-b*x^2+a)^(1/2)*b^(1/2)*x)*a^2-32*arctan(b^(1/2
)*x/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b)^(1/2)))^(1/2))*a^2)/(b*x^2+a)^(1/2)/(-
b*x^2+a)^(1/2)/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{5/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.200 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=110

$$\frac{3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\,\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

```
[Out] -1/2*x*(-b*x^2+a)*(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/2*a*arctan(x*b^(1/
2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2
)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1152, 388, 217, 203}

$$\frac{3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\,\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -(x*(a - b*x^2)*Sqrt[a + b*x^2])/(2*Sqrt[a^2 - b^2*x^4]) + (3*a*Sqrt[a - b*
x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(2*Sqrt[b]*Sqrt[a
^2 - b^2*x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{a+bx^2}{\sqrt{a-bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$
$$= -\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{\left(3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}}\,dx}{2\sqrt{a^2-b^2x^4}}$$
$$= -\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{\left(3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{2\sqrt{a^2-b^2x^4}}$$
$$= -\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Mathematica [C] time = 0.07, size = 86, normalized size = 0.78

$$-\frac{x\sqrt{a^2-b^2x^4}}{2\sqrt{a+bx^2}} + \frac{3ia\log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}} - 2i\sqrt{b}\,x\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -1/2*(x*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2] + (((3*I)/2)*a*Log[(-2*I)*Sqrt
[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]
```

fricas [A] time = 0.72, size = 223, normalized size = 2.03

$$\left[-\frac{2\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,bx + 3\left(abx^2+a^2\right)\sqrt{-b}\,\log\left(-\frac{2b^2x^4+abx^2-2\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{-b}\,x-a^2}{bx^2+a}\right)}{4\left(b^2x^2+ab\right)}, -\frac{\sqrt{-b^2x^4+a^2}\,\sqrt{b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt(
-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt
(-b)*x - a^2)/(b*x^2 + a)))/(b^2*x^2 + a*b), -1/2*(sqrt(-b^2*x^4 + a^2)*sqr
t(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2)*sq
rt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(b^2*x^2 + a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

maple [A] time = 0.02, size = 107, normalized size = 0.97

$$-\frac{\sqrt{-b^2x^4+a^2}\,\left(-4a\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{\left(-bx+\sqrt{ab}\right)\left(bx+\sqrt{ab}\right)}{b}}}\right) + a\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+a}}\right) + \sqrt{-b\,x^2+a}\,\sqrt{b}\,x\right)}{2\sqrt{b\,x^2+a}\,\sqrt{-b\,x^2+a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/2/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(x*(-b*x^2+a)^(1/2)*b^(1/2)+a*arc
tan(1/(-b*x^2+a)^(1/2)*b^(1/2)*x)-4*arctan(1/((-b*x+(a*b)^(1/2))*(b*x+(a*b)
^(1/2))/b)^(1/2)*b^(1/2)*x)*a)/(-b*x^2+a)^(1/2)/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.201 $\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}} \, dx$

Optimal. Leaf size=65

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)
/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1152, 217, 203}

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt
[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}} \, dx &= \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}} \, dx}{\sqrt{a^2-b^2x^4}}\\
&= \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1+bx^2} \, dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}}\\
&= \frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\,\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 50, normalized size = 0.77

$$\frac{i\log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}-2i\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
[Out] (I*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]
```

fricas [A] time = 0.85, size = 121, normalized size = 1.86

$$\left[-\frac{\sqrt{-b}\log\left(-\frac{2\,b^2x^4+abx^2-2\,\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{-b}\,x-a^2}{bx^2+a}\right)}{2\,b},-\frac{\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{b}}{b^2x^3+abx}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^
2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a))/b, -arctan(sqrt(-b^2*x^4 + a^2)*sqrt(
b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x))/sqrt(b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

maple [A] time = 0.02, size = 69, normalized size = 1.06

$$\frac{\sqrt{-b^2x^4+a^2}\,\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{\left(-bx+\sqrt{ab}\right)\left(bx+\sqrt{ab}\right)}{b}}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{-b\,x^2+a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
[Out] 1/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*arctan(1/((-b*x+(a*b)^(1/2))*(b*x+(a
*b)^(1/2))/b)^(1/2)*b^(1/2)*x)/(-b*x^2+a)^(1/2)/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.202 $\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/2*arctan(x*2^(1/2)*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(
1/2)/a*2^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1152, 377, 205}

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]
])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}\left(a+bx^2\right)}\,dx}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a+2abx^2}\,dx,x,\frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\,\sqrt{a^2-b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 1.00

$$\frac{\sqrt{a^2-b^2x^4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt[2]*
a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

fricas [A] time = 0.99, size = 152, normalized size = 1.95

$$\left[-\frac{\sqrt{2}\sqrt{-b}\log\left(-\frac{3b^2x^4+2abx^2-2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{b^2x^4+2abx^2+a^2}\right)}{4ab}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{2(b^2x^3+abx)}\right)}{2a\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^
4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*
b), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sq
rt(b)/(b^2*x^3 + a*b*x))/(a*sqrt(b))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)
```

maple [B] time = 0.06, size = 249, normalized size = 3.19

$$\frac{\sqrt{-b^2x^4+a^2}\left(\sqrt{2}\sqrt{a}\sqrt{b}\ln\left(\frac{2\left(a-\sqrt{-ab}\,x+\sqrt{2}\sqrt{-bx^2+a}\sqrt{a}\right)b}{bx-\sqrt{-ab}}\right)-\sqrt{2}\sqrt{a}\sqrt{b}\ln\left(\frac{2\left(a+\sqrt{-ab}\,x+\sqrt{2}\sqrt{-bx^2+a}\sqrt{a}\right)b}{bx+\sqrt{-ab}}\right)-2\sqrt{-\cdots}\right.}{2\sqrt{bx^2+a}\sqrt{-bx^2+a}\left(\sqrt{-ab}+\sqrt{ab}\right)\left(\sqrt{-ab}-\sqrt{ab}\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/2*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(
-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*b^(1/2)-a^(1/2)*2^(1/
2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(
1/2)))*b^(1/2)-2*(-a*b)^(1/2)*arctan(1/((-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2)
)/b)^(1/2)*b^(1/2)*x)+2*(-a*b)^(1/2)*arctan(1/(-b*x^2+a)^(1/2)*b^(1/2)*x))/
(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))/((-a*b)^(1/2)-(
a*b)^(1/2))/(-a*b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\,\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a + b*x**2)), x)
```

3.203 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=125

$$\frac{x\left(a-bx^2\right)}{4a^2\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a+bx^2}\,\sqrt{a-bx^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/4*x*(-b*x^2+a)/a^2/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/8*arctan(x*2^(1
/2)*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^2*2^(1/2)/
b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1152, 382, 377, 205}

$$\frac{x\left(a-bx^2\right)}{4a^2\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a+bx^2}\,\sqrt{a-bx^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a - b*x^2))/(4*a^2*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b
*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(4*Sqrt[
2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2} \sqrt{a^2-b^2x^4}} \, dx = \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}\left(a+bx^2\right)^2} \, dx}{\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a-bx^2\right)}{4a^2\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a-bx^2}\left(a+bx^2\right)} \, dx}{4a\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a-bx^2\right)}{4a^2\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{a+2abx^2} \, dx, x, \right.}{4a\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a-bx^2\right)}{4a^2\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 0.89

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}\,x\sqrt{a-bx^2}+3\sqrt{2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)\right)}{8a^2\sqrt{b}\,\sqrt{a-bx^2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2] + 3*Sqrt[2]*(a + b*x^2)*A
rcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(8*a^2*Sqrt[b]*Sqrt[a - b*x^2]
*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.91, size = 297, normalized size = 2.38

$$\left[\frac{4\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,bx-3\sqrt{2}\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-b}\,\log\left(-\frac{3\,b^2x^4+2\,abx^2-2\,\sqrt{2}\,\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{-b}\,x-a^2}{b^2x^4+2\,abx^2+a^2}\right)}{16\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2*
a*b*x^2 + a^2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x
^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(
a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^
2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b)*arctan(1/2*sqrt(
2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(a^2*b^
3*x^4 + 2*a^3*b^2*x^2 + a^4*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\left(bx^2+a\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

maple [B] time = 0.06, size = 488, normalized size = 3.90

$$-\frac{\sqrt{-b^2x^4+a^2}\left(3\sqrt{2}\,\sqrt{a}\,b^{\frac{3}{2}}x^2\ln\left(\frac{2\left(a-\sqrt{-ab}\,x+\sqrt{2}\,\sqrt{-b\,x^2+a}\,\sqrt{a}\right)b}{bx-\sqrt{-ab}}\right)-3\sqrt{2}\,\sqrt{a}\,b^{\frac{3}{2}}x^2\ln\left(\frac{2\left(a+\sqrt{-ab}\,x+\sqrt{2}\,\sqrt{-b\,x^2+a}\,\sqrt{a}\right)b}{bx+\sqrt{-ab}}\right)-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/4*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(3*2^(1/2)*ln(2*(a-(-a*b)^(1/2)*x+2^(1/2)
*(-b*x^2+a)^(1/2)*a^(1/2))/(b*x-(-a*b)^(1/2))*b)*x^2*b^(3/2)*a^(1/2)-3*2^(1
/2)*ln(2*(a+(-a*b)^(1/2)*x+2^(1/2)*(-b*x^2+a)^(1/2)*a^(1/2))/(b*x+(-a*b)^(1
/2))*b)*x^2*b^(3/2)*a^(1/2)+3*2^(1/2)*ln(2*(a-(-a*b)^(1/2)*x+2^(1/2)*(-b*x^
2+a)^(1/2)*a^(1/2))/(b*x-(-a*b)^(1/2))*b)*a^(3/2)*b^(1/2)-3*2^(1/2)*ln(2*(a
+(-a*b)^(1/2)*x+2^(1/2)*(-b*x^2+a)^(1/2)*a^(1/2))/(b*x+(-a*b)^(1/2))*b)*a^(
3/2)*b^(1/2)+4*arctan(1/(-b*x^2+a)^(1/2)*b^(1/2)*x)*x^2*b*(-a*b)^(1/2)-4*ar
ctan(1/((-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2))/b)^(1/2)*b^(1/2)*x)*x^2*b*(-a*
b)^(1/2)-4*b^(1/2)*(-a*b)^(1/2)*(-b*x^2+a)^(1/2)*x+4*arctan(1/(-b*x^2+a)^(1
/2)*b^(1/2)*x)*a*(-a*b)^(1/2)-4*arctan(1/((-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/
2))/b)^(1/2)*b^(1/2)*x)*a*(-a*b)^(1/2))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/((
-a*b)^(1/2)+(a*b)^(1/2))^2/((-a*b)^(1/2)-(a*b)^(1/2))^2/(-a*b)^(1/2)/(b*x+(
-a*b)^(1/2))/(b*x-(-a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(3/2)),x)
```

```
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\left(a+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(3/2)), x)
```

3.204 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=168

$$\frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/8*x*(-b*x^2+a)/a^2/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)+9/32*x*(-b*x^2+a)
/a^3/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/64*arctan(x*2^(1/2)*b^(1/2)/(-
b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^3*2^(1/2)/b^(1/2)/(-b^2*
x^4+a^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1152, 414, 527, 12, 377, 205}

$$\frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a - b*x^2))/(8*a^2*(a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a - b
*x^2))/(32*a^3*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*S
qrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(32*Sqrt[2]*a^3
*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}\left(a+bx^2\right)^3}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} - \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{-7ab+2b^2x^2}{\sqrt{a-bx^2}\left(a+bx^2\right)^2}\,dx}{8a^2b\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx\right.}{32a^4b^2\sqrt{}}$$

$$= \frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\,\sqrt{a+\right.}{32a^2\sqrt{}}$$

$$= \frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\,\sqrt{a+\right.}{}$$

$$= \frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2}\,\sqrt{a+b}}{32\sqrt{2}\,a^3\sqrt{b}}$$

Mathematica [A] time = 0.11, size = 123, normalized size = 0.73

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}\,x\sqrt{a-bx^2}\left(13a+9bx^2\right)+19\sqrt{2}\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)\right)}{64a^3\sqrt{b}\sqrt{a-bx^2}\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2]*(13*a + 9*b*x^2) + 19*Sqr
t[2]*(a + b*x^2)^2*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(64*a^3*Sq
rt[b]*Sqrt[a - b*x^2]*(a + b*x^2)^(5/2))
```

fricas [A] time = 0.93, size = 365, normalized size = 2.17

$$\left[-\frac{19\sqrt{2}\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{-b}\log\left(-\frac{3\,b^2x^4+2\,abx^2-2\,\sqrt{2}\,\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{-b}\,x-a^2}{b^2x^4+2\,abx^2+a^2}\right)-4\,\sqrt{-b^2x^4+a^2}}{128\left(a^3b^4x^6+3\,a^4b^3x^4+3\,a^5b^2x^2+a^6b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(19*sqrt(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-b)*lo
g(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*
sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(-b^2*x^4 + a^2)*(9*
b^2*x^3 + 13*a*b*x)*sqrt(b*x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b
^2*x^2 + a^6*b), -1/64*(19*sqrt(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3)*sqrt(b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)
/(b^2*x^3 + a*b*x)) - 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 + 13*a*b*x)*sqrt(b*
x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 + a^6*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-b^2x^4+a^2}\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

maple [B] time = 0.06, size = 711, normalized size = 4.23

$$-\frac{\sqrt{-b^2x^4+a^2}\left(19\sqrt{2}\,\sqrt{a}\,b^{\frac{5}{2}}x^4\ln\left(\frac{2\left(a-\sqrt{-ab}\,x+\sqrt{2}\,\sqrt{-b\,x^2+a}\,\sqrt{a}\right)b}{bx-\sqrt{-ab}}\right)-19\sqrt{2}\,\sqrt{a}\,b^{\frac{5}{2}}x^4\ln\left(\frac{2\left(a+\sqrt{-ab}\,x+\sqrt{2}\,\sqrt{-b\,x^2+a}\,\sqrt{a}\right.}{bx+\sqrt{-ab}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/16*(-b^2*x^4+a^2)^(1/2)*b^(9/2)*(19*2^(1/2)*ln(2*(a-(-a*b)^(1/2)*x+2^(1/
2)*(-b*x^2+a)^(1/2)*a^(1/2))/(b*x-(-a*b)^(1/2))*b)*x^4*b^(5/2)*a^(1/2)-19*2
^(1/2)*ln(2*(a+(-a*b)^(1/2)*x+2^(1/2)*(-b*x^2+a)^(1/2)*a^(1/2))/(b*x+(-a*b)
^(1/2))*b)*x^4*b^(5/2)*a^(1/2)+38*2^(1/2)*ln(2*(a-(-a*b)^(1/2)*x+2^(1/2)*(-
b*x^2+a)^(1/2)*a^(1/2))/(b*x-(-a*b)^(1/2))*b)*x^2*a^(3/2)*b^(3/2)-38*2^(1/2
)*ln(2*(a+(-a*b)^(1/2)*x+2^(1/2)*(-b*x^2+a)^(1/2)*a^(1/2))/(b*x+(-a*b)^(1/2
))*b)*x^2*a^(3/2)*b^(3/2)+16*arctan(1/(-b*x^2+a)^(1/2)*b^(1/2)*x)*x^4*b^2*(
-a*b)^(1/2)-16*arctan(1/((-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2))/b)^(1/2)*b^(1
/2)*x)*x^4*b^2*(-a*b)^(1/2)-36*b^(3/2)*(-a*b)^(1/2)*(-b*x^2+a)^(1/2)*x^3+19
*2^(1/2)*ln(2*(a-(-a*b)^(1/2)*x+2^(1/2)*(-b*x^2+a)^(1/2)*a^(1/2))/(b*x-(-a*
b)^(1/2))*b)*a^(5/2)*b^(1/2)-19*2^(1/2)*ln(2*(a+(-a*b)^(1/2)*x+2^(1/2)*(-b*
x^2+a)^(1/2)*a^(1/2))/(b*x+(-a*b)^(1/2))*b)*a^(5/2)*b^(1/2)+32*arctan(1/(-b
*x^2+a)^(1/2)*b^(1/2)*x)*x^2*a*b*(-a*b)^(1/2)-32*arctan(1/((-b*x+(a*b)^(1/2
))*(b*x+(a*b)^(1/2))/b)^(1/2)*b^(1/2)*x)*x^2*a*b*(-a*b)^(1/2)-52*a*(-a*b)^(
1/2)*b^(1/2)*(-b*x^2+a)^(1/2)*x+16*arctan(1/(-b*x^2+a)^(1/2)*b^(1/2)*x)*a^2
*(-a*b)^(1/2)-16*arctan(1/((-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2))/b)^(1/2)*b^
(1/2)*x)*a^2*(-a*b)^(1/2))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/(-a*b)^(1/2)/((
```

```
-a*b)^(1/2)+(a*b)^(1/2))^3/(-(-a*b)^(1/2)+(a*b)^(1/2))^3/(b*x+(-a*b)^(1/2))
^2/(b*x-(-a*b)^(1/2))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\left(bx^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2 - b^2\,x^4}\,\left(b\,x^2 + a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(5/2)),x)
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a + bx^2\right)\left(a + bx^2\right)}\,\left(a + bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(5/2)), x)
```

3.205 $\int \frac{\left(a-bx^2\right)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=152

$$-\frac{9ax\sqrt{a-bx^2}\left(a+bx^2\right)}{8\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] -1/4*x*(-b*x^2+a)^(3/2)*(b*x^2+a)/(-b^2*x^4+a^2)^(1/2)-9/8*a*x*(b*x^2+a)*(-
b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1152, 416, 388, 217, 206}

$$-\frac{9ax\sqrt{a-bx^2}\left(a+bx^2\right)}{8\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-9*a*x*Sqrt[a - b*x^2]*(a + b*x^2))/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^
2)^(3/2)*(a + b*x^2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqr
t[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^
2*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(a-bx^2\right)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{\left(a-bx^2\right)^2}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= -\frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{5a^2b-9ab^2x^2}{\sqrt{a+bx^2}}\,dx}{4b\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax\sqrt{a-bx^2}\left(a+bx^2\right)}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}}\,d\ldots}{8\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax\sqrt{a-bx^2}\left(a+bx^2\right)}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{\ldots}{1\ldots}\right.}{8\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax\sqrt{a-bx^2}\left(a+bx^2\right)}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}\ldots}{\sqrt{a+b\ldots}}\right.}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [A] time = 0.20, size = 123, normalized size = 0.81

$$\frac{1}{8}\left(\frac{x\left(2bx^2-11a\right)\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}} + \frac{19a^2\log\left(\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}+abx-b^2x^3\right)}{\sqrt{b}} - \frac{19a^2\log\left(bx^2-a\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] ((x*(-11*a + 2*b*x^2)*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2] - (19*a^2*Log[-a
 + b*x^2])/Sqrt[b] + (19*a^2*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*
Sqrt[a^2 - b^2*x^4]])/Sqrt[b])/8
```

fricas [A] time = 0.95, size = 265, normalized size = 1.74

$$\left[\frac{19\left(a^2bx^2-a^3\right)\sqrt{b}\log\left(\frac{2b^2x^4-abx^2-2\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{bx^2-a}\right)-2\sqrt{-b^2x^4+a^2}\left(2b^2x^3-11abx\right)\sqrt{-bx^2+a}}{16\left(b^2x^2-ab\right)}, \frac{19\left(\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(19*(a^2*b*x^2 - a^3)*sqrt(b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*
x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b*x^2 - a)) - 2*sqrt(-b^2*x^4
 + a^2)*(2*b^2*x^3 - 11*a*b*x)*sqrt(-b*x^2 + a))/(b^2*x^2 - a*b), 1/8*(19*(
a^2*b*x^2 - a^3)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt
(-b)/(b^2*x^3 - a*b*x)) - sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 - 11*a*b*x)*sqrt(
-b*x^2 + a))/(b^2*x^2 - a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

maple [A] time = 0.02, size = 105, normalized size = 0.69

$$-\frac{\sqrt{-b\,x^2+a}\,\sqrt{-b^2x^4+a^2}\,\left(2\sqrt{b\,x^2+a}\,b^{\frac{3}{2}}x^3+19a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)-11\sqrt{b\,x^2+a}\,a\sqrt{b}\,x\right)}{8\left(b\,x^2-a\right)\sqrt{b\,x^2+a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
[Out] -1/8*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(2*x^3*b^(3/2)*(b*x^2+a)^(1/2)-1
1*x*a*b^(1/2)*(b*x^2+a)^(1/2)+19*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*a^2)/(b*x^2-
a)/(b*x^2+a)^(1/2)/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^2\right)^{5/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a - b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a-bx^2\right)^{\frac{5}{2}}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral((a - b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.206 $\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=109

$$\frac{3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\,\sqrt{a^2-b^2x^4}}-\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

```
[Out] -1/2*x*(b*x^2+a)*(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/2*a*arctanh(x*b^(1
/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2
)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1152, 388, 217, 206}

$$\frac{3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\,\sqrt{a^2-b^2x^4}}-\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -(x*Sqrt[a - b*x^2]*(a + b*x^2))/(2*Sqrt[a^2 - b^2*x^4]) + (3*a*Sqrt[a - b*
x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]*Sqrt[
a^2 - b^2*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{a-bx^2}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$
$$= -\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}} + \frac{\left(3a\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2\sqrt{a^2-b^2x^4}}$$
$$= -\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}} + \frac{\left(3a\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2\sqrt{a^2-b^2x^4}}$$
$$= -\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}} + \frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [A] time = 0.11, size = 110, normalized size = 1.01

$$\frac{1}{2}\left(-\frac{x\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}} + \frac{3a\log\left(\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}+abx-b^2x^3\right)}{\sqrt{b}} - \frac{3a\log\left(bx^2-a\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
[Out] (-((x*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2]) - (3*a*Log[-a + b*x^2])/Sqrt[b]
 + (3*a*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]])
/Sqrt[b])/2
```

fricas [A] time = 1.01, size = 236, normalized size = 2.17

$$\left[\frac{2\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\,bx + 3\left(abx^2-a^2\right)\sqrt{b}\log\left(\frac{2b^2x^4-abx^2-2\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}\,x-a^2}{bx^2-a}\right)}{4\left(b^2x^2-ab\right)}, \frac{\sqrt{-b^2x^4+a^2}\sqrt{-b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
[Out] [1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt(
b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(
b)*x - a^2)/(b*x^2 - a)))/(b^2*x^2 - a*b), 1/2*(sqrt(-b^2*x^4 + a^2)*sqrt(-
b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqr
t(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/(b^2*x^2 - a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

maple [A] time = 0.01, size = 85, normalized size = 0.78

$$-\frac{\sqrt{-b\,x^{2}+a}\,\sqrt{-b^{2}x^{4}+a^{2}}\,\left(3a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)-\sqrt{b\,x^{2}+a}\,\sqrt{b}\,x\right)}{2\left(b\,x^{2}-a\right)\sqrt{b\,x^{2}+a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/2*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(-x*(b*x^2+a)^(1/2)*b^(1/2)+3*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))*a)/(b*x^2-a)/(b*x^2+a)^(1/2)/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^{2}+a\right)^{\frac{3}{2}}}{\sqrt{-b^{2}x^{4}+a^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^{2}\right)^{3/2}}{\sqrt{a^{2}-b^{2}\,x^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2),x)
```

```
[Out] int((a - b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a-bx^{2}\right)^{\frac{3}{2}}}{\sqrt{-\left(-a+bx^{2}\right)\left(a+bx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a - b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.207 $\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)
/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1152, 217, 206}

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqr
t[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\,\sqrt{a^2-b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 1.05

$$\frac{\log \left(\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}+abx-b^2x^3\right)-\log \left(bx^2-a\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-Log[-a + b*x^2] + Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2
- b^2*x^4]])/Sqrt[b]
```

fricas [A] time = 0.94, size = 125, normalized size = 1.95

$$\left[\frac{\log \left(\frac{2\,b^2x^4-abx^2-2\,\sqrt{-b^2x^4+a^2}\,\sqrt{-bx^2+a}\,\sqrt{b}\,x-a^2}{bx^2-a}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\,\sqrt{-bx^2+a}\,\sqrt{-b}}{b^2x^3-abx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqr
t(b)*x - a^2)/(b*x^2 - a))/sqrt(b), sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sq
rt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

maple [A] time = 0.01, size = 54, normalized size = 0.84

$$\frac{\sqrt{-b^2x^4+a^2}\;\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{-b\,x^2+a}\,\sqrt{b\,x^2+a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/(-b*x^2+a)^(1/2)/(b*x^2+a)^(1/2)/b^(1/2)*(-b^2*x^4+a^2)^(1/2)*ln(b^(1/2)*
x+(b*x^2+a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a-b\,x^2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a - b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral(sqrt(a - b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.208 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(
1/2)/a*2^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1152, 377, 208}

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a-bx^2\right)\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a-2abx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}}\\
&= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 1.00

$$\frac{\sqrt{a^2-b^2x^4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[2]
*a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

fricas [A] time = 0.74, size = 155, normalized size = 2.01

$$\left[\frac{\sqrt{2}\log\left(-\frac{3b^2x^4-2abx^2-2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{b^2x^4-2abx^2+a^2}\right)}{4a\sqrt{b}}, \frac{\sqrt{2}\sqrt{-b}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{-b}}{2(b^2x^3-abx)}\right)}{2ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*s
qrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2))/(a*sqrt(b)),
1/2*sqrt(2)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 +
a)*sqrt(-b)/(b^2*x^3 - a*b*x))/(a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)
```

maple [B] time = 0.06, size = 267, normalized size = 3.47

$$\frac{\sqrt{-bx^2+a}\sqrt{-b^2x^4+a^2}\left(\sqrt{2}\sqrt{a}\sqrt{b}\ln\left(\frac{2\left(a-\sqrt{ab}\,x+\sqrt{2}\sqrt{bx^2+a}\sqrt{a}\right)b}{bx+\sqrt{ab}}\right)-\sqrt{2}\sqrt{a}\sqrt{b}\ln\left(\frac{2\left(a+\sqrt{ab}\,x+\sqrt{2}\sqrt{bx^2+a}\sqrt{a}\right.}{bx-\sqrt{ab}}\right.\right.}{2\left(bx^2-a\right)\sqrt{bx^2+a}\left(\sqrt{-ab}+\sqrt{ab}\right)\left(-\sqrt{-ab}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/2*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(
2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*b^(1/2)
-a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(
b*x-(a*b)^(1/2)))*b^(1/2)-2*(a*b)^(1/2)*ln((b^(1/2)*(-(b*x+(-a*b)^(1/2))/b*
(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))+2*(a*b)^(1/2)*ln((b^(1/2)*(b*x^2+a
)^(1/2)+b*x)/b^(1/2)))/(b*x^2-a)/(b*x^2+a)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))
/(-(-a*b)^(1/2)+(a*b)^(1/2))/(a*b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,\sqrt{a-b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(1/2)),x)
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\,\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a - b*x**2)), x)
```

3.209 $\int \frac{1}{\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=124

$$\frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/4*x*(b*x^2+a)/a^2/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/8*arctanh(x*2^(
1/2)*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^2*2^(1/2)/
b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1152, 382, 377, 208}

$$\frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2))/(4*a^2*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b
*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(4*Sqrt
[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a-bx^2\right)^2\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a-bx^2\right)\sqrt{a+bx^2}}\,dx}{4a\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a-2abx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right.}{4a\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [A] time = 0.08, size = 110, normalized size = 0.89

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}\,x\sqrt{a+bx^2}+3\sqrt{2}\left(a-bx^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\right)}{8a^2\sqrt{b}\left(a-bx^2\right)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a + b*x^2] + 3*Sqrt[2]*(a - b*x^2)*A
rcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(8*a^2*Sqrt[b]*(a - b*x^2)^(3
/2)*Sqrt[a + b*x^2])
```

fricas [A] time = 0.65, size = 302, normalized size = 2.44

$$\left[\frac{4\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\,bx+3\sqrt{2}\left(b^2x^4-2\,abx^2+a^2\right)\sqrt{b}\log\left(-\frac{3\,b^2x^4-2\,abx^2-2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}\,x-a^2}{b^2x^4-2\,abx^2+a^2}\right)}{16\left(a^2b^3x^4-2\,a^3b^2x^2+a^4b\right)}, \frac{2\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 - 2
*a*b*x^2 + a^2)*sqrt(b)*log(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x
^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2)))/(
a^2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x
^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(-b)*arctan(1/2*sqr
t(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/(a^
2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\left(-bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

maple [B] time = 0.04, size = 510, normalized size = 4.11

$$\sqrt{-b\,x^{2}+a}\,\sqrt{-b^{2}x^{4}+a^{2}}\,\left(3\sqrt{2}\,\sqrt{a}\,b^{\frac{3}{2}}x^{2}\ln\left(\frac{2\left(a-\sqrt{a b}\,x+\sqrt{2}\,\sqrt{b\,x^{2}+a}\,\sqrt{a}\right)b}{b x+\sqrt{a b}}\right)-3\sqrt{2}\,\sqrt{a}\,b^{\frac{3}{2}}x^{2}\ln\left(\frac{2\left(a+\sqrt{a b}\,x+\sqrt{2}\,\sqrt{b\,x^{2}+\right.}{b x-\sqrt{a b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/4*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(3*2^(1/2)*ln(2*(a-(a*b)^
(1/2)*x+2^(1/2)*(b*x^2+a)^(1/2)*a^(1/2))/(b*x+(a*b)^(1/2))*b)*x^2*b^(3/2)*a
^(1/2)-3*2^(1/2)*ln(2*(a+(a*b)^(1/2)*x+2^(1/2)*(b*x^2+a)^(1/2)*a^(1/2))/(b*
x-(a*b)^(1/2))*b)*x^2*b^(3/2)*a^(1/2)-3*2^(1/2)*ln(2*(a-(a*b)^(1/2)*x+2^(1/
2)*(b*x^2+a)^(1/2)*a^(1/2))/(b*x+(a*b)^(1/2))*b)*a^(3/2)*b^(1/2)+3*2^(1/2)*
ln(2*(a+(a*b)^(1/2)*x+2^(1/2)*(b*x^2+a)^(1/2)*a^(1/2))/(b*x-(a*b)^(1/2))*b)
*a^(3/2)*b^(1/2)+4*ln((b*x+(b*x^2+a)^(1/2)*b^(1/2))/b^(1/2))*x^2*b*(a*b)^(1
/2)-4*ln((b*x+(-(b*x+(-a*b)^(1/2))*(-b*x+(-a*b)^(1/2))/b)^(1/2)*b^(1/2))/b^
(1/2))*x^2*b*(a*b)^(1/2)+4*b^(1/2)*(a*b)^(1/2)*(b*x^2+a)^(1/2)*x-4*ln((b*x+
(b*x^2+a)^(1/2)*b^(1/2))/b^(1/2))*a*(a*b)^(1/2)+4*ln((b*x+(-(b*x+(-a*b)^(1/
2))*(-b*x+(-a*b)^(1/2))/b)^(1/2)*b^(1/2))/b^(1/2))*a*(a*b)^(1/2))/(b*x^2-a)
/(b*x^2+a)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))^2/((-a*b)^(1/2)-(a*b)^(1/2))^2/
(a*b)^(1/2)/(b*x-(a*b)^(1/2))/(b*x+(a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^{2}x^{4}+a^{2}}\left(-bx^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^{2}-b^{2}\,x^{4}}\,{\left(a-b\,x^{2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(3/2)),x)
```

```
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a+bx^{2}\right)\left(a+bx^{2}\right)}\left(a-bx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(3/2)), x)
```

3.210 $\int \frac{1}{\left(a-bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=167

$$\frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/8*x*(b*x^2+a)/a^2/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)+9/32*x*(b*x^2+a)/
a^3/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/64*arctanh(x*2^(1/2)*b^(1/2)/(
b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^3*2^(1/2)/b^(1/2)/(-b^2*
x^4+a^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1152, 414, 527, 12, 377, 208}

$$\frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2))/(8*a^2*(a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a + b
*x^2))/(32*a^3*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*S
qrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32*Sqrt[2]*a^
3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\left(a-bx^2\right)^3\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{7ab+2b^2x^2}{\left(a-bx^2\right)^2\sqrt{a+bx^2}}\,dx}{8a^2b\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+\right.}{32a^4b}$$

$$= \frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\,\sqrt{a\right.}{32a}$$

$$= \frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\,\sqrt{a\right.}{}$$

$$= \frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2}\,\sqrt{a+}{32\sqrt{2}\,a^3}$$

Mathematica [A] time = 0.11, size = 122, normalized size = 0.73

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}\,x\left(13a-9bx^2\right)\sqrt{a+bx^2}+19\sqrt{2}\left(a-bx^2\right)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\right)}{64a^3\sqrt{b}\left(a-bx^2\right)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*(13*a - 9*b*x^2)*Sqrt[a + b*x^2] + 19*Sqr
t[2]*(a - b*x^2)^2*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(64*a^3*S
qrt[b]*(a - b*x^2)^(5/2)*Sqrt[a + b*x^2])
```

fricas [A] time = 0.96, size = 376, normalized size = 2.25

$$\left[\frac{19\sqrt{2}\left(b^{3}x^{6}-3\,ab^{2}x^{4}+3\,a^{2}bx^{2}-a^{3}\right)\sqrt{b}\log\left(-\frac{3\,b^{2}x^{4}-2\,abx^{2}-2\,\sqrt{2}\,\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{-bx^{2}+a}\,\sqrt{b}\,x-a^{2}}{b^{2}x^{4}-2\,abx^{2}+a^{2}}\right)+4\,\sqrt{-b^{2}x^{4}+a^{2}}\left(9\,b^{2}x\right.}{128\left(a^{3}b^{4}x^{6}-3\,a^{4}b^{3}x^{4}+3\,a^{5}b^{2}x^{2}-a^{6}b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(19*sqrt(2)*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3)*sqrt(b)*log(
-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*s
qrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2)) + 4*sqrt(-b^2*x^4 + a^2)*(9*b^
2*x^3 - 13*a*b*x)*sqrt(-b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^5*b^
2*x^2 - a^6*b), 1/64*(19*sqrt(2)*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3
)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b
)/(b^2*x^3 - a*b*x)) + 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 - 13*a*b*x)*sqrt(-
b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 - a^6*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^{2}x^{4}+a^{2}}\left(-bx^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

maple [B] time = 0.05, size = 739, normalized size = 4.43

$$\frac{\sqrt{-b\,x^{2}+a}\,\sqrt{-b^{2}x^{4}+a^{2}}\,\left(19\sqrt{2}\,\sqrt{a}\,b^{\frac{5}{2}}x^{4}\ln\left(\frac{2\left(a-\sqrt{ab}\,x+\sqrt{2}\,\sqrt{b\,x^{2}+a}\,\sqrt{a}\right)b}{bx+\sqrt{ab}}\right)-19\sqrt{2}\,\sqrt{a}\,b^{\frac{5}{2}}x^{4}\ln\left(\frac{2\left(a+\sqrt{ab}\,x+\sqrt{2}\,\sqrt{b\,x^{2}+a}\right.}{bx-\sqrt{ab}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/16*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*b^(9/2)*(19*ln(2*(a-(a*b)^(1/2)*
x+2^(1/2)*(b*x^2+a)^(1/2)*a^(1/2))/(b*x+(a*b)^(1/2))*b)*2^(1/2)*x^4*b^(5/2)
*a^(1/2)-19*ln(2*(a+(a*b)^(1/2)*x+2^(1/2)*(b*x^2+a)^(1/2)*a^(1/2))/(b*x-(a*
b)^(1/2))*b)*2^(1/2)*x^4*b^(5/2)*a^(1/2)+16*ln((b*x+(b*x^2+a)^(1/2)*b^(1/2)
)/b^(1/2))*x^4*b^2*(a*b)^(1/2)-16*ln((b*x+(-(b*x+(-a*b)^(1/2))*(-b*x+(-a*b)
^(1/2))/b)^(1/2)*b^(1/2))/b^(1/2))*x^4*b^2*(a*b)^(1/2)-38*ln(2*(a-(a*b)^(1/
2)*x+2^(1/2)*(b*x^2+a)^(1/2)*a^(1/2))/(b*x+(a*b)^(1/2))*b)*2^(1/2)*x^2*a^(3
/2)*b^(3/2)+38*ln(2*(a+(a*b)^(1/2)*x+2^(1/2)*(b*x^2+a)^(1/2)*a^(1/2))/(b*x-
(a*b)^(1/2))*b)*2^(1/2)*x^2*a^(3/2)*b^(3/2)+36*b^(3/2)*(a*b)^(1/2)*(b*x^2+a
)^(1/2)*x^3-32*ln((b*x+(b*x^2+a)^(1/2)*b^(1/2))/b^(1/2))*x^2*a*b*(a*b)^(1/2
)+32*ln((b*x+(-(b*x+(-a*b)^(1/2))*(-b*x+(-a*b)^(1/2))/b)^(1/2)*b^(1/2))/b^(
1/2))*x^2*a*b*(a*b)^(1/2)+19*ln(2*(a-(a*b)^(1/2)*x+2^(1/2)*(b*x^2+a)^(1/2)*
a^(1/2))/(b*x+(a*b)^(1/2))*b)*2^(1/2)*a^(5/2)*b^(1/2)-19*ln(2*(a+(a*b)^(1/2
)*x+2^(1/2)*(b*x^2+a)^(1/2)*a^(1/2))/(b*x-(a*b)^(1/2))*b)*2^(1/2)*a^(5/2)*b
^(1/2)-52*a*(a*b)^(1/2)*(b*x^2+a)^(1/2)*b^(1/2)*x+16*ln((b*x+(b*x^2+a)^(1/2
)*b^(1/2))/b^(1/2))*a^2*(a*b)^(1/2)-16*ln((b*x+(-(b*x+(-a*b)^(1/2))*(-b*x+(
-a*b)^(1/2))/b)^(1/2)*b^(1/2))/b^(1/2))*a^2*(a*b)^(1/2))/(b*x^2-a)/(b*x^2+a
```

```
)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))^3/(-(-a*b)^(1/2)+(a*b)^(1/2))^3/(a*b)^(1
/2)/(b*x-(a*b)^(1/2))^2/(b*x+(a*b)^(1/2))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\left(-bx^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2 - b^2\,x^4}\,\left(a - b\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(5/2)),x)
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a + bx^2\right)\left(a + bx^2\right)}\,\left(a - bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(5/2)), x)
```

3.211 $\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} \, dx$

Optimal. Leaf size=30

$$\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

```
[Out] arcsinh(x)*(x^2-1)^(1/2)*(x^2+1)^(1/2)/(x^4-1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1152, 215}

$$\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}} \, dx &= \frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1+x^2}} \, dx}{\sqrt{-1+x^4}} \\ &= \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,\sinh^{-1}(x)}{\sqrt{-1+x^4}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.27

$$\log\left(x^3+\sqrt{x^2-1}\,\sqrt{x^4-1}-x\right)-\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]
[Out] -Log[1 - x^2] + Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]]
```

fricas [B] time = 0.95, size = 73, normalized size = 2.43

$$\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\,\sqrt{x^2-1}-x}{x^3-x}\right)-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\,\sqrt{x^2-1}-x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{\sqrt{x^4-1}\ \operatorname{arcsinh}(x)}{\sqrt{x^2-1}\ \sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)/(x^4-1)^(1/2),x)
```

```
[Out] 1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2),x)
```

```
[Out] int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)}}{\sqrt{(x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.212 $\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=24

$$-\frac{\sqrt{x^4-1}\,\sin^{-1}(x)}{\sqrt{1-x^4}}$$

```
[Out] -arcsin(x)*(x^4-1)^(1/2)/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.67, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1152, 217, 206}

$$\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx &= \frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^4}}\\
&= \frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}\\
&= \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.42

$$\log\left(x^3+\sqrt{x^2+1}\,\sqrt{x^4-1}+x\right)-\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
```

```
[Out] -Log[1 + x^2] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

fricas [B] time = 0.88, size = 65, normalized size = 2.71

$$\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\,\sqrt{x^2+1}+x}{x^3+x}\right)-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\,\sqrt{x^2+1}+x}{x^3+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

maple [A] time = 0.01, size = 33, normalized size = 1.38

$$\frac{\sqrt{x^4-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}\,\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(x^4-1)^(1/2),x)
```

```
[Out] 1/(x^2+1)^(1/2)*(x^4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{(x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.213 $\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt{x^2-1}\,\sqrt{x^4-1}\,\sinh^{-1}(x)}{(1-x^2)\,\sqrt{x^2+1}}-\frac{\sqrt{x^4-1}\,\sin^{-1}(x)}{\sqrt{1-x^2}\,\sqrt{x^2+1}}$$

```
[Out] -arcsin(x)*(x^4-1)^(1/2)/(-x^2+1)^(1/2)/(x^2+1)^(1/2)+arcsinh(x)*(x^2-1)^(1
/2)*(x^4-1)^(1/2)/(-x^2+1)/(x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {6742, 1152, 215, 217, 206}

$$\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

```
[Out] -((Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]) + (Sqrt[-1 + x^
2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \left(-\frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}+\frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\right)dx$$
$$= -\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx + \int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$$
$$= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^4}} - \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{\sqrt{-1+x^4}}$$
$$= -\frac{\sqrt{-1+x^2}\sqrt{1+x^2}\sinh^{-1}(x)}{\sqrt{-1+x^4}} + \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^{}}}\right.}{\sqrt{-1+x^4}}$$
$$= -\frac{\sqrt{-1+x^2}\sqrt{1+x^2}\sinh^{-1}(x)}{\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^2}\sqrt{1+x^2}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.97

$$\log\left(1-x^2\right)-\log\left(x^2+1\right)-\log\left(x^3+\sqrt{x^2-1}\sqrt{x^4-1}-x\right)+\log\left(x^3+\sqrt{x^2+1}\sqrt{x^4-1}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

```
[Out] Log[1 - x^2] - Log[1 + x^2] - Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]]
 + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

fricas [B] time = 1.38, size = 137, normalized size = 1.88

$$\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)-\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log((x^3 + sqrt(x^4 - 1
)*sqrt(x^2 - 1) - x)/(x^3 - x)) + 1/2*log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2 -
1) - x)/(x^3 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

maple [A] time = 0.00, size = 59, normalized size = 0.81

$$-\frac{\sqrt{x^4-1}\,\operatorname{arcsinh}(x)}{\sqrt{x^2-1}\,\sqrt{x^2+1}}+\frac{\sqrt{x^4-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}\,\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x)
```

```
[Out] -1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)+1/(x^2+1)^(1/2)*(x^
4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{x^2-1}-\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2),x)
```

```
[Out] int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{x^2-1}+\sqrt{x^2+1}}{\sqrt{(x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x**2-1)**(1/2)+(x**2+1)**(1/2))/(x**4-1)**(1/2),x)
```

```
[Out] Integral((-sqrt(x**2 - 1) + sqrt(x**2 + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)
), x)
```

3.214 $\int \frac{\left(d+ex^2\right)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=121

$$\frac{x\left(b^2e^2-5bcde+7c^2d^2\right)}{c^3}-\frac{(2cd-be)^3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}+\frac{ex^3(4cd-be)}{3c^2}+\frac{e^2x^5}{5c}$$

```
[Out] (b^2*e^2-5*b*c*d*e+7*c^2*d^2)*x/c^3+1/3*e*(-b*e+4*c*d)*x^3/c^2+1/5*e^2*x^5/
c-(-b*e+2*c*d)^3*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(7/2)/e^(1/2
)/(-b*e+c*d)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1149, 390, 208}

$$\frac{x\left(b^2e^2-5bcde+7c^2d^2\right)}{c^3}+\frac{ex^3(4cd-be)}{3c^2}-\frac{(2cd-be)^3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}+\frac{e^2x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ((7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)/c^3 + (e*(4*c*d - b*e)*x^3)/(3*c^2) +
 (e^2*x^5)/(5*c) - ((2*c*d - b*e)^3*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d -
b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(d+ex^2)^3}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$
$$= \int \left(\frac{7c^2d^2-5bcde+b^2e^2}{c^3}+\frac{e(4cd-be)x^2}{c^2}+\frac{e^2x^4}{c}+\frac{8c^3d^3-12bc^2d^2e+6\ldots}{c^3\left(-cd+be+\ldots\right.}\right.$$
$$= \frac{(7c^2d^2-5bcde+b^2e^2)\,x}{c^3}+\frac{e(4cd-be)x^3}{3c^2}+\frac{e^2x^5}{5c}+\frac{(2cd-be)^3\int\frac{1}{-cd+be+c\ldots}}{c^3}$$
$$= \frac{(7c^2d^2-5bcde+b^2e^2)\,x}{c^3}+\frac{e(4cd-be)x^3}{3c^2}+\frac{e^2x^5}{5c}-\frac{(2cd-be)^3\tanh^{-1}\left(\ldots\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}$$

Mathematica [A] time = 0.07, size = 121, normalized size = 1.00

$$-\frac{x\left(-b^2e^2+5bcde-7c^2d^2\right)}{c^3}-\frac{(be-2cd)^3\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{7/2}\sqrt{e}\sqrt{be-cd}}-\frac{ex^3(be-4cd)}{3c^2}+\frac{e^2x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((-7*c^2*d^2 + 5*b*c*d*e - b^2*e^2)*x)/c^3) - (e*(-4*c*d + b*e)*x^3)/(3*c
^2) + (e^2*x^5)/(5*c) - ((-2*c*d + b*e)^3*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-
(c*d) + b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

fricas [B] time = 0.75, size = 446, normalized size = 3.69

$$\left[\frac{6\left(c^4de^3-bc^3e^4\right)x^5+10\left(4c^4d^2e^2-5bc^3de^3+b^2c^2e^4\right)x^3-15\left(8c^3d^3-12bc^2d^2e+6b^2cde^2-b^3e^3\right)\sqrt{c^2de-b\ldots}}{30\left(c^5de-bc^4e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/30*(6*(c^4*d*e^3 - b*c^3*e^4)*x^5 + 10*(4*c^4*d^2*e^2 - 5*b*c^3*d*e^3 +
b^2*c^2*e^4)*x^3 - 15*(8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3
)*sqrt(c^2*d*e - b*c*e^2)*log((c*e*x^2 + c*d - b*e + 2*sqrt(c^2*d*e - b*c*e
^2)*x)/(c*e*x^2 - c*d + b*e)) + 30*(7*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*
c^2*d*e^3 - b^3*c*e^4)*x)/(c^5*d*e - b*c^4*e^2), 1/15*(3*(c^4*d*e^3 - b*c^3
*e^4)*x^5 + 5*(4*c^4*d^2*e^2 - 5*b*c^3*d*e^3 + b^2*c^2*e^4)*x^3 - 15*(8*c^3
*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*sqrt(-c^2*d*e + b*c*e^2)*a
rctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) + 15*(7*c^4*d^3*e - 12*b*c^3
*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e^4)*x)/(c^5*d*e - b*c^4*e^2)]
```

giac [B] time = 5.85, size = 10312, normalized size = 85.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] -1/8*(128*b*c^10*d^6*e^10 - 64*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b
^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b
*c^8*d^6*e^6 - 384*b^2*c^9*d^5*e^11 + 192*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*
c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^
4)*c*e^2)*b^2*c^7*d^5*e^7 + 480*b^3*c^8*d^4*e^12 - 240*sqrt(2)*sqrt(4*c^2*d
^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d
*e^3 + b^2*e^4)*c*e^2)*b^3*c^6*d^4*e^8 + 32*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*
b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*
e^4)*c*e^2)*b^2*c^7*d^4*e^8 - 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
 b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)
*b*c^8*d^4*e^8 - 320*b^4*c^7*d^3*e^13 - 32*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b
^2*e^4)*b*c^8*d^4*e^8 + 160*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*
e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*
c^5*d^3*e^9 - 64*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b
*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^6*d^3*e^9
 + 32*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sq
rt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^7*d^3*e^9 + 120*b^5*
c^6*d^2*e^14 + 64*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^7*d^3*e^9 -
 60*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt
(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^4*d^2*e^10 + 48*sqrt(2
)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2
*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^5*d^2*e^10 - 24*sqrt(2)*sqrt(4*c
^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b
*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^6*d^2*e^10 - 24*b^6*c^5*d*e^15 - 48*(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^6*d^2*e^10 + 12*sqrt(2)*sqrt(4*c^2*
d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*
d*e^3 + b^2*e^4)*c*e^2)*b^6*c^3*d*e^11 - 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*
b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*
e^4)*c*e^2)*b^5*c^4*d*e^11 + 8*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b
^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b
^4*c^5*d*e^11 + 2*b^7*c^4*e^16 + 16*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*b^4*c^5*d*e^11 - sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(
b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^7*c^2*e^12 +
 2*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(
4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^6*c^3*e^12 - sqrt(2)*sqrt(4
*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4
*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^4*e^12 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3
+ b^2*e^4)*b^5*c^4*e^12 + (256*c^9*d^7*e^9 - 128*sqrt(2)*sqrt(4*c^2*d^2*e^2
 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
 b^2*e^4)*c*e^2)*c^7*d^7*e^5 - 896*b*c^8*d^6*e^10 + 448*sqrt(2)*sqrt(4*c^2*
d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*
d*e^3 + b^2*e^4)*c*e^2)*b*c^6*d^6*e^6 + 1344*b^2*c^7*d^5*e^11 - 672*sqrt(2)
*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^5*d^5*e^7 + 64*sqrt(2)*sqrt(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c
*d*e^3 + b^2*e^4)*c*e^2)*b*c^6*d^5*e^7 - 32*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*
b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*
e^4)*c*e^2)*c^7*d^5*e^7 - 1120*b^3*c^6*d^4*e^12 - 64*(4*c^2*d^2*e^2 - 4*b*c
*d*e^3 + b^2*e^4)*c^7*d^5*e^7 + 560*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^
3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e
^2)*b^3*c^4*d^4*e^8 - 160*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^
4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^
5*d^4*e^8 + 80*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c
*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^6*d^4*e^8 + 5
60*b^4*c^5*d^3*e^13 + 160*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^6*d^4
*e^8 - 280*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4
 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^3*d^3*e^9 + 160
*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*
c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^4*d^3*e^9 - 80*sqrt(2)*sq
```

rt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^5*d^3*e^9 - 168*b^5*c^4*d^2*e^14 - 160*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^5*d^3*e^9 + 84*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^2*d^2*e^10 - 80*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^3*d^2*e^10 + 40*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^4*d^2*e^10 + 28*b^6*c^3*d*e^15 + 80*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^4*d^2*e^10 - 14*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^6*c*d*e^11 + 20*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^2*d*e^11 - 10*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^3*d*e^11 - 2*b^7*c^2*e^16 - 20*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^4*c^3*d*e^11 + sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^7*e^12 - 2*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^6*c*e^12 + sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^2*e^12 + 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^5*c^2*e^12)*c^2 - 2*(256*c^10*d^8*e^8 - 128*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^9*d^8*e^6 - 896*b*c^9*d^7*e^9 + 448*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^8*d^7*e^7 + 1344*b^2*c^8*d^6*e^10 - 672*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^7*d^6*e^8 + 64*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^8*d^6*e^8 - 32*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^9*d^6*e^8 - 1120*b^3*c^7*d^5*e^11 + 560*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^6*d^5*e^9 - 160*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^7*d^5*e^9 + 80*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^8*d^5*e^9 - 64*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^8*d^6*e^6 + 560*b^4*c^6*d^4*e^12 - 280*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^5*d^4*e^10 + 160*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^6*d^4*e^10 - 80*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^7*d^4*e^10 + 160*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^7*d^5*e^7 - 168*b^5*c^5*d^3*e^13 + 84*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^4*d^3*e^11 - 80*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^5*d^3*e^11 + 40*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^6*d^3*e^11 - 160*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^6*d^4*e^8 + 28*b^6*c^4*d^2*e^14 - 14*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^6*c^3*d^2*e^12 + 20*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^4*d^2*e^12 - 10*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^5*d^2*e^12 + 80*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^5*d^3*e^9 - 2*b^7*c^3*d*e^15 + sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^7*c^2*d*e^13 - 2*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^6*c^3*d*e^13 + sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^4*d*e^13 - 20*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^4*c^4*d^2*e^10 + 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^5*c^3*d*e^11)*abs(c))*arctan(2*sqrt(1/2)*x*e^6/sqrt((b*c^5*e^12 + sqrt(b^2*c^10*e^24 + 4*(c^6*d^2*e^10 - b*c^5*d*e^11)*c^6*e^12))/c^6))/((16*c^10*d^6*e^8 - 48*b*c^9*d^5*e^9 + 56*b^2*c^8*d^4*e^10 - 8*b*c^9*d^4*e^10 + 4*c^10*d^4*e^10 - 32*b^3*c^7*d^3*e^11 + 16*b^2*c^8*d^3*e^11 - 8*b*c^9*d^3*e^11 + 9*b^4*c^6*d^2*e^12 - 10*b^3*c^7*d^2*e^12 + 5*b^2*c^8*d

^2*e^12 - b^5*c^5*d*e^13 + 2*b^4*c^6*d*e^13 - b^3*c^7*d*e^13)*c^2) + 1/8*(1
28*b*c^10*d^6*e^10 - 64*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^8*d^
6*e^6 - 384*b^2*c^9*d^5*e^11 + 192*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^
2)*b^2*c^7*d^5*e^7 + 480*b^3*c^8*d^4*e^12 - 240*sqrt(2)*sqrt(4*c^2*d^2*e^2
 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
b^2*e^4)*c*e^2)*b^3*c^6*d^4*e^8 + 32*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e
^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*
e^2)*b^2*c^7*d^4*e^8 - 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^
4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^8*
d^4*e^8 - 320*b^4*c^7*d^3*e^13 - 32*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*b*c^8*d^4*e^8 + 160*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sq
rt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^5*d^3
*e^9 - 64*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4
 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^6*d^3*e^9 + 32*s
qrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^
2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^7*d^3*e^9 + 120*b^5*c^6*d^2
e^14 + 64(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^7*d^3*e^9 - 60*sqr
t(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*
d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^4*d^2*e^10 + 48*sqrt(2)*sqrt(
4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 -
4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^5*d^2*e^10 - 24*sqrt(2)*sqrt(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^
3 + b^2*e^4)*c*e^2)*b^3*c^6*d^2*e^10 - 24*b^6*c^5*d*e^15 - 48*(4*c^2*d^2*e^
2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^6*d^2*e^10 + 12*sqrt(2)*sqrt(4*c^2*d^2*e^2
 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
 b^2*e^4)*c*e^2)*b^6*c^3*d*e^11 - 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e
^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*
e^2)*b^5*c^4*d*e^11 + 8*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^5*
d*e^11 + 2*b^7*c^4*e^16 + 16*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^4*c^
5*d*e^11 - sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4
 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^7*c^2*e^12 + 2*sqrt
(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d
^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^6*c^3*e^12 - sqrt(2)*sqrt(4*c^2*d^
2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*
e^3 + b^2*e^4)*c*e^2)*b^5*c^4*e^12 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e
^4)*b^5*c^4*e^12 + (256*c^9*d^7*e^9 - 128*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*
c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^
4)*c*e^2)*c^7*d^7*e^5 - 896*b*c^8*d^6*e^10 + 448*sqrt(2)*sqrt(4*c^2*d^2*e^2
 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
 b^2*e^4)*c*e^2)*b*c^6*d^6*e^6 + 1344*b^2*c^7*d^5*e^11 - 672*sqrt(2)*sqrt(4
*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4
*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^5*d^5*e^7 + 64*sqrt(2)*sqrt(4*c^2*d^2*e^
2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
+ b^2*e^4)*c*e^2)*b*c^6*d^5*e^7 - 32*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e
^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*
e^2)*c^7*d^5*e^7 - 1120*b^3*c^6*d^4*e^12 - 64*(4*c^2*d^2*e^2 - 4*b*c*d*e^3
+ b^2*e^4)*c^7*d^5*e^7 + 560*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2
*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3
*c^4*d^4*e^8 - 160*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt
(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^5*d^4*e
^8 + 80*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 -
sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^6*d^4*e^8 + 560*b^4*
c^5*d^3*e^13 + 160*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^6*d^4*e^8 -
280*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt
(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^3*d^3*e^9 + 160*sqrt(2
)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2

*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^4*d^3*e^9 - 80*sqrt(2)*sqrt(4*c^
2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*
c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^5*d^3*e^9 - 168*b^5*c^4*d^2*e^14 - 160*(4*c
^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^5*d^3*e^9 + 84*sqrt(2)*sqrt(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c
*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^2*d^2*e^10 - 80*sqrt(2)*sqrt(4*c^2*d^2*e^2 -
 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b
^2*e^4)*c*e^2)*b^4*c^3*d^2*e^10 + 40*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e
^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*
e^2)*b^3*c^4*d^2*e^10 + 28*b^6*c^3*d*e^15 + 80*(4*c^2*d^2*e^2 - 4*b*c*d*e^3
 + b^2*e^4)*b^3*c^4*d^2*e^10 - 14*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
+ b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2
)*b^6*c*d*e^11 + 20*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqr
t(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^2*d*e^
11 - 10*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 -
sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^3*d*e^11 - 2*b^7*c
^2*e^16 - 20*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^4*c^3*d*e^11 + sqrt(
2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^
2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^7*e^12 - 2*sqrt(2)*sqrt(4*c^2*d^2*e
^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
 + b^2*e^4)*c*e^2)*b^6*c*e^12 + sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*
b^5*c^2*e^12 + 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^5*c^2*e^12)*c^2
- 2*(256*c^10*d^8*e^8 + 128*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b
*c*d*e^3 + b^2*e^4)*c*e^2)*c^9*d^8*e^6 - 896*b*c^9*d^7*e^9 - 448*sqrt(2)*sq
rt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^8*d^7*e
^7 + 1344*b^2*c^8*d^6*e^10 + 672*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2
- 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^7*d^6*e^8 - 64*sqrt(2)*sqrt(b*c*e^4 -
 sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^8*d^6*e^8 + 32*sqrt
(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^9*d
^6*e^8 - 1120*b^3*c^7*d^5*e^11 - 560*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^6*d^5*e^9 + 160*sqrt(2)*sqrt(b*c*
e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^7*d^5*e^9 -
80*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2
)*b*c^8*d^5*e^9 - 64*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^8*d^6*e^6 +
560*b^4*c^6*d^4*e^12 + 280*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*
c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^5*d^4*e^10 - 160*sqrt(2)*sqrt(b*c*e^4 - sqr
t(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^6*d^4*e^10 + 80*sqrt(
2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^
7*d^4*e^10 + 160*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^7*d^5*e^7 - 16
8*b^5*c^5*d^3*e^13 - 84*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d
*e^3 + b^2*e^4)*c*e^2)*b^5*c^4*d^3*e^11 + 80*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*
c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^5*d^3*e^11 - 40*sqrt(2)*s
qrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^6*d^
3*e^11 - 160*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^6*d^4*e^8 + 28*b
^6*c^4*d^2*e^14 + 14*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^
3 + b^2*e^4)*c*e^2)*b^6*c^3*d^2*e^12 - 20*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^4*d^2*e^12 + 10*sqrt(2)*sqrt
(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^5*d^2*e
^12 + 80*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^5*d^3*e^9 - 2*b^7*c^
3*d*e^15 - sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^
4)*c*e^2)*b^7*c^2*d*e^13 + 2*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*
b*c*d*e^3 + b^2*e^4)*c*e^2)*b^6*c^3*d*e^13 - sqrt(2)*sqrt(b*c*e^4 - sqrt(4*
c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^4*d*e^13 - 20*(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^4*c^4*d^2*e^10 + 2*(4*c^2*d^2*e^2 - 4*b*c*d*
e^3 + b^2*e^4)*b^5*c^3*d*e^11)*abs(c))*arctan(2*sqrt(1/2)*x*e^6/sqrt((b*c^5
*e^12 - sqrt(b^2*c^10*e^24 + 4*(c^6*d^2*e^10 - b*c^5*d*e^11)*c^6*e^12))/c^6
))/((16*c^10*d^6*e^8 - 48*b*c^9*d^5*e^9 + 56*b^2*c^8*d^4*e^10 - 8*b*c^9*d^4
*e^10 + 4*c^10*d^4*e^10 - 32*b^3*c^7*d^3*e^11 + 16*b^2*c^8*d^3*e^11 - 8*b*c

```
^9*d^3*e^11 + 9*b^4*c^6*d^2*e^12 - 10*b^3*c^7*d^2*e^12 + 5*b^2*c^8*d^2*e^12
 - b^5*c^5*d*e^13 + 2*b^4*c^6*d*e^13 - b^3*c^7*d*e^13)*c^2) + 1/15*(3*c^4*x
^5*e^12 + 20*c^4*d*x^3*e^11 - 5*b*c^3*x^3*e^12 + 105*c^4*d^2*x*e^10 - 75*b*
c^3*d*x*e^11 + 15*b^2*c^2*x*e^12)*e^(-10)/c^5
```

maple [B] time = 0.01, size = 226, normalized size = 1.87

$$\frac{e^{2}x^{5}}{5c}-\frac{b^{3}e^{3}\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)\,ce}\;c^{3}}+\frac{6b^{2}d\,e^{2}\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)\,ce}\;c^{2}}-\frac{12b\,d^{2}e\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)\,ce}\;c}-\frac{b\,e^{2}x^{3}}{3c^{2}}+\frac{4de\,x^{3}}{3c}+\frac{8d^{3}\arctan\ldots}{\sqrt{(b\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/5*e^2*x^5/c-1/3/c^2*x^3*b*e^2+4/3/c*x^3*d*e+1/c^3*b^2*e^2*x-5/c^2*b*d*e*x
+7/c*d^2*x-1/c^3/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*
b^3*e^3+6/c^2/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*b^2
*d*e^2-12/c/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*b*d^2
*e+8/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*d^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for more
 details)Is b*e-c*d positive or negative?
```

mupad [B] time = 4.53, size = 182, normalized size = 1.50

$$x\left(\frac{3\,d^{2}}{c}+\frac{\left(\frac{e\,(b\,e-c\,d)}{c^{2}}-\frac{3\,d\,e}{c}\right)\,(b\,e-c\,d)}{c\,e}\right)-x^{3}\,\left(\frac{e\,(b\,e-c\,d)}{3\,c^{2}}-\frac{d\,e}{c}\right)+\frac{e^{2}\,x^{5}}{5\,c}-\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,e\,x\,(b\,e-2\,c\,d)^{3}}{\sqrt{b\,e^{2}-c\,d\,e}\,\left(b^{3}\,e^{3}-6\,b^{2}\,c\,d\,e^{2}+12\,b\,c^{2}\,d^{2}\,e-8\,c^{3}\,d\ldots\right.}\right.\ldots}{c^{7/2}\,\sqrt{b\,e^{2}-c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] x*((3*d^2)/c + (((e*(b*e - c*d))/c^2 - (3*d*e)/c)*(b*e - c*d))/(c*e)) - x^3
*((e*(b*e - c*d))/(3*c^2) - (d*e)/c) + (e^2*x^5)/(5*c) - (atan((c^(1/2)*e*x
*(b*e - 2*c*d)^3)/((b*e^2 - c*d*e)^(1/2)*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^
2*e - 6*b^2*c*d*e^2)))*(b*e - 2*c*d)^3)/(c^(7/2)*(b*e^2 - c*d*e)^(1/2))
```

sympy [B] time = 1.00, size = 345, normalized size = 2.85

$$x^{3}\left(-\frac{be^{2}}{3c^{2}}+\frac{4de}{3c}\right)+x\left(\frac{b^{2}e^{2}}{c^{3}}-\frac{5bde}{c^{2}}+\frac{7d^{2}}{c}\right)+\frac{\sqrt{-\frac{1}{c^{7}e(be-cd)}}\,(be-2cd)^{3}\log\left(x+\frac{-bc^{3}e\sqrt{-\frac{1}{c^{7}e(be-cd)}}\,(be-2cd)^{3}+c^{4}d\sqrt{-\frac{1}{c^{7}e(be-cd)}}\ldots}{b^{3}e^{3}-6b^{2}cde^{2}+12bc^{2}d^{2}e-8c^{3}d^{3}}\right.\ldots}{2}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] x**3*(-b*e**2/(3*c**2) + 4*d*e/(3*c)) + x*(b**2*e**2/c**3 - 5*b*d*e/c**2 +
7*d**2/c) + sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log(x + (-b*c**3
*e*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 + c**4*d*sqrt(-1/(c**7*e*
(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 + 12*b*c**2*d*
*2*e - 8*c**3*d**3))/2 - sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log
(x + (b*c**3*e*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 - c**4*d*sqrt
(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 +
12*b*c**2*d**2*e - 8*c**3*d**3))/2 + e**2*x**5/(5*c)
```

3.215 $\int \frac{\left(d+ex^2\right)^3}{-cd^2+bde+be^2x^2+ce^2x^4} \, dx$

Optimal. Leaf size=86

$$-\frac{(2cd-be)^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}+\frac{x(3cd-be)}{c^2}+\frac{ex^3}{3c}$$

```
[Out] (-b*e+3*c*d)*x/c^2+1/3*e*x^3/c-(-b*e+2*c*d)^2*arctanh(x*c^(1/2)*e^(1/2)/(-b
*e+c*d)^(1/2))/c^(5/2)/e^(1/2)/(-b*e+c*d)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1149, 390, 208}

$$\frac{x(3cd-be)}{c^2}-\frac{(2cd-be)^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ((3*c*d - b*e)*x)/c^2 + (e*x^3)/(3*c) - ((2*c*d - b*e)^2*ArcTanh[(Sqrt[c]*S
qrt[e]*x)/Sqrt[c*d - b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{\left(d+ex^2\right)^2}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$
$$= \int \left(\frac{3cd-be}{c^2}+\frac{ex^2}{c}+\frac{4c^2d^2-4bcde+b^2e^2}{c^2\left(-cd+be+cex^2\right)}\right)dx$$
$$= \frac{(3cd-be)x}{c^2}+\frac{ex^3}{3c}+\frac{(2cd-be)^2\int \frac{1}{-cd+be+cex^2}\,dx}{c^2}$$
$$= \frac{(3cd-be)x}{c^2}+\frac{ex^3}{3c}-\frac{(2cd-be)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}$$

Mathematica [A] time = 0.05, size = 84, normalized size = 0.98

$$\frac{(be-2cd)^2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{5/2}\sqrt{e}\sqrt{be-cd}}-\frac{x(be-3cd)}{c^2}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((-3*c*d + b*e)*x)/c^2) + (e*x^3)/(3*c) + ((-2*c*d + b*e)^2*ArcTan[(Sqrt[
c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

fricas [A] time = 0.95, size = 311, normalized size = 3.62

$$\left[\frac{2\left(c^3de^2-bc^2e^3\right)x^3+3\left(4c^2d^2-4bcde+b^2e^2\right)\sqrt{c^2de-bce^2}\log\left(\frac{cex^2+cd-be-2\sqrt{c^2de-bce^2}x}{cex^2-cd+be}\right)+6\left(3c^3d^2e-4bc^2\right.}{6\left(c^4de-bc^3e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/6*(2*(c^3*d*e^2 - b*c^2*e^3)*x^3 + 3*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*s
qrt(c^2*d*e - b*c*e^2)*log((c*e*x^2 + c*d - b*e - 2*sqrt(c^2*d*e - b*c*e^2)
*x)/(c*e*x^2 - c*d + b*e)) + 6*(3*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*x)
/(c^4*d*e - b*c^3*e^2), 1/3*((c^3*d*e^2 - b*c^2*e^3)*x^3 - 3*(4*c^2*d^2 - 4
*b*c*d*e + b^2*e^2)*sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-c^2*d*e + b*c*e^
2)*x/(c*d - b*e)) + 3*(3*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*x)/(c^4*d*e
 - b*c^3*e^2)]
```

giac [B] time = 5.30, size = 8680, normalized size = 100.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] -1/8*(64*b*c^9*d^5*e^8 - 32*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*
e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^
7*d^5*e^4 - 160*b^2*c^8*d^4*e^9 + 80*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e
^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*
```

e^2)*b^2*c^6*d^4*e^5 + 160*b^3*c^7*d^3*e^10 - 80*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^5*d^3*e^6 + 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^6*d^3*e^6 - 8*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^7*d^3*e^6 - 80*b^4*c^6*d^2*e^11 - 16*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^7*d^3*e^6 + 40*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^4*d^2*e^7 - 24*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^5*d^2*e^7 + 12*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^6*d^2*e^7 + 20*b^5*c^5*d*e^12 + 24*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^6*d^2*e^7 - 10*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^3*d*e^8 + 12*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^4*d*e^8 - 6*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^5*d*e^8 - 2*b^6*c^4*e^13 - 12*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^5*d*e^8 + sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^6*c^2*e^9 - 2*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^3*e^9 + sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^4*e^9 + 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^4*c^4*e^9 + (128*c^8*d^6*e^7 - 64*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^6*d^6*e^3 - 384*b*c^7*d^5*e^8 + 192*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^5*d^5*e^4 + 480*b^2*c^6*d^4*e^9 - 240*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^4*d^4*e^5 + 32*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^5*d^4*e^5 - 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^6*d^4*e^5 - 320*b^3*c^5*d^3*e^10 - 32*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^6*d^4*e^5 + 160*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^3*d^3*e^6 - 64*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^4*d^3*e^6 + 32*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^5*d^3*e^6 + 120*b^4*c^4*d^2*e^11 + 64*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^5*d^3*e^6 - 60*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^2*d^2*e^7 + 48*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^3*d^2*e^7 - 24*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^4*d^2*e^7 - 24*b^5*c^3*d*e^12 - 48*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^4*d^2*e^7 + 12*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c*d*e^8 - 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^2*d*e^8 + 8*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^3*d*e^8 + 2*b^6*c^2*e^13 + 16*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^3*d*e^8 - sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^6*e^9 + 2*sqrt(2)*sq

```
rt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2
 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c*e^9 - sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4
*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2
*e^4)*c*e^2)*b^4*c^2*e^9 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^4*c^
2*e^9)*c^2 - 2*(128*c^9*d^7*e^6 - 64*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^8*d^7*e^4 - 384*b*c^8*d^6*e^7 + 192*s
qrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*
c^7*d^6*e^5 + 480*b^2*c^7*d^5*e^8 - 240*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d
^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^6*d^5*e^6 + 32*sqrt(2)*sqrt(b*
c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^7*d^5*e^6 -
16*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2
)*c^8*d^5*e^6 - 320*b^3*c^6*d^4*e^9 + 160*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^5*d^4*e^7 - 64*sqrt(2)*sqrt(
b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^6*d^4*e^
7 + 32*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c
*e^2)*b*c^7*d^4*e^7 - 32*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^7*d^5*e^
4 + 120*b^4*c^5*d^3*e^10 - 60*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4
*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^4*d^3*e^8 + 48*sqrt(2)*sqrt(b*c*e^4 + sq
rt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^5*d^3*e^8 - 24*sqrt(
2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^
6*d^3*e^8 + 64*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^6*d^4*e^5 - 24*b
^5*c^4*d^2*e^11 + 12*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^
3 + b^2*e^4)*c*e^2)*b^5*c^3*d^2*e^9 - 16*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*
d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^4*d^2*e^9 + 8*sqrt(2)*sqrt(b*
c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^5*d^2*e^9
- 48*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^5*d^3*e^6 + 2*b^6*c^3*d*
e^12 - sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c
*e^2)*b^6*c^2*d*e^10 + 2*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*
d*e^3 + b^2*e^4)*c*e^2)*b^5*c^3*d*e^10 - sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*
d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^4*d*e^10 + 16*(4*c^2*d^2*e^2
- 4*b*c*d*e^3 + b^2*e^4)*b^3*c^4*d^2*e^7 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
 b^2*e^4)*b^4*c^3*d*e^8)*abs(c))*arctan(2*sqrt(1/2)*x*e^4/sqrt((b*c^3*e^8 +
 sqrt(b^2*c^6*e^16 + 4*(c^4*d^2*e^6 - b*c^3*d*e^7)*c^4*e^8))/c^4))/((16*c^9
*d^6*e^6 - 48*b*c^8*d^5*e^7 + 56*b^2*c^7*d^4*e^8 - 8*b*c^8*d^4*e^8 + 4*c^9*
d^4*e^8 - 32*b^3*c^6*d^3*e^9 + 16*b^2*c^7*d^3*e^9 - 8*b*c^8*d^3*e^9 + 9*b^4
*c^5*d^2*e^10 - 10*b^3*c^6*d^2*e^10 + 5*b^2*c^7*d^2*e^10 - b^5*c^4*d*e^11 +
 2*b^4*c^5*d*e^11 - b^3*c^6*d*e^11)*c^2) + 1/8*(64*b*c^9*d^5*e^8 - 32*sqrt(
2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^
2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^7*d^5*e^4 - 160*b^2*c^8*d^4*e^9 +
 80*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt
(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^6*d^4*e^5 + 160*b^3*c^
7*d^3*e^10 - 80*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*
c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^5*d^3*e^6
+ 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqr
t(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^6*d^3*e^6 - 8*sqrt(2)
*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^7*d^3*e^6 - 80*b^4*c^6*d^2*e^11 - 1
6*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^7*d^3*e^6 + 40*sqrt(2)*sqrt(4
*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4
*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^4*d^2*e^7 - 24*sqrt(2)*sqrt(4*c^2*d^2*e^
2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
+ b^2*e^4)*c*e^2)*b^3*c^5*d^2*e^7 + 12*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d
*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*
c*e^2)*b^2*c^6*d^2*e^7 + 20*b^5*c^5*d*e^12 + 24*(4*c^2*d^2*e^2 - 4*b*c*d*e^
3 + b^2*e^4)*b^2*c^6*d^2*e^7 - 10*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
+ b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2
)*b^5*c^3*d*e^8 + 12*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sq
rt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^4*d*e
^8 - 6*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - s
```

qrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^5*d*e^8 - 2*b^6*c^4
e^13 - 12(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^5*d*e^8 + sqrt(2)*
sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e
^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^6*c^2*e^9 - 2*sqrt(2)*sqrt(4*c^2*d^2*e
^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
 + b^2*e^4)*c*e^2)*b^5*c^3*e^9 + sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
 b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)
*b^4*c^4*e^9 + 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^4*c^4*e^9 + (128
*c^8*d^6*e^7 - 64*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(
b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^6*d^6*e^3 -
384*b*c^7*d^5*e^8 + 192*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^5*d^
5*e^4 + 480*b^2*c^6*d^4*e^9 - 240*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2
)*b^2*c^4*d^4*e^5 + 32*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*
sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^5*d^4
*e^5 - 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4
 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^6*d^4*e^5 - 320*b^3*
c^5*d^3*e^10 - 32*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^6*d^4*e^5 + 160
*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*
c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^3*d^3*e^6 - 64*sqrt(2)*sq
rt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2
 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^4*d^3*e^6 + 32*sqrt(2)*sqrt(4*c^2*d^
2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*
e^3 + b^2*e^4)*c*e^2)*b*c^5*d^3*e^6 + 120*b^4*c^4*d^2*e^11 + 64*(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^5*d^3*e^6 - 60*sqrt(2)*sqrt(4*c^2*d^2*e^2
 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
b^2*e^4)*c*e^2)*b^4*c^2*d^2*e^7 + 48*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e
^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*
e^2)*b^3*c^3*d^2*e^7 - 24*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^
4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^
4*d^2*e^7 - 24*b^5*c^3*d*e^12 - 48*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*
b^2*c^4*d^2*e^7 + 12*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sq
rt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c*d*e^8
 - 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sq
rt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^2*d*e^8 + 8*sqrt(2)*
sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e
^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^3*d*e^8 + 2*b^6*c^2*e^13 + 16*(4*c
^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^3*d*e^8 - sqrt(2)*sqrt(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^
3 + b^2*e^4)*c*e^2)*b^6*e^9 + 2*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*
b^5*c*e^9 - sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^
4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^2*e^9 - 2*(4*c
^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^4*c^2*e^9)*c^2 - 2*(128*c^9*d^7*e^6 +
 64*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^
2)*c^8*d^7*e^4 - 384*b*c^8*d^6*e^7 - 192*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*
d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^7*d^6*e^5 + 480*b^2*c^7*d^5*e^8
 + 240*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c
*e^2)*b^2*c^6*d^5*e^6 - 32*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*
c*d*e^3 + b^2*e^4)*c*e^2)*b*c^7*d^5*e^6 + 16*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*
c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^8*d^5*e^6 - 320*b^3*c^6*d^4*e
^9 - 160*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*c*e^2)*b^3*c^5*d^4*e^7 + 64*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*
b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^6*d^4*e^7 - 32*sqrt(2)*sqrt(b*c*e^4 - sqr
t(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^7*d^4*e^7 - 32*(4*c^2*d
^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^7*d^5*e^4 + 120*b^4*c^5*d^3*e^10 + 60*sqr
t(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*
c^4*d^3*e^8 - 48*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +

```
b^2*e^4)*c*e^2)*b^3*c^5*d^3*e^8 + 24*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^6*d^3*e^8 + 64*(4*c^2*d^2*e^2 - 4
*b*c*d*e^3 + b^2*e^4)*b*c^6*d^4*e^5 - 24*b^5*c^4*d^2*e^11 - 12*sqrt(2)*sqrt
(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^3*d^2*e
^9 + 16*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*
c*e^2)*b^4*c^4*d^2*e^9 - 8*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*
c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^5*d^2*e^9 - 48*(4*c^2*d^2*e^2 - 4*b*c*d*e^3
 + b^2*e^4)*b^2*c^5*d^3*e^6 + 2*b^6*c^3*d*e^12 + sqrt(2)*sqrt(b*c*e^4 - sqr
t(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^6*c^2*d*e^10 - 2*sqrt(2)*
sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^3*d
*e^10 + sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*
c*e^2)*b^4*c^4*d*e^10 + 16*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^4*
d^2*e^7 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^4*c^3*d*e^8)*abs(c))*
arctan(2*sqrt(1/2)*x*e^4/sqrt((b*c^3*e^8 - sqrt(b^2*c^6*e^16 + 4*(c^4*d^2*e
^6 - b*c^3*d*e^7)*c^4*e^8))/c^4))/((16*c^9*d^6*e^6 - 48*b*c^8*d^5*e^7 + 56*
b^2*c^7*d^4*e^8 - 8*b*c^8*d^4*e^8 + 4*c^9*d^4*e^8 - 32*b^3*c^6*d^3*e^9 + 16
*b^2*c^7*d^3*e^9 - 8*b*c^8*d^3*e^9 + 9*b^4*c^5*d^2*e^10 - 10*b^3*c^6*d^2*e^
10 + 5*b^2*c^7*d^2*e^10 - b^5*c^4*d*e^11 + 2*b^4*c^5*d*e^11 - b^3*c^6*d*e^1
1)*c^2) + 1/3*(c^2*x^3*e^7 + 9*c^2*d*x*e^6 - 3*b*c*x*e^7)*e^(-6)/c^3
```

maple [A] time = 0.00, size = 142, normalized size = 1.65

$$\frac{b^{2}e^{2}\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)\,ce}\;c^{2}}-\frac{4bde\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)\,ce}\;c}+\frac{e\,x^{3}}{3c}+\frac{4d^{2}\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)\,ce}}-\frac{bex}{c^{2}}+\frac{3dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/3*e*x^3/c-1/c^2*b*e*x+3/c*d*x+1/c^2/((b*e-c*d)*c*e)^(1/2)*arctan(1/((b*e-
c*d)*c*e)^(1/2)*c*e*x)*b^2*e^2-4/c/((b*e-c*d)*c*e)^(1/2)*arctan(1/((b*e-c*d
)*c*e)^(1/2)*c*e*x)*b*d*e+4/((b*e-c*d)*c*e)^(1/2)*arctan(1/((b*e-c*d)*c*e)^
(1/2)*c*e*x)*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for more
 details)Is b*e-c*d positive or negative?
```

mupad [B] time = 4.52, size = 113, normalized size = 1.31

$$x\left(\frac{2\,d}{c}-\frac{b\,e-c\,d}{c^2}\right)+\frac{e\,x^3}{3\,c}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,e\,x\,{\left(b\,e-2\,c\,d\right)}^2}{\sqrt{b\,e^2-c\,d\,e}\,\left(b^2\,e^2-4\,b\,c\,d\,e+4\,c^2\,d^2\right)}\right)\,{\left(b\,e-2\,c\,d\right)}^2}{c^{5/2}\,\sqrt{b\,e^2-c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] x*((2*d)/c - (b*e - c*d)/c^2) + (e*x^3)/(3*c) + (atan((c^(1/2)*e*x*(b*e - 2
*c*d)^2)/((b*e^2 - c*d*e)^(1/2)*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)))*(b*e -
2*c*d)^2)/(c^(5/2)*(b*e^2 - c*d*e)^(1/2))
```

<u>**sympy**</u> **[B]** time = 0.72, size = 275, normalized size = 3.20

$$x\left(-\frac{be}{c^2}+\frac{3d}{c}\right)-\frac{\sqrt{-\frac{1}{c^5e(be-cd)}}\,(be-2cd)^2\log\left(x+\frac{-bc^2e\sqrt{-\frac{1}{c^5e(be-cd)}}\,(be-2cd)^2+c^3d\sqrt{-\frac{1}{c^5e(be-cd)}}\,(be-2cd)^2}{b^2e^2-4bcde+4c^2d^2}\right)}{2}+\frac{\sqrt{-\frac{1}{c^5e(be-cd)}}\,(be-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] x*(-b*e/c**2 + 3*d/c) - sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(
x + (-b*c**2*e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 + c**3*d*sqrt
(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2
*d**2))/2 + sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(x + (b*c**2*
e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 - c**3*d*sqrt(-1/(c**5*e*(
b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2*d**2))/2 + e
*x**3/(3*c)
```

3.216 $\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=64

$$\frac{x}{c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

```
[Out] x/c-(-b*e+2*c*d)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(3/2)/e^(1/2
)/(-b*e+c*d)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1149, 388, 208}

$$\frac{x}{c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] x/c - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^(3/2)
*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx &= \int \frac{d+ex^2}{\frac{-cd^2+bde}{d}+cex^2}\,dx\\
&= \frac{x}{c}-\frac{\left(-cde+\frac{e(-cd^2+bde)}{d}\right)\int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx}{ce}\\
&= \frac{x}{c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 63, normalized size = 0.98

$$\frac{x}{c}-\frac{(be-2cd)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{3/2}\sqrt{e}\sqrt{be-cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] x/c - ((-2*c*d + b*e)*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(3
/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

fricas [A] time = 1.01, size = 210, normalized size = 3.28

$$\left[-\frac{\sqrt{c^2de-bce^2}\,(2\,cd-be)\log\left(\frac{cex^2+cd-be+2\,\sqrt{c^2de-bce^2}\,x}{cex^2-cd+be}\right)-2\left(c^2de-bce^2\right)x}{2\left(c^3de-bc^2e^2\right)},-\frac{\sqrt{-c^2de+bce^2}\,(2\,cd-be)\arctan\left(-\right.}{c^3de-bc^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [-1/2*(sqrt(c^2*d*e - b*c*e^2)*(2*c*d - b*e)*log((c*e*x^2 + c*d - b*e + 2*s
qrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*d + b*e)) - 2*(c^2*d*e - b*c*e^2)*x)
/(c^3*d*e - b*c^2*e^2), -(sqrt(-c^2*d*e + b*c*e^2)*(2*c*d - b*e)*arctan(-sq
rt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) - (c^2*d*e - b*c*e^2)*x)/(c^3*d*e - b
*c^2*e^2)]
```

giac [B] time = 4.82, size = 7051, normalized size = 110.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] x/c - 1/8*(32*b*c^8*d^4*e^8 - 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
 b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)
*b*c^6*d^4*e^4 - 64*b^2*c^7*d^3*e^9 + 32*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c
*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4
)*c*e^2)*b^2*c^5*d^3*e^5 + 48*b^3*c^6*d^2*e^10 - 24*sqrt(2)*sqrt(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^
3 + b^2*e^4)*c*e^2)*b^3*c^4*d^2*e^6 + 8*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*
d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*c*e^2)*b^2*c^5*d^2*e^6 - 4*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*
e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^
6*d^2*e^6 - 16*b^4*c^5*d*e^11 - 8*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b
*c^6*d^2*e^6 + 8*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b
*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^3*d*e^7 -
 8*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(
4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^4*d*e^7 + 4*sqrt(2)*sqr
t(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2
- 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^5*d*e^7 + 2*b^5*c^4*e^12 + 8*(4*c^2*d
^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^5*d*e^7 - sqrt(2)*sqrt(4*c^2*d^2*e^2
- 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
b^2*e^4)*c*e^2)*b^5*c^2*e^8 + 2*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*
```

b^4*c^3*e^8 - sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*
e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^4*e^8 - 2*(4
*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^4*e^8 + (64*c^7*d^5*e^7 - 32*sq
rt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^5*d^5*e^3 - 160*b*c^6*d^4*e^8 +
80*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(
4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^4*d^4*e^4 + 160*b^2*c^5*d
^3*e^9 - 80*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^
4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^3*d^3*e^5 + 16
*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*
c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^4*d^3*e^5 - 8*sqrt(2)*sqrt(
4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 -
4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^5*d^3*e^5 - 80*b^3*c^4*d^2*e^10 - 16*(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^5*d^3*e^5 + 40*sqrt(2)*sqrt(4*c^2*d^2*e
^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
 + b^2*e^4)*c*e^2)*b^3*c^2*d^2*e^6 - 24*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*
d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*c*e^2)*b^2*c^3*d^2*e^6 + 12*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2
*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c
^4*d^2*e^6 + 20*b^4*c^3*d*e^11 + 24*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*b*c^4*d^2*e^6 - 10*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqr
t(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c*d*e^7
+ 12*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqr
t(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^2*d*e^7 - 6*sqrt(2)*s
qrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^
2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^3*d*e^7 - 2*b^5*c^2*e^12 - 12*(4*c^
2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^3*d*e^7 + sqrt(2)*sqrt(4*c^2*d^2*e
^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
 + b^2*e^4)*c*e^2)*b^5*e^8 - 2*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b
^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b
^4*c*e^8 + sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4
 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^2*e^8 + 2*(4*c^
2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^2*e^8)*c^2 - 2*(64*c^8*d^6*e^6 - 3
2*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)
*c^7*d^6*e^4 - 160*b*c^7*d^5*e^7 + 80*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2
*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^6*d^5*e^5 + 160*b^2*c^6*d^4*e^8 -
80*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2
)*b^2*c^5*d^4*e^6 + 16*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*
e^3 + b^2*e^4)*c*e^2)*b*c^6*d^4*e^6 - 8*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d
^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^7*d^4*e^6 - 80*b^3*c^5*d^3*e^9 + 4
0*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)
*b^3*c^4*d^3*e^7 - 24*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e
^3 + b^2*e^4)*c*e^2)*b^2*c^5*d^3*e^7 + 12*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^6*d^3*e^7 - 16*(4*c^2*d^2*e^2
- 4*b*c*d*e^3 + b^2*e^4)*c^6*d^4*e^4 + 20*b^4*c^4*d^2*e^10 - 10*sqrt(2)*sqr
t(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^3*d^2*
e^8 + 12*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*c*e^2)*b^3*c^4*d^2*e^8 - 6*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b
*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^5*d^2*e^8 + 24*(4*c^2*d^2*e^2 - 4*b*c*d*e^
3 + b^2*e^4)*b*c^5*d^3*e^5 - 2*b^5*c^3*d*e^11 + sqrt(2)*sqrt(b*c*e^4 + sqrt
(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^2*d*e^9 - 2*sqrt(2)*sq
rt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^3*d*e
^9 + sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e
^2)*b^3*c^4*d*e^9 - 12*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^4*d^2*
e^6 + 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^3*d*e^7)*abs(c))*arct
an(2*sqrt(1/2)*x*e^2/sqrt((b*c*e^4 + sqrt(b^2*c^2*e^8 + 4*(c^2*d^2*e^2 - b*
c*d*e^3)*c^2*e^4))/c^2))/((16*c^8*d^6*e^6 - 48*b*c^7*d^5*e^7 + 56*b^2*c^6*d
^4*e^8 - 8*b*c^7*d^4*e^8 + 4*c^8*d^4*e^8 - 32*b^3*c^5*d^3*e^9 + 16*b^2*c^6*
d^3*e^9 - 8*b*c^7*d^3*e^9 + 9*b^4*c^4*d^2*e^10 - 10*b^3*c^5*d^2*e^10 + 5*b^

```
2*c^6*d^2*e^10 - b^5*c^3*d*e^11 + 2*b^4*c^4*d*e^11 - b^3*c^5*d*e^11)*c^2) +
 1/8*(32*b*c^8*d^4*e^8 - 16*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*
e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^
6*d^4*e^4 - 64*b^2*c^7*d^3*e^9 + 32*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^
3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e
^2)*b^2*c^5*d^3*e^5 + 48*b^3*c^6*d^2*e^10 - 24*sqrt(2)*sqrt(4*c^2*d^2*e^2 -
 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b
^2*e^4)*c*e^2)*b^3*c^4*d^2*e^6 + 8*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^
2)*b^2*c^5*d^2*e^6 - 4*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*
sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^6*d^2
*e^6 - 16*b^4*c^5*d*e^11 - 8*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^6*
d^2*e^6 + 8*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^
4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^3*d*e^7 - 8*sq
rt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^4*d*e^7 + 4*sqrt(2)*sqrt(4*c
^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b
*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^5*d*e^7 + 2*b^5*c^4*e^12 + 8*(4*c^2*d^2*e^
2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^5*d*e^7 - sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b
*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e
^4)*c*e^2)*b^5*c^2*e^8 + 2*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e
^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c
^3*e^8 - sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 -
 sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^4*e^8 - 2*(4*c^2*
d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^4*e^8 + (64*c^7*d^5*e^7 - 32*sqrt(2)
*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^5*d^5*e^3 - 160*b*c^6*d^4*e^8 + 80*sq
rt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^4*d^4*e^4 + 160*b^2*c^5*d^3*e^
9 - 80*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - s
qrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^3*d^3*e^5 + 16*sqrt
(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d
^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^4*d^3*e^5 - 8*sqrt(2)*sqrt(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c
*d*e^3 + b^2*e^4)*c*e^2)*c^5*d^3*e^5 - 80*b^3*c^4*d^2*e^10 - 16*(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^5*d^3*e^5 + 40*sqrt(2)*sqrt(4*c^2*d^2*e^2 -
4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^
2*e^4)*c*e^2)*b^3*c^2*d^2*e^6 - 24*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^
2)*b^2*c^3*d^2*e^6 + 12*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^4*d^
2*e^6 + 20*b^4*c^3*d*e^11 + 24*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^
4*d^2*e^6 - 10*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c
*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c*d*e^7 + 12*
sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c
^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^2*d*e^7 - 6*sqrt(2)*sqrt(4
*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4
*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^3*d*e^7 - 2*b^5*c^2*e^12 - 12*(4*c^2*d^2
*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^3*d*e^7 + sqrt(2)*sqrt(4*c^2*d^2*e^2 -
4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^
2*e^4)*c*e^2)*b^5*e^8 - 2*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^
4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c*
e^8 + sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sq
rt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c^2*e^8 + 2*(4*c^2*d^2
*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^2*e^8)*c^2 - 2*(64*c^8*d^6*e^6 + 32*sqr
t(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^7*
d^6*e^4 - 160*b*c^7*d^5*e^7 - 80*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2
- 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^6*d^5*e^5 + 160*b^2*c^6*d^4*e^8 + 80*sq
rt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2
```

```
*c^5*d^4*e^6 - 16*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
 b^2*e^4)*c*e^2)*b*c^6*d^4*e^6 + 8*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^
2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^7*d^4*e^6 - 80*b^3*c^5*d^3*e^9 - 40*sqr
t(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*
c^4*d^3*e^7 + 24*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
b^2*e^4)*c*e^2)*b^2*c^5*d^3*e^7 - 12*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^6*d^3*e^7 - 16*(4*c^2*d^2*e^2 - 4*b
*c*d*e^3 + b^2*e^4)*c^6*d^4*e^4 + 20*b^4*c^4*d^2*e^10 + 10*sqrt(2)*sqrt(b*c
*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^3*d^2*e^8 -
 12*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^
2)*b^3*c^4*d^2*e^8 + 6*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*
e^3 + b^2*e^4)*c*e^2)*b^2*c^5*d^2*e^8 + 24*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b
^2*e^4)*b*c^5*d^3*e^5 - 2*b^5*c^3*d*e^11 - sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^
2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^5*c^2*d*e^9 + 2*sqrt(2)*sqrt(b*
c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*c^3*d*e^9 -
sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b
^3*c^4*d*e^9 - 12*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c^4*d^2*e^6 +
 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b^3*c^3*d*e^7)*abs(c))*arctan(2*
sqrt(1/2)*x*e^2/sqrt((b*c*e^4 - sqrt(b^2*c^2*e^8 + 4*(c^2*d^2*e^2 - b*c*d*e
^3)*c^2*e^4))/c^2))/((16*c^8*d^6*e^6 - 48*b*c^7*d^5*e^7 + 56*b^2*c^6*d^4*e^
8 - 8*b*c^7*d^4*e^8 + 4*c^8*d^4*e^8 - 32*b^3*c^5*d^3*e^9 + 16*b^2*c^6*d^3*e
^9 - 8*b*c^7*d^3*e^9 + 9*b^4*c^4*d^2*e^10 - 10*b^3*c^5*d^2*e^10 + 5*b^2*c^6
*d^2*e^10 - b^5*c^3*d*e^11 + 2*b^4*c^4*d*e^11 - b^3*c^5*d*e^11)*c^2)
```

maple [A] time = 0.00, size = 79, normalized size = 1.23

$$-\frac{be\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)\,ce}\;c}+\frac{2d\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)\,ce}}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/c*x-1/c/((b*e-c*d)*c*e)^(1/2)*arctan(1/((b*e-c*d)*c*e)^(1/2)*c*e*x)*b*e+2
/((b*e-c*d)*c*e)^(1/2)*arctan(1/((b*e-c*d)*c*e)^(1/2)*c*e*x)*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for more
 details)Is b*e-c*d positive or negative?
```

mupad [B] time = 0.07, size = 52, normalized size = 0.81

$$\frac{x}{c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,e\,x}{\sqrt{b\,e^2-c\,d\,e}}\right)\,\left(b\,e-2\,c\,d\right)}{c^{3/2}\,\sqrt{b\,e^2-c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] x/c - (atan((c^(1/2)*e*x)/(b*e^2 - c*d*e)^(1/2))*(b*e - 2*c*d))/(c^(3/2)*(b
*e^2 - c*d*e)^(1/2))
```

sympy [B] time = 0.49, size = 212, normalized size = 3.31

$$\frac{\sqrt{-\frac{1}{c^3e(be-cd)}}\,(be-2cd)\log\left(x+\frac{-bce\sqrt{-\frac{1}{c^3e(be-cd)}}\,(be-2cd)+c^2d\sqrt{-\frac{1}{c^3e(be-cd)}}\,(be-2cd)}{be-2cd}\right)}{2}-\frac{\sqrt{-\frac{1}{c^3e(be-cd)}}\,(be-2cd)\log\left(x+\frac{bce\sqrt{}}{}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(x + (-b*c*e*sqrt(-1/(c**3*e
*(b*e - c*d)))*(b*e - 2*c*d) + c**2*d*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e -
2*c*d))/(b*e - 2*c*d))/2 - sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(
x + (b*c*e*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d) - c**2*d*sqrt(-1/(c*
*3*e*(b*e - c*d)))*(b*e - 2*c*d))/(b*e - 2*c*d))/2 + x/c
```

3.217 $\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

```
[Out] -arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(1/2)/e^(1/2)/(-b*e+c*d)^(1/
2)
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {1149, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[c*d -
b*e]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{1}{\frac{-cd^2+bde}{d}+cex^2} dx\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{c}\sqrt{e}\sqrt{be-cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[-(c*d)
 + b*e])
```

fricas [A] time = 0.90, size = 134, normalized size = 2.73

$$\left[\frac{\log\left(\frac{cex^2+cd-be-2\sqrt{c^2de-bce^2}x}{cex^2-cd+be}\right)}{2\sqrt{c^2de-bce^2}}, -\frac{\sqrt{-c^2de+bce^2}\arctan\left(-\frac{\sqrt{-c^2de+bce^2}x}{cd-be}\right)}{c^2de-bce^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas"
)
```

```
[Out] [1/2*log((c*e*x^2 + c*d - b*e - 2*sqrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*d
 + b*e))/sqrt(c^2*d*e - b*c*e^2), -sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-c
^2*d*e + b*c*e^2)*x/(c*d - b*e))/(c^2*d*e - b*c*e^2)]
```

giac [B] time = 6.09, size = 3276, normalized size = 66.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")
```

```
[Out] 1/4*(32*c^5*d^4*e^4 - 16*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*
d*e^3 + b^2*e^4)*c*e^2)*c^4*d^4*e^2 - 64*b*c^4*d^3*e^5 - 16*c^5*d^3*e^5 + 3
2*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)
*b*c^3*d^3*e^3 + 8*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt
(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^3*d^3*e + 4
8*b^2*c^3*d^2*e^6 + 24*b*c^4*d^2*e^6 - 24*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2
*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^2*d^2*e^4 + 8*sqrt(2)*sqrt(b
*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^3*d^2*e^4 -
 4*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2
)*c^4*d^2*e^4 - 12*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt
(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^2*d^2*e^2
 - 16*b^3*c^2*d*e^7 - 12*b^2*c^3*d*e^7 + 8*sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^
2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c*d*e^5 - 8*sqrt(2)*sqrt(b*c*
e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^2*d*e^5 + 4*
sqrt(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b
*c^3*d*e^5 - 8*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^3*d^2*e^2 + 6*sqrt
(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d
^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c*d*e^3 - 4*sqrt(2)*sqrt(4*c^2*d
^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d
*e^3 + b^2*e^4)*c*e^2)*b*c^2*d*e^3 + 2*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d
*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*
c*e^2)*c^3*d*e^3 + 2*b^4*c*e^8 + 2*b^3*c^2*e^8 - sqrt(2)*sqrt(b*c*e^4 + sqr
t(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^4*e^6 + 2*sqrt(2)*sqrt(b*
c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c*e^6 - sqrt
(2)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c
^2*e^6 + 8*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^2*d*e^3 + 4*(4*c^2*d
^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^3*d*e^3 - sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*
b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*
e^4)*c*e^2)*b^3*e^4 + 2*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)
*sqrt(b*c*e^4 + sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c*e^
4 - sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 + sqrt
(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^2*e^4 - 2*(4*c^2*d^2*e^2
 - 4*b*c*d*e^3 + b^2*e^4)*b^2*c*e^4 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*
e^4)*b*c^2*e^4)*arctan(2*sqrt(1/2)*x*e/sqrt((b*e^2 + sqrt(b^2*e^4 + 4*(c*d^
2 - b*d*e)*c*e^2))/c))/((16*c^5*d^5*e^4 - 48*b*c^4*d^4*e^5 + 56*b^2*c^3*d^3
*e^6 - 8*b*c^4*d^3*e^6 + 4*c^5*d^3*e^6 - 32*b^3*c^2*d^2*e^7 + 16*b^2*c^3*d^
2*e^7 - 8*b*c^4*d^2*e^7 + 9*b^4*c*d*e^8 - 10*b^3*c^2*d*e^8 + 5*b^2*c^3*d*e^
```

```
8 - b^5*e^9 + 2*b^4*c*e^9 - b^3*c^2*e^9)*abs(c)) - 1/4*(32*c^5*d^4*e^4 + 16
*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*
c^4*d^4*e^2 - 64*b*c^4*d^3*e^5 - 16*c^5*d^3*e^5 - 32*sqrt(2)*sqrt(b*c*e^4 -
 sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^3*d^3*e^3 + 8*sqrt(
2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^
2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^3*d^3*e + 48*b^2*c^3*d^2*e^6 + 24*b
*c^4*d^2*e^6 + 24*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 +
 b^2*e^4)*c*e^2)*b^2*c^2*d^2*e^4 - 8*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*
e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^3*d^2*e^4 + 4*sqrt(2)*sqrt(b*c*e^4
- sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^4*d^2*e^4 - 12*sqrt(
2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^
2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^2*d^2*e^2 - 16*b^3*c^2*d*e^7 - 12
*b^2*c^3*d*e^7 - 8*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3
+ b^2*e^4)*c*e^2)*b^3*c*d*e^5 + 8*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2
 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^2*d*e^5 - 4*sqrt(2)*sqrt(b*c*e^4 - s
qrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b*c^3*d*e^5 - 8*(4*c^2*d^
2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c^3*d^2*e^2 + 6*sqrt(2)*sqrt(4*c^2*d^2*e^2 -
 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b
^2*e^4)*c*e^2)*b^2*c*d*e^3 - 4*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b
^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b
*c^2*d*e^3 + 2*sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c
*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*c^3*d*e^3 + 2*b^4
*c*e^8 + 2*b^3*c^2*e^8 + sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*
d*e^3 + b^2*e^4)*c*e^2)*b^4*e^6 - 2*sqrt(2)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e
^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*c*e^6 + sqrt(2)*sqrt(b*c*e^4 - sqrt(
4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c^2*e^6 + 8*(4*c^2*d^2*e^
2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^2*d*e^3 + 4*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b
^2*e^4)*c^3*d*e^3 - sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqr
t(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^3*e^4 + 2*
sqrt(2)*sqrt(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c
^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*c*e^2)*b^2*c*e^4 - sqrt(2)*sqrt(4*c^2*d
^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*sqrt(b*c*e^4 - sqrt(4*c^2*d^2*e^2 - 4*b*c*d
*e^3 + b^2*e^4)*c*e^2)*b*c^2*e^4 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4
)*b^2*c*e^4 - 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*b*c^2*e^4)*arctan(2
*sqrt(1/2)*x*e/sqrt((b*e^2 - sqrt(b^2*e^4 + 4*(c*d^2 - b*d*e)*c*e^2))/c))/(
(16*c^5*d^5*e^4 - 48*b*c^4*d^4*e^5 + 56*b^2*c^3*d^3*e^6 - 8*b*c^4*d^3*e^6 +
 4*c^5*d^3*e^6 - 32*b^3*c^2*d^2*e^7 + 16*b^2*c^3*d^2*e^7 - 8*b*c^4*d^2*e^7
+ 9*b^4*c*d*e^8 - 10*b^3*c^2*d*e^8 + 5*b^2*c^3*d*e^8 - b^5*e^9 + 2*b^4*c*e^
9 - b^3*c^2*e^9)*abs(c))
```

maple [A] time = 0.00, size = 33, normalized size = 0.67

$$\frac{\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{\sqrt{(be-cd)\,ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/((b*e-c*d)*c*e)^(1/2)*arctan(1/((b*e-c*d)*c*e)^(1/2)*c*e*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for more
 details)Is b*e-c*d positive or negative?
```

mupad [B] time = 4.49, size = 38, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{c\,e\,x}{\sqrt{b\,c\,e^2-c^2\,d\,e}}\right)}{\sqrt{b\,c\,e^2-c^2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] atan((c*e*x)/(b*c*e^2 - c^2*d*e)^(1/2))/(b*c*e^2 - c^2*d*e)^(1/2)
```

sympy [B] time = 0.32, size = 124, normalized size = 2.53

$$-\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(-be\sqrt{-\frac{1}{ce(be-cd)}}+cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(be\sqrt{-\frac{1}{ce(be-cd)}}-cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] -sqrt(-1/(c*e*(b*e - c*d)))*log(-b*e*sqrt(-1/(c*e*(b*e - c*d))) + c*d*sqrt(
-1/(c*e*(b*e - c*d))) + x)/2 + sqrt(-1/(c*e*(b*e - c*d)))*log(b*e*sqrt(-1/(
c*e*(b*e - c*d))) - c*d*sqrt(-1/(c*e*(b*e - c*d))) + x)/2
```

3.218 $\int \frac{1}{\left(d+ex^2\right)\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$

Optimal. Leaf size=136

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2}-\frac{(4cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2}-\frac{x}{2d\left(d+ex^2\right)(2cd-be)}$$

```
[Out] -1/2*x/d/(-b*e+2*c*d)/(e*x^2+d)-1/2*(-b*e+4*c*d)*arctan(x*e^(1/2)/d^(1/2))/
d^(3/2)/(-b*e+2*c*d)^2/e^(1/2)-c^(3/2)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)
^(1/2))/(-b*e+2*c*d)^2/e^(1/2)/(-b*e+c*d)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1149, 414, 522, 205, 208}

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2}-\frac{(4cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2}-\frac{x}{2d\left(d+ex^2\right)(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -x/(2*d*(2*c*d - b*e)*(d + e*x^2)) - ((4*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[
e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

```
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \int \frac{1}{\left(d+ex^2\right)^2\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx$$

$$= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)} + \frac{\int \frac{e(3cd-be)-ce^2x^2}{\left(d+ex^2\right)\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2de(2cd-be)}$$

$$= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)} + \frac{c^2\int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx}{(2cd-be)^2} - \frac{(4cd-be)\int \frac{1}{d+e}}{2d(2cd-be)}$$

$$= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)} - \frac{(4cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2} - \frac{c^{3/2}\tanh^{-1}}{\sqrt{e}\sqrt{cd-be}}$$

Mathematica [A] time = 0.20, size = 133, normalized size = 0.98

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{e}(be-2cd)^2\sqrt{be-cd}} + \frac{(be-4cd)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2} - \frac{x}{2d\left(d+ex^2\right)(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -1/2*x/(d*(2*c*d - b*e)*(d + e*x^2)) + ((-4*c*d + b*e)*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) + (c^(3/2)*ArcTan[(Sqrt[c]*Sqr
t[e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*(-2*c*d + b*e)^2*Sqrt[-(c*d) + b*e])
```

fricas [A] time = 1.53, size = 895, normalized size = 6.58

$$\left[\frac{2\left(cd^2e^2x^2+cd^3e\right)\sqrt{\frac{c}{cde-be^2}}\log\left(\frac{cex^2-2\left(cde-be^2\right)x\sqrt{\frac{c}{cde-be^2}}+cd-be}{cex^2-cd+be}\right)+\left(4cd^2-bde+\left(4cde-be^2\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{}}{ex^2}\right.}{4\left(4c^2d^5e-4bcd^4e^2+b^2d^3e^3+\left(4c^2d^4e^2-4bcd^3e^3+b^2d^2e^4\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/4*(2*(c*d^2*e^2*x^2 + c*d^3*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 - 2*
(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e
)) + (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sq
rt(-d*e)*x - d)/(e*x^2 + d)) - 2*(2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*
b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x
^2), -1/2*((4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(d*e)*arctan(sqrt(
d*e)*x/d) - (c*d^2*e^2*x^2 + c*d^3*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2
- 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d +
 b*e)) + (2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^
3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2), 1/4*(4*(c*d^2*e^2*x
^2 + c*d^3*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2)))
```

```
 + (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt
(-d*e)*x - d)/(e*x^2 + d)) - 2*(2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*
c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2
), 1/2*(2*(c*d^2*e^2*x^2 + c*d^3*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqr
t(-c/(c*d*e - b*e^2))) - (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(d*e
)*arctan(sqrt(d*e)*x/d) - (2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*c*d^4
*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [b,c,d,exp(1),exp(2)]=[-95,-68,60,-66,8]Warning, need to choose a
branch for the root of a polynomial with parameters. This might be wrong.Th
e choice was done assuming [b,c,d,exp(1),exp(2)]=[79,32,2,-92,39]sym2poly/r
2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument
 Valuesym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Err
or: Bad Argument Valuesym2poly/r2sym(const gen & e,const index_m & i,const
vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 10.7
7Done
```

maple [A] time = 0.01, size = 155, normalized size = 1.14

$$\frac{c^2\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)}{(be-2cd)^2\sqrt{(be-cd)ce}}+\frac{bex}{2(be-2cd)^2\left(ex^2+d\right)d}+\frac{be\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2(be-2cd)^2\sqrt{de}\,d}-\frac{cx}{(be-2cd)^2\left(ex^2+d\right)}-\frac{2c\arctan\left(\cdots\right.}{(be-2cd)^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] c^2/(b*e-2*c*d)^2/((b*e-c*d)*c*e)^(1/2)*arctan(1/((b*e-c*d)*c*e)^(1/2)*c*e*
x)+1/2/(b*e-2*c*d)^2/d*x/(e*x^2+d)*b*e-1/(b*e-2*c*d)^2*x/(e*x^2+d)*c+1/2/(b
*e-2*c*d)^2/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b*e-2/(b*e-2*c*d)^2/(d*
e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for more
 details)Is b*e-c*d positive or negative?
```

mupad [B] time = 5.40, size = 3901, normalized size = 28.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

```
[Out] - x/(2*(d + e*x^2)*(2*c*d^2 - b*d*e)) - (atan(((((((96*c^7*d^6*e^6 - 224*b*
c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 +
 22*b^4*c^3*d^2*e^10)/(2*(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*
c^2*d^4*e)) - (x*(-c^3*e*(b*e - c*d))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b^2*c^
5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^
12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e + 8*
b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e^4 -
4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)) - (x*(b^2*c^3*e^8 + 20*c^5*
d^2*e^6 - 8*b*c^4*d*e^7))/(4*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))*(-c^
3*e*(b*e - c*d))^(1/2)*1i)/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2
*c*d*e^3) - (((((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 20
8*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(2*(8*c^3*d^5
 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e)) + (x*(-c^3*e*(b*e - c*d
))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 -
128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 -
4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-
c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b
^2*c*d*e^3)) + (x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(4*(4*c^2
*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))*(-c^3*e*(b*e - c*d))^(1/2)*1i)/(b^3*e^4
 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3))/((((((96*c^7*d^6*e^6 - 2
24*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*
e^9 + 22*b^4*c^3*d^2*e^10)/(2*(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 -
12*b*c^2*d^4*e)) - (x*(-c^3*e*(b*e - c*d))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b
^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d
^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e
 + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e
^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)) - (x*(b^2*c^3*e^8 + 20
*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(4*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))
*(-c^3*e*(b*e - c*d))^(1/2))/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b
^2*c*d*e^3) - ((b*c^4*e^6)/2 - 2*c^5*d*e^5)/(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^
2*c*d^3*e^2 - 12*b*c^2*d^4*e) + (((((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2
*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2
*e^10)/(2*(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e)) + (
x*(-c^3*e*(b*e - c*d))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384
*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*
d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 -
 5*b^2*c*d*e^3)))*(-c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e^4 - 4*c^3*d^3*e + 8
*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)) + (x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c
^4*d*e^7))/(4*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))*(-c^3*e*(b*e - c*d)
)^(1/2))/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-c^3*
e*(b*e - c*d))^(1/2)*1i)/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c
*d*e^3) - (atan(((((x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(2*(4
*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)) - ((-d^3*e)^(1/2)*((96*c^7*d^6*e^6 -
 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^
3*e^9 + 22*b^4*c^3*d^2*e^10)/(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 1
2*b*c^2*d^4*e) - (x*(-d^3*e)^(1/2)*(b*e - 4*c*d)*(256*b*c^6*d^6*e^8 - 512*b
^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d
^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(4*c^2*d^5*e + b^2*d^3
*e^3 - 4*b*c*d^4*e^2)))*(b*e - 4*c*d))/(4*(4*c^2*d^5*e + b^2*d^3*e^3 - 4*b*
c*d^4*e^2)))*(-d^3*e)^(1/2)*(b*e - 4*c*d)*1i)/(4*(4*c^2*d^5*e + b^2*d^3*e^3
 - 4*b*c*d^4*e^2)) + (((x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(
2*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)) + ((-d^3*e)^(1/2)*((96*c^7*d^6*e
^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^
4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2
 - 12*b*c^2*d^4*e) + (x*(-d^3*e)^(1/2)*(b*e - 4*c*d)*(256*b*c^6*d^6*e^8 - 5
12*b^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c
```

```
^2*d^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(4*c^2*d^5*e + b^2
*d^3*e^3 - 4*b*c*d^4*e^2)))*(b*e - 4*c*d))/(4*(4*c^2*d^5*e + b^2*d^3*e^3 -
4*b*c*d^4*e^2)))*(-d^3*e)^(1/2)*(b*e - 4*c*d)*1i)/(4*(4*c^2*d^5*e + b^2*d^3
*e^3 - 4*b*c*d^4*e^2)))/(((b*c^4*e^6)/2 - 2*c^5*d*e^5)/(8*c^3*d^5 - b^3*d^2
*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e) + (((x*(b^2*c^3*e^8 + 20*c^5*d^2*e
^6 - 8*b*c^4*d*e^7))/(2*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)) - ((-d^3*e
)^(1/2)*((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c
^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(8*c^3*d^5 - b^3*d^2
*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e) - (x*(-d^3*e)^(1/2)*(b*e - 4*c*d)*
(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c
^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3
*e)*(4*c^2*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2)))*(b*e - 4*c*d))/(4*(4*c^2*
d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2)))*(-d^3*e)^(1/2)*(b*e - 4*c*d))/(4*(4*
c^2*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2)) - (((x*(b^2*c^3*e^8 + 20*c^5*d^2*
e^6 - 8*b*c^4*d*e^7))/(2*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)) + ((-d^3*
e)^(1/2)*((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*
c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(8*c^3*d^5 - b^3*d^
2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e) + (x*(-d^3*e)^(1/2)*(b*e - 4*c*d)
*(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*
c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^
3*e)*(4*c^2*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2)))*(b*e - 4*c*d))/(4*(4*c^2
*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2)))*(-d^3*e)^(1/2)*(b*e - 4*c*d))/(4*(4
*c^2*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2))))*(-d^3*e)^(1/2)*(b*e - 4*c*d)*1
i)/(2*(4*c^2*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Timed out
```

3.219 $\int \frac{1}{\left(d+ex^2\right)^2\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$

Optimal. Leaf size=187

$$-\frac{\left(3b^2e^2-16bcde+28c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}\,(2cd-be)^3}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^3}-\frac{x(10cd-3be)}{8d^2\left(d+ex^2\right)(2cd-be)^2}-\frac{x}{4d\left(d+ex^2\right)^2(2cd}$$

```
[Out] -1/4*x/d/(-b*e+2*c*d)/(e*x^2+d)^2-1/8*(-3*b*e+10*c*d)*x/d^2/(-b*e+2*c*d)^2/
(e*x^2+d)-1/8*(3*b^2*e^2-16*b*c*d*e+28*c^2*d^2)*arctan(x*e^(1/2)/d^(1/2))/d
^(5/2)/(-b*e+2*c*d)^3/e^(1/2)-c^(5/2)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^
(1/2))/(-b*e+2*c*d)^3/e^(1/2)/(-b*e+c*d)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1149, 414, 527, 522, 205, 208}

$$-\frac{\left(3b^2e^2-16bcde+28c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}\,(2cd-be)^3}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^3}-\frac{x(10cd-3be)}{8d^2\left(d+ex^2\right)(2cd-be)^2}-\frac{x}{4d\left(d+ex^2\right)^2(2cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -x/(4*d*(2*c*d - b*e)*(d + e*x^2)^2) - ((10*c*d - 3*b*e)*x)/(8*d^2*(2*c*d -
 b*e)^2*(d + e*x^2)) - ((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]*(2*c*d - b*e)^3) - (c^(5/2)*ArcTanh[(Sqr
t[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^3\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx$$

$$=-\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}+\frac{\int \frac{e(7cd-3be)-3ce^2x^2}{\left(d+ex^2\right)^2\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{4de(2cd-be)}$$

$$=-\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2\left(d+ex^2\right)}+\frac{\int \frac{e^2(1}{}}{}$$

$$=-\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2\left(d+ex^2\right)}+\frac{c^3\int}{(2}$$

$$=-\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2\left(d+ex^2\right)}-\frac{\left(28c^2\right.}{}$$

Mathematica [A] time = 0.41, size = 177, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{\left(3b^2e^2-16bcde+28c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}(2cd-be)^3}-\frac{\frac{8c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{e}\sqrt{be-cd}}+\frac{x(be-2cd)\left(2cd\left(7d+5ex^2\right)-be\left(5d+3ex^2\right)\right)}{d^2\left(d+ex^2\right)^2}}{(be-2cd)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] (-(((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(
5/2)*Sqrt[e]*(2*c*d - b*e)^3)) - (((-2*c*d + b*e)*x*(-(b*e*(5*d + 3*e*x^2))
 + 2*c*d*(7*d + 5*e*x^2)))/(d^2*(d + e*x^2)^2) + (8*c^(5/2)*ArcTan[(Sqrt[c]
*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*Sqrt[-(c*d) + b*e]))/(-2*c*d + b*
e)^3)/8
```

fricas [B] time = 2.91, size = 1765, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fri
cas")
```

```
[Out] [-1/16*(2*(20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 + 8*(c^2*d^3*
e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x
^2 + 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*
d + b*e)) - (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 -
16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*
e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(28*
c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*
e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6
*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6
*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/8*((20*c^2*d^3*e^2 - 16*b*c*d^2*e^3
+ 3*b^2*d*e^4)*x^3 + (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d
^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 +
 3*b^2*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 4*(c^2*d^3*e^3*x^4 + 2
*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 + 2*(c*d
*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e)) +
 (28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2
*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^
4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^
3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/16*(2*(20*c^2*d^3*e^2 - 16*b*c*
d^2*e^3 + 3*b^2*d*e^4)*x^3 - 16*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*
d^5*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2))) - (28*
c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3
*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*sqrt(-
d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(28*c^2*d^4*e - 24*b
*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^
6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 -
 b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 -
 b^3*d^4*e^5)*x^2), -1/8*((20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x
^3 - 8*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(-c/(c*d*e - b
*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2))) + (28*c^2*d^4 - 16*b*c*d^3*e +
3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2
*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d)
 + (28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c
^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*
e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*
e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="gia
c")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (4*b^
5*c*exp(1)*exp(2)^5+2*b^5*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c
^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)*exp(2)^4-36*b^4*c^2*d*e
xp(1)^2*exp(2)^4-4*b^4*c^2*d*exp(2)^5-4*b^4*c^2*exp(1)*exp(2)^5-18*b^4*c*d*
sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(
1)*exp(2))*exp(2))*exp(1)^2*exp(2)^3-2*b^4*c*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*
sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(2)^4-
4*b^4*c*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*
c*d*exp(1)*exp(2))*exp(2))*exp(1)*exp(2)^4+2*b^4*sqrt(2)*sqrt(b*c*exp(2)^2-
c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^
```

2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)^3+96*b^3*c
^3*d^2*exp(1)^3*exp(2)^3+64*b^3*c^3*d^2*exp(1)*exp(2)^4+28*b^3*c^3*d*exp(1)
^2*exp(2)^4+4*b^3*c^3*d*exp(2)^5+48*b^3*c^2*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c
*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^3
*exp(2)^2+32*b^3*c^2*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^
2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)*exp(2)^3+20*b^3*c^2*d*sq
rt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)
*exp(2))*exp(2))*exp(1)^2*exp(2)^3+4*b^3*c^2*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*
sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(2)^4+
2*b^3*c^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*
b*c*d*exp(1)*exp(2))*exp(2))*exp(1)*exp(2)^4-14*b^3*c*d*sqrt(2)*sqrt(b*c*ex
p(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*
sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^2*exp(2)^2
-2*b^3*c*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4
*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*ex
p(1)*exp(2))*exp(2)^3-4*b^3*c*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2
+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^
2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)^3-4*b^3*c*(b^2*exp(2)^2+4*c^2
*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)^3-64*b^2*c^4*d^3*exp(1)^4*
exp(2)^2-224*b^2*c^4*d^3*exp(1)^2*exp(2)^3-32*b^2*c^4*d^3*exp(2)^4-48*b^2*c
^4*d^2*exp(1)^3*exp(2)^3-48*b^2*c^4*d^2*exp(1)*exp(2)^4-32*b^2*c^3*d^3*sqrt
(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*e
xp(2))*exp(2))*exp(1)^4*exp(2)-112*b^2*c^3*d^3*sqrt(2)*sqrt(b*c*exp(2)^2-c*
sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^2*
exp(2)^2-16*b^2*c^3*d^3*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2
*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(2)^3-16*b^2*c^3*d^2*sqrt(2)*
sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2
))*exp(2))*exp(1)^3*exp(2)^2-32*b^2*c^3*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqr
t(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)*exp(2
)^3-10*b^2*c^3*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*ex
p(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^2*exp(2)^3-2*b^2*c^3*d*sqrt(2)*s
qrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2)
)*exp(2))*exp(2)^4+24*b^2*c^2*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(
2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^
2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^3*exp(2)+24*b^2*c^2*d^2*sqrt(2)*
sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2
))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)
*exp(2)^2+12*b^2*c^2*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*
d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2
)-4*b*c*d*exp(1)*exp(2))*exp(1)^2*exp(2)^2+4*b^2*c^2*d*sqrt(2)*sqrt(b*c*exp
(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*s
qrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2)^3+20*b^2*c^
2*d*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^2*exp(2)^2
+4*b^2*c^2*d*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2)^3
+2*b^2*c^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4
*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*ex
p(1)*exp(2))*exp(1)*exp(2)^3+4*b^2*c^2*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c
*d*exp(1)*exp(2))*exp(1)*exp(2)^3+128*b*c^5*d^4*exp(1)^3*exp(2)^2+192*b*c^5
*d^4*exp(1)*exp(2)^3+112*b*c^5*d^3*exp(1)^2*exp(2)^3+16*b*c^5*d^3*exp(2)^4+
64*b*c^4*d^4*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)
-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^3*exp(2)+96*b*c^4*d^4*sqrt(2)*sqrt(b
*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp
(2))*exp(1)*exp(2)^2+16*b*c^4*d^3*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(
2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^2*exp(2)^2+16*b
*c^4*d^3*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b
*c*d*exp(1)*exp(2))*exp(2))*exp(2)^3+8*b*c^4*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-
c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^
3*exp(2)^2+16*b*c^4*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2
*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)*exp(2)^3-56*b*c^3*d^3*sqr

t(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^2*exp(2)-8*b*c^3*d^3*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2)^2-16*b*c^3*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)^2-16*b*c^3*d^2*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^3*exp(2)-32*b*c^3*d^2*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)^2-6*b*c^3*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^2*exp(2)^2-2*b*c^3*d*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2)^3-12*b*c^3*d*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^2*exp(2)^2-4*b*c^3*d*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2)^3-64*c^6*d^5*exp(1)^2*exp(2)^2-64*c^6*d^5*exp(2)^3-64*c^6*d^4*exp(1)*exp(2)^3-32*c^5*d^5*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^2*exp(2)-32*c^5*d^5*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(2)^2-8*c^5*d^3*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^2*exp(2)^2-8*c^5*d^3*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(2)^3+32*c^4*d^4*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)+16*c^4*d^3*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^2*exp(2)+16*c^4*d^3*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2)^2+8*c^4*d^2*sqrt(2)*sqrt(b*c*exp(2)^2-c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)^2+16*c^4*d^2*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)^2)/(8*b^6*d^3*exp(1)^6*exp(2)^3-16*b^6*d^3*exp(1)^4*exp(2)^4+8*b^6*d^3*exp(1)^2*exp(2)^5-64*b^5*c*d^4*exp(1)^7*exp(2)^2+112*b^5*c*d^4*exp(1)^5*exp(2)^3-32*b^5*c*d^4*exp(1)^3*exp(2)^4-16*b^5*c*d^4*exp(1)*exp(2)^5-16*b^5*c*d^3*exp(1)^6*exp(2)^3+32*b^5*c*d^3*exp(1)^4*exp(2)^4-16*b^5*c*d^3*exp(1)^2*exp(2)^5+128*b^4*c^2*d^5*exp(1)^8*exp(2)-64*b^4*c^2*d^5*exp(1)^6*exp(2)^2-248*b^4*c^2*d^5*exp(1)^4*exp(2)^3+176*b^4*c^2*d^5*exp(1)^2*exp(2)^4+8*b^4*c^2*d^5*exp(2)^5+64*b^4*c^2*d^4*exp(1)^7*exp(2)^2-96*b^4*c^2*d^4*exp(1)^5*exp(2)^3+32*b^4*c^2*d^4*exp(1)*exp(2)^5+8*b^4*c^2*d^3*exp(1)^6*exp(2)^3-16*b^4*c^2*d^3*exp(1)^4*exp(2)^4+8*b^4*c^2*d^3*exp(1)^2*exp(2)^5-512*b^3*c^3*d^6*exp(1)^7*exp(2)+832*b^3*c^3*d^6*exp(1)^5*exp(2)^2-128*b^3*c^3*d^6*exp(1)^3*exp(2)^3-192*b^3*c^3*d^6*exp(1)*exp(2)^4-192*b^3*c^3*d^5*exp(1)^6*exp(2)^2+368*b^3*c^3*d^5*exp(1)^4*exp(2)^3-160*b^3*c^3*d^5*exp(1)^2*exp(2)^4-16*b^3*c^3*d^5*exp(2)^5-32*b^3*c^3*d^4*exp(1)^7*exp(2)^2+48*b^3*c^3*d^4*exp(1)^5*exp(2)^3-16*b^3*c^3*d^4*exp(1)*exp(2)^5+768*b^2*c^4*d^7*exp(1)^6*exp(2)-1472*b^2*c^4*d^7*exp(1)^4*exp(2)^2+640*b^2*c^4*d^7*exp(1)^2*exp(2)^3+64*b^2*c^4*d^7*exp(2)^4+192*b^2*c^4*d^6*exp(1)^5*exp(2)^2-384*b^2*c^4*d^6*exp(1)^3*exp(2)^3+192*b^2*c^4*d^6*exp(1)*exp(2)^4+96*b^2*c^4*d^5*exp(1)^6*exp(2)^2-184*b^2*c^4*d^5*exp(1)^4*exp(2)^3+80*b^2*c^4*d^5*exp(1)^2*exp(2)^4+8*b^2*c^4*d^5*exp(2)^5-512*b*c^5*d^8*exp(1)^5*exp(2)+1024*b*c^5*d^8*exp(1)^3*exp(2)^2-512*b*c^5*d^8*exp(1)*exp(2)^3-64*b*c^5*d^7*exp(1)^4*exp(2)^2+128*b*c^5*d^7*exp(1)^2*exp(2)^3-64*b*c^5*d^7*exp(2)^4-96*b*c^5*d^6*exp(1)^5*exp(2)^2+192*b*c^5*d^6*exp(1)^3*exp(2)^3-96*b*c^5*d^6*exp(1)*exp(2)^4+128*c^6*d^9*exp(1)^4*exp(2)-256*c^6*d^9*exp(1)^2*exp(2)^2+128*c^6*d^9*exp(2)^3+32*c^6*d^7*exp(1)^4*exp(2)^2-64*c^6*d^7*exp(1)^2*exp(2)^3+32*c^6*d^7*exp(2)^4)/abs(c)*atan(x/sqrt(-(c^2*exp(2)^3*b*d^4-2*c^2*exp(2)^2*b*d^4*exp(1)^2+c^2*exp(2)*b*d^4*exp(1)^4-2*c*exp(2)^3*b^2*d^3*exp(1)+4*c*exp(2)^2*b^2*d^3*exp(1)^3-2*c*exp(2)*b^2*d^3*exp(1)^5+exp(2)^3*b^3*d^2*exp(1)^2-2*exp(2)^2*b^3*d^2*exp(1)^4+exp(2)*b^3*d^2*exp(1)^6+sqrt((-c^2*exp(2)^3*b*d^4+2*c^2*exp(2)^2*b*d^4*exp(1

)^2-c^2*exp(2)*b*d^4*exp(1)^4+2*c*exp(2)^3*b^2*d^3*exp(1)-4*c*exp(2)^2*b^2*
d^3*exp(1)^3+2*c*exp(2)*b^2*d^3*exp(1)^5-exp(2)^3*b^3*d^2*exp(1)^2+2*exp(2)
^2*b^3*d^2*exp(1)^4-exp(2)*b^3*d^2*exp(1)^6)*(-c^2*exp(2)^3*b*d^4+2*c^2*exp
(2)^2*b*d^4*exp(1)^2-c^2*exp(2)*b*d^4*exp(1)^4+2*c*exp(2)^3*b^2*d^3*exp(1)-
4*c*exp(2)^2*b^2*d^3*exp(1)^3+2*c*exp(2)*b^2*d^3*exp(1)^5-exp(2)^3*b^3*d^2*
exp(1)^2+2*exp(2)^2*b^3*d^2*exp(1)^4-exp(2)*b^3*d^2*exp(1)^6)-4*(-c^3*exp(2
)^3*d^4+2*c^3*exp(2)^2*d^4*exp(1)^2-c^3*exp(2)*d^4*exp(1)^4+2*c^2*exp(2)^3*
b*d^3*exp(1)-4*c^2*exp(2)^2*b*d^3*exp(1)^3+2*c^2*exp(2)*b*d^3*exp(1)^5-c*ex
p(2)^3*b^2*d^2*exp(1)^2+2*c*exp(2)^2*b^2*d^2*exp(1)^4-c*exp(2)*b^2*d^2*exp(
1)^6)*(c^3*exp(2)^2*d^6-2*c^3*exp(2)*d^6*exp(1)^2+c^3*d^6*exp(1)^4-3*c^2*ex
p(2)^2*b*d^5*exp(1)+6*c^2*exp(2)*b*d^5*exp(1)^3-3*c^2*b*d^5*exp(1)^5+3*c*ex
p(2)^2*b^2*d^4*exp(1)^2-6*c*exp(2)*b^2*d^4*exp(1)^4+3*c*b^2*d^4*exp(1)^6-ex
p(2)^2*b^3*d^3*exp(1)^3+2*exp(2)*b^3*d^3*exp(1)^5-b^3*d^3*exp(1)^7)))/2/(-c
^3*exp(2)^3*d^4+2*c^3*exp(2)^2*d^4*exp(1)^2-c^3*exp(2)*d^4*exp(1)^4+2*c^2*e
xp(2)^3*b*d^3*exp(1)-4*c^2*exp(2)^2*b*d^3*exp(1)^3+2*c^2*exp(2)*b*d^3*exp(1
)^5-c*exp(2)^3*b^2*d^2*exp(1)^2+2*c*exp(2)^2*b^2*d^2*exp(1)^4-c*exp(2)*b^2*
d^2*exp(1)^6)))-(4*b^5*c*exp(1)*exp(2)^5-2*b^5*sqrt(2)*sqrt(b*c*exp(2)^2+c*
sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)*ex
p(2)^4-36*b^4*c^2*d*exp(1)^2*exp(2)^4-4*b^4*c^2*d*exp(2)^5-4*b^4*c^2*exp(1)
*exp(2)^5+18*b^4*c*d*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^
2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^2*exp(2)^3+2*b^4*c*d*sqrt(2)
*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(
2))*exp(2))*exp(2)^4+4*b^4*c*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+
4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)*exp(2)^4+2*b^4*sqrt(
2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*ex
p(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp
(1)*exp(2)^3+96*b^3*c^3*d^2*exp(1)^3*exp(2)^3+64*b^3*c^3*d^2*exp(1)*exp(2)^
4+28*b^3*c^3*d*exp(1)^2*exp(2)^4+4*b^3*c^3*d*exp(2)^5-48*b^3*c^2*d^2*sqrt(2
)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp
(2))*exp(2))*exp(1)^3*exp(2)^2-32*b^3*c^2*d^2*sqrt(2)*sqrt(b*c*exp(2)^2+c*s
qrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)*exp
(2)^3-20*b^3*c^2*d*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*
exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^2*exp(2)^3-4*b^3*c^2*d*sqrt(2)
*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(
2))*exp(2))*exp(2)^4-2*b^3*c^2*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^
2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)*exp(2)^4-14*b^3*c*
d*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*ex
p(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(
2))*exp(1)^2*exp(2)^2-2*b^3*c*d*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)
^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*
d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2)^3-4*b^3*c*sqrt(2)*sqrt(b*c*exp(2)^
2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(
b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)^3-4*b^3*
c*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)^3-64*
b^2*c^4*d^3*exp(1)^4*exp(2)^2-224*b^2*c^4*d^3*exp(1)^2*exp(2)^3-32*b^2*c^4*
d^3*exp(2)^4-48*b^2*c^4*d^2*exp(1)^3*exp(2)^3-48*b^2*c^4*d^2*exp(1)*exp(2)^
4+32*b^2*c^3*d^3*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*ex
p(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^4*exp(2)+112*b^2*c^3*d^3*sqrt(2)
*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(
2))*exp(2))*exp(1)^2*exp(2)^2+16*b^2*c^3*d^3*sqrt(2)*sqrt(b*c*exp(2)^2+c*sq
rt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(2)^3+16
*b^2*c^3*d^2*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)
-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^3*exp(2)^2+32*b^2*c^3*d^2*sqrt(2)*sq
rt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))
*exp(2))*exp(1)*exp(2)^3+10*b^2*c^3*d*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*
exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^2*exp(2)^3+
2*b^2*c^3*d*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-
4*b*c*d*exp(1)*exp(2))*exp(2))*exp(2)^4+24*b^2*c^2*d^2*sqrt(2)*sqrt(b*c*exp
(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*s

qrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^3*exp(2)+24
*b^2*c^2*d^2*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)
-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*
exp(1)*exp(2))*exp(1)*exp(2)^2+12*b^2*c^2*d*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqr
t(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp
(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^2*exp(2)^2+4*b^2*c^2*d
*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp
(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2
))*exp(2)^3+20*b^2*c^2*d*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(
2))*exp(1)^2*exp(2)^2+4*b^2*c^2*d*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*ex
p(1)*exp(2))*exp(2)^3+2*b^2*c^2*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)
^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*
d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)^3+4*b^2*c^2*(b^2*exp(2)^2+4
*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)^3+128*b*c^5*d^4*exp(1)
^3*exp(2)^2+192*b*c^5*d^4*exp(1)*exp(2)^3+112*b*c^5*d^3*exp(1)^2*exp(2)^3+1
6*b*c^5*d^3*exp(2)^4-64*b*c^4*d^4*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(
2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^3*exp(2)-96*b*c
^4*d^4*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c
*d*exp(1)*exp(2))*exp(2))*exp(1)*exp(2)^2-16*b*c^4*d^3*sqrt(2)*sqrt(b*c*exp
(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*e
xp(1)^2*exp(2)^2-16*b*c^4*d^3*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2
+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(2)^3-8*b*c^4*d^2*sqrt(
2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*ex
p(2))*exp(2))*exp(1)^3*exp(2)^2-16*b*c^4*d^2*sqrt(2)*sqrt(b*c*exp(2)^2+c*sq
rt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)*exp(
2)^3-56*b*c^3*d^3*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*e
xp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b
*c*d*exp(1)*exp(2))*exp(1)^2*exp(2)-8*b*c^3*d^3*sqrt(2)*sqrt(b*c*exp(2)^2+c
*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2
*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2)^2-16*b*c^3*d^2*sqr
t(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*
exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*e
xp(1)*exp(2)^2-16*b*c^3*d^2*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*e
xp(2))*exp(1)^3*exp(2)-32*b*c^3*d^2*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*
exp(1)*exp(2))*exp(1)*exp(2)^2-6*b*c^3*d*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b
^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)
^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^2*exp(2)^2-2*b*c^3*d*sqrt
(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*e
xp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*ex
p(2)^3-12*b*c^3*d*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp
(1)^2*exp(2)^2-4*b*c^3*d*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(
2))*exp(2)^3-64*c^6*d^5*exp(1)^2*exp(2)^2-64*c^6*d^5*exp(2)^3-64*c^6*d^4*ex
p(1)*exp(2)^3+32*c^5*d^5*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^
2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^2*exp(2)+32*c^5*d^5*sqrt
(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*e
xp(2))*exp(2))*exp(2)^2+8*c^5*d^3*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(
2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*exp(1)^2*exp(2)^2+8*c^
5*d^3*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*
d*exp(1)*exp(2))*exp(2))*exp(2)^3+32*c^4*d^4*sqrt(2)*sqrt(b*c*exp(2)^2+c*sq
rt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2))*sqrt(b^2*ex
p(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)*exp(2)+16*c^4*d^3*(b^
2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(1)^2*exp(2)+16*c^4*d
^3*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2))*exp(2)^2+8*c^4*d^2
*sqrt(2)*sqrt(b*c*exp(2)^2+c*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp
(1)*exp(2))*exp(2))*sqrt(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)*exp(2
))*exp(1)*exp(2)^2+16*c^4*d^2*(b^2*exp(2)^2+4*c^2*d^2*exp(2)-4*b*c*d*exp(1)
*exp(2))*exp(1)*exp(2)^2)/(8*b^6*d^3*exp(1)^6*exp(2)^3-16*b^6*d^3*exp(1)^4*
exp(2)^4+8*b^6*d^3*exp(1)^2*exp(2)^5-64*b^5*c*d^4*exp(1)^7*exp(2)^2+112*b^5
*c*d^4*exp(1)^5*exp(2)^3-32*b^5*c*d^4*exp(1)^3*exp(2)^4-16*b^5*c*d^4*exp(1)

```
*exp(2)^5-16*b^5*c*d^3*exp(1)^6*exp(2)^3+32*b^5*c*d^3*exp(1)^4*exp(2)^4-16*
b^5*c*d^3*exp(1)^2*exp(2)^5+128*b^4*c^2*d^5*exp(1)^8*exp(2)-64*b^4*c^2*d^5*
exp(1)^6*exp(2)^2-248*b^4*c^2*d^5*exp(1)^4*exp(2)^3+176*b^4*c^2*d^5*exp(1)^
2*exp(2)^4+8*b^4*c^2*d^5*exp(2)^5+64*b^4*c^2*d^4*exp(1)^7*exp(2)^2-96*b^4*c
^2*d^4*exp(1)^5*exp(2)^3+32*b^4*c^2*d^4*exp(1)*exp(2)^5+8*b^4*c^2*d^3*exp(1
)^6*exp(2)^3-16*b^4*c^2*d^3*exp(1)^4*exp(2)^4+8*b^4*c^2*d^3*exp(1)^2*exp(2)
^5-512*b^3*c^3*d^6*exp(1)^7*exp(2)+832*b^3*c^3*d^6*exp(1)^5*exp(2)^2-128*b^
3*c^3*d^6*exp(1)^3*exp(2)^3-192*b^3*c^3*d^6*exp(1)*exp(2)^4-192*b^3*c^3*d^5
*exp(1)^6*exp(2)^2+368*b^3*c^3*d^5*exp(1)^4*exp(2)^3-160*b^3*c^3*d^5*exp(1)
^2*exp(2)^4-16*b^3*c^3*d^5*exp(2)^5-32*b^3*c^3*d^4*exp(1)^7*exp(2)^2+48*b^3
*c^3*d^4*exp(1)^5*exp(2)^3-16*b^3*c^3*d^4*exp(1)*exp(2)^5+768*b^2*c^4*d^7*e
xp(1)^6*exp(2)-1472*b^2*c^4*d^7*exp(1)^4*exp(2)^2+640*b^2*c^4*d^7*exp(1)^2*
exp(2)^3+64*b^2*c^4*d^7*exp(2)^4+192*b^2*c^4*d^6*exp(1)^5*exp(2)^2-384*b^2*
c^4*d^6*exp(1)^3*exp(2)^3+192*b^2*c^4*d^6*exp(1)*exp(2)^4+96*b^2*c^4*d^5*ex
p(1)^6*exp(2)^2-184*b^2*c^4*d^5*exp(1)^4*exp(2)^3+80*b^2*c^4*d^5*exp(1)^2*e
xp(2)^4+8*b^2*c^4*d^5*exp(2)^5-512*b*c^5*d^8*exp(1)^5*exp(2)+1024*b*c^5*d^8
*exp(1)^3*exp(2)^2-512*b*c^5*d^8*exp(1)*exp(2)^3-64*b*c^5*d^7*exp(1)^4*exp(
2)^2+128*b*c^5*d^7*exp(1)^2*exp(2)^3-64*b*c^5*d^7*exp(2)^4-96*b*c^5*d^6*exp
(1)^5*exp(2)^2+192*b*c^5*d^6*exp(1)^3*exp(2)^3-96*b*c^5*d^6*exp(1)*exp(2)^4
+128*c^6*d^9*exp(1)^4*exp(2)-256*c^6*d^9*exp(1)^2*exp(2)^2+128*c^6*d^9*exp(
2)^3+32*c^6*d^7*exp(1)^4*exp(2)^2-64*c^6*d^7*exp(1)^2*exp(2)^3+32*c^6*d^7*e
xp(2)^4)/abs(c)*atan(x/sqrt(-(c^2*exp(2)^3*b*d^4-2*c^2*exp(2)^2*b*d^4*exp(1
)^2+c^2*exp(2)*b*d^4*exp(1)^4-2*c*exp(2)^3*b^2*d^3*exp(1)+4*c*exp(2)^2*b^2*
d^3*exp(1)^3-2*c*exp(2)*b^2*d^3*exp(1)^5+exp(2)^3*b^3*d^2*exp(1)^2-2*exp(2)
^2*b^3*d^2*exp(1)^4+exp(2)*b^3*d^2*exp(1)^6-sqrt((-c^2*exp(2)^3*b*d^4+2*c^2
*exp(2)^2*b*d^4*exp(1)^2-c^2*exp(2)*b*d^4*exp(1)^4+2*c*exp(2)^3*b^2*d^3*exp
(1)-4*c*exp(2)^2*b^2*d^3*exp(1)^3+2*c*exp(2)*b^2*d^3*exp(1)^5-exp(2)^3*b^3*
d^2*exp(1)^2+2*exp(2)^2*b^3*d^2*exp(1)^4-exp(2)*b^3*d^2*exp(1)^6)*(-c^2*exp
(2)^3*b*d^4+2*c^2*exp(2)^2*b*d^4*exp(1)^2-c^2*exp(2)*b*d^4*exp(1)^4+2*c*exp
(2)^3*b^2*d^3*exp(1)-4*c*exp(2)^2*b^2*d^3*exp(1)^3+2*c*exp(2)*b^2*d^3*exp(1
)^5-exp(2)^3*b^3*d^2*exp(1)^2+2*exp(2)^2*b^3*d^2*exp(1)^4-exp(2)*b^3*d^2*ex
p(1)^6)-4*(-c^3*exp(2)^3*d^4+2*c^3*exp(2)^2*d^4*exp(1)^2-c^3*exp(2)*d^4*exp
(1)^4+2*c^2*exp(2)^3*b*d^3*exp(1)-4*c^2*exp(2)^2*b*d^3*exp(1)^3+2*c^2*exp(2
)*b*d^3*exp(1)^5-c*exp(2)^3*b^2*d^2*exp(1)^2+2*c*exp(2)^2*b^2*d^2*exp(1)^4-
c*exp(2)*b^2*d^2*exp(1)^6)*(c^3*exp(2)^2*d^6-2*c^3*exp(2)*d^6*exp(1)^2+c^3*
d^6*exp(1)^4-3*c^2*exp(2)^2*b*d^5*exp(1)+6*c^2*exp(2)*b*d^5*exp(1)^3-3*c^2*
b*d^5*exp(1)^5+3*c*exp(2)^2*b^2*d^4*exp(1)^2-6*c*exp(2)*b^2*d^4*exp(1)^4+3*
c*b^2*d^4*exp(1)^6-exp(2)^2*b^3*d^3*exp(1)^3+2*exp(2)*b^3*d^3*exp(1)^5-b^3*
d^3*exp(1)^7)))/2/(-c^3*exp(2)^3*d^4+2*c^3*exp(2)^2*d^4*exp(1)^2-c^3*exp(2)
*d^4*exp(1)^4+2*c^2*exp(2)^3*b*d^3*exp(1)-4*c^2*exp(2)^2*b*d^3*exp(1)^3+2*c
^2*exp(2)*b*d^3*exp(1)^5-c*exp(2)^3*b^2*d^2*exp(1)^2+2*c*exp(2)^2*b^2*d^2*e
xp(1)^4-c*exp(2)*b^2*d^2*exp(1)^6)))+(-5*c*exp(2)*d*exp(1)^2+c*d*exp(1)^4+3
*exp(2)*b*exp(1)^3-b*exp(1)^5)*1/2/(-c^2*exp(2)^2*d^4+2*c^2*exp(2)*d^4*exp(
1)^2-c^2*d^4*exp(1)^4+2*c*exp(2)^2*b*d^3*exp(1)-4*c*exp(2)*b*d^3*exp(1)^3+2
*c*b*d^3*exp(1)^5-exp(2)^2*b^2*d^2*exp(1)^2+2*exp(2)*b^2*d^2*exp(1)^4-b^2*d
^2*exp(1)^6)/sqrt(d*exp(1))*atan(x*exp(1)/sqrt(d*exp(1)))-x*exp(1)^2/(-2*c*
exp(2)*d^3+2*c*d^3*exp(1)^2+2*exp(2)*b*d^2*exp(1)-2*b*d^2*exp(1)^3)/(x^2*ex
p(1)+d)
```

maple [A] time = 0.01, size = 319, normalized size = 1.71

$$\frac{3b^2e^3x^3}{8\left(be-2cd\right)^3\left(ex^2+d\right)^2d^2}-\frac{2bc\,e^2x^3}{\left(be-2cd\right)^3\left(ex^2+d\right)^2d}+\frac{5c^2e\,x^3}{2\left(be-2cd\right)^3\left(ex^2+d\right)^2}+\frac{5b^2e^2x}{8\left(be-2cd\right)^3\left(ex^2+d\right)^2d}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] -c^3/(b*e-2*c*d)^3/((b*e-c*d)*c*e)^(1/2)*arctan(1/((b*e-c*d)*c*e)^(1/2)*c*e
*x)+3/8/(b*e-2*c*d)^3/(e*x^2+d)^2*e^3/d^2*x^3*b^2-2/(b*e-2*c*d)^3/(e*x^2+d)
```

```
^2*e^2/d*x^3*b*c+5/2/(b*e-2*c*d)^3/(e*x^2+d)^2*e*x^3*c^2+5/8/(b*e-2*c*d)^3/
(e*x^2+d)^2/d*x*b^2*e^2-3/(b*e-2*c*d)^3/(e*x^2+d)^2*x*b*c*e+7/2/(b*e-2*c*d)
^3/(e*x^2+d)^2*d*x*c^2+3/8/(b*e-2*c*d)^3/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/
2)*e*x)*b^2*e^2-2/(b*e-2*c*d)^3/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b*c
*e+7/2/(b*e-2*c*d)^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for more
 details)Is b*e-c*d positive or negative?
```

mupad [**B**] time = 6.45, size = 6267, normalized size = 33.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^2*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

```
[Out] ((x*(5*b*e - 14*c*d))/(8*d*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)) + (e*x^3*(3*b
*e - 10*c*d))/(8*d^2*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)))/(d^2 + e^2*x^4 + 2
*d*e*x^2) - (atan(((((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e
^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8 + b^4*d^4*e^4
 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - (((576*c^10*d^
10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 +
 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*
b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^6*d^10 + b^6*d^4*e^6
 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^
2*d^6*e^4 - 192*b*c^5*d^9*e)) - (x*(-c^5*e*(b*e - c*d))^(1/2)*(16384*b*c^8*
d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*b^4*c^5*d
^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7*c^2*d^4*
e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^
2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*
d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(2*(b^4*e^5 + 8*c^4*
d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b
*e - c*d))^(1/2)*1i)/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2
*d^2*e^3 - 7*b^3*c*d*e^4) + (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*
c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8 + b^
4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) + (((57
6*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*
d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^
12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^6*d^10 + b^
6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 +
60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) + (x*(-c^5*e*(b*e - c*d))^(1/2)*(163
84*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*
b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7
*c^2*d^4*e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c
^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18
*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(2*(b^4*e^5
 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(
-c^5*e*(b*e - c*d))^(1/2)*1i)/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 1
8*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4))/(((9*b^3*c^5*e^8)/32 - (35*c^8*d^3*e^5)
/4 + (61*b*c^7*d^2*e^6)/8 - (39*b^2*c^6*d*e^7)/16)/(64*c^6*d^10 + b^6*d^4*e
```

```
^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*
c^2*d^6*e^4 - 192*b*c^5*d^9*e) + (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 8
96*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8
 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) -
(((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3
*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d
^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^6*d^10
 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e
^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) - (x*(-c^5*e*(b*e - c*d))^(1/2)
*(16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 4
0960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 25
6*b^7*c^2*d^4*e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*
b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2
 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(2*(b^
4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4
)))*(-c^5*e*(b*e - c*d))^(1/2))/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 +
 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4) - (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e
^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c
^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*
e)) + (((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 32
88*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6
*c^4*d^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^
6*d^10 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3
*d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) + (x*(-c^5*e*(b*e - c*d))
^(1/2)*(16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^
10 - 40960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^1
3 + 256*b^7*c^2*d^4*e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3
 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d
^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/
(2*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c
*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3
*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2)*1i)
/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d
*e^4) - (atan(((((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 -
 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(32*(16*c^4*d^8 + b^4*d^4*e^4 - 8
*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - (((576*c^10*d^10*e
^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 194
0*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*b^7*
c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(64*c^6*d^10 + b^6*d^4*e^6 - 12*b
^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e
^4 - 192*b*c^5*d^9*e) - (x*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*
d*e)*(16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10
 - 40960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13
+ 256*b^7*c^2*d^4*e^14))/(512*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2
 + 6*b^2*c*d^6*e^3)*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^
2*d^6*e^2 - 32*b*c^3*d^7*e)))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b
*c*d*e))/(16*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^
3)))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e)*1i)/(16*(8*c^3*d^
8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)) + (((x*(9*b^4*c^3*
e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5
*d^2*e^8))/(32*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6
*e^2 - 32*b*c^3*d^7*e)) + (((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*
b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^
5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^
2*e^14)/2)/(64*c^6*d^10 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*
e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e) + (x*(-d^
5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e)*(16384*b*c^8*d^10*e^8 - 49
152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*b^4*c^5*d^7*e^11 + 153
60*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7*c^2*d^4*e^14))/(512*(
```

```
8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)*(16*c^4*d^8
 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)))*(
-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e))/(16*(8*c^3*d^8*e - b^3
*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)))*(-d^5*e)^(1/2)*(3*b^2*e^2
+ 28*c^2*d^2 - 16*b*c*d*e)*1i)/(16*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^
7*e^2 + 6*b^2*c*d^6*e^3)))/(((9*b^3*c^5*e^8)/32 - (35*c^8*d^3*e^5)/4 + (61*
b*c^7*d^2*e^6)/8 - (39*b^2*c^6*d*e^7)/16)/(64*c^6*d^10 + b^6*d^4*e^6 - 12*b
^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e
^4 - 192*b*c^5*d^9*e) + (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*
d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(32*(16*c^4*d^8 + b^4*d^
4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - (((576*c^
10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*
e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 -
 (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(64*c^6*d^10 + b^6*d^4*e
^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*
c^2*d^6*e^4 - 192*b*c^5*d^9*e) - (x*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2
- 16*b*c*d*e)*(16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6
*d^8*e^10 - 40960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*
d^5*e^13 + 256*b^7*c^2*d^4*e^14))/(512*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^
2*d^7*e^2 + 6*b^2*c*d^6*e^3)*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 +
24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d
^2 - 16*b*c*d*e))/(16*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2
*c*d^6*e^3)))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e))/(16*(8*
c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)) - (((x*(9*b^
4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b
^2*c^5*d^2*e^8))/(32*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c
^2*d^6*e^2 - 32*b*c^3*d^7*e)) + (((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 +
 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*
b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*
c^2*d^2*e^14)/2)/(64*c^6*d^10 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^
4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e) + (
x*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e)*(16384*b*c^8*d^10*e^
8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*b^4*c^5*d^7*e^11
 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7*c^2*d^4*e^14))/
(512*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)*(16*c
^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*
e)))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e))/(16*(8*c^3*d^8*e
 - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)))*(-d^5*e)^(1/2)*(3*b^
2*e^2 + 28*c^2*d^2 - 16*b*c*d*e))/(16*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2
*d^7*e^2 + 6*b^2*c*d^6*e^3))))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*
b*c*d*e)*1i)/(8*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6
*e^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Timed out
```

3.220 $\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=139

$$-\frac{(2cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c^2\sqrt{e}\sqrt{cd-be}}+\frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

```
[Out] 1/2*(-2*b*e+5*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^2/e^(1/2)-(-b*e+2*c
*d)^(3/2)*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(
1/2))/c^2/e^(1/2)/(-b*e+c*d)^(1/2)+1/2*x*(e*x^2+d)^(1/2)/c
```

Rubi [A] time = 0.28, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1149, 416, 523, 217, 206, 377, 208}

$$-\frac{(2cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c^2\sqrt{e}\sqrt{cd-be}}+\frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) + ((5*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e
*x^2]])/(2*c^2*Sqrt[e]) - ((2*c*d - b*e)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[2*c*d
- b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c^2*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{\left(d+ex^2\right)^{3/2}}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{\int \frac{de(3cd-be)+e^2(5cd-2be)x^2}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2ce}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c^2} + \frac{(2cd-be)^2\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} + \frac{(2cd-be)^2\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{(2cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{2cd-\cdots}}{\sqrt{cd-be}\,\sqrt{d+\cdots}}\right.}{c^2\sqrt{e}\,\sqrt{cd-be}}$$

Mathematica [A] time = 0.26, size = 134, normalized size = 0.96

$$-\frac{\frac{(2be-5cd)\log\left(\sqrt{e}\,\sqrt{d+ex^2}+ex\right)}{\sqrt{e}} - \frac{2(be-2cd)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x\,\sqrt{be-2cd}}{\sqrt{d+ex^2}\,\sqrt{be-cd}}\right)}{\sqrt{e}\,\sqrt{be-cd}} - cx\sqrt{d+ex^2}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -1/2*(-(c*x*Sqrt[d + e*x^2]) - (2*(-2*c*d + b*e)^(3/2)*ArcTanh[(Sqrt[e]*Sqr
t[-2*c*d + b*e]*x)/(Sqrt[-(c*d) + b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[-(c
*d) + b*e]) + ((-5*c*d + 2*b*e)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/Sqrt[e]
)/c^2
```

fricas [A] time = 1.97, size = 1079, normalized size = 7.76

$$\left[\frac{2\sqrt{ex^2+d}\,cex - (5cd-2be)\sqrt{e}\log\left(-2ex^2+2\sqrt{ex^2+d}\,\sqrt{e}\,x-d\right) - \left(2cde-be^2\right)\sqrt{\frac{2cd-be}{cde-be^2}}\log\left(\frac{c^2d^4-2bcd^3e+b^2d^2\cdots}{\cdots}\right.}{4c\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*(2*sqrt(e*x^2 + d)*c*e*x - (5*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*e
- b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b
*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*
x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c
*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)
))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2
)))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x - 2*(5*c*d - 2*b*e)*sqrt(-e)*arct
an(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*
e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24
*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3
)*x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b
*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^
2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x
^2)))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x + 2*(2*c*d*e - b*e^2)*sqrt(-(2*
c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)
*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))/((2*c*d*e - b*e^
2)*x^3 + (2*c*d^2 - b*d*e)*x)) - (5*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*s
qrt(e*x^2 + d)*sqrt(e)*x - d))/(c^2*e), 1/2*(sqrt(e*x^2 + d)*c*e*x - (5*c*d
 - 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (2*c*d*e - b*e^2)*s
qrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e -
2*b*e^2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))/((2*c*d*
e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)))/(c^2*e)]
```

giac [A] time = 2.39, size = 54, normalized size = 0.39

$$-\frac{(5\,cd-2\,be)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{4\,c^2}+\frac{\sqrt{x^2e+d}\,x}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] -1/4*(5*c*d - 2*b*e)*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^2 + 1/
2*sqrt(x^2*e + d)*x/c
```

maple [B] time = 0.06, size = 7043, normalized size = 50.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{5}{2}}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate((e*x^2 + d)^(5/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^{5/2}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(5/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
[Out] int((d + e*x^2)^(5/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^{\frac{3}{2}}}{be-cd+cex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
[Out] Integral((d + e*x**2)**(3/2)/(b*e - c*d + c*e*x**2), x)
```

3.221 $\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=108

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\,\sqrt{cd-be}}\right)}{c\sqrt{e}\,\sqrt{cd-be}}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(1/2)-arctanh(x*e^(1/2)*(-b*e+2*c*d)
^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))*(-b*e+2*c*d)^(1/2)/c/e^(1/2)/(-b*e
+c*d)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1149, 402, 217, 206, 377, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\,\sqrt{cd-be}}\right)}{c\sqrt{e}\,\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(c*Sqrt[e]) - (Sqrt[2*c*d - b*e]*ArcTa
nh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c*Sqr
t[e]*Sqrt[c*d - b*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{\sqrt{d+ex^2}}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$

$$= \frac{\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c} - \frac{\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{ce}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c} - \frac{\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)\text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e}{\cdots}\right.}\right.}{ce}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\sqrt{2cd-be}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{2cd-be}\,x}{\sqrt{cd-be}\,\sqrt{d+ex^2}}\right)}{c\sqrt{e}\,\sqrt{cd-be}}$$

Mathematica [A] time = 0.09, size = 103, normalized size = 0.95

$$-\frac{\frac{\sqrt{be-2cd}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x\,\sqrt{be-2cd}}{\sqrt{d+ex^2}\,\sqrt{be-cd}}\right)}{\sqrt{be-cd}}-\log\left(\sqrt{e}\,\sqrt{d+ex^2}+ex\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((Sqrt[-2*c*d + b*e]*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d + b*e]*x)/(Sqrt[-(c*d)
+ b*e]*Sqrt[d + e*x^2])])/Sqrt[-(c*d) + b*e] - Log[e*x + Sqrt[e]*Sqrt[d + e
*x^2]])/(c*Sqrt[e]))
```

fricas [A] time = 1.22, size = 940, normalized size = 8.70

$$\left[\frac{e\sqrt{\frac{2\,cd-be}{cde-be^2}}\,\log\left(\frac{c^2d^4-2\,bcd^3e+b^2d^2e^2+\left(17\,c^2d^2e^2-24\,bcde^3+8\,b^2e^4\right)x^4+2\left(7\,c^2d^3e-11\,bcd^2e^2+4\,b^2de^3\right)x^2-4\left(\left(3\,c^2d^2e^2-5\,bcde^3+2\,b^2e^4\right)x^3+\left(c^2d^3e-\cdots\right.\right.}{c^2e^2x^4+c^2d^2-2\,bcde+b^2e^2-2\left(c^2de-bce^2\right)x^2}\right.}{4\,ce}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*(e*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^
2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*
e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2
*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*
sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b
```

```
^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) + 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^
2 + d)*sqrt(e)*x - d))/(c*e), 1/4*(e*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*lo
g((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8
*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*
c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b
^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*
x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) - 4*sqrt(
-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*e), 1/2*(e*sqrt(-(2*c*d - b*e)/(
c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e
*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))/((2*c*d*e - b*e^2)*x^3 + (2*
c*d^2 - b*d*e)*x)) + sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d
))/(c*e), 1/2*(e*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b
*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e
- b*e^2))/((2*c*d*e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)) - 2*sqrt(-e)*arcta
n(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*e)]
```

giac [A] time = 2.39, size = 27, normalized size = 0.25

$$-\frac{e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^{2}e+d}\right)^{2}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] -1/2*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

maple [B] time = 0.02, size = 4308, normalized size = 39.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] -1/6*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-
c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e
-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(
3/2)+1/4*c*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*
e-c*d)*c*e)^(1/2))*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(
1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*x+5/4*c*e^(1/2)/
((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(
1/2))*ln((-(-(b*e-c*d)*c*e)^(1/2)/c+(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/
2)+((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e
-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))*d+1/2*c*e^2/((-d*e)^(1/2)*c+(-(
b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c
*e)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x
+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*b-c^2*e/((-d*e)^(1/2)*c+(
-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)
*c*e)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*
(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*d-1/2*e^(3/2)/((-d*e)^(
1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*ln(
(-(-(b*e-c*d)*c*e)^(1/2)/c+(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/2)+((x+(-
(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e
)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))*b+1/2*e^3/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*
e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(
-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-
(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/
2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-
```

2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))*b^2-2*c*e^2/((-d*e)^(1/2)*
c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c
*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d)*c*
e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x+(-(b
*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^
(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))*b*d+2*c^2*
e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)
^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-
2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c
)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(
-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/
c/e))*d^2-1/6*c*e/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-
d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)
*(x-(-d*e)^(1/2)/e))^(3/2)-1/4*c*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/
(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(
1/2)*(x-(-d*e)^(1/2)/e))^(1/2)*x-1/4*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*
c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*d*ln(((x-(-d*e)^(1/2)/
e)*e+(-d*e)^(1/2))/e^(1/2)+((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)
^(1/2)/e))^(1/2))+1/6*c*e/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/
2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x+(-d*e)^(1/2)/e)^2*e-2*(-d*
e)^(1/2)*(x+(-d*e)^(1/2)/e))^(3/2)-1/4*c*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)
^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x+(-d*e)^(1/2)/e)^2*e-2*
(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)*x-1/4*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b
*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*d*ln(((x+(-d*e
)^(1/2)/e)*e-(-d*e)^(1/2))/e^(1/2)+((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(
x+(-d*e)^(1/2)/e))^(1/2))+1/6*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))
/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x-(-(b*e
-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1
/2)/c/e)-(b*e-2*c*d)/c)^(3/2)+1/4*c*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2
))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)
^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/
c)^(1/2)*x+5/4*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(
1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*ln(((-(b*e-c*d)*c*e)^(1/2)/c+(x-(-(b*e-c*d)*
c*e)^(1/2)/c/e)*e)/e^(1/2)+((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d
)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))*d-1/2*c
*e^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c
*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(
b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*b
+c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)
*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(
-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)
*d-1/2*e^(3/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(
b*e-c*d)*c*e)^(1/2))*ln(((-(b*e-c*d)*c*e)^(1/2)/c+(x-(-(b*e-c*d)*c*e)^(1/2)
/c/e)*e)/e^(1/2)+((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/
2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))*b-1/2*e^3/((-d*e)
^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(
-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-
c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*(
(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d
)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))*b^2
+2*c*e^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*
d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*
c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2
*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)
/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)
^(1/2)/c/e))*b*d-2*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(
1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2
)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)
/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e

```
-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-
(-(b*e-c*d)*c*e)^(1/2)/c/e))*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^{3/2}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] int((d + e*x^2)^(3/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{be-cd+cex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(b*e - c*d + c*e*x**2), x)
```

3.222 $\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$

Optimal. Leaf size=76

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

```
[Out] -arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/e^(
1/2)/(-b*e+c*d)^(1/2)/(-b*e+2*c*d)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {1149, 377, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])]/
(Sqrt[e]*Sqrt[c*d - b*e]*Sqrt[2*c*d - b*e]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx &= \int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)} dx \\
&= \text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)x^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 76, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])]/
(Sqrt[e]*Sqrt[c*d - b*e]*Sqrt[2*c*d - b*e]))
```

fricas [B] time = 0.80, size = 432, normalized size = 5.68

$$\left[\frac{\log\left(\frac{c^2d^4-2bcd^3e+b^2d^2e^2+(17c^2d^2e^2-24bcde^3+8b^2e^4)x^4+2(7c^2d^3e-11bcd^2e^2+4b^2de^3)x^2-4\sqrt{2c^2d^2e-3bcde^2+b^2e^3}\left((3cde-2be^2)x^3+(cd^2-\right.}{c^2e^2x^4+c^2d^2-2bcde+b^2e^2-2(c^2de-bce^2)x^2}\right.}{4\sqrt{2c^2d^2e-3bcde^2+b^2e^3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*
e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 -
 4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*((3*c*d*e - 2*b*e^2)*x^3 + (c*
d^2 - b*d*e)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e
^2 - 2*(c^2*d*e - b*c*e^2)*x^2))/sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3),
 -1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*arctan(-1/2*sqrt(-2*c^2*d^
2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt
(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^2*d^3*e - 3
*b*c*d^2*e^2 + b^2*d*e^3)*x))/(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 2252, normalized size = 29.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] -1/2*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-
c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e
-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(
1/2)+1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c
+(-(b*e-c*d)*c*e)^(1/2))*ln((-(-(b*e-c*d)*c*e)^(1/2)/c+(x+(-(b*e-c*d)*c*e)^
(1/2)/c/e)*e)/e^(1/2)+((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e
)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))-1/2*c*e^2/((
-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/
```

```
2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-
(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1
/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*
e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)
)*b+c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c
*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2
*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-
2*c*d)/c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2
)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e
)^(1/2)/c/e))*d-1/2*c*e/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2)
)/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)
^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)-1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*
c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*ln(((x-(-d*e)^(1/2)/e)
*e+(-d*e)^(1/2))/e^(1/2)+((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(
1/2)/e))^(1/2))+1/2*c*e/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2)
)/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)
^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)-1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*
c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*ln(((x+(-d*e)^(1/2)/e)
*e-(-d*e)^(1/2))/e^(1/2)+((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(
1/2)/e))^(1/2))+1/2*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^
(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x-(-(b*e-c*d)*c*e)
^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(
b*e-2*c*d)/c)^(1/2)+1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(
-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*ln(((-(b*e-c*d)*c*e)^(1/2)/c+(x-(-(
b*e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/2)+((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(
-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)
)+1/2*c*e^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e
-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e
-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*
e-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1
/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c
*e)^(1/2)/c/e))*b-c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1
/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)
*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/
c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-
c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(
-(b*e-c*d)*c*e)^(1/2)/c/e))*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}}{ce^2x^4 + be^2x^2 - cd^2 + bde}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,x^2+d}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] int((d + e*x^2)^(1/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\left(be-cd+cex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
[Out] Integral(1/(sqrt(d + e*x**2)*(b*e - c*d + c*e*x**2)), x)
```

3.223 $\int \frac{1}{\sqrt{d+ex^2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$

Optimal. Leaf size=106

$$-\frac{x}{d\sqrt{d+ex^2}\,(2cd-be)}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\,\sqrt{cd-be}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^{3/2}}$$

```
[Out] -c*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/(
-b*e+2*c*d)^(3/2)/e^(1/2)/(-b*e+c*d)^(1/2)-x/d/(-b*e+2*c*d)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {1149, 382, 377, 208}

$$-\frac{x}{d\sqrt{d+ex^2}\,(2cd-be)}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\,\sqrt{cd-be}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -(x/(d*(2*c*d - b*e)*Sqrt[d + e*x^2])) - (c*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b
*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d
- b*e)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \int \frac{1}{\left(d+ex^2\right)^{3/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx$$

$$= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} + \frac{c\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2cd-be}$$

$$= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)x^2}\right.}{2cd-be}$$

$$= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} - \frac{c\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-be}\,x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}\,(2cd-be)^{3/2}}$$

Mathematica [C] time = 1.05, size = 418, normalized size = 3.94

$$-\frac{x\left(-\frac{2cex^2\left(\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}\right)^{5/2}{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)}\right)}{cd-be}+2\left(\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}\right)^{5/2}{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)}\right)+\frac{10cex^2\sqrt{\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}}\cdots}{cd-be}\cdots\right.}{5\left(d+ex^2\right)^{3/2}(cd-be)\left(\frac{ex^2(be-2cd)}{(d+ex^2)(be-\cdots)}\cdots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -1/5*(x*(-15*Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))] + (1
0*c*e*x^2*Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))])/(c*d -
 b*e) + 15*ArcTanh[Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))
]] - (10*c*e*x^2*ArcTanh[Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e
*x^2))]])/(c*d - b*e) + 2*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^
2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (e*(-2*c*d + b*e)*x^2)/((-(c*d) +
 b*e)*(d + e*x^2))] - (2*c*e*x^2*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d
 + e*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (e*(-2*c*d + b*e)*x^2)/((-
(c*d) + b*e)*(d + e*x^2))])/(c*d - b*e)))/((c*d - b*e)*((e*(-2*c*d + b*e)*x
^2)/((-(c*d) + b*e)*(d + e*x^2)))^(3/2)*(d + e*x^2)^(3/2))
```

fricas [B] time = 1.17, size = 701, normalized size = 6.61

$$\left[-\frac{4\left(2c^2d^2e-3bcde^2+b^2e^3\right)\sqrt{ex^2+d}\,x+\sqrt{2c^2d^2e-3bcde^2+b^2e^3}\left(cdex^2+cd^2\right)\log\left(\frac{c^2d^4-2bcd^3e+b^2d^2e^2+\left(17c^2\cdots\right.}{\cdots}\right.}{4\left(4c^3d^5e-8bc^2d^4e^2+5b^2cd^3e^3-b^3d^2e^4+\left(4c^3d\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"fricas")
```

```
[Out] [-1/4*(4*(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x + sqrt(2*c
^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*(c*d*e*x^2 + c*d^2)*log((c^2*d^4 - 2*b*c*
d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(
7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 + 4*sqrt(2*c^2*d^2*e - 3*b*
c*d*e^2 + b^2*e^3)*((3*c*d*e - 2*b*e^2)*x^3 + (c*d^2 - b*d*e)*x)*sqrt(e*x^2
```

```
 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)
*x^2)))/(4*c^3*d^5*e - 8*b*c^2*d^4*e^2 + 5*b^2*c*d^3*e^3 - b^3*d^2*e^4 + (4
*c^3*d^4*e^2 - 8*b*c^2*d^3*e^3 + 5*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2), -1/2*(2
*(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x + sqrt(-2*c^2*d^2*
e + 3*b*c*d*e^2 - b^2*e^3)*(c*d*e*x^2 + c*d^2)*arctan(-1/2*sqrt(-2*c^2*d^2*
e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e
*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^2*d^3*e - 3*b
*c*d^2*e^2 + b^2*d*e^3)*x)))/(4*c^3*d^5*e - 8*b*c^2*d^4*e^2 + 5*b^2*c*d^3*e
^3 - b^3*d^2*e^4 + (4*c^3*d^4*e^2 - 8*b*c^2*d^3*e^3 + 5*b^2*c*d^2*e^4 - b^3
*d*e^5)*x^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 771, normalized size = 7.27

$$-\frac{c^{2} e \ln\left(\frac{-\frac{2(be-2cd)}{c}+\frac{2\sqrt{-(be-cd)ce}\left(x-\frac{\sqrt{-(be-cd)ce}}{ce}\right)}{c}+2\sqrt{-\frac{be-2cd}{c}}\sqrt{\left(x-\frac{\sqrt{-(be-cd)ce}}{ce}\right)^{2} e+\frac{2\sqrt{-(be-cd)ce}\left(x-\frac{\sqrt{-(be-cd)ce}}{ce}\right)}{c}-\frac{be-2cd}{c}}}{x-\frac{\sqrt{-(be-cd)ce}}{ce}}\right)}{2\left(\sqrt{-de}\, c+\sqrt{-(be-cd)\, ce}\right)\left(-\sqrt{-de}\, c+\sqrt{-(be-cd)\, ce}\right)\sqrt{-(be-cd)\, ce}\, \sqrt{-\frac{be-2cd}{c}}}+\frac{c^{2} e \ln\left(-\frac{2(be-2cd)}{c}\cdots\right)}{2\left(\sqrt{-de}\, c\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/2*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c
*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2
*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)/c+2*(-(b*e-
2*c*d)/c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2
)*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)/c-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e
)^(1/2)/c/e))-1/2*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2
)*c+(-(b*e-c*d)*c*e)^(1/2))/(x-(-d*e)^(1/2)/e)*((x-(-d*e)^(1/2)/e)^2*e+2*(-
d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)-1/2*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*
e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(x+(-d*e)^(1/2)/e)*((x+(
-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)-1/2*c^2*e/((-d*
e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))
/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*
e-c*d)*c*e)^(1/2)*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)/c+2*(-(b*e-2*c*d)/c)^(1/2)
*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)*(x-(-(b*e-c*d
)*c*e)^(1/2)/c/e)/c-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ce^{2}x^{4}+be^{2}x^{2}-cd^{2}+bde\right)\sqrt{ex^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\,x^2+d}\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^(1/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
[Out] int(1/((d + e*x^2)^(1/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^{\frac{3}{2}}\left(be-cd+cex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
[Out] Integral(1/((d + e*x**2)**(3/2)*(b*e - c*d + c*e*x**2)), x)
```

3.224 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$

Optimal. Leaf size=149

$$-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\,\sqrt{cd-be}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^{5/2}}-\frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}\,(2cd-be)^2}-\frac{x}{3d\left(d+ex^2\right)^{3/2}(2cd-be)}$$

```
[Out] -1/3*x/d/(-b*e+2*c*d)/(e*x^2+d)^(3/2)-c^2*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1
/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/(-b*e+2*c*d)^(5/2)/e^(1/2)/(-b*e+c*d)
^(1/2)-1/3*(-2*b*e+7*c*d)*x/d^2/(-b*e+2*c*d)^2/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {1149, 414, 527, 12, 377, 208}

$$-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\,\sqrt{cd-be}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^{5/2}}-\frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}\,(2cd-be)^2}-\frac{x}{3d\left(d+ex^2\right)^{3/2}(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -x/(3*d*(2*c*d - b*e)*(d + e*x^2)^(3/2)) - ((7*c*d - 2*b*e)*x)/(3*d^2*(2*c*
d - b*e)^2*Sqrt[d + e*x^2]) - (c^2*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(S
qrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^(5
/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \int \frac{1}{\left(d+ex^2\right)^{5/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx$$

$$= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}} + \frac{\int \frac{e(5cd-2be)-2ce^2x^2}{\left(d+ex^2\right)^{3/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{3de(2cd-be)}$$

$$= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} - \ldots$$

Mathematica [C] time = 4.14, size = 1058, normalized size = 7.10

$$-\frac{x\left(-\frac{56c^2e^2\left(\frac{e(be-2cd)x^2}{(be-cd)\left(ex^2+d\right)}\right)^{3/2}x^4}{(cd-be)^2} + \frac{168c^2e^2\tanh^{-1}\left(\sqrt{\frac{e(be-2cd)x^2}{(be-cd)\left(ex^2+d\right)}}\right)x^4}{(cd-be)^2} + \frac{36c^2e^2\left(\frac{e(be-2cd)x^2}{(be-cd)\left(ex^2+d\right)}\right)^{7/2}{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{e(be-2cd)x^2}{(be-cd)\left(ex^2+d\right)}\right)x^4}{(cd-be)^2} + \frac{12c^2e^2(\ldots}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),
x]
```

```
[Out] -1/63*(x*(-315*Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))] +
(420*c*e*x^2*Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))])/(c*
```

```
d - b*e) - (168*c^2*e^2*x^4*Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d
+ e*x^2))])/(c*d - b*e)^2 - 105*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d
+ e*x^2)))^(3/2) + (140*c*e*x^2*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d
+ e*x^2)))^(3/2))/(c*d - b*e) - (56*c^2*e^2*x^4*((e*(-2*c*d + b*e)*x^2)/((-
(c*d) + b*e)*(d + e*x^2)))^(3/2))/(c*d - b*e)^2 + 315*ArcTanh[Sqrt[(e*(-2*c
*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))]] - (420*c*e*x^2*ArcTanh[Sqrt[(
e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))]])/(c*d - b*e) + (168*c^
2*e^2*x^4*ArcTanh[Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))]
])/(c*d - b*e)^2 + 48*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2)))
^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e
)*(d + e*x^2))] - (84*c*e*x^2*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d +
e*x^2)))^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (e*(-2*c*d + b*e)*x^2)/((-(c*
d) + b*e)*(d + e*x^2))])/(c*d - b*e) + (36*c^2*e^2*x^4*((e*(-2*c*d + b*e)*x
^2)/((-(c*d) + b*e)*(d + e*x^2)))^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (e*(
-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))])/(c*d - b*e)^2 + 12*((e*(-
2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2)))^(7/2)*HypergeometricPFQ[{2,
 2, 7/2}, {1, 9/2}, (e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))] -
(24*c*e*x^2*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2)))^(7/2)*Hyp
ergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, (e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e
)*(d + e*x^2))])/(c*d - b*e) + (12*c^2*e^2*x^4*((e*(-2*c*d + b*e)*x^2)/((-(
c*d) + b*e)*(d + e*x^2)))^(7/2)*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, (e
*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))])/(c*d - b*e)^2))/((c*d -
 b*e)*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2)))^(5/2)*(d + e*x^
2)^(5/2))
```

fricas [B] time = 2.64, size = 1063, normalized size = 7.13

$$\left[\frac{3\left(c^2d^2e^2x^4+2\,c^2d^3ex^2+c^2d^4\right)\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}\,\log\left(\frac{c^2d^4-2\,bcd^3e+b^2d^2e^2+\left(17\,c^2d^2e^2-24\,bcde^3+8\,b^2e^4\right)x^4+2\left(7\,c^2d^3e-\cdots\right.}{c^2e^2x^4+c^2\cdots}\right.}{12\left(8\,c^4d^8e-20\,bc^3d^7e^2+18\,b^2c^2d^6e^3-7\,b^3cd^5e^4+b^4d^4e^5+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"fricas")
```

```
[Out] [1/12*(3*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*sqrt(2*c^2*d^2*e - 3
*b*c*d*e^2 + b^2*e^3)*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^
2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4
*b^2*d*e^3)*x^2 - 4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*((3*c*d*e - 2
*b*e^2)*x^3 + (c*d^2 - b*d*e)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 -
2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) - 4*((14*c^3*d^3*e^2 - 25
*b*c^2*d^2*e^3 + 13*b^2*c*d*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e - 11*b*c^
2*d^3*e^2 + 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(8*c^4*d^8*e -
 20*b*c^3*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c*d^5*e^4 + b^4*d^4*e^5 + (8
*c^4*d^6*e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c*d^3*e^6 + b^
4*d^2*e^7)*x^4 + 2*(8*c^4*d^7*e^2 - 20*b*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 -
 7*b^3*c*d^4*e^5 + b^4*d^3*e^6)*x^2), -1/6*(3*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*
e*x^2 + c^2*d^4)*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*arctan(-1/2*sqr
t(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2
)*x^2)*sqrt(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^
2*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x)) + 2*((14*c^3*d^3*e^2 - 25*b*c^2*d^
2*e^3 + 13*b^2*c*d*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e - 11*b*c^2*d^3*e^2
 + 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(8*c^4*d^8*e - 20*b*c^3
*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c*d^5*e^4 + b^4*d^4*e^5 + (8*c^4*d^6*
e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c*d^3*e^6 + b^4*d^2*e^7
)*x^4 + 2*(8*c^4*d^7*e^2 - 20*b*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 - 7*b^3*c*
d^4*e^5 + b^4*d^3*e^6)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [b,c,d,exp(1),exp(2)]=[-21,-18,-46,11,70]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.T
he choice was done assuming [b,c,d,exp(1),exp(2)]=[72,91,-18,-31,46]Evaluat
ion time: 2.06Unable to transpose Error: Bad Argument Value
```

maple [B] time = 0.02, size = 1637, normalized size = 10.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/2*c^3*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c
*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(b*e-2*c*d)/((x+(-(b*e-c*d)*c*e)^(1/
2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)/c-(b*e-
2*c*d)/c)^(1/2)+1/2*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^
(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(b*e-2*c*d)/d/((x+(-(b*e-c*d)*c*e)^(1/2)/c/
e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)/c-(b*e-2*c*d
)/c)^(1/2)*x-1/2*c^3*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/
2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(b*e-2*c*d)/(-(b*e-2*c*
d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)*(x+(-(b*e-c*d)*c*
e)^(1/2)/c/e)/c+2*(-(b*e-2*c*d)/c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*
e-2*(-(b*e-c*d)*c*e)^(1/2)*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)/c-(b*e-2*c*d)/c)^
(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))-1/6*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)*
c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(x-(-d*e)^(1/2)/e)/((x
-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)-1/3*c*e/d^2/(
(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1
/2))/((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)*x-1/6
*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c
*e)^(1/2))/(x+(-d*e)^(1/2)/e)/((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d
*e)^(1/2)/e))^(1/2)-1/3*c*e/d^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(
-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2
)*(x+(-d*e)^(1/2)/e))^(1/2)*x-1/2*c^3*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1
/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(b*e-2
*c*d)/((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)*(x-(-(b*
e-c*d)*c*e)^(1/2)/c/e)/c-(b*e-2*c*d)/c)^(1/2)+1/2*c^2*e/((-d*e)^(1/2)*c+(-(
b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(b*e-2*c*d)/d
/((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)*(x-(-(b*e-c*d
)*c*e)^(1/2)/c/e)/c-(b*e-2*c*d)/c)^(1/2)*x+1/2*c^3*e/((-d*e)^(1/2)*c+(-(b*e
-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)
^(1/2)/(b*e-2*c*d)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d
)*c*e)^(1/2)*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)/c+2*(-(b*e-2*c*d)/c)^(1/2)*((x-
(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)*(x-(-(b*e-c*d)*c*e
)^(1/2)/c/e)/c-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ce^2x^4 + be^2x^2 - cd^2 + bde\right)\left(ex^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^(3/2)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(e\,x^2+d\right)^{3/2}\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^(3/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

```
[Out] int(1/((d + e*x^2)^(3/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^{\frac{5}{2}}\left(be-cd+cex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/((d + e*x**2)**(5/2)*(b*e - c*d + c*e*x**2)), x)
```

3.225 $\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\, dx$

Optimal. Leaf size=183

$$\frac{1}{3}\left(x^4+x^2+1\right)^{3/2}x+\frac{2}{45}\left(6x^2+7\right)\sqrt{x^4+x^2+1}\,x+\frac{26\sqrt{x^4+x^2+1}\,x}{45\left(x^2+1\right)}+\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}}-\ldots$$

```
[Out] 1/3*x*(x^4+x^2+1)^(3/2)+1/9*x^3*(x^4+x^2+1)^(3/2)+26/45*x*(x^4+x^2+1)^(1/2)
/(x^2+1)+2/45*x*(6*x^2+7)*(x^4+x^2+1)^(1/2)-26/45*(x^2+1)*(cos(2*arctan(x))
^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^
2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+7/15*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos
(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)
/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3+\frac{1}{3}\left(x^4+x^2+1\right)^{3/2}x+\frac{2}{45}\left(6x^2+7\right)\sqrt{x^4+x^2+1}\,x+\frac{26\sqrt{x^4+x^2+1}\,x}{45\left(x^2+1\right)}+\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+x^2+\ldots}{\left(x^2+\ldots\right.}}\ldots}{15\sqrt{x^4\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (26*x*Sqrt[1 + x^2 + x^4])/(45*(1 + x^2)) + (2*x*(7 + 6*x^2)*Sqrt[1 + x^2 +
 x^4])/45 + (x*(1 + x^2 + x^4)^(3/2))/3 + (x^3*(1 + x^2 + x^4)^(3/2))/9 - (
26*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])
/(45*Sqrt[1 + x^2 + x^4]) + (7*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*
EllipticF[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\,dx &= \frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}\int \sqrt{1+x^2+x^4}\left(9+24x^2+21x^4\right)dx\\ &= \frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}+\frac{1}{63}\int \left(42+84x^2\right)\sqrt{1+x^2+x^4}\,dx\\ &= \frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}+\frac{1}{945}\\ &= \frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}-\frac{26}{45}\int\\ &= \frac{26x\sqrt{1+x^2+x^4}}{45\left(1+x^2\right)}+\frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}x^3\left(1-\right.\end{aligned}$$

Mathematica [C] time = 0.32, size = 169, normalized size = 0.92

$$\frac{2(-1)^{5/6}\left(4\sqrt{3}+9i\right)\sqrt{\sqrt[3]{-1}\,x^2+1}\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+26\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}\,x^2+1}\sqrt{1-(-1)}}{45\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(29 + 61*x^2 + 81*x^4 + 57*x^6 + 25*x^8 + 5*x^10) + 26*(-1)^(1/3)*Sqrt[1
 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*
```

```
x], (-1)^(2/3)] + 2*(-1)^(5/6)*(9*I + 4*Sqrt[3])*Sqrt[1 + (-1)^(1/3)*x^2]*S
qrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(45
*Sqrt[1 + x^2 + x^4])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(x^6+3\,x^4+3\,x^2+1\right)\sqrt{x^4+x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)*sqrt(x^4 + x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+x^2+1}\left(x^2+1\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

maple [C] time = 0.16, size = 263, normalized size = 1.44

$$\frac{\sqrt{x^4+x^2+1}\,x^7}{9}+\frac{4\sqrt{x^4+x^2+1}\,x^5}{9}+\frac{32\sqrt{x^4+x^2+1}\,x^3}{45}+\frac{29\sqrt{x^4+x^2+1}\,x}{45}+\frac{32\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-(}}{45\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^3*(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/9*x^7*(x^4+x^2+1)^(1/2)+4/9*x^5*(x^4+x^2+1)^(1/2)+32/45*x^3*(x^4+x^2+1)^(
1/2)+29/45*x*(x^4+x^2+1)^(1/2)+32/45/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*
3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*El
lipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-104/45/(-2
+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/
2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(
1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+x^2+1}\left(x^2+1\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+1\right)^3\sqrt{x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(x^2 - x + 1)(x^2 + x + 1)}\,(x^2 + 1)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**3*(x**4+x**2+1)**(1/2),x)
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3, x)
```

3.226 $\int \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}\,dx$

Optimal. Leaf size=164

$$\frac{1}{7}x\left(x^4+x^2+1\right)^{3/2}+\frac{2}{21}x\left(3x^2+4\right)\sqrt{x^4+x^2+1}+\frac{2x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}+\frac{4\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{7\sqrt{x^4+x^2+1}} - \frac{2(\cdots}{\cdots}$$

```
[Out] 1/7*x*(x^4+x^2+1)^(3/2)+2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+2/21*x*(3*x^2+4)*(x
^4+x^2+1)^(1/2)-2/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Ell
ipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2
)+4/7*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*a
rctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1206, 1176, 1197, 1103, 1195}

$$\frac{1}{7}x\left(x^4+x^2+1\right)^{3/2}+\frac{2}{21}x\left(3x^2+4\right)\sqrt{x^4+x^2+1}+\frac{2x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}+\frac{4\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{7\sqrt{x^4+x^2+1}} - \frac{2(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (2*x*(4 + 3*x^2)*Sqrt[1 + x^2 + x
^4])/21 + (x*(1 + x^2 + x^4)^(3/2))/7 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(
1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (4*(1 +
x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(7*Sqrt
[1 + x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}\, dx = \frac{1}{7}x\left(1+x^2+x^4\right)^{3/2} + \frac{1}{7}\int \left(6+10x^2\right)\sqrt{1+x^2+x^4}\, dx$$
$$= \frac{2}{21}x\left(4+3x^2\right)\sqrt{1+x^2+x^4} + \frac{1}{7}x\left(1+x^2+x^4\right)^{3/2} + \frac{1}{105}\int \frac{50+70x^2}{\sqrt{1+x^2+x^4}}\, dx$$
$$= \frac{2}{21}x\left(4+3x^2\right)\sqrt{1+x^2+x^4} + \frac{1}{7}x\left(1+x^2+x^4\right)^{3/2} - \frac{2}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\, dx + \frac{8}{7}\int$$
$$= \frac{2x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{2}{21}x\left(4+3x^2\right)\sqrt{1+x^2+x^4} + \frac{1}{7}x\left(1+x^2+x^4\right)^{3/2} - \frac{2\left(1+x^2\right.}{}$$

Mathematica [C] time = 0.16, size = 162, normalized size = 0.99

$$\frac{2\sqrt[3]{-1}\left(5\sqrt[3]{-1}-7\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+14\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/}}}{21\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(11 + 20*x^2 + 23*x^4 + 12*x^6 + 3*x^8) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(1
/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(
2/3)] + 2*(-1)^(1/3)*(-7 + 5*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
(-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(21*Sqrt[1
+ x^2 + x^4])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(x^4+2x^2+1\right)\sqrt{x^4+x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 2*x^2 + 1)*sqrt(x^4 + x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+x^2+1}\left(x^2+1\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

maple [C] time = 0.01, size = 248, normalized size = 1.51

$$\frac{\sqrt{x^4+x^2+1}\,x^5}{7}+\frac{3\sqrt{x^4+x^2+1}\,x^3}{7}+\frac{11\sqrt{x^4+x^2+1}\,x}{21}+\frac{20\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2+1}\ \mathrm{El}}{21\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2*(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/7*(x^4+x^2+1)^(1/2)*x^5+3/7*(x^4+x^2+1)^(1/2)*x^3+11/21*(x^4+x^2+1)^(1/2)
*x+20/21/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2
-1/2*I*3^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*(-2+2*I*3^(1/2
))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1
/2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^
(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(
1/2))^(1/2))-EllipticE(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1
/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+x^2+1}\left(x^2+1\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+1\right)^2\sqrt{x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\left(x^2+1\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2, x)
```

3.227 $\int \left(1+x^2\right) \sqrt{1+x^2+x^4} \, dx$

Optimal. Leaf size=145

$$\frac{1}{5}\left(x^2+2\right)\sqrt{x^4+x^2+1}\,x+\frac{3\sqrt{x^4+x^2+1}\,x}{5\left(x^2+1\right)}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan\right.}{5\sqrt{x^4+x^2+1}}$$

```
[Out] 3/5*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/5*x*(x^2+2)*(x^4+x^2+1)^(1/2)-3/5*(x^2+1)
*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2
)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+3/5*(x^2+1)*(cos(2*arctan
(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)
/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1176, 1197, 1103, 1195}

$$\frac{1}{5}\left(x^2+2\right)\sqrt{x^4+x^2+1}\,x+\frac{3\sqrt{x^4+x^2+1}\,x}{5\left(x^2+1\right)}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan\right.}{5\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (3*x*Sqrt[1 + x^2 + x^4])/(5*(1 + x^2)) + (x*(2 + x^2)*Sqrt[1 + x^2 + x^4])
/5 - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
1/4])/(5*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \left(1+x^2\right) \sqrt{1+x^2+x^4}\, dx = \frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4} + \frac{1}{15}\int \frac{9+9x^2}{\sqrt{1+x^2+x^4}}\, dx$$
$$= \frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4} - \frac{3}{5}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\, dx + \frac{6}{5}\int \frac{1}{\sqrt{1+x^2+x^4}}\, dx$$
$$= \frac{3x\sqrt{1+x^2+x^4}}{5\left(1+x^2\right)} + \frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4} - \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\, E\left(2\tan^{-1}(\right.}{5\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.18, size = 168, normalized size = 1.16

$$\frac{x^7+3x^5+3x^3+\frac{3}{2}\sqrt{2+\left(1-i\sqrt{3}\right)x^2}\sqrt{2+\left(1+i\sqrt{3}\right)x^2}\,F\left(\sin^{-1}\left(\frac{1}{2}\left(i\sqrt{3}x+x\right)\right)|\frac{1}{2}i\left(i+\sqrt{3}\right)\right)+3\sqrt[3]{-1}\sqrt{\sqrt[3]{-}}}{5\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (2*x + 3*x^3 + 3*x^5 + x^7 + 3*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
 (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (3*Sqrt[2
 + (1 - I*Sqrt[3])*x^2]*Sqrt[2 + (1 + I*Sqrt[3])*x^2]*EllipticF[ArcSin[(x +
 I*Sqrt[3]*x)/2], (I/2)*(I + Sqrt[3])])/2)/(5*Sqrt[1 + x^2 + x^4])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{x^4+x^2+1}\left(x^2+1\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+x^2+1}\left(x^2+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

maple [C] time = 0.00, size = 233, normalized size = 1.61

$$\frac{\sqrt{x^4+x^2+1}\,x^3}{5}+\frac{2\sqrt{x^4+x^2+1}\,x}{5}+\frac{6\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2+1}\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2}, \right.}{5\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/5*(x^4+x^2+1)^(1/2)*x^3+2/5*(x^4+x^2+1)^(1/2)*x+6/5/(-2+2*I*3^(1/2))^(1/2
)*(-(-1/2+1/2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3^(1/2))*x^2+1)^(1/2)/(
x^4+x^2+1)^(1/2)*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2)
)^(1/2))-12/5/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/2*I*3^(1/2))*x^2+1)^(1/2)*(-
(-1/2-1/2*I*3^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(Elliptic
F(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*(-
2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4 + x^2 + 1}\left(x^2 + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2 + 1\right)\sqrt{x^4 + x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)*(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)*(x^2 + x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)}\left(x^2 + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1), x)
```

3.228 $\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx$

Optimal. Leaf size=137

$$\frac{\sqrt{x^4+x^2+1}\,x}{x^2+1}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan\right.}{\sqrt{x^4+x^2+1}}$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))+x*(x^4+x^2+1)^(1/2)/(x^2+1)-(x^2+1)*(cos(2*
arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+
x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+3/4*(x^2+1)*(cos(2*arctan(x))^2)^
(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)
^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1208, 1139, 1103, 1195, 1210, 1698, 203}

$$\frac{\sqrt{x^4+x^2+1}\,x}{x^2+1}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan\right.}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - ((1 +
 x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1
 + x^2 + x^4] + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*
ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx = \int \frac{x^2}{\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\sqrt{1+x^2+x^4}}\,dx - \int \frac{1-}{\sqrt{1+}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\right.}{4\sqrt{1+x^2+x^4}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{3\left(1+\right.}{}$$

Mathematica [C] time = 0.09, size = 117, normalized size = 0.85

$$\frac{\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,\left(-F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\sqrt[3]{-1}\,\Pi\left(\sqrt[3]{}\right.\right.}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

```
[Out] ((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1
)^(2/3)] + (-1)^(1/3)*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^
(2/3)]))/Sqrt[1 + x^2 + x^4]
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

maple [C] time = 0.11, size = 293, normalized size = 2.14

$$\frac{4\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{\frac{x^2}{2}+\frac{i\sqrt{3}\,x^2}{2}+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}-\frac{4\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{\frac{x^2}{2}+\frac{i\sqrt{3}\,x^2}{2}+1}}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1),x)
```

```
[Out] -4/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/
2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*(-2+2*
I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))+4/(-2+2*I*3^(1/2))^(1/2)*(1+
1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x
^2+1)^(1/2)/(1+I*3^(1/2))*EllipticE(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*
I*3^(1/2))^(1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2)
)^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-
1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/
2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1),x)
```

```
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)}}{x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1), x)
```

3.229 $\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=49

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

```
[Out] 1/2*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arc
tan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1225}

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

```
[Out] ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(
2*Sqrt[1 + x^2 + x^4])
```

Rule 1225

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> With[{q = Rt[e/d, 2]}, Simp[(c*(d + e*x^2)*Sqrt[(e^2*(a + b*x^2 + c
*x^4))/(c*(d + e*x^2)^2)]*EllipticE[2*ArcTan[q*x], (2*c*d - b*e)/(4*c*d)])/
(2*d*e^2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e}, x] && Ne
Q[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
 && PosQ[e/d]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^2} \, dx = \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.35, size = 164, normalized size = 3.35

$$\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\sqrt[3]{-1}\,\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,\left(F\left(i\sin\right.\right.}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

```
[Out] ((x + x^3 + x^5)/(1 + x^2) + (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*
```

```
Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1
)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/
(2*Sqrt[1 + x^2 + x^4])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^4+2\,x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^4 + 2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

maple [C] time = 0.02, size = 224, normalized size = 4.57

$$\frac{\sqrt{x^4+x^2+1}\;x}{2x^2+2}+\frac{\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2+1}\;\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-2+2i\sqrt{3}}\;x}{2}, \frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\;\sqrt{x^4+x^2+1}}+\frac{2\sqrt{-\left(-\frac{1}{2}+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^2,x)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/2*I*3^(1
/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*Ell
ipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*3
^(1/2))^(1/2)*(-(-1/2+1/2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3^(1/2))*x^
2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*(-2+2*I*3^(1/2))^
(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*(-2+2*I*3^(1/2))^(1/2)*x,
1/2*(-2+2*I*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^2,x)
```

```
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)}}{\left(x^2 + 1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**2,x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**2, x)
```

3.230 $\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^3} \, dx$

Optimal. Leaf size=93

$$\frac{\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

```
[Out] 1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/4*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {1228, 1223, 1696, 1593, 1712, 1195, 1700, 1103, 1698, 203, 12, 1317, 1210}

$$\frac{\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
 ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/
(4*Sqrt[1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
```

```
 0] && EqQ[B*d + A*e, 0]
```

Rule 1700

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^3}\,dx = \int \left(\frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}} - \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}} + \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\right) dx$$

$$= \int \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx - \int \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} - \frac{1}{4}\int \frac{-3+2x^2-x^4}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{8}\int \frac{-1}{\left(1+x^2\right.}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\right.}{2\sqrt{1+x^2+}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\right.}{2\sqrt{1+x^2+}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{\left(1-\right.}{}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{}{\sqrt{1+}}\right.$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.30, size = 176, normalized size = 1.89

$$\frac{2(-1)^{2/3}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right) + \sqrt[3]{-1}\,\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

```
[Out] ((x*(2 + x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 + (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)
*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2
/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) + 2*(-1)^(2/3)*Sqrt[
1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSi
nh[(-1)^(5/6)*x], (-1)^(2/3)])/(4*Sqrt[1 + x^2 + x^4])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^6+3\,x^4+3\,x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^6 + 3*x^4 + 3*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

maple [C] time = 0.02, size = 333, normalized size = 3.58

$$\frac{\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)^2}+\frac{\sqrt{x^4+x^2+1}\,x}{4x^2+4}-\frac{\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{\frac{x^2}{2}+\frac{i\sqrt{3}\,x^2}{2}+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}+\frac{\sqrt{\frac{x^2}{2}-\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^3,x)
```

```
[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/4*(x^4+x^2+1)^(1/2)/(x^2+1)*x+1/(-2+2*I
*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*
x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*(-2+2*I*3^(1/2))
^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-1/(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*
I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2
)/(1+I*3^(1/2))*EllipticE(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))
^(1/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*
(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*
I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2
+1/2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^3,x)
```

```
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}{\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**3,x)
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**3, x)
```

3.231 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{x^4+x^2+1}\,x}{6(x^2+1)^2}+\frac{\sqrt{x^4+x^2+1}\,x}{6(x^2+1)^3}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{8\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2}{(x^2+1}}}{3\sqrt{x^4}}$$

```
[Out] 1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^3+1/6*x*(x^
4+x^2+1)^(1/2)/(x^2+1)^2+1/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arcta
n(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^
2+1)^(1/2)-1/8*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Elliptic
F(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {1228, 1223, 1696, 1586, 1197, 1103, 1195, 1593, 1712, 1700, 1698, 203, 12, 1317}

$$\frac{\sqrt{x^4+x^2+1}\,x}{6(x^2+1)^2}+\frac{\sqrt{x^4+x^2+1}\,x}{6(x^2+1)^3}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{8\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2}{(x^2+1}}}{3\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^3) + (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x
^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/
(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) - ((1 + x
^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(8*Sqrt[
1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
```

```
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
```

```
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1700

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx = \int \left(\frac{1}{\left(1+x^2\right)^4\sqrt{1+x^2+x^4}} - \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}} + \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\right)dx$$

$$= \int \frac{1}{\left(1+x^2\right)^4\sqrt{1+x^2+x^4}}\,dx - \int \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} - \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} - \frac{1}{6}\int \frac{-5+2x^2-3x^4}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx +$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{24}\int \frac{10-8x^2+10x^4}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\right.}{2\sqrt{1+x^2+x^4}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\right.}{2\sqrt{1+x^2+x^4}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} + \frac{7x\sqrt{1+x^2+x^4}}{12\left(1+x^2\right)} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\right.}{4\sqrt{1+x^2+x^4}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\right.}{3\sqrt{1+x^2+x}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\right.}{3\sqrt{1+x^2+x}}$$

Mathematica [C] time = 0.41, size = 240, normalized size = 1.45

$$-(-1)^{2/3}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right) + 3(-1)^{2/3}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```

```
[Out] ((x*(1 + x^2 + x^4)*(4 + 5*x^2 + 2*x^4))/(1 + x^2)^3 - 2*(-1)^(1/3)*Sqrt[1
+ (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*
x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - (-1)^(2
/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(
-1)^(5/6)*x], (-1)^(2/3)] + 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
(-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]
)/(6*Sqrt[1 + x^2 + x^4])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^8+4\,x^6+6\,x^4+4\,x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="fricas")
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

maple [C] time = 0.03, size = 438, normalized size = 2.64

$$\frac{\sqrt{x^4+x^2+1}\,x}{6\left(x^2+1\right)^3}+\frac{\sqrt{x^4+x^2+1}\,x}{6\left(x^2+1\right)^2}+\frac{\sqrt{x^4+x^2+1}\,x}{3x^2+3}-\frac{4\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{\frac{x^2}{2}+\frac{i\sqrt{3}\,x^2}{2}+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^4,x)
[Out] 1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^3+1/6*(x^4+x^2+1)^(1/2)/(x^2+1)^2*x+1/3*(x^
4+x^2+1)^(1/2)/(x^2+1)*x-1/3/(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*
x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*Elliptic
F(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))+4/3/(-2+2*I*3^(1
/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1
)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2
)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-4/3/(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3
^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(
1+I*3^(1/2))*EllipticE(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1
/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/
2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3
^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/
2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^4,x)
```

```
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^4, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}{\left(x^2+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**4,x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**4, x)
```

3.232 $\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}} \, dx$

Optimal. Leaf size=159

$$\frac{14\sqrt{x^4+x^2+1}\,x}{15\left(x^2+1\right)}+\frac{11}{15}\sqrt{x^4+x^2+1}\,x+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{14\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\right.}{15\sqrt{x^4+x^2+1}}$$

```
[Out] 11/15*x*(x^4+x^2+1)^(1/2)+1/5*x^3*(x^4+x^2+1)^(1/2)+14/15*x*(x^4+x^2+1)^(1/
2)/(x^2+1)-14/15*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Ellipt
icE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+3
/5*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arct
an(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1206, 1679, 1197, 1103, 1195}

$$\frac{1}{5}\sqrt{x^4+x^2+1}\,x^3+\frac{14\sqrt{x^4+x^2+1}\,x}{15\left(x^2+1\right)}+\frac{11}{15}\sqrt{x^4+x^2+1}\,x+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{14\left(x^2+1\right)\sqrt{\frac{x^4+}{(x^2}}}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (11*x*Sqrt[1 + x^2 + x^4])/15 + (x^3*Sqrt[1 + x^2 + x^4])/5 + (14*x*Sqrt[1
+ x^2 + x^4])/(15*(1 + x^2)) - (14*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticE[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sq
rt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^
2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+x^2\right)^3}{\sqrt{1+x^2+x^4}}\,dx &= \frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{1}{5}\int \frac{5+12x^2+11x^4}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{1}{15}\int \frac{4+14x^2}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}-\frac{14}{15}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+\frac{6}{5}\int \frac{1}{\sqrt{1+x^2+x^4}}\,d\\ &= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{14x\sqrt{1+x^2+x^4}}{15\left(1+x^2\right)}-\frac{14\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\right.}{15\sqrt{1+x^2+x^4}}\end{aligned}$$

Mathematica [C] time = 0.17, size = 157, normalized size = 0.99

$$\frac{2\sqrt[3]{-1}\left(2\sqrt[3]{-1}-7\right)\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+14\sqrt[3]{-1}\,\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-}{15\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(11 + 14*x^2 + 14*x^4 + 3*x^6) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*
Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2
*(-1)^(1/3)*(-7 + 2*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3
)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(15*Sqrt[1 + x^2 + x
^4])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6+3\,x^4+3\,x^2+1}{\sqrt{x^4+x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^3}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

maple [C] time = 0.03, size = 233, normalized size = 1.47

$$\frac{\sqrt{x^4+x^2+1}\,x^3}{5}+\frac{11\sqrt{x^4+x^2+1}\,x}{15}+\frac{8\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\frac{\sqrt{-2}}{}\right.}{15\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^3/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/5*(x^4+x^2+1)^(1/2)*x^3+11/15*(x^4+x^2+1)^(1/2)*x+8/15/(-2+2*I*3^(1/2))^(
1/2)*(-(-1/2+1/2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3^(1/2))*x^2+1)^(1/2
)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1
/2))^(1/2))-56/15/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/2*I*3^(1/2))*x^2+1)^(1/2
)*(-(-1/2-1/2*I*3^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(Elli
pticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/
2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^3}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+1\right)^3}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^3}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)**3/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.233 $\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=137

$$\frac{4\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}+\frac{1}{3}\sqrt{x^4+x^2+1}\,x+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{4\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] 1/3*x*(x^4+x^2+1)^(1/2)+4/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)-4/3*(x^2+1)*(cos(2*
arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+
x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+(x^2+1)*(cos(2*arctan(x))^2)^(1/2
)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^
(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1206, 1197, 1103, 1195}

$$\frac{4\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}+\frac{1}{3}\sqrt{x^4+x^2+1}\,x+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{4\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/3 + (4*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) - (4*(1
 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*S
qrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Elliptic
F[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{\left(1+x^2\right)^2}{\sqrt{1+x^2+x^4}}\,dx = \frac{1}{3}x\sqrt{1+x^2+x^4}+\frac{1}{3}\int \frac{2+4x^2}{\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{1}{3}x\sqrt{1+x^2+x^4}-\frac{4}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+2\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{1}{3}x\sqrt{1+x^2+x^4}+\frac{4x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}-\frac{4\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\ldots}}{\ldots}$$

Mathematica [C] time = 0.14, size = 143, normalized size = 1.04

$$\frac{x^5+x^3+2\sqrt[3]{-1}\left(\sqrt[3]{-1}-2\right)\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+4\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1\ldots}}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x + x^3 + x^5 + 4*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*
x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(1/3)*(-2 + (-
1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*Arc
Sinh[(-1)^(5/6)*x], (-1)^(2/3)])/(3*Sqrt[1 + x^2 + x^4])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4+2\,x^2+1}{\sqrt{x^4+x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 2*x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

maple [C] time = 0.01, size = 218, normalized size = 1.59

$$\frac{\sqrt{x^4+x^2+1}\,x}{3}+\frac{4\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{16\sqrt{-\left(\cdots\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/3*(x^4+x^2+1)^(1/2)*x+4/3/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/2*I*3^(1/2))*x
^2+1)^(1/2)*(-(-1/2-1/2*I*3^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF
(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-16/3/(-2+2*I*3^(1
/2))^(1/2)*(-(-1/2+1/2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3^(1/2))*x^2+1
)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*(-2+2*I*3^(1/2))^(1/
2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2
*(-2+2*I*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)**2/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.234 $\int \frac{1+x^2}{\sqrt{1+x^2+x^4}} \, dx$

Optimal. Leaf size=115

$$\frac{\sqrt{x^4+x^2+1}\,x}{x^2+1}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

```
[Out] x*(x^4+x^2+1)^(1/2)/(x^2+1)-(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan
(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2
+1)^(1/2)+(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin
(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt{x^4+x^2+1}\,x}{x^2+1}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^
2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] + ((1 + x^2)*Sqrt[(1
 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = 2\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx - \int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x\right.}{\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.07, size = 94, normalized size = 0.82

$$\frac{\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,\left(\left(\sqrt[3]{-1}-1\right)F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] ((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1 + (-1)^(1/3))*EllipticF[I*ArcSinh[
(-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4]
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2+1}{\sqrt{x^4+x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

maple [C] time = 0.01, size = 205, normalized size = 1.78

$$\frac{2\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2}, \frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}} - \frac{4\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2+1}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] -4/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I
*3^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*(-2+2
*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*(-2+2*I*3^(1/
2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2)))+2/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1
```

```
/2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^
(1/2)*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)/(x^2 + x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.235 $\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))+1/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(
2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/
(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1210, 1103, 1698, 203}

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.07, size = 72, normalized size = 1.04

$$\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-
1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/Sqrt[1 + x^2 + x^4]
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^6+2\,x^4+2\,x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^6 + 2*x^4 + 2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

maple [C] time = 0.02, size = 104, normalized size = 1.51

$$\frac{\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{\frac{x^2}{2}+\frac{i\sqrt{3}\,x^2}{2}+1}\;\text{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x, -\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}, \frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1
/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))
^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(
1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+x^2+1}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2+1\right)\,\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)), x)
```

3.236 $\int \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}} \, dx$

Optimal. Leaf size=118

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))+1/2*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(
2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/
(x^4+x^2+1)^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*E
llipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1
/2)
```

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1223, 1712, 1195, 12, 1317, 1103, 1698, 203}

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticE[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[
(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 +
 x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}}\,dx = \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{-1+2x^2+x^4}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx - \frac{1}{2}\int \frac{2x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \int \frac{x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1-x}{\left(1+x^2\right)\sqrt{1\ldots}}$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2}\mathrm{Su}\ldots$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2\ldots}{\left(1+x\ldots\right.}}}{4\sqrt{1\ldots}}$$

Mathematica [C] time = 0.40, size = 226, normalized size = 1.92

$$\frac{-(-1)^{2/3}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+2(-1)^{2/3}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{-}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((x + x^3 + x^5)/(1 + x^2) - (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*
Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1
)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) +
 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[
(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(2*Sqrt[1 + x^2 + x^4])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^8+3\,x^6+4\,x^4+3\,x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^8 + 3*x^6 + 4*x^4 + 3*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

maple [C] time = 0.02, size = 397, normalized size = 3.36

$$\frac{\sqrt{x^4+x^2+1}\,x}{2x^2+2}-\frac{2\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{\frac{x^2}{2}+\frac{i\sqrt{3}\,x^2}{2}+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}-\frac{\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/2*(x^4+x^2+1)^(1/2)/(x^2+1)*x-1/(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(
1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*Ell
ipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*3
^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^
2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*(-2+2*I*3^(1/2))^(
1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-2/(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*
3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/
(1+I*3^(1/2))*EllipticE(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(
1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2
*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^
(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2
*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2+1\right)^2\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2), x)
```

3.237 $\int \frac{1}{\left(1+x^2\right)^3 \sqrt{1+x^2+x^4}} \, dx$

Optimal. Leaf size=142

$$\frac{\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x\right.}{4\sqrt{x^4+x^2+1}}$$

```
[Out] 1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+3/4*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)-1/2*(x^2+1)*(cos(2*arct
an(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+
1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1223, 1696, 1593, 1712, 1195, 1700, 1103, 1698, 203}

$$\frac{\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x\right.}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
 (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4]
)/(4*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
```

```
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1700

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx = \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{1}{4}\int \frac{-3+2x^2-x^4}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{8}\int \frac{-10x^2-6x^4}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{8}\int \frac{x^2\left(-10-6x^2\right)}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{8}\int \frac{-6-10x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx + \frac{3}{4}\int \frac{1}{\sqrt{1+}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{4}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,F\left(2\right.}{2\sqrt{1+x^2+}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right.}{4\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.34, size = 235, normalized size = 1.65

$$-2(-1)^{2/3}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right) + 2(-1)^{2/3}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((x*(4 + 3*x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 - 3*(-1)^(1/3)*Sqrt[1 + (-1)^(
1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)
^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - 2*(-1)^(2/3)*Sq
rt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5
/6)*x], (-1)^(2/3)] + 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(
2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(4*S
qrt[1 + x^2 + x^4])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^{10}+4\,x^8+7\,x^6+7\,x^4+4\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^10 + 4*x^8 + 7*x^6 + 7*x^4 + 4*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+x^2+1}\,(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

maple [C] time = 0.02, size = 418, normalized size = 2.94

$$\frac{\sqrt{x^4+x^2+1}\,x}{4\,(x^2+1)^2}+\frac{3\sqrt{x^4+x^2+1}\,x}{4\,(x^2+1)}-\frac{3\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{\frac{x^2}{2}+\frac{i\sqrt{3}\,x^2}{2}+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x)
[Out] 1/4*(x^4+x^2+1)^(1/2)/(x^2+1)^2*x+3/4*(x^4+x^2+1)^(1/2)/(x^2+1)*x-1/(-2+2*I
*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*
x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(
-2+2*I*3^(1/2))^(1/2))+3/(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+
1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2)
)*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-3/(-2+
2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/
2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticE(1/2*(-2+2*I*3^(1/
2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1/2
*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^
2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),
(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+x^2+1}\,(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^2+1)^3\,\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2)),x)
[Out] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3), x)
```

3.238 $\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=144

$$\frac{2\sqrt{x^4+x^2+1}\,x}{3(x^2+1)}-\frac{(1-x^2)\,x}{3\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] -1/3*x*(-x^2+1)/(x^4+x^2+1)^(1/2)+2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)-2/3*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+(x^2+1)*(cos(2*arctan(x
))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(
x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1205, 1197, 1103, 1195}

$$\frac{2\sqrt{x^4+x^2+1}\,x}{3(x^2+1)}-\frac{(1-x^2)\,x}{3\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] -(x*(1 - x^2))/(3*Sqrt[1 + x^2 + x^4]) + (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 +
x^2)) - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x
], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2
)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+x^2\right)^3}{\left(1+x^2+x^4\right)^{3/2}}\,dx &= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{1}{3}\int \frac{4+2x^2}{\sqrt{1+x^2+x^4}}\,dx\\ &= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}}-\frac{2}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+2\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx\\ &= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}-\frac{2\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right)\ldots}{\ldots}\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(x^6+3\,x^4+3\,x^2+1\right)\sqrt{x^4+x^2+1}}{x^8+2\,x^6+3\,x^4+2\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)*sqrt(x^4 + x^2 + 1)/(x^8 + 2*x^6 + 3*x^4
 + 2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^3}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)
```

maple [C] time = 0.04, size = 268, normalized size = 1.86

$$\frac{8\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2+1}\,\,\mathrm{EllipticF}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{4\left(\frac{1}{6}x^3-\frac{1}{6}x\right)}{\sqrt{x^4+x^2+1}}-\frac{8\sqrt{-\left(-\frac{1}{2}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^3/(x^4+x^2+1)^(3/2),x)
```

```
[Out] -4*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^(1/2)+8/3/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/
2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(
1/2)*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-8/3
/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3
^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*(-2+2*I
*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*(-2+2*I*3^(1/2)
)^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2)))-6*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)-
6*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^3}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+1\right)^3}{\left(x^4+x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(3/2),x)
```

```
[Out] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^3}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)**3/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

3.239 $\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=98

$$-\frac{2\sqrt{x^4+x^2+1}\,x}{3(x^2+1)}+\frac{(2x^2+1)\,x}{3\sqrt{x^4+x^2+1}}+\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] 1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)-2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+2/3*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1205, 1195}

$$-\frac{2\sqrt{x^4+x^2+1}\,x}{3(x^2+1)}+\frac{(2x^2+1)\,x}{3\sqrt{x^4+x^2+1}}+\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) - (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 +
 x^2)) + (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[
x], 1/4])/(3*Sqrt[1 + x^2 + x^4])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx = \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3}\int \frac{2-2x^2}{\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} - \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(x^4+2x^2+1)\sqrt{x^4+x^2+1}}{x^8+2x^6+3x^4+2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral((x^4 + 2*x^2 + 1)*sqrt(x^4 + x^2 + 1)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2
 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+1)^2}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

maple [C] time = 0.01, size = 268, normalized size = 2.73

$$\frac{4\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2}, \frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}} - \frac{2\left(\frac{1}{6}x^3+\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}} + \frac{8\sqrt{-\left(-\frac{1}{2}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(x^4+x^2+1)^(3/2),x)
[Out] -2*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)+4/3/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/2
*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(1
/2)*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))+8/3/
(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3^
```

```
(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*(-2+2*I*
3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*(-2+2*I*3^(1/2))
^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2)))-4*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)-
2*(1/6*x^3-1/6*x)/(x^4+x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+1\right)^2}{\left(x^4+x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(3/2),x)
[Out] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(x**4+x**2+1)**(3/2),x)
[Out] Integral((x**2 + 1)**2/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

3.240 $\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}+\frac{\left(x^2+2\right)x}{3\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] 1/3*x*(x^2+2)/(x^4+x^2+1)^(1/2)-1/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/3*(x^2+1)
*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2
)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1178, 1195}

$$-\frac{\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}+\frac{\left(x^2+2\right)x}{3\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2
)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/
4])/(3*Sqrt[1 + x^2 + x^4])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx &= \frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{1}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}}-\frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}\,(x^2+1)}{x^8+2\,x^6+3\,x^4+2\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)*(x^2 + 1)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

maple [C] time = 0.01, size = 247, normalized size = 2.57

$$\frac{2\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{2\left(-\frac{1}{3}x^3-\frac{1}{6}x\right)}{\sqrt{x^4+x^2+1}}+\frac{4\sqrt{-\left(-\frac{1}{2}+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+x^2+1)^(3/2),x)
```

```
[Out] -2*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/
2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(
1/2)*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))+4/3
/(-2+2*I*3^(1/2))^(1/2)*(-(-1/2+1/2*I*3^(1/2))*x^2+1)^(1/2)*(-(-1/2-1/2*I*3
^(1/2))*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*(-2+2*I
*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*(-2+2*I*3^(1/2)
)^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2)))-2*(1/6*x^3-1/6*x)/(x^4+x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\left(x^4+x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^2 + x^4 + 1)^(3/2),x)
[Out] int((x^2 + 1)/(x^2 + x^4 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+x**2+1)**(3/2),x)
[Out] Integral((x**2 + 1)/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

3.241 $\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}} dx$

Optimal. Leaf size=166

$$\frac{2\sqrt{x^4+x^2+1}\,x}{3(x^2+1)}-\frac{(2x^2+1)x}{3\sqrt{x^4+x^2+1}}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{}}{3\sqrt{}}$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)+2/3*x*(x^
4+x^2+1)^(1/2)/(x^2+1)-2/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(
x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+
1)^(1/2)+3/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(
sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1221, 1119, 1197, 1103, 1195, 1210, 1698, 203}

$$\frac{2\sqrt{x^4+x^2+1}\,x}{3(x^2+1)}-\frac{(2x^2+1)x}{3\sqrt{x^4+x^2+1}}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{}}{3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) + (2*x*Sqrt[1 + x^2 + x^4])/(3*(1
+ x^2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^
4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (3*(
1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*
Sqrt[1 + x^2 + x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
```

```
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)\left(1+x^2+x^4\right)^{3/2}}\,dx = -\int \frac{x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx + \int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= -\frac{x\left(1+2x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3}\int \frac{1+2x^2}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{}{(1}$$

$$= -\frac{x\left(1+2x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\right.$$

$$= -\frac{x\left(1+2x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{2x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{2\left(1+x^2\right)}{}$$

Mathematica [C] time = 0.21, size = 204, normalized size = 1.23

$$\frac{-2x^3+\sqrt[3]{-1}\left(\sqrt[3]{-1}-2\right)\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+2\sqrt[3]{-1}\,\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(\cdots}{3\sqrt{x^{\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (-x - 2*x^3 + 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2
]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*(-2 + (-1)^(1
/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[
(-1)^(5/6)*x], (-1)^(2/3)] + 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
 (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)
])/(3*Sqrt[1 + x^2 + x^4])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^{10}+3\,x^8+5\,x^6+5\,x^4+3\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^10 + 3*x^8 + 5*x^6 + 5*x^4 + 3*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

maple [C] time = 0.02, size = 398, normalized size = 2.40

$$\frac{8\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{\frac{x^2}{2}+\frac{i\sqrt{3}\,x^2}{2}+1}\,\text{EllipticE}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}+\frac{2\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{\frac{x^2}{2}+\frac{i\sqrt{3}\,x^2}{2}+1}\,\text{E}\cdots}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^4+x^2+1)^(3/2),x)
```

```
[Out] -2*(1/3*x^3+1/6*x)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/
2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1
/2)*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/
(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3
^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*(-2+2*I*3
^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))+8/3/(-2+2*I*3^(1/2))^(1/2)*(1/2
*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^
2+1)^(1/2)/(1+I*3^(1/2))*EllipticE(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I
*3^(1/2))^(1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)
^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1
```

```
/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2
)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2+1\right)\,\left(x^4+x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(3/2)),x)
```

```
[Out] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)), x)
```

3.242 $\int \frac{1}{(1+x^2)^2 (1+x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=111

$$\frac{\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}-\frac{\left(x^2+2\right)x}{3\sqrt{x^4+x^2+1}}+\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{6\sqrt{x^4+x^2+1}}$$

```
[Out] arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(x^2+2)/(x^4+x^2+1)^(1/2)+1/3*x*(x^4+x^2+
1)^(1/2)/(x^2+1)+1/6*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*El
lipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/
2)
```

Rubi [A] time = 0.28, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {1228, 1178, 1195, 1223, 1712, 12, 1317, 1103, 1698, 203, 1210}

$$\frac{\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}-\frac{\left(x^2+2\right)x}{3\sqrt{x^4+x^2+1}}+\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{6\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) + (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^
2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]] + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 +
x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(6*Sqrt[1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^2\left(1+x^2+x^4\right)^{3/2}}\,dx = \int \left(\frac{-1-x^2}{\left(1+x^2+x^4\right)^{3/2}} + \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}} + \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\right)dx$$

$$= \int \frac{-1-x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx + \int \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+\cdots}}$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} + \frac{1}{3}\int \frac{-1+x^2}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+\cdots}}$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{5x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \cdots$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+\cdots}{(1\cdots}}}{6\sqrt{\cdots}}$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+\cdots}{(1\cdots}}}{6\sqrt{\cdots}}$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+\cdots}{(1\cdots}}}{6\sqrt{\cdots}}$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2\cdots}{(1+x\cdots}}}{6\sqrt{1\cdots}}$$

Mathematica [C] time = 0.38, size = 168, normalized size = 1.51

$$\frac{-2x\left(x^2+1\right)\left(x^2+2\right) - \sqrt[3]{-1}\left(x^2+1\right)\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\left(\left(5\sqrt[3]{-1}-1\right)F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\cdots\right.}{6\left(x^2+1\right)\sqrt{x^4+x^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (-2*x*(1 + x^2)*(2 + x^2) + 3*x*(1 + x^2 + x^4) - (-1)^(1/3)*(1 + x^2)*Sqrt
[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/
6)*x], (-1)^(2/3)] + (-1 + 5*(-1)^(1/3))*EllipticF[I*ArcSinh[(-1)^(5/6)*x],
 (-1)^(2/3)] - 12*(-1)^(1/3)*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x]
, (-1)^(2/3)]))/(6*(1 + x^2)*Sqrt[1 + x^2 + x^4])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^{12}+4\,x^{10}+8\,x^8+10\,x^6+8\,x^4+4\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^12 + 4*x^10 + 8*x^8 + 10*x^6 + 8*x^4 + 4*x^
2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

maple [C] time = 0.03, size = 419, normalized size = 3.77

$$\frac{\sqrt{x^4+x^2+1}\,x}{2x^2+2}-\frac{2\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{\frac{x^2}{2}+\frac{i\sqrt{3}\,x^2}{2}+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}-\frac{5\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x)
```

```
[Out] 1/2*(x^4+x^2+1)^(1/2)/(x^2+1)*x-2*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)-5/3/(-2
+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1
/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1
/2*(-2+2*I*3^(1/2))^(1/2))+2/3/(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2
)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3
^(1/2))*EllipticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-
2/3/(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2
*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticE(1/2*(-2+2
*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-1/2+1/2*I*3^(1/2))^(1/2
)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(
x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(
1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^2+1\right)}^2\,{\left(x^4+x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(3/2)),x)
```

```
[Out] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)\right)^{\frac{3}{2}}\left(x^2 + 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**4+x**2+1)**(3/2),x)
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**2), x)
```

3.243 $\int \frac{1}{\left(1+x^2\right)^3\left(1+x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}+\frac{\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)^2}-\frac{\left(1-x^2\right)x}{3\sqrt{x^4+x^2+1}}+\frac{3}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(\right.}{4\sqrt{x^4+x^2+1}}$$

```
[Out] 3/4*arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(-x^2+1)/(x^4+x^2+1)^(1/2)+1/4*x*(x^4
+x^2+1)^(1/2)/(x^2+1)^2-1/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+19/12*(x^2+1)*(cos(
2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^
4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)-5/4*(x^2+1)*(cos(2*arctan(x))^2
)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+
1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {1228, 1092, 1197, 1103, 1195, 1223, 1696, 1593, 1712, 1700, 1698, 203, 12, 1317}

$$-\frac{\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}+\frac{\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)^2}-\frac{\left(1-x^2\right)x}{3\sqrt{x^4+x^2+1}}+\frac{3}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(\right.}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(1 - x^2))/(3*Sqrt[1 + x^2 + x^4]) + (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^
2)^2) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (3*ArcTan[x/Sqrt[1 + x^2 +
x^4]])/4 + (19*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcT
an[x], 1/4])/(12*Sqrt[1 + x^2 + x^4]) - (5*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(
1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
```

```
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
```

```
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1700

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^3\left(1+x^2+x^4\right)^{3/2}}\,dx = \int\left(-\frac{1}{\left(1+x^2+x^4\right)^{3/2}}+\frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}+\frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\right)dx$$

$$= -\int \frac{1}{\left(1+x^2+x^4\right)^{3/2}}\,dx + \int \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\left(1+x^2\right)^2\sqrt{1+x}}$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} - \frac{1}{4}\int \frac{-3+2x^2-x}{\left(1+x^2\right)^2\sqrt{1+x^2}}$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{5x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{8}\int \frac{-10x^2-6x}{\left(1+x^2\right)\sqrt{1+x^2}}$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{5x\sqrt{1+x^2+x^4}}{12\left(1+x^2\right)} + \frac{5\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E}{6\sqrt{1+x^2}}$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{5x\sqrt{1+x^2+x^4}}{12\left(1+x^2\right)} + \frac{5\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E}{6\sqrt{1+x^2}}$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{19\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E}{12\sqrt{1+x^2}}$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right.$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{3}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right.$$

Mathematica [C] time = 0.34, size = 192, normalized size = 1.01

$$\frac{4x\left(x^2-1\right)\left(x^2+1\right)^2-\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}\,x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(x^2+1\right)^2\left(\left(-9+10i\sqrt{3}\right)F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right.}{12\left(x\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (4*x*(-1 + x^2)*(1 + x^2)^2 + 3*x*(1 + x^2 + x^4) + 15*x*(1 + x^2)*(1 + x^2
 + x^4) - (-1)^(1/3)*(1 + x^2)^2*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/
3)*x^2]*(19*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-9 + (10*I)*S
qrt[3])*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 18*(-1)^(1/3)*Elli
pticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/(12*(1 + x^2)^2*S
qrt[1 + x^2 + x^4])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^{14}+5\,x^{12}+12\,x^{10}+18\,x^8+18\,x^6+12\,x^4+5\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^14 + 5*x^12 + 12*x^10 + 18*x^8 + 18*x^6 + 1
2*x^4 + 5*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

maple [C] time = 0.03, size = 439, normalized size = 2.31

$$\frac{\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)^2}+\frac{5\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)}-\frac{19\sqrt{\frac{x^2}{2}-\frac{i\sqrt{3}\,x^2}{2}+1}\,\sqrt{\frac{x^2}{2}+\frac{i\sqrt{3}\,x^2}{2}+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{-2+2i\sqrt{3}}\,x}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x)
```

```
[Out] 1/4*(x^4+x^2+1)^(1/2)/(x^2+1)^2*x+5/4*(x^4+x^2+1)^(1/2)/(x^2+1)*x-2*(1/6*x-
1/6*x^3)/(x^4+x^2+1)^(1/2)-10/3/(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/
2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*Ellip
ticF(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))+19/3/(-2+2*I*
3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x
^2+1)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*(-2+2*I*3^(1/2))^
(1/2)*x,1/2*(-2+2*I*3^(1/2))^(1/2))-19/3/(-2+2*I*3^(1/2))^(1/2)*(1/2*x^2-1/
2*I*3^(1/2)*x^2+1)^(1/2)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1
/2)/(1+I*3^(1/2))*EllipticE(1/2*(-2+2*I*3^(1/2))^(1/2)*x,1/2*(-2+2*I*3^(1/2
))^(1/2))+3/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1/2*x^2-1/2*I*3^(1/2)*x^2+1)^(1/2
)*(1/2*x^2+1/2*I*3^(1/2)*x^2+1)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/
2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1
/2+1/2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^2+1\right)}^3\,{\left(x^4+x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(3/2)),x)
```

```
[Out] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**3), x)
```

3.244 $\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=135

$$\frac{1}{9}e^2x^9\left(e(ae+4bd)+6cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right)+\frac{2}{7}dex^7\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{3}d^3x^3(4ae+bd)+ad^4x+$$

```
[Out] a*d^4*x+1/3*d^3*(4*a*e+b*d)*x^3+1/5*d^2*(6*a*e^2+4*b*d*e+c*d^2)*x^5+2/7*d*e
*(2*c*d^2+e*(2*a*e+3*b*d))*x^7+1/9*e^2*(6*c*d^2+e*(a*e+4*b*d))*x^9+1/11*e^3
*(b*e+4*c*d)*x^11+1/13*c*e^4*x^13
```

Rubi [A] time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{9}e^2x^9\left(e(ae+4bd)+6cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right)+\frac{2}{7}dex^7\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{3}d^3x^3(4ae+bd)+ad^4x+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)
/5 + (2*d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^7)/7 + (e^2*(6*c*d^2 + e*(4*b*d
 + a*e))*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(d+ex^2\right)^4\left(a+bx^2+cx^4\right)dx = \int \left(ad^4+d^3(bd+4ae)x^2+d^2\left(cd^2+4bde+6ae^2\right)x^4+2de\left(2cd^2+e(3bd\right.\right.$$
$$= ad^4x+\frac{1}{3}d^3(bd+4ae)x^3+\frac{1}{5}d^2\left(cd^2+4bde+6ae^2\right)x^5+\frac{2}{7}de\left(2cd^2+e(3bd\right.$$

Mathematica [A] time = 0.04, size = 135, normalized size = 1.00

$$\frac{1}{9}e^2x^9\left(ae^2+4bde+6cd^2\right)+\frac{2}{7}dex^7\left(2ae^2+3bde+2cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right)+\frac{1}{3}d^3x^3(4ae+bd)+ad^4x+$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)
/5 + (2*d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + 4*b*d*e
+ a*e^2)*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13
```

fricas [A] time = 0.68, size = 148, normalized size = 1.10

$$\frac{1}{13}x^{13}e^4c+\frac{4}{11}x^{11}e^3dc+\frac{1}{11}x^{11}e^4b+\frac{2}{3}x^9e^2d^2c+\frac{4}{9}x^9e^3db+\frac{1}{9}x^9e^4a+\frac{4}{7}x^7ed^3c+\frac{6}{7}x^7e^2d^2b+\frac{4}{7}x^7e^3da+\frac{1}{5}x^5d^4c+\frac{4}{5}x^5ed^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^4*c + 4/11*x^11*e^3*d*c + 1/11*x^11*e^4*b + 2/3*x^9*e^2*d^2*c +
 4/9*x^9*e^3*d*b + 1/9*x^9*e^4*a + 4/7*x^7*e*d^3*c + 6/7*x^7*e^2*d^2*b + 4/
7*x^7*e^3*d*a + 1/5*x^5*d^4*c + 4/5*x^5*e*d^3*b + 6/5*x^5*e^2*d^2*a + 1/3*x
^3*d^4*b + 4/3*x^3*e*d^3*a + x*d^4*a
```

giac [A] time = 0.15, size = 142, normalized size = 1.05

$$\frac{1}{13}\,cx^{13}e^4+\frac{4}{11}\,cdx^{11}e^3+\frac{1}{11}\,bx^{11}e^4+\frac{2}{3}\,cd^2x^9e^2+\frac{4}{9}\,bdx^9e^3+\frac{4}{7}\,cd^3x^7e+\frac{1}{9}\,ax^9e^4+\frac{6}{7}\,bd^2x^7e^2+\frac{1}{5}\,cd^4x^5+\frac{4}{7}\,adx^7e^3+\frac{4}{5}\,bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^4 + 4/11*c*d*x^11*e^3 + 1/11*b*x^11*e^4 + 2/3*c*d^2*x^9*e^2 +
 4/9*b*d*x^9*e^3 + 4/7*c*d^3*x^7*e + 1/9*a*x^9*e^4 + 6/7*b*d^2*x^7*e^2 + 1/
5*c*d^4*x^5 + 4/7*a*d*x^7*e^3 + 4/5*b*d^3*x^5*e + 6/5*a*d^2*x^5*e^2 + 1/3*b
*d^4*x^3 + 4/3*a*d^3*x^3*e + a*d^4*x
```

maple [A] time = 0.00, size = 136, normalized size = 1.01

$$\frac{c\,e^4x^{13}}{13}+\frac{\left(e^4b+4d\,e^3c\right)x^{11}}{11}+\frac{\left(e^4a+4d\,e^3b+6d^2e^2c\right)x^9}{9}+\frac{\left(4d\,e^3a+6d^2e^2b+4d^3ec\right)x^7}{7}+a\,d^4x+\frac{\left(6d^2e^2a+4d^3eb+\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/13*c*e^4*x^13+1/11*(b*e^4+4*c*d*e^3)*x^11+1/9*(a*e^4+4*b*d*e^3+6*c*d^2*e^
2)*x^9+1/7*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^7+1/5*(6*a*d^2*e^2+4*b*d^3*e
+c*d^4)*x^5+1/3*(4*a*d^3*e+b*d^4)*x^3+a*d^4*x
```

maxima [A] time = 0.96, size = 135, normalized size = 1.00

$$\frac{1}{13}\,ce^4x^{13}+\frac{1}{11}\left(4\,cde^3+be^4\right)x^{11}+\frac{1}{9}\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^9+\frac{2}{7}\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^7+ad^4x+\frac{1}{5}\left(cd^4+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*e^4*x^13 + 1/11*(4*c*d*e^3 + b*e^4)*x^11 + 1/9*(6*c*d^2*e^2 + 4*b*d*
e^3 + a*e^4)*x^9 + 2/7*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^7 + a*d^4*x
+ 1/5*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^5 + 1/3*(b*d^4 + 4*a*d^3*e)*x^3
```

mupad [B] time = 0.06, size = 131, normalized size = 0.97

$$x^3\left(\frac{b\,d^4}{3}+\frac{4\,a\,e\,d^3}{3}\right)+x^{11}\left(\frac{b\,e^4}{11}+\frac{4\,c\,d\,e^3}{11}\right)+x^5\left(\frac{c\,d^4}{5}+\frac{4\,b\,d^3\,e}{5}+\frac{6\,a\,d^2\,e^2}{5}\right)+x^9\left(\frac{2\,c\,d^2\,e^2}{3}+\frac{4\,b\,d\,e^3}{9}+\frac{a\,e^4}{9}\right)+\frac{c\,e^4\,x}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((b*d^4)/3 + (4*a*d^3*e)/3) + x^11*((b*e^4)/11 + (4*c*d*e^3)/11) + x^5*
((c*d^4)/5 + (6*a*d^2*e^2)/5 + (4*b*d^3*e)/5) + x^9*((a*e^4)/9 + (2*c*d^2*e
^2)/3 + (4*b*d*e^3)/9) + (c*e^4*x^13)/13 + a*d^4*x + (2*d*e*x^7*(2*a*e^2 +
2*c*d^2 + 3*b*d*e))/7
```

sympy [A] time = 0.11, size = 156, normalized size = 1.16

$$ad^4x+\frac{ce^4x^{13}}{13}+x^{11}\left(\frac{be^4}{11}+\frac{4cde^3}{11}\right)+x^9\left(\frac{ae^4}{9}+\frac{4bde^3}{9}+\frac{2cd^2e^2}{3}\right)+x^7\left(\frac{4ade^3}{7}+\frac{6bd^2e^2}{7}+\frac{4cd^3e}{7}\right)+x^5\left(\frac{6ad^2e^2}{5}+\frac{4bd^3}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**13/13 + x**11*(b*e**4/11 + 4*c*d*e**3/11) + x**9*(a*e*
*4/9 + 4*b*d*e**3/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 6*b*d**2*e**2
/7 + 4*c*d**3*e/7) + x**5*(6*a*d**2*e**2/5 + 4*b*d**3*e/5 + c*d**4/5) + x**
3*(4*a*d**3*e/3 + b*d**4/3)
```

3.245 $\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=103

$$\frac{1}{7}ex^7\left(e(ae+3bd)+3cd^2\right)+\frac{1}{5}dx^5\left(3e(ae+bd)+cd^2\right)+\frac{1}{3}d^2x^3(3ae+bd)+ad^3x+\frac{1}{9}e^2x^9(be+3cd)+\frac{1}{11}ce^3x^{11}$$

```
[Out] a*d^3*x+1/3*d^2*(3*a*e+b*d)*x^3+1/5*d*(c*d^2+3*e*(a*e+b*d))*x^5+1/7*e*(3*c*
d^2+e*(a*e+3*b*d))*x^7+1/9*e^2*(b*e+3*c*d)*x^9+1/11*c*e^3*x^11
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{7}ex^7\left(e(ae+3bd)+3cd^2\right)+\frac{1}{5}dx^5\left(3e(ae+bd)+cd^2\right)+\frac{1}{3}d^2x^3(3ae+bd)+ad^3x+\frac{1}{9}e^2x^9(be+3cd)+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^5)/5 +
 (e*(3*c*d^2 + e*(3*b*d + a*e))*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e^3
*x^11)/11
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^3\left(a+bx^2+cx^4\right)dx &= \int \left(ad^3+d^2(bd+3ae)x^2+d\left(cd^2+3e(bd+ae)\right)x^4+e\left(3cd^2+e(3bd+ae)\right)x\right.\\ &= ad^3x+\frac{1}{3}d^2(bd+3ae)x^3+\frac{1}{5}d\left(cd^2+3e(bd+ae)\right)x^5+\frac{1}{7}e\left(3cd^2+e(3bd+ae)\right.\end{aligned}$$

Mathematica [A] time = 0.03, size = 104, normalized size = 1.01

$$\frac{1}{7}ex^7\left(ae^2+3bde+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+3bde+cd^2\right)+\frac{1}{3}d^2x^3(3ae+bd)+ad^3x+\frac{1}{9}e^2x^9(be+3cd)+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^5)/5
 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e
^3*x^11)/11
```

fricas [A] time = 0.59, size = 111, normalized size = 1.08

$$\frac{1}{11}x^{11}e^3c+\frac{1}{3}x^9e^2dc+\frac{1}{9}x^9e^3b+\frac{3}{7}x^7ed^2c+\frac{3}{7}x^7e^2db+\frac{1}{7}x^7e^3a+\frac{1}{5}x^5d^3c+\frac{3}{5}x^5ed^2b+\frac{3}{5}x^5e^2da+\frac{1}{3}x^3d^3b+x^3ed^2a+xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^3*c + 1/3*x^9*e^2*d*c + 1/9*x^9*e^3*b + 3/7*x^7*e*d^2*c + 3/7*x
^7*e^2*d*b + 1/7*x^7*e^3*a + 1/5*x^5*d^3*c + 3/5*x^5*e*d^2*b + 3/5*x^5*e^2*
d*a + 1/3*x^3*d^3*b + x^3*e*d^2*a + x*d^3*a
```

giac [A] time = 0.16, size = 108, normalized size = 1.05

$$\frac{1}{11}cx^{11}e^3+\frac{1}{3}cdx^9e^2+\frac{1}{9}bx^9e^3+\frac{3}{7}cd^2x^7e+\frac{3}{7}bdx^7e^2+\frac{1}{5}cd^3x^5+\frac{1}{7}ax^7e^3+\frac{3}{5}bd^2x^5e+\frac{3}{5}adx^5e^2+\frac{1}{3}bd^3x^3+ad^2x^3e+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^3 + 1/3*c*d*x^9*e^2 + 1/9*b*x^9*e^3 + 3/7*c*d^2*x^7*e + 3/7*b
*d*x^7*e^2 + 1/5*c*d^3*x^5 + 1/7*a*x^7*e^3 + 3/5*b*d^2*x^5*e + 3/5*a*d*x^5*
e^2 + 1/3*b*d^3*x^3 + a*d^2*x^3*e + a*d^3*x
```

maple [A] time = 0.00, size = 103, normalized size = 1.00

$$\frac{c\,e^3x^{11}}{11}+\frac{\left(e^3b+3d\,e^2c\right)x^9}{9}+\frac{\left(a\,e^3+3d\,e^2b+3c\,d^2e\right)x^7}{7}+a\,d^3x+\frac{\left(3d\,e^2a+3d^2eb+d^3c\right)x^5}{5}+\frac{\left(3d^2ea+d^3b\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/11*c*e^3*x^11+1/9*(b*e^3+3*c*d*e^2)*x^9+1/7*(a*e^3+3*b*d*e^2+3*c*d^2*e)*x
^7+1/5*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^5+1/3*(3*a*d^2*e+b*d^3)*x^3+a*d^3*x
```

maxima [A] time = 1.03, size = 102, normalized size = 0.99

$$\frac{1}{11}ce^3x^{11}+\frac{1}{9}\left(3\,cde^2+be^3\right)x^9+\frac{1}{7}\left(3\,cd^2e+3\,bde^2+ae^3\right)x^7+\frac{1}{5}\left(cd^3+3\,bd^2e+3\,ade^2\right)x^5+ad^3x+\frac{1}{3}\left(bd^3+3\,ad^2e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/11*c*e^3*x^11 + 1/9*(3*c*d*e^2 + b*e^3)*x^9 + 1/7*(3*c*d^2*e + 3*b*d*e^2
+ a*e^3)*x^7 + 1/5*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^5 + a*d^3*x + 1/3*(b*d
^3 + 3*a*d^2*e)*x^3
```

mupad [B] time = 4.63, size = 101, normalized size = 0.98

$$x^3\left(\frac{b\,d^3}{3}+a\,e\,d^2\right)+x^9\left(\frac{b\,e^3}{9}+\frac{c\,d\,e^2}{3}\right)+x^5\left(\frac{c\,d^3}{5}+\frac{3\,b\,d^2\,e}{5}+\frac{3\,a\,d\,e^2}{5}\right)+x^7\left(\frac{3\,c\,d^2\,e}{7}+\frac{3\,b\,d\,e^2}{7}+\frac{a\,e^3}{7}\right)+\frac{c\,e^3\,x^{11}}{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((b*d^3)/3 + a*d^2*e) + x^9*((b*e^3)/9 + (c*d*e^2)/3) + x^5*((c*d^3)/5
+ (3*a*d*e^2)/5 + (3*b*d^2*e)/5) + x^7*((a*e^3)/7 + (3*b*d*e^2)/7 + (3*c*d^
2*e)/7) + (c*e^3*x^11)/11 + a*d^3*x
```

sympy [A] time = 0.29, size = 112, normalized size = 1.09

$$ad^3x+\frac{ce^3x^{11}}{11}+x^9\left(\frac{be^3}{9}+\frac{cde^2}{3}\right)+x^7\left(\frac{ae^3}{7}+\frac{3bde^2}{7}+\frac{3cd^2e}{7}\right)+x^5\left(\frac{3ade^2}{5}+\frac{3bd^2e}{5}+\frac{cd^3}{5}\right)+x^3\left(ad^2e+\frac{bd^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**11/11 + x**9*(b*e**3/9 + c*d*e**2/3) + x**7*(a*e**3/7
+ 3*b*d*e**2/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + 3*b*d**2*e/5 + c*d**3
/5) + x**3*(a*d**2*e + b*d**3/3)
```

3.246 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=73

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x+1/3*d*(2*a*e+b*d)*x^3+1/5*(c*d^2+e*(a*e+2*b*d))*x^5+1/7*e*(b*e+2*c*
d)*x^7+1/9*c*e^2*x^9
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx &= \int \left(ad^2+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^4+e(2cd+be)x^6+ce^2x^8\right)dx\\ &= ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5+\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9\end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.00

$$\frac{1}{5}x^5\left(ae^2+2bde+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

fricas [A] time = 0.49, size = 76, normalized size = 1.04

$$\frac{1}{9}x^9e^2c+\frac{2}{7}x^7edc+\frac{1}{7}x^7e^2b+\frac{1}{5}x^5d^2c+\frac{2}{5}x^5edb+\frac{1}{5}x^5e^2a+\frac{1}{3}x^3d^2b+\frac{2}{3}x^3eda+xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/7*x^7*e^2*b + 1/5*x^5*d^2*c + 2/5*x^5*e*d
*b + 1/5*x^5*e^2*a + 1/3*x^3*d^2*b + 2/3*x^3*e*d*a + x*d^2*a
```

giac [A] time = 0.15, size = 76, normalized size = 1.04

$$\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{7}bx^7e^2 + \frac{1}{5}cd^2x^5 + \frac{2}{5}bdx^5e + \frac{1}{5}ax^5e^2 + \frac{1}{3}bd^2x^3 + \frac{2}{3}adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/7*b*x^7*e^2 + 1/5*c*d^2*x^5 + 2/5*b*d*x^5
*e + 1/5*a*x^5*e^2 + 1/3*b*d^2*x^3 + 2/3*a*d*x^3*e + a*d^2*x
```

maple [A] time = 0.00, size = 70, normalized size = 0.96

$$\frac{c\,e^2x^9}{9} + \frac{\left(b\,e^2 + 2dce\right)x^7}{7} + \frac{\left(a\,e^2 + 2bde + c\,d^2\right)x^5}{5} + a\,d^2x + \frac{\left(2dea + b\,d^2\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x^5+1/3*(2*
a*d*e+b*d^2)*x^3+a*d^2*x
```

maxima [A] time = 1.07, size = 69, normalized size = 0.95

$$\frac{1}{9}ce^2x^9 + \frac{1}{7}\left(2\,cde + be^2\right)x^7 + \frac{1}{5}\left(cd^2 + 2\,bde + ae^2\right)x^5 + ad^2x + \frac{1}{3}\left(bd^2 + 2\,ade\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x
^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

mupad [B] time = 4.59, size = 70, normalized size = 0.96

$$x^5\left(\frac{c\,d^2}{5} + \frac{2\,b\,d\,e}{5} + \frac{a\,e^2}{5}\right) + x^3\left(\frac{b\,d^2}{3} + \frac{2\,a\,e\,d}{3}\right) + x^7\left(\frac{b\,e^2}{7} + \frac{2\,c\,d\,e}{7}\right) + \frac{c\,e^2\,x^9}{9} + a\,d^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^5*((a*e^2)/5 + (c*d^2)/5 + (2*b*d*e)/5) + x^3*((b*d^2)/3 + (2*a*d*e)/3) +
 x^7*((b*e^2)/7 + (2*c*d*e)/7) + (c*e^2*x^9)/9 + a*d^2*x
```

sympy [A] time = 0.11, size = 78, normalized size = 1.07

$$ad^2x + \frac{ce^2x^9}{9} + x^7\left(\frac{be^2}{7} + \frac{2cde}{7}\right) + x^5\left(\frac{ae^2}{5} + \frac{2bde}{5} + \frac{cd^2}{5}\right) + x^3\left(\frac{2ade}{3} + \frac{bd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2
*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

3.247 $\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=42

$$\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

```
[Out] a*d*x+1/3*(a*e+b*d)*x^3+1/5*(b*e+c*d)*x^5+1/7*c*e*x^7
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1153}

$$\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)\,dx &= \int \left(ad+(bd+ae)x^2+(cd+be)x^4+cex^6\right)\,dx\\ &= adx+\frac{1}{3}(bd+ae)x^3+\frac{1}{5}(cd+be)x^5+\frac{1}{7}cex^7\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

fricas [A] time = 0.48, size = 40, normalized size = 0.95

$$\frac{1}{7}x^7ec+\frac{1}{5}x^5dc+\frac{1}{5}x^5eb+\frac{1}{3}x^3db+\frac{1}{3}x^3ea+xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e*c + 1/5*x^5*d*c + 1/5*x^5*e*b + 1/3*x^3*d*b + 1/3*x^3*e*a + x*d*a
```

giac [A] time = 0.15, size = 43, normalized size = 1.02

$$\frac{1}{7}\,cx^7e+\frac{1}{5}\,cdx^5+\frac{1}{5}\,bx^5e+\frac{1}{3}\,bdx^3+\frac{1}{3}\,ax^3e+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e + 1/5*c*d*x^5 + 1/5*b*x^5*e + 1/3*b*d*x^3 + 1/3*a*x^3*e + a*d*x
```

maple [A] time = 0.00, size = 37, normalized size = 0.88

$$\frac{ce\,x^7}{7}+\frac{(be+cd)\,x^5}{5}+adx+\frac{(ae+bd)\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] a*d*x+1/3*(a*e+b*d)*x^3+1/5*(b*e+c*d)*x^5+1/7*c*e*x^7
```

maxima [A] time = 0.90, size = 36, normalized size = 0.86

$$\frac{1}{7}\,cex^7+\frac{1}{5}\,(cd+be)x^5+\frac{1}{3}\,(bd+ae)x^3+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*c*e*x^7 + 1/5*(c*d + b*e)*x^5 + 1/3*(b*d + a*e)*x^3 + a*d*x
```

mupad [B] time = 0.04, size = 38, normalized size = 0.90

$$\frac{c\,e\,x^7}{7}+\left(\frac{b\,e}{5}+\frac{c\,d}{5}\right)\,x^5+\left(\frac{a\,e}{3}+\frac{b\,d}{3}\right)\,x^3+a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((a*e)/3 + (b*d)/3) + x^5*((b*e)/5 + (c*d)/5) + a*d*x + (c*e*x^7)/7
```

sympy [A] time = 0.10, size = 39, normalized size = 0.93

$$adx+\frac{cex^7}{7}+x^5\left(\frac{be}{5}+\frac{cd}{5}\right)+x^3\left(\frac{ae}{3}+\frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + c*e*x**7/7 + x**5*(b*e/5 + c*d/5) + x**3*(a*e/3 + b*d/3)
```

3.248 $\int \frac{a+bx^2+cx^4}{d+ex^2} dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{d}\,e^{5/2}}-\frac{x(cd-be)}{e^2}+\frac{cx^3}{3e}$$

```
[Out] -(-b*e+c*d)*x/e^2+1/3*c*x^3/e+(a*e^2-b*d*e+c*d^2)*arctan(x*e^(1/2)/d^(1/2))
/e^(5/2)/d^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1153, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{d}\,e^{5/2}}-\frac{x(cd-be)}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
```

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx &= \int \left(-\frac{cd-be}{e^2}+\frac{cx^2}{e}+\frac{cd^2-bde+ae^2}{e^2\left(d+ex^2\right)}\right)dx\\ &= -\frac{(cd-be)x}{e^2}+\frac{cx^3}{3e}+\frac{\left(cd^2-bde+ae^2\right)\int \frac{1}{d+ex^2}\,dx}{e^2}\\ &= -\frac{(cd-be)x}{e^2}+\frac{cx^3}{3e}+\frac{\left(cd^2-bde+ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{d}\,e^{5/2}}+\frac{x(be-cd)}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
```

```
[Out] ((-(c*d) + b*e)*x)/e^2 + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

fricas [A] time = 0.61, size = 159, normalized size = 2.41

$$\left[\frac{2\,cde^2x^3 - 3\left(cd^2 - bde + ae^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right) - 6\left(cd^2e - bde^2\right)x}{6\,de^3},\ \frac{cde^2x^3 + 3\left(cd^2 - bde + ae^2\right)\sqrt{de}\ \ldots}{3\,d\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c*d*e^2*x^3 - 3*(c*d^2 - b*d*e + a*e^2)*sqrt(-d*e)*log((e*x^2 - 2*s
qrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(c*d^2*e - b*d*e^2)*x)/(d*e^3), 1/3*(c*d*
e^2*x^3 + 3*(c*d^2 - b*d*e + a*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*
d^2*e - b*d*e^2)*x)/(d*e^3)]
```

giac [A] time = 0.15, size = 56, normalized size = 0.85

$$\frac{\left(cd^2 - bde + ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}} + \frac{1}{3}\left(cx^3e^2 - 3\,cdxe + 3\,bxe^2\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + 1/3*(c
*x^3*e^2 - 3*c*d*x*e + 3*b*x*e^2)*e^(-3)
```

maple [A] time = 0.00, size = 84, normalized size = 1.27

$$\frac{c\,x^3}{3e} + \frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - \frac{bd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e} + \frac{c\,d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^2} + \frac{bx}{e} - \frac{cdx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d),x)
```

```
[Out] 1/3*c/e*x^3+1/e*b*x-c*d/e^2*x+1/(d*e)^(1/2)*a*arctan(1/(d*e)^(1/2)*e*x)-1/e
/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b*d+1/(d*e)^(1/2)*c*d^2/e^2*arctan(1
/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.41, size = 58, normalized size = 0.88

$$\frac{\left(cd^2 - bde + ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^2} + \frac{cex^3 - 3\,(cd - be)x}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) + 1/3*(c*e*x^
3 - 3*(c*d - b*e)*x)/e^2
```

mupad [B] time = 0.09, size = 57, normalized size = 0.86

$$x\left(\frac{b}{e} - \frac{c\,d}{e^2}\right) + \frac{c\,x^3}{3\,e} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{\sqrt{d}\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2),x)
```

```
[Out] x*(b/e - (c*d)/e^2) + (c*x^3)/(3*e) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c
*d^2 - b*d*e))/(d^(1/2)*e^(5/2))
```

sympy [B] time = 0.73, size = 117, normalized size = 1.77

$$\frac{cx^3}{3e} + x\left(\frac{b}{e} - \frac{cd}{e^2}\right) - \frac{\sqrt{-\frac{1}{de^5}}\left(ae^2 - bde + cd^2\right)\log\left(-de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}}\left(ae^2 - bde + cd^2\right)\log\left(de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d),x)
```

```
[Out] c*x**3/(3*e) + x*(b/e - c*d/e**2) - sqrt(-1/(d*e**5))*(a*e**2 - b*d*e + c*d
**2)*log(-d*e**2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 - b*d
*e + c*d**2)*log(d*e**2*sqrt(-1/(d*e**5)) + x)/2
```

3.249 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}$$

$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\left(3cd^2-e(bd+ae)\right)\int \frac{1}{d+ex^2}\,dx}{2de^2}$$

$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\left(3cd^2-e(bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

fricas [A] time = 0.75, size = 268, normalized size = 3.23

$$\left[\frac{4\,cd^2e^2x^3 + \left(3\,cd^3 - bd^2e - ade^2 + \left(3\,cd^2e - bde^2 - ae^3\right)x^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right) + 2\left(3\,cd^3e - bd^2e^2 + ade^3\right)}{4\left(d^2e^4x^2 + d^3e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

giac [A] time = 0.17, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-2)}}{2\left(x^2e + d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

maple [A] time = 0.01, size = 118, normalized size = 1.42

$$\frac{ax}{2\left(e\,x^2+d\right)d} + \frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d} - \frac{bx}{2\left(e\,x^2+d\right)e} + \frac{b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e} + \frac{cdx}{2\left(e\,x^2+d\right)e^2} - \frac{3cd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^2} + \frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] c/e^2*x+1/2/(e*x^2+d)*a/d*x-1/2/e*x/(e*x^2+d)*b+1/2/(e*x^2+d)*c*d/e^2*x+1/2
/(d*e)^(1/2)*a/d*arctan(1/(d*e)^(1/2)*e*x)+1/2/e/(d*e)^(1/2)*arctan(1/(d*e)
^(1/2)*e*x)*b-3/2/(d*e)^(1/2)*c*d/e^2*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.25, size = 84, normalized size = 1.01

$$\frac{\left(cd^2 - bde + ae^2\right)x}{2\left(de^3x^2 + d^2e^2\right)} + \frac{cx}{e^2} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*d^2 - b*d*e + a*e^2)*x/(d*e^3*x^2 + d^2*e^2) + c*x/e^2 - 1/2*(3*c*d^
2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^2)
```

mupad [B] time = 4.67, size = 77, normalized size = 0.93

$$\frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(-3\,c\,d^2 + b\,d\,e + a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{2\,d\,\left(e^3\,x^2 + d\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

sympy [B] time = 1.23, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x\left(ae^2 - bde + cd^2\right)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2 + bde - 3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2 + bde - 3cd^2\right)\log(}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

3.250 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3} dx$

Optimal. Leaf size=115

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

```
[Out] 1/4*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^2-1/8*(5*c*d^2-e*(3*a*e+b*d))*x/d^2/
e^2/(e*x^2+d)+1/8*(3*c*d^2+e*(3*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)
/e^(5/2)
```

Rubi [A] time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 385, 205}

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d + 3*
a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx=\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2}-\frac{\int \frac{-3a+\frac{d(cd-be)}{e^2}-\frac{4cdx^2}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}$$

$$=\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2}-\frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)}-\frac{\left(-\frac{4cd^2}{e}+e\left(-3a+\frac{d(cd-be)}{e^2}\right)\right)\int \frac{1}{d+ex^2}\,dx}{8d^2e}$$

$$=\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2}-\frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)}+\frac{\left(3cd^2+e(bd+3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Mathematica [A] time = 0.10, size = 110, normalized size = 0.96

$$\frac{x\left(e\left(ae\left(5d+3ex^2\right)+bd\left(ex^2-d\right)\right)-cd^2\left(3d+5ex^2\right)\right)}{8d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2))))
/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

fricas [A] time = 0.59, size = 391, normalized size = 3.40

$$\left[-\frac{2\left(5\,cd^3e^2-bd^2e^3-3\,ade^4\right)x^3+\left(3\,cd^4+bd^3e+3\,ad^2e^2+\left(3\,cd^2e^2+bde^3+3\,ae^4\right)x^4+2\left(3\,cd^3e+bd^2e^2+3\right.\right.}{16\left(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 + (3*c*d^4 + b*d^3*e +
3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*
e^2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 +
d)) + 2*(3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x
^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 - (3*c*d^4 +
 b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3
*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^4
*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3)]
```

giac [A] time = 0.23, size = 101, normalized size = 0.88

$$\frac{\left(3\,cd^2+bde+3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(5\,cd^2x^3e-bdx^3e^2+3\,cd^3x-3\,ax^3e^3+bd^2xe-5\,adxe^2\right)e^{(-2)}}{8\left(x^2e+d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2)
- 1/8*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^3*e^3 + b*d^2*x*e -
5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2)
```

maple [A] time = 0.01, size = 131, normalized size = 1.14

$$\frac{3a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^2}+\frac{b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,de}+\frac{3c\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^2}+\frac{\frac{\left(3a\,e^2+bde-5c\,d^2\right)x^3}{8d^2e}+\frac{\left(5a\,e^2-bde-3c\,d^2\right)x}{8d\,e^2}}{\left(e\,x^2+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/d/e^2*x)
/(e*x^2+d)^2+3/8/(d*e)^(1/2)*a/d^2*arctan(1/(d*e)^(1/2)*e*x)+1/8/d/e/(d*e)^
(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b+3/8/(d*e)^(1/2)*c/e^2*arctan(1/(d*e)^(1/2
)*e*x)
```

maxima [A] time = 2.25, size = 121, normalized size = 1.05

$$-\frac{\left(5\,cd^2e-bde^2-3\,ae^3\right)x^3+\left(3\,cd^3+bd^2e-5\,ade^2\right)x}{8\left(d^2e^4x^4+2\,d^3e^3x^2+d^4e^2\right)}+\frac{\left(3\,cd^2+bde+3\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*c*d^2*e - b*d*e^2 - 3*a*e^3)*x^3 + (3*c*d^3 + b*d^2*e - 5*a*d*e^2)
*x)/(d^2*e^4*x^4 + 2*d^3*e^3*x^2 + d^4*e^2) + 1/8*(3*c*d^2 + b*d*e + 3*a*e^
2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^2)
```

mupad [B] time = 4.85, size = 112, normalized size = 0.97

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}}-\frac{\frac{x\left(3\,c\,d^2+b\,d\,e-5\,a\,e^2\right)}{8\,d\,e^2}-\frac{x^3\left(-5\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^2\,e}}{d^2+2\,d\,e\,x^2+e^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^3,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 3*c*d^2 + b*d*e))/(8*d^(5/2)*e^(5/2))
 - ((x*(3*c*d^2 - 5*a*e^2 + b*d*e))/(8*d*e^2) - (x^3*(3*a*e^2 - 5*c*d^2 + b
*d*e))/(8*d^2*e))/(d^2 + e^2*x^4 + 2*d*e*x^2)
```

sympy [A] time = 2.27, size = 196, normalized size = 1.70

$$-\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2+bde+3cd^2\right)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2+bde+3cd^2\right)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{x^3(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1/
(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*l
og(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*
c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e*
*3*x**2 + 8*d**2*e**4*x**4)
```

3.251 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx$

Optimal. Leaf size=150

$$-\frac{x\left(7cd^2-e(5ae+bd)\right)}{24d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(5ae+bd)+cd^2\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(e(5ae+bd)+cd^2\right)}{16d^3e^2\left(d+ex^2\right)}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{6d\left(d+ex^2\right)^3}$$

```
[Out] 1/6*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^3-1/24*(7*c*d^2-e*(5*a*e+b*d))*x/d^2
/e^2/(e*x^2+d)^2+1/16*(c*d^2+e*(5*a*e+b*d))*x/d^3/e^2/(e*x^2+d)+1/16*(c*d^2
+e*(5*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(5/2)
```

Rubi [A] time = 0.21, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1157, 385, 199, 205}

$$\frac{x\left(e(5ae+bd)+cd^2\right)}{16d^3e^2\left(d+ex^2\right)}-\frac{x\left(7cd^2-e(5ae+bd)\right)}{24d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(5ae+bd)+cd^2\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{6d\left(d+ex^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(6*d*(d + e*x^2)^3) - ((7*c*d^2 - e*(b*d + 5*
a*e))*x)/(24*d^2*e^2*(d + e*x^2)^2) + ((c*d^2 + e*(b*d + 5*a*e))*x)/(16*d^3
*e^2*(d + e*x^2)) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(16*d^(7/2)*e^(5/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
```

```
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^4}dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\left(d+ex^2\right)^3} - \frac{\int \frac{-5a+\frac{d(cd-be)}{e^2}-\frac{6cdx^2}{e}}{\left(d+ex^2\right)^3}dx}{6d}$$
$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\left(d+ex^2\right)^3} - \frac{\left(7cd^2-e(bd+5ae)\right)x}{24d^2e^2\left(d+ex^2\right)^2} + \frac{\left(cd^2+e(bd+5ae)\right)\int \frac{1}{\left(d+ex^2\right)^2}dx}{8d^2e^2}$$
$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\left(d+ex^2\right)^3} - \frac{\left(7cd^2-e(bd+5ae)\right)x}{24d^2e^2\left(d+ex^2\right)^2} + \frac{\left(cd^2+e(bd+5ae)\right)x}{16d^3e^2\left(d+ex^2\right)} + \frac{\left(cd^2+e(bd+5ae)\right)\int \frac{1}{d}}{16d^3e^2}$$
$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\left(d+ex^2\right)^3} - \frac{\left(7cd^2-e(bd+5ae)\right)x}{24d^2e^2\left(d+ex^2\right)^2} + \frac{\left(cd^2+e(bd+5ae)\right)x}{16d^3e^2\left(d+ex^2\right)} + \frac{\left(cd^2+e(bd+5ae)\right)\tan}{16d^{7/2}e^{5/2}}$$

Mathematica [A] time = 0.13, size = 142, normalized size = 0.95

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(5ae+bd)+cd^2\right)}{16d^{7/2}e^{5/2}} + \frac{x\left(e\left(ae\left(33d^2+40dex^2+15e^2x^4\right)+bd\left(-3d^2+8dex^2+3e^2x^4\right)\right)+cd^2\left(-3d^2-\right.\right.}{48d^3e^2\left(d+ex^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + e*(b*d*(-3*d^2 + 8*d*e*x^2 + 3
*e^2*x^4) + a*e*(33*d^2 + 40*d*e*x^2 + 15*e^2*x^4))))/(48*d^3*e^2*(d + e*x^
2)^3) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)
*e^(5/2))
```

fricas [A] time = 0.57, size = 530, normalized size = 3.53

$$\left[\frac{6\left(cd^3e^3+bd^2e^4+5ade^5\right)x^5-16\left(cd^4e^2-bd^3e^3-5ad^2e^4\right)x^3-3\left(\left(cd^2e^3+bde^4+5ae^5\right)x^6+cd^5+bd^4e+5ad^3\right.}{96\left(d^4e^6x^6+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^5 - 16*(c*d^4*e^2 - b*d^3*e^
3 - 5*a*d^2*e^4)*x^3 - 3*((c*d^2*e^3 + b*d*e^4 + 5*a*e^5)*x^6 + c*d^5 + b*d
^4*e + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e
 + b*d^3*e^2 + 5*a*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d
)/(e*x^2 + d)) - 6*(c*d^5*e + b*d^4*e^2 - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6 + 3
*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3), 1/48*(3*(c*d^3*e^3 + b*d^2*e^4 + 5
*a*d*e^5)*x^5 - 8*(c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^3 + 3*((c*d^2*e^3
 + b*d*e^4 + 5*a*e^5)*x^6 + c*d^5 + b*d^4*e + 5*a*d^3*e^2 + 3*(c*d^3*e^2 +
b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + b*d^3*e^2 + 5*a*d^2*e^3)*x^2)*sqr
t(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*d^5*e + b*d^4*e^2 - 11*a*d^3*e^3)*x)/(d
^4*e^6*x^6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]
```

giac [A] time = 0.16, size = 134, normalized size = 0.89

$$\frac{\left(cd^2+bde+5\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{16\,d^{\frac{7}{2}}}+\frac{\left(3\,cd^2x^5e^2+3\,bdx^5e^3-8\,cd^3x^3e+15\,ax^5e^4+8\,bd^2x^3e^2-3\,cd^4x+40\right.}{48\left(x^2e+d\right)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] 1/16*(c*d^2 + b*d*e + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(7/2) +
 1/48*(3*c*d^2*x^5*e^2 + 3*b*d*x^5*e^3 - 8*c*d^3*x^3*e + 15*a*x^5*e^4 + 8*b
*d^2*x^3*e^2 - 3*c*d^4*x + 40*a*d*x^3*e^3 - 3*b*d^3*x*e + 33*a*d^2*x*e^2)*e
^(-2)/((x^2*e + d)^3*d^3)
```

maple [A] time = 0.01, size = 158, normalized size = 1.05

$$\frac{5a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de}\,d^3}+\frac{b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de}\,d^2e}+\frac{c\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de}\,d\,e^2}+\frac{\frac{\left(5a\,e^2+bde+c\,d^2\right)x^5}{16d^3}+\frac{\left(5a\,e^2+bde-c\,d^2\right)x^3}{6d^2e}+\frac{\left(11a\,e^2-bde-c\,d^2\right)x}{16d\,e^2}}{\left(e\,x^2+d\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^4,x)
```

```
[Out] (1/16*(5*a*e^2+b*d*e+c*d^2)/d^3*x^5+1/6*(5*a*e^2+b*d*e-c*d^2)/d^2/e*x^3+1/1
6*(11*a*e^2-b*d*e-c*d^2)/d/e^2*x)/(e*x^2+d)^3+5/16/(d*e)^(1/2)*a/d^3*arctan
(1/(d*e)^(1/2)*e*x)+1/16/d^2/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b+1/16
/(d*e)^(1/2)*c/d/e^2*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.51, size = 162, normalized size = 1.08

$$\frac{3\left(cd^2e^2+bde^3+5\,ae^4\right)x^5-8\left(cd^3e-bd^2e^2-5\,ade^3\right)x^3-3\left(cd^4+bd^3e-11\,ad^2e^2\right)x}{48\left(d^3e^5x^6+3\,d^4e^4x^4+3\,d^5e^3x^2+d^6e^2\right)}+\frac{\left(cd^2+bde+5\,ae^2\right)\mathrm{arct}}{16\,\sqrt{de}\,d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^5 - 8*(c*d^3*e - b*d^2*e^2 - 5*a*
d*e^3)*x^3 - 3*(c*d^4 + b*d^3*e - 11*a*d^2*e^2)*x)/(d^3*e^5*x^6 + 3*d^4*e^4
*x^4 + 3*d^5*e^3*x^2 + d^6*e^2) + 1/16*(c*d^2 + b*d*e + 5*a*e^2)*arctan(e*x
/sqrt(d*e))/(sqrt(d*e)*d^3*e^2)
```

mupad [B] time = 4.51, size = 144, normalized size = 0.96

$$\frac{\frac{x^5\left(c\,d^2+b\,d\,e+5\,a\,e^2\right)}{16\,d^3}-\frac{x\left(c\,d^2+b\,d\,e-11\,a\,e^2\right)}{16\,d\,e^2}+\frac{x^3\left(-c\,d^2+b\,d\,e+5\,a\,e^2\right)}{6\,d^2\,e}}{d^3+3\,d^2\,e\,x^2+3\,d\,e^2\,x^4+e^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(c\,d^2+b\,d\,e+5\,a\,e^2\right)}{16\,d^{7/2}\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^4,x)
```

```
[Out] ((x^5*(5*a*e^2 + c*d^2 + b*d*e))/(16*d^3) - (x*(c*d^2 - 11*a*e^2 + b*d*e))/
(16*d*e^2) + (x^3*(5*a*e^2 - c*d^2 + b*d*e))/(6*d^2*e))/(d^3 + e^3*x^6 + 3*
d^2*e*x^2 + 3*d*e^2*x^4) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2 + b*
d*e))/(16*d^(7/2)*e^(5/2))
```

sympy [A] time = 4.41, size = 241, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+bde+cd^2\right)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+bde+cd^2\right)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{x^5\left(15a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**4,x)
```

```
[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(-d**4*e**2*sqrt(-1/(d
**7*e**5)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(d
**4*e**2*sqrt(-1/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*b*d*e**3 + 3*c
*d**2*e**2) + x**3*(40*a*d*e**3 + 8*b*d**2*e**2 - 8*c*d**3*e) + x*(33*a*d**
2*e**2 - 3*b*d**3*e - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*e**3*x**2 + 144*d
**4*e**4*x**4 + 48*d**3*e**5*x**6)
```

3.252 $\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=223

$$\frac{1}{7}x^7\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3x+\frac{1}{11}ex^{11}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{1}{5}dx^5\left(6abde\ -\right.$$

```
[Out] a^2*d^3*x+1/3*a*d^2*(3*a*e+2*b*d)*x^3+1/5*d*(b^2*d^2+6*a*b*d*e+a*(3*a*e^2+2
*c*d^2))*x^5+1/7*(a^2*e^3+6*a*b*d*e^2+6*a*c*d^2*e+3*b^2*d^2*e+2*b*c*d^3)*x^
7+1/9*(c^2*d^3+6*c*d*e*(a*e+b*d)+b*e^2*(2*a*e+3*b*d))*x^9+1/11*e*(3*c^2*d^2
+b^2*e^2+2*c*e*(a*e+3*b*d))*x^11+1/13*c*e^2*(2*b*e+3*c*d)*x^13+1/15*c^2*e^3
*x^15
```

Rubi [A] time = 0.20, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1153}

$$\frac{1}{7}x^7\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3x+\frac{1}{11}ex^{11}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{1}{5}dx^5\left(6abde\ -\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*
c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d +
2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 +
(c*e^2*(3*c*d + 2*b*e)*x^13)/13 + (c^2*e^3*x^15)/15
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^3\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2d^3+ad^2(2bd+3ae)x^2+d\left(b^2d^2+6abde+a\left(2cd^2+3ae^2\right)\right)x^4+\left(2\right.\right.\\ &= a^2d^3x+\frac{1}{3}ad^2(2bd+3ae)x^3+\frac{1}{5}d\left(b^2d^2+6abde+a\left(2cd^2+3ae^2\right)\right)x^5+\frac{1}{7}\end{aligned}$$

Mathematica [A] time = 0.09, size = 223, normalized size = 1.00

$$\frac{1}{7}x^7\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3x+\frac{1}{11}ex^{11}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{1}{5}dx^5\left(6abde\ -\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*
c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d +
2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 +
(c*e^2*(3*c*d + 2*b*e)*x^13)/13 + (c^2*e^3*x^15)/15
```

fricas [A] time = 0.44, size = 261, normalized size = 1.17

$$\frac{1}{15}x^{15}e^3c^2+\frac{3}{13}x^{13}e^2dc^2+\frac{2}{13}x^{13}e^3cb+\frac{3}{11}x^{11}ed^2c^2+\frac{6}{11}x^{11}e^2dcb+\frac{1}{11}x^{11}e^3b^2+\frac{2}{11}x^{11}e^3ca+\frac{1}{9}x^9d^3c^2+\frac{2}{3}x^9ed^2cb+\frac{1}{3}x^9e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*e^3*c^2 + 3/13*x^13*e^2*d*c^2 + 2/13*x^13*e^3*c*b + 3/11*x^11*e*d
^2*c^2 + 6/11*x^11*e^2*d*c*b + 1/11*x^11*e^3*b^2 + 2/11*x^11*e^3*c*a + 1/9*
x^9*d^3*c^2 + 2/3*x^9*e*d^2*c*b + 1/3*x^9*e^2*d*b^2 + 2/3*x^9*e^2*d*c*a + 2
/9*x^9*e^3*b*a + 2/7*x^7*d^3*c*b + 3/7*x^7*e*d^2*b^2 + 6/7*x^7*e*d^2*c*a +
6/7*x^7*e^2*d*b*a + 1/7*x^7*e^3*a^2 + 1/5*x^5*d^3*b^2 + 2/5*x^5*d^3*c*a + 6
/5*x^5*e*d^2*b*a + 3/5*x^5*e^2*d*a^2 + 2/3*x^3*d^3*b*a + x^3*e*d^2*a^2 + x*
d^3*a^2
```

giac [A] time = 0.16, size = 255, normalized size = 1.14

$$\frac{1}{15}c^2x^{15}e^3+\frac{3}{13}c^2dx^{13}e^2+\frac{2}{13}bcx^{13}e^3+\frac{3}{11}c^2d^2x^{11}e+\frac{6}{11}bcdx^{11}e^2+\frac{1}{9}c^2d^3x^9+\frac{1}{11}b^2x^{11}e^3+\frac{2}{11}acx^{11}e^3+\frac{2}{3}bcd^2x^9e+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*c^2*x^15*e^3 + 3/13*c^2*d*x^13*e^2 + 2/13*b*c*x^13*e^3 + 3/11*c^2*d^2*
x^11*e + 6/11*b*c*d*x^11*e^2 + 1/9*c^2*d^3*x^9 + 1/11*b^2*x^11*e^3 + 2/11*a
*c*x^11*e^3 + 2/3*b*c*d^2*x^9*e + 1/3*b^2*d*x^9*e^2 + 2/3*a*c*d*x^9*e^2 + 2
/7*b*c*d^3*x^7 + 2/9*a*b*x^9*e^3 + 3/7*b^2*d^2*x^7*e + 6/7*a*c*d^2*x^7*e +
6/7*a*b*d*x^7*e^2 + 1/5*b^2*d^3*x^5 + 2/5*a*c*d^3*x^5 + 1/7*a^2*x^7*e^3 + 6
/5*a*b*d^2*x^5*e + 3/5*a^2*d*x^5*e^2 + 2/3*a*b*d^3*x^3 + a^2*d^2*x^3*e + a^
2*d^3*x
```

maple [A] time = 0.00, size = 219, normalized size = 0.98

$$\frac{c^2e^3x^{15}}{15}+\frac{\left(2e^3bc+3d\,e^2c^2\right)x^{13}}{13}+\frac{\left(6bcd\,e^2+3c^2d^2e+\left(2ac+b^2\right)e^3\right)x^{11}}{11}+\frac{\left(2ab\,e^3+6bc\,d^2e+c^2d^3+3\left(2ac+b^2\right)d\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/15*c^2*e^3*x^15+1/13*(2*b*c*e^3+3*c^2*d*e^2)*x^13+1/11*(3*d^2*e*c^2+6*d*e
^2*b*c+e^3*(2*a*c+b^2))*x^11+1/9*(c^2*d^3+6*d^2*e*b*c+3*d*e^2*(2*a*c+b^2)+2
*e^3*a*b)*x^9+1/7*(2*b*c*d^3+3*d^2*e*(2*a*c+b^2)+6*a*b*d*e^2+a^2*e^3)*x^7+1
/5*(d^3*(2*a*c+b^2)+6*d^2*e*a*b+3*d*e^2*a^2)*x^5+1/3*(3*a^2*d^2*e+2*a*b*d^3
)*x^3+a^2*d^3*x
```

maxima [A] time = 1.04, size = 218, normalized size = 0.98

$$\frac{1}{15}c^2e^3x^{15}+\frac{1}{13}\left(3\,c^2de^2+2\,bce^3\right)x^{13}+\frac{1}{11}\left(3\,c^2d^2e+6\,bcde^2+\left(b^2+2\,ac\right)e^3\right)x^{11}+\frac{1}{9}\left(c^2d^3+6\,bcd^2e+2\,abe^3+3\left(b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*c^2*e^3*x^15 + 1/13*(3*c^2*d*e^2 + 2*b*c*e^3)*x^13 + 1/11*(3*c^2*d^2*e
 + 6*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^11 + 1/9*(c^2*d^3 + 6*b*c*d^2*e + 2*a
*b*e^3 + 3*(b^2 + 2*a*c)*d*e^2)*x^9 + 1/7*(2*b*c*d^3 + 6*a*b*d*e^2 + a^2*e^
3 + 3*(b^2 + 2*a*c)*d^2*e)*x^7 + a^2*d^3*x + 1/5*(6*a*b*d^2*e + 3*a^2*d*e^2
 + (b^2 + 2*a*c)*d^3)*x^5 + 1/3*(2*a*b*d^3 + 3*a^2*d^2*e)*x^3
```

mupad [B] time = 4.48, size = 220, normalized size = 0.99

$$x^7\left(\frac{a^2e^3}{7}+\frac{6abde^2}{7}+\frac{6cad^2e}{7}+\frac{3b^2d^2e}{7}+\frac{2cbd^3}{7}\right)+x^9\left(\frac{b^2de^2}{3}+\frac{2bcd^2e}{3}+\frac{2abe^3}{9}+\frac{c^2d^3}{9}+\frac{2acde^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^7*((a^2*e^3)/7 + (3*b^2*d^2*e)/7 + (2*b*c*d^3)/7 + (6*a*b*d*e^2)/7 + (6*a
*c*d^2*e)/7) + x^9*((c^2*d^3)/9 + (b^2*d*e^2)/3 + (2*a*b*e^3)/9 + (2*a*c*d*
e^2)/3 + (2*b*c*d^2*e)/3) + x^5*((b^2*d^3)/5 + (3*a^2*d*e^2)/5 + (2*a*c*d^3
)/5 + (6*a*b*d^2*e)/5) + x^11*((b^2*e^3)/11 + (3*c^2*d^2*e)/11 + (2*a*c*e^3
)/11 + (6*b*c*d*e^2)/11) + a^2*d^3*x + (c^2*e^3*x^15)/15 + (a*d^2*x^3*(3*a*
e + 2*b*d))/3 + (c*e^2*x^13*(2*b*e + 3*c*d))/13
```

sympy [A] time = 0.22, size = 272, normalized size = 1.22

$$a^2d^3x+\frac{c^2e^3x^{15}}{15}+x^{13}\left(\frac{2bce^3}{13}+\frac{3c^2de^2}{13}\right)+x^{11}\left(\frac{2ace^3}{11}+\frac{b^2e^3}{11}+\frac{6bcde^2}{11}+\frac{3c^2d^2e}{11}\right)+x^9\left(\frac{2abe^3}{9}+\frac{2acde^2}{3}+\frac{b^2de^2}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d**3*x + c**2*e**3*x**15/15 + x**13*(2*b*c*e**3/13 + 3*c**2*d*e**2/13)
 + x**11*(2*a*c*e**3/11 + b**2*e**3/11 + 6*b*c*d*e**2/11 + 3*c**2*d**2*e/11
) + x**9*(2*a*b*e**3/9 + 2*a*c*d*e**2/3 + b**2*d*e**2/3 + 2*b*c*d**2*e/3 +
c**2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*b*d*e**2/7 + 6*a*c*d**2*e/7 + 3*b**2
*d**2*e/7 + 2*b*c*d**3/7) + x**5*(3*a**2*d*e**2/5 + 6*a*b*d**2*e/5 + 2*a*c*
d**3/5 + b**2*d**3/5) + x**3*(a**2*d**2*e + 2*a*b*d**3/3)
```

3.253 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=155

$$a^2d^2x+\frac{1}{9}x^9\left(2ce(ae+2bd)+b^2e^2+c^2d^2\right)+\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right)+\frac{1}{5}x^5\left(4abde+a\left(ae^2+2cd^2\right)+b^2d^2\right.$$

```
[Out] a^2*d^2*x+2/3*a*d*(a*e+b*d)*x^3+1/5*(b^2*d^2+4*a*b*d*e+a*(a*e^2+2*c*d^2))*x
^5+2/7*(a*b*e^2+2*a*c*d*e+b^2*d*e+b*c*d^2)*x^7+1/9*(c^2*d^2+b^2*e^2+2*c*e*(
a*e+2*b*d))*x^9+2/11*c*e*(b*e+c*d)*x^11+1/13*c^2*e^2*x^13
```

Rubi [A] time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1153}

$$a^2d^2x+\frac{1}{9}x^9\left(2ce(ae+2bd)+b^2e^2+c^2d^2\right)+\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right)+\frac{1}{5}x^5\left(4abde+a\left(ae^2+2cd^2\right)+b^2d^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 4*a*b*d*e + a*(2*c*d^2
+ a*e^2))*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((
c^2*d^2 + b^2*e^2 + 2*c*e*(2*b*d + a*e))*x^9)/9 + (2*c*e*(c*d + b*e)*x^11)/
11 + (c^2*e^2*x^13)/13
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2d^2+2ad(bd+ae)x^2+\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)x^4+2\left(bcd^2+\right.\right.\\ &= a^2d^2x+\frac{2}{3}ad(bd+ae)x^3+\frac{1}{5}\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)x^5+\frac{2}{7}\left(bcd^2+\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 156, normalized size = 1.01

$$\frac{1}{5}x^5\left(a^2e^2+4abde+2acd^2+b^2d^2\right)+a^2d^2x+\frac{1}{9}x^9\left(2ace^2+b^2e^2+4bcde+c^2d^2\right)+\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 2*a*c*d^2 + 4*a*b*d*e +
 a^2*e^2)*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((
c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 2*a*c*e^2)*x^9)/9 + (2*c*e*(c*d + b*e)*x^11
)/11 + (c^2*e^2*x^13)/13
```

fricas [A] time = 0.50, size = 181, normalized size = 1.17

$$\frac{1}{13}x^{13}e^2c^2+\frac{2}{11}x^{11}edc^2+\frac{2}{11}x^{11}e^2cb+\frac{1}{9}x^9d^2c^2+\frac{4}{9}x^9edcb+\frac{1}{9}x^9e^2b^2+\frac{2}{9}x^9e^2ca+\frac{2}{7}x^7d^2cb+\frac{2}{7}x^7edb^2+\frac{4}{7}x^7edca+\frac{2}{7}x^7e^2bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^2*c^2 + 2/11*x^11*e*d*c^2 + 2/11*x^11*e^2*c*b + 1/9*x^9*d^2*c^2
 + 4/9*x^9*e*d*c*b + 1/9*x^9*e^2*b^2 + 2/9*x^9*e^2*c*a + 2/7*x^7*d^2*c*b +
2/7*x^7*e*d*b^2 + 4/7*x^7*e*d*c*a + 2/7*x^7*e^2*b*a + 1/5*x^5*d^2*b^2 + 2/5
*x^5*d^2*c*a + 4/5*x^5*e*d*b*a + 1/5*x^5*e^2*a^2 + 2/3*x^3*d^2*b*a + 2/3*x^
3*e*d*a^2 + x*d^2*a^2
```

giac [A] time = 0.17, size = 181, normalized size = 1.17

$$\frac{1}{13}c^2x^{13}e^2+\frac{2}{11}c^2dx^{11}e+\frac{2}{11}bcx^{11}e^2+\frac{1}{9}c^2d^2x^9+\frac{4}{9}bcdx^9e+\frac{1}{9}b^2x^9e^2+\frac{2}{9}acx^9e^2+\frac{2}{7}bcd^2x^7+\frac{2}{7}b^2dx^7e+\frac{4}{7}acdx^7e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13*e^2 + 2/11*c^2*d*x^11*e + 2/11*b*c*x^11*e^2 + 1/9*c^2*d^2*x^9
 + 4/9*b*c*d*x^9*e + 1/9*b^2*x^9*e^2 + 2/9*a*c*x^9*e^2 + 2/7*b*c*d^2*x^7 +
2/7*b^2*d*x^7*e + 4/7*a*c*d*x^7*e + 2/7*a*b*x^7*e^2 + 1/5*b^2*d^2*x^5 + 2/5
*a*c*d^2*x^5 + 4/5*a*b*d*x^5*e + 1/5*a^2*x^5*e^2 + 2/3*a*b*d^2*x^3 + 2/3*a^
2*d*x^3*e + a^2*d^2*x
```

maple [A] time = 0.00, size = 155, normalized size = 1.00

$$\frac{c^2e^2x^{13}}{13}+\frac{\left(2bce^2+2c^2de\right)x^{11}}{11}+\frac{\left(4bcde+c^2d^2+\left(2ac+b^2\right)e^2\right)x^9}{9}+\frac{\left(2abe^2+2bcd^2+2\left(2ac+b^2\right)de\right)x^7}{7}+a^2d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/13*c^2*e^2*x^13+1/11*(2*b*c*e^2+2*c^2*d*e)*x^11+1/9*(c^2*d^2+4*d*e*b*c+e^
2*(2*a*c+b^2))*x^9+1/7*(2*b*c*d^2+2*d*e*(2*a*c+b^2)+2*a*b*e^2)*x^7+1/5*(d^2
*(2*a*c+b^2)+4*a*b*d*e+e^2*a^2)*x^5+1/3*(2*a^2*d*e+2*a*b*d^2)*x^3+a^2*d^2*x
```

maxima [A] time = 1.14, size = 147, normalized size = 0.95

$$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}\left(c^2de+bce^2\right)x^{11}+\frac{1}{9}\left(c^2d^2+4\,bcde+\left(b^2+2\,ac\right)e^2\right)x^9+\frac{2}{7}\left(bcd^2+abe^2+\left(b^2+2\,ac\right)de\right)x^7+\frac{1}{5}(4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*(c^2*d*e + b*c*e^2)*x^11 + 1/9*(c^2*d^2 + 4*b*c*d*
e + (b^2 + 2*a*c)*e^2)*x^9 + 2/7*(b*c*d^2 + a*b*e^2 + (b^2 + 2*a*c)*d*e)*x^
7 + 1/5*(4*a*b*d*e + a^2*e^2 + (b^2 + 2*a*c)*d^2)*x^5 + a^2*d^2*x + 2/3*(a*
b*d^2 + a^2*d*e)*x^3
```

mupad [B] time = 4.52, size = 148, normalized size = 0.95

$$x^5\left(\frac{a^2e^2}{5}+\frac{4\,a\,b\,d\,e}{5}+\frac{2\,c\,a\,d^2}{5}+\frac{b^2d^2}{5}\right)+x^9\left(\frac{b^2e^2}{9}+\frac{4\,b\,c\,d\,e}{9}+\frac{c^2d^2}{9}+\frac{2\,a\,c\,e^2}{9}\right)+x^7\left(\frac{2\,b^2\,d\,e}{7}+\frac{2\,c\,b\,d^2}{7}+\frac{2\,a}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5*((a^2*e^2)/5 + (b^2*d^2)/5 + (2*a*c*d^2)/5 + (4*a*b*d*e)/5) + x^9*((b^2
*e^2)/9 + (c^2*d^2)/9 + (2*a*c*e^2)/9 + (4*b*c*d*e)/9) + x^7*((2*a*b*e^2)/7
```

```
 + (2*b*c*d^2)/7 + (2*b^2*d*e)/7 + (4*a*c*d*e)/7) + a^2*d^2*x + (c^2*e^2*x^
13)/13 + (2*a*d*x^3*(a*e + b*d))/3 + (2*c*e*x^11*(b*e + c*d))/11
```

sympy [A] time = 0.16, size = 192, normalized size = 1.24

$$a^2d^2x+\frac{c^2e^2x^{13}}{13}+x^{11}\left(\frac{2bce^2}{11}+\frac{2c^2de}{11}\right)+x^9\left(\frac{2ace^2}{9}+\frac{b^2e^2}{9}+\frac{4bcde}{9}+\frac{c^2d^2}{9}\right)+x^7\left(\frac{2abe^2}{7}+\frac{4acde}{7}+\frac{2b^2de}{7}+\frac{2bcd^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d**2*x + c**2*e**2*x**13/13 + x**11*(2*b*c*e**2/11 + 2*c**2*d*e/11) +
x**9*(2*a*c*e**2/9 + b**2*e**2/9 + 4*b*c*d*e/9 + c**2*d**2/9) + x**7*(2*a*b
*e**2/7 + 4*a*c*d*e/7 + 2*b**2*d*e/7 + 2*b*c*d**2/7) + x**5*(a**2*e**2/5 +
4*a*b*d*e/5 + 2*a*c*d**2/5 + b**2*d**2/5) + x**3*(2*a**2*d*e/3 + 2*a*b*d**2
/3)
```

3.254 $\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=96

$$a^2dx+\frac{1}{7}x^7\left(2ace+b^2e+2bcd\right)+\frac{1}{5}x^5\left(2abe+2acd+b^2d\right)+\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

```
[Out] a^2*d*x+1/3*a*(a*e+2*b*d)*x^3+1/5*(2*a*b*e+2*a*c*d+b^2*d)*x^5+1/7*(2*a*c*e+
b^2*e+2*b*c*d)*x^7+1/9*c*(2*b*e+c*d)*x^9+1/11*c^2*e*x^11
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$a^2dx+\frac{1}{7}x^7\left(2ace+b^2e+2bcd\right)+\frac{1}{5}x^5\left(2abe+2acd+b^2d\right)+\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^11)
/11
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\int \left(a^2d+a(2bd+ae)x^2+\left(b^2d+2acd+2abe\right)x^4+\left(2bcd+b^2e+2ace\right)x^6$$
$$=a^2dx+\frac{1}{3}a(2bd+ae)x^3+\frac{1}{5}\left(b^2d+2acd+2abe\right)x^5+\frac{1}{7}\left(2bcd+b^2e+2ace\right.$$

Mathematica [A] time = 0.02, size = 96, normalized size = 1.00

$$a^2dx+\frac{1}{7}x^7\left(2ace+b^2e+2bcd\right)+\frac{1}{5}x^5\left(2abe+2acd+b^2d\right)+\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^11)
/11
```

fricas [A] time = 0.53, size = 100, normalized size = 1.04

$$\frac{1}{11}x^{11}ec^2+\frac{1}{9}x^9dc^2+\frac{2}{9}x^9ecb+\frac{2}{7}x^7dcb+\frac{1}{7}x^7eb^2+\frac{2}{7}x^7eca+\frac{1}{5}x^5db^2+\frac{2}{5}x^5dca+\frac{2}{5}x^5eba+\frac{2}{3}x^3dba+\frac{1}{3}x^3ea^2+xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e*c^2 + 1/9*x^9*d*c^2 + 2/9*x^9*e*c*b + 2/7*x^7*d*c*b + 1/7*x^7*e
*b^2 + 2/7*x^7*e*c*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a + 2/5*x^5*e*b*a + 2/3*
x^3*d*b*a + 1/3*x^3*e*a^2 + x*d*a^2
```

giac [A] time = 0.17, size = 106, normalized size = 1.10

$$\frac{1}{11}c^2x^{11}e+\frac{1}{9}c^2dx^9+\frac{2}{9}bcx^9e+\frac{2}{7}bcdx^7+\frac{1}{7}b^2x^7e+\frac{2}{7}acx^7e+\frac{1}{5}b^2dx^5+\frac{2}{5}acdx^5+\frac{2}{5}abx^5e+\frac{2}{3}abdx^3+\frac{1}{3}a^2x^3e+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11*e + 1/9*c^2*d*x^9 + 2/9*b*c*x^9*e + 2/7*b*c*d*x^7 + 1/7*b^2*x
^7*e + 2/7*a*c*x^7*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*x^5*e + 2/3*
a*b*d*x^3 + 1/3*a^2*x^3*e + a^2*d*x
```

maple [A] time = 0.00, size = 91, normalized size = 0.95

$$\frac{c^2e\,x^{11}}{11}+\frac{\left(2ebc+d\,c^2\right)x^9}{9}+\frac{\left(2bcd+\left(2ac+b^2\right)e\right)x^7}{7}+\frac{\left(2abe+\left(2ac+b^2\right)d\right)x^5}{5}+a^2dx+\frac{\left(e\,a^2+2dab\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/11*c^2*e*x^11+1/9*(2*b*c*e+c^2*d)*x^9+1/7*(2*b*c*d+e*(2*a*c+b^2))*x^7+1/5
*(d*(2*a*c+b^2)+2*a*b*e)*x^5+1/3*(a^2*e+2*a*b*d)*x^3+a^2*d*x
```

maxima [A] time = 1.01, size = 90, normalized size = 0.94

$$\frac{1}{11}c^2ex^{11}+\frac{1}{9}\left(c^2d+2\,bce\right)x^9+\frac{1}{7}\left(2\,bcd+\left(b^2+2\,ac\right)e\right)x^7+\frac{1}{5}\left(2\,abe+\left(b^2+2\,ac\right)d\right)x^5+a^2dx+\frac{1}{3}\left(2\,abd+a^2e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*(c^2*d + 2*b*c*e)*x^9 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*
e)*x^7 + 1/5*(2*a*b*e + (b^2 + 2*a*c)*d)*x^5 + a^2*d*x + 1/3*(2*a*b*d + a^2
*e)*x^3
```

mupad [B] time = 0.04, size = 90, normalized size = 0.94

$$x^5\left(\frac{d\,b^2}{5}+\frac{2\,a\,e\,b}{5}+\frac{2\,a\,c\,d}{5}\right)+x^7\left(\frac{e\,b^2}{7}+\frac{2\,c\,d\,b}{7}+\frac{2\,a\,c\,e}{7}\right)+x^3\left(\frac{e\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)+x^9\left(\frac{d\,c^2}{9}+\frac{2\,b\,e\,c}{9}\right)+\frac{c^2\,e\,x^{11}}{11}+a^2\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5*((b^2*d)/5 + (2*a*b*e)/5 + (2*a*c*d)/5) + x^7*((b^2*e)/7 + (2*a*c*e)/7
+ (2*b*c*d)/7) + x^3*((a^2*e)/3 + (2*a*b*d)/3) + x^9*((c^2*d)/9 + (2*b*c*e)
/9) + (c^2*e*x^11)/11 + a^2*d*x
```

sympy [A] time = 0.25, size = 107, normalized size = 1.11

$$a^2dx+\frac{c^2ex^{11}}{11}+x^9\left(\frac{2bce}{9}+\frac{c^2d}{9}\right)+x^7\left(\frac{2ace}{7}+\frac{b^2e}{7}+\frac{2bcd}{7}\right)+x^5\left(\frac{2abe}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right)+x^3\left(\frac{a^2e}{3}+\frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + c**2*e*x**11/11 + x**9*(2*b*c*e/9 + c**2*d/9) + x**7*(2*a*c*e/7
+ b**2*e/7 + 2*b*c*d/7) + x**5*(2*a*b*e/5 + 2*a*c*d/5 + b**2*d/5) + x**3*(a
**2*e/3 + 2*a*b*d/3)
```

3.255 $\int \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=49

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1090}

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2+2abx^2+b^2\left(1+\frac{2ac}{b^2}\right)x^4+2bcx^6+c^2x^8\right) dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

fricas [A] time = 0.47, size = 43, normalized size = 0.88

$$\frac{1}{9}x^9c^2+\frac{2}{7}x^7cb+\frac{1}{5}x^5b^2+\frac{2}{5}x^5ca+\frac{2}{3}x^3ba+xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/7*x^7*c*b + 1/5*x^5*b^2 + 2/5*x^5*c*a + 2/3*x^3*b*a + x*a^2
```

giac [A] time = 0.14, size = 43, normalized size = 0.88

$$\frac{1}{9}c^2x^9+\frac{2}{7}bcx^7+\frac{1}{5}b^2x^5+\frac{2}{5}acx^5+\frac{2}{3}abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A] time = 0.00, size = 42, normalized size = 0.86

$$\frac{c^{2} x^{9}}{9}+\frac{2 b c \,x^{7}}{7}+\frac{2 a b \,x^{3}}{3}+\frac{\left(2 a c +b^{2}\right) x^{5}}{5}+a^{2} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/9*c^2*x^9+2/7*b*c*x^7+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+a^2*x
```

maxima [A] time = 1.10, size = 45, normalized size = 0.92

$$\frac{1}{9} c^{2} x^{9} + \frac{2}{7} b c x^{7} + \frac{1}{5} b^{2} x^{5} + a^{2} x + \frac{2}{15} \left(3 c x^{5} + 5 b x^{3}\right) a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)*
a
```

mupad [B] time = 0.02, size = 42, normalized size = 0.86

$$a^2\,x+x^5\,\left(\frac{b^2}{5}+\frac{2\,a\,c}{5}\right)+\frac{c^2\,x^9}{9}+\frac{2\,a\,b\,x^3}{3}+\frac{2\,b\,c\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2,x)
```

```
[Out] a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7)
/7
```

sympy [A] time = 0.15, size = 48, normalized size = 0.98

$$a^{2} x + \frac{2 a b x^{3}}{3} + \frac{2 b c x^{7}}{7} + \frac{c^{2} x^{9}}{9} + x^{5} \left(\frac{2 a c}{5} + \frac{b^{2}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

3.256 $\int \frac{\left(a+bx^2+cx^4\right)^2}{d+ex^2}\,dx$

Optimal. Leaf size=143

$$\frac{x^3\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{3e^3}-\frac{x(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)^2}{\sqrt{d}\,e^{9/2}}-\frac{cx^5(cd-2b}{5e^2}$$

```
[Out] -(-b*e+c*d)*(c*d^2-e*(-2*a*e+b*d))*x/e^4+1/3*(c^2*d^2+b^2*e^2-2*c*e*(-a*e+b
*d))*x^3/e^3-1/5*c*(-2*b*e+c*d)*x^5/e^2+1/7*c^2*x^7/e+(a*e^2-b*d*e+c*d^2)^2
*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1153, 205}

$$\frac{x^3\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{3e^3}-\frac{x(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)^2}{\sqrt{d}\,e^{9/2}}-\frac{cx^5(cd-2b}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] -(((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^2*e^2 - 2*
c*e*(b*d - a*e))*x^3)/(3*e^3) - (c*(c*d - 2*b*e)*x^5)/(5*e^2) + (c^2*x^7)/(
7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(
9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{d+ex^2}\,dx=\int \left(-\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}-\frac{c(cd-2be)x^4}{e^2}\right.$$

$$=-\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)x}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^3}{3e^3}-\frac{c(cd-2be)x^5}{5e^2}+$$

$$=-\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)x}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^3}{3e^3}-\frac{c(cd-2be)x^5}{5e^2}+$$

Mathematica [A] time = 0.07, size = 144, normalized size = 1.01

$$\frac{x^3\left(2ace^2+b^2e^2-2bcde+c^2d^2\right)}{3e^3}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)^2}{\sqrt{d}\,e^{9/2}}+\frac{x(be-cd)\left(2ae^2-bde+cd^2\right)}{e^4}+\frac{cx^5(2be-c}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] ((-(c*d) + b*e)*(c*d^2 - b*d*e + 2*a*e^2)*x)/e^4 + ((c^2*d^2 - 2*b*c*d*e +
b^2*e^2 + 2*a*c*e^2)*x^3)/(3*e^3) + (c*(-(c*d) + 2*b*e)*x^5)/(5*e^2) + (c^2
*x^7)/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[d]*e^(9/2))
```

fricas [A] time = 0.69, size = 406, normalized size = 2.84

$$\left[\frac{30\,c^2de^4x^7 - 42\left(c^2d^2e^3 - 2\,bcde^4\right)x^5 + 70\left(c^2d^3e^2 - 2\,bcd^2e^3 + \left(b^2 + 2\,ac\right)de^4\right)x^3 - 105\left(c^2d^4 - 2\,bcd^3e - 2\,abde^3\right.}{210\,de^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/210*(30*c^2*d*e^4*x^7 - 42*(c^2*d^2*e^3 - 2*b*c*d*e^4)*x^5 + 70*(c^2*d^3
*e^2 - 2*b*c*d^2*e^3 + (b^2 + 2*a*c)*d*e^4)*x^3 - 105*(c^2*d^4 - 2*b*c*d^3*
e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*sqrt(-d*e)*log((e*x^2 -
2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 210*(c^2*d^4*e - 2*b*c*d^3*e^2 - 2*a*b*d
*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)/(d*e^5), 1/105*(15*c^2*d*e^4*x^7 - 21*(c^2
*d^2*e^3 - 2*b*c*d*e^4)*x^5 + 35*(c^2*d^3*e^2 - 2*b*c*d^2*e^3 + (b^2 + 2*a*
c)*d*e^4)*x^3 + 105*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 +
 2*a*c)*d^2*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 105*(c^2*d^4*e - 2*b*c*d
^3*e^2 - 2*a*b*d*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)/(d*e^5)]
```

giac [A] time = 0.16, size = 185, normalized size = 1.29

$$\frac{\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,abde^3 + a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} + \frac{1}{105}\left(15\,c^2x^7e^6 - 21\,c^2dx^5e^5 + 42\,bcx^5e^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^
4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt(d) + 1/105*(15*c^2*x^7*e^6 - 21*
c^2*d*x^5*e^5 + 42*b*c*x^5*e^6 + 35*c^2*d^2*x^3*e^4 - 70*b*c*d*x^3*e^5 - 10
5*c^2*d^3*x*e^3 + 35*b^2*x^3*e^6 + 70*a*c*x^3*e^6 + 210*b*c*d^2*x*e^4 - 105
*b^2*d*x*e^5 - 210*a*c*d*x*e^5 + 210*a*b*x*e^6)*e^(-7)
```

maple [B] time = 0.00, size = 267, normalized size = 1.87

$$\frac{c^2x^7}{7e} + \frac{2bc\,x^5}{5e} - \frac{c^2d\,x^5}{5e^2} + \frac{2ac\,x^3}{3e} + \frac{b^2x^3}{3e} - \frac{2bcd\,x^3}{3e^2} + \frac{c^2d^2x^3}{3e^3} + \frac{a^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - \frac{2abd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e} + \frac{2ac\,d^2\arctan\left(\frac{ex}{\sqrt{de}}\right.}{\sqrt{de}\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d),x)
```

```
[Out] 1/7*c^2/e*x^7+2/5/e*x^5*b*c-1/5*c^2*d/e^2*x^5+2/3*a*c/e*x^3+1/3/e*x^3*b^2-2
/3/e^2*x^3*b*c*d+1/3*c^2*d^2/e^3*x^3+2/e*a*b*x-2*a*c*d/e^2*x-1/e^2*b^2*d*x+
2/e^3*b*c*d^2*x-c^2*d^3/e^4*x+1/(d*e)^(1/2)*a^2*arctan(1/(d*e)^(1/2)*e*x)-2
/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a*b*d+2/(d*e)^(1/2)*a*c*d^2/e^2*ar
ctan(1/(d*e)^(1/2)*e*x)+1/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b^2*d^2
-2/e^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b*c*d^3+1/(d*e)^(1/2)*c^2*d^4/
e^4*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.38, size = 176, normalized size = 1.23

$$\frac{\left(c^2d^4-2\,bcd^3e-2\,abde^3+a^2e^4+\left(b^2+2\,ac\right)d^2e^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^4}+\frac{15\,c^2e^3x^7-21\left(c^2de^2-2\,bce^3\right)x^5+35\left(c^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*arc
tan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) + 1/105*(15*c^2*e^3*x^7 - 21*(c^2*d*e^2
- 2*b*c*e^3)*x^5 + 35*(c^2*d^2*e - 2*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^3 - 1
05*(c^2*d^3 - 2*b*c*d^2*e - 2*a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)/e^4
```

mupad [B] time = 4.47, size = 229, normalized size = 1.60

$$x^3\left(\frac{b^2+2\,a\,c}{3\,e}+\frac{d\left(\frac{c^2\,d}{e^2}-\frac{2\,b\,c}{e}\right)}{3\,e}\right)-x\left(\frac{d\left(\frac{b^2+2\,a\,c}{e}+\frac{d\left(\frac{c^2\,d}{e^2}-\frac{2\,b\,c}{e}\right)}{e}\right)}{e}-\frac{2\,a\,b}{e}\right)-x^5\left(\frac{c^2\,d}{5\,e^2}-\frac{2\,b\,c}{5\,e}\right)+\frac{c^2\,x^7}{7\,e}+\frac{\mathrm{atan}\left(\frac{}{\sqrt{d}\,\left(a^2\,e^{\cdot}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2),x)
```

```
[Out] x^3*((2*a*c + b^2)/(3*e) + (d*((c^2*d)/e^2 - (2*b*c)/e))/(3*e)) - x*((d*((2
*a*c + b^2)/e + (d*((c^2*d)/e^2 - (2*b*c)/e))/e))/e - (2*a*b)/e) - x^5*((c^
2*d)/(5*e^2) - (2*b*c)/(5*e)) + (c^2*x^7)/(7*e) + (atan((e^(1/2)*x*(a*e^2 +
 c*d^2 - b*d*e)^2)/(d^(1/2)*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3
- 2*b*c*d^3*e + 2*a*c*d^2*e^2)))*(a*e^2 + c*d^2 - b*d*e)^2)/(d^(1/2)*e^(9/2
))
```

sympy [B] time = 1.53, size = 371, normalized size = 2.59

$$\frac{c^2x^7}{7e}+x^5\left(\frac{2bc}{5e}-\frac{c^2d}{5e^2}\right)+x^3\left(\frac{2ac}{3e}+\frac{b^2}{3e}-\frac{2bcd}{3e^2}+\frac{c^2d^2}{3e^3}\right)+x\left(\frac{2ab}{e}-\frac{2acd}{e^2}-\frac{b^2d}{e^2}+\frac{2bcd^2}{e^3}-\frac{c^2d^3}{e^4}\right)-\frac{\sqrt{-\frac{1}{de^9}}\left(ae^2-b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d),x)
```

```
[Out] c**2*x**7/(7*e) + x**5*(2*b*c/(5*e) - c**2*d/(5*e**2)) + x**3*(2*a*c/(3*e)
+ b**2/(3*e) - 2*b*c*d/(3*e**2) + c**2*d**2/(3*e**3)) + x*(2*a*b/e - 2*a*c*
d/e**2 - b**2*d/e**2 + 2*b*c*d**2/e**3 - c**2*d**3/e**4) - sqrt(-1/(d*e**9)
)*(a*e**2 - b*d*e + c*d**2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 - b*d*
e + c*d**2)**2/(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2
 - 2*b*c*d**3*e + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c
*d**2)**2*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c*d**2)**2/(a**2*e
**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2
*d**4) + x)/2
```

3.257 $\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^2} dx$

Optimal. Leaf size=166

$$\frac{x\left(-2ce(2bd-ae)+b^2e^2+3c^2d^2\right)}{e^4}+\frac{x\left(ae^2-bde+cd^2\right)^2}{2de^4\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)\left(7cd^2-e(ae+3bd)\right)}{2d^{3/2}e^{9/2}}-2c$$

```
[Out] (3*c^2*d^2+b^2*e^2-2*c*e*(-a*e+2*b*d))*x/e^4-2/3*c*(-b*e+c*d)*x^3/e^3+1/5*c
^2*x^5/e^2+1/2*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)-1/2*(a*e^2-b*d*e+c*d
^2)*(7*c*d^2-e*(a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(9/2)
```

Rubi [A] time = 0.30, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1157, 1810, 205}

$$\frac{x\left(-2ce(2bd-ae)+b^2e^2+3c^2d^2\right)}{e^4}+\frac{x\left(ae^2-bde+cd^2\right)^2}{2de^4\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)\left(7cd^2-e(ae+3bd)\right)}{2d^{3/2}e^{9/2}}-2c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] ((3*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d - a*e))*x)/e^4 - (2*c*(c*d - b*e)*x^3)
/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 - b*d*e + a*e^2)^2*x)/(2*d*e^4*(d +
e*x^2)) - ((c*d^2 - b*d*e + a*e^2)*(7*c*d^2 - e*(3*b*d + a*e))*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx = \frac{\left(cd^2-bde+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-a^2e^2\right)}{e^4}-\frac{2d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{2cd(cd-2be)x^4}{e^2}}{d+ex^2}}{2d}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{\int \left(-\frac{2d\left(3c^2d^2+b^2e^2-2ce(2bd-ae)\right)}{e^4}+\frac{4cd(cd-be)x^2}{e^3}-\frac{2c^2dx^4}{e^2}+\frac{7c^2d^4-10bc}{}\right.}{2d}$$

$$= \frac{\left(3c^2d^2+b^2e^2-2ce(2bd-ae)\right)x}{e^4} - \frac{2c(cd-be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{(7}{}$$

$$= \frac{\left(3c^2d^2+b^2e^2-2ce(2bd-ae)\right)x}{e^4} - \frac{2c(cd-be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{(7}{}$$

Mathematica [A] time = 0.10, size = 183, normalized size = 1.10

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(a^2e^2+2abde-3b^2d^2\right)+2cd^2e(3ae-5bd)+7c^2d^4\right)}{2d^{3/2}e^{9/2}}+\frac{x\left(2ce(ae-2bd)+b^2e^2+3c^2d^2\right)}{e^4}+\frac{x(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] ((3*c^2*d^2 + b^2*e^2 + 2*c*e*(-2*b*d + a*e))*x)/e^4 + (2*c*(-(c*d) + b*e)*
x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(2*d*e^
4*(d + e*x^2)) - ((7*c^2*d^4 + 2*c*d^2*e*(-5*b*d + 3*a*e) - e^2*(-3*b^2*d^2
 + 2*a*b*d*e + a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

fricas [A] time = 0.79, size = 600, normalized size = 3.61

$$\left[\frac{12\,c^2d^2e^4x^7-4\left(7\,c^2d^3e^3-10\,bcd^2e^4\right)x^5+20\left(7\,c^2d^4e^2-10\,bcd^3e^3+3\left(b^2+2\,ac\right)d^2e^4\right)x^3+15\left(7\,c^2d^5-10\,bc\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^2*d^2*e^4*x^7 - 4*(7*c^2*d^3*e^3 - 10*b*c*d^2*e^4)*x^5 + 20*(7*
c^2*d^4*e^2 - 10*b*c*d^3*e^3 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^3 + 15*(7*c^2*d^5
 - 10*b*c*d^4*e - 2*a*b*d^2*e^3 - a^2*d*e^4 + 3*(b^2 + 2*a*c)*d^3*e^2 + (7*
c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b*d*e^4 - a^2*e^5 + 3*(b^2 + 2*a*c)*d^2*e^
3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 30*(7*c^
2*d^5*e - 10*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*
e^3)*x)/(d^2*e^6*x^2 + d^3*e^5), 1/30*(6*c^2*d^2*e^4*x^7 - 2*(7*c^2*d^3*e^3
 - 10*b*c*d^2*e^4)*x^5 + 10*(7*c^2*d^4*e^2 - 10*b*c*d^3*e^3 + 3*(b^2 + 2*a*
c)*d^2*e^4)*x^3 - 15*(7*c^2*d^5 - 10*b*c*d^4*e - 2*a*b*d^2*e^3 - a^2*d*e^4
+ 3*(b^2 + 2*a*c)*d^3*e^2 + (7*c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b*d*e^4 - a
^2*e^5 + 3*(b^2 + 2*a*c)*d^2*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 15
*(7*c^2*d^5*e - 10*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + 3*(b^2 + 2*a*c
)*d^3*e^3)*x)/(d^2*e^6*x^2 + d^3*e^5)]
```

giac [A] time = 0.18, size = 207, normalized size = 1.25

$$\frac{1}{15}\left(3\,c^2x^5e^8-10\,c^2dx^3e^7+10\,bcx^3e^8+45\,c^2d^2xe^6-60\,bcdxe^7+15\,b^2xe^8+30\,acxe^8\right)e^{(-10)}-\frac{\left(7\,c^2d^4-10\,bcd^3e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*c^2*x^5*e^8 - 10*c^2*d*x^3*e^7 + 10*b*c*x^3*e^8 + 45*c^2*d^2*x*e^6
- 60*b*c*d*x*e^7 + 15*b^2*x*e^8 + 30*a*c*x*e^8)*e^(-10) - 1/2*(7*c^2*d^4 -
10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 - 2*a*b*d*e^3 - a^2*e^4)*arcta
n(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(3/2) + 1/2*(c^2*d^4*x - 2*b*c*d^3*x*e + b^
2*d^2*x*e^2 + 2*a*c*d^2*x*e^2 - 2*a*b*d*x*e^3 + a^2*x*e^4)*e^(-4)/((x^2*e +
 d)*d)
```

maple [B] time = 0.01, size = 320, normalized size = 1.93

$$\frac{c^2x^5}{5e^2}+\frac{2bc\,x^3}{3e^2}-\frac{2c^2d\,x^3}{3e^3}+\frac{a^2x}{2\left(e\,x^2+d\right)d}+\frac{a^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d}-\frac{abx}{\left(e\,x^2+d\right)e}+\frac{ab\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e}+\frac{acdx}{\left(e\,x^2+d\right)e^2}-\frac{3acd\arcta}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x)
```

```
[Out] 1/5*c^2/e^2*x^5+2/3/e^2*x^3*b*c-2/3*c^2*d/e^3*x^3+2*a*c/e^2*x+1/e^2*b^2*x-4
/e^3*b*c*d*x+3*c^2*d^2/e^4*x+1/2/(e*x^2+d)*a^2/d*x-1/e*x/(e*x^2+d)*a*b+1/(e
*x^2+d)*a*c*d/e^2*x+1/2/e^2*d*x/(e*x^2+d)*b^2-1/e^3*d^2*x/(e*x^2+d)*b*c+1/2
/(e*x^2+d)*c^2*d^3/e^4*x+1/2/(d*e)^(1/2)*a^2/d*arctan(1/(d*e)^(1/2)*e*x)+1/
e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a*b-3/(d*e)^(1/2)*a*c*d/e^2*arctan(
1/(d*e)^(1/2)*e*x)-3/2/e^2*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b^2+5/e^
3*d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b*c-7/2/(d*e)^(1/2)*c^2*d^3/e^4
*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.42, size = 205, normalized size = 1.23

$$\frac{\left(c^2d^4-2\,bcd^3e-2\,abde^3+a^2e^4+\left(b^2+2\,ac\right)d^2e^2\right)x}{2\left(de^5x^2+d^2e^4\right)}+\frac{3\,c^2e^2x^5-10\left(c^2de-bce^2\right)x^3+15\left(3\,c^2d^2-4\,bcde+\left(b^2+\right.\right.}{15\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)
*x/(d*e^5*x^2 + d^2*e^4) + 1/15*(3*c^2*e^2*x^5 - 10*(c^2*d*e - b*c*e^2)*x^3
 + 15*(3*c^2*d^2 - 4*b*c*d*e + (b^2 + 2*a*c)*e^2)*x)/e^4 - 1/2*(7*c^2*d^4 -
 10*b*c*d^3*e - 2*a*b*d*e^3 - a^2*e^4 + 3*(b^2 + 2*a*c)*d^2*e^2)*arctan(e*x
/sqrt(d*e))/(sqrt(d*e)*d*e^4)
```

mupad [B] time = 4.56, size = 293, normalized size = 1.77

$$x\left(\frac{b^2+2\,a\,c}{e^2}+\frac{2\,d\left(\frac{2\,c^2\,d}{e^3}-\frac{2\,b\,c}{e^2}\right)}{e}-\frac{c^2\,d^2}{e^4}\right)-x^3\left(\frac{2\,c^2\,d}{3\,e^3}-\frac{2\,b\,c}{3\,e^2}\right)+\frac{c^2\,x^5}{5\,e^2}+\frac{x\left(a^2\,e^4-2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2-\right.}{2\,d\left(e^5\,x^2+d\,e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x)
```

```
[Out] x*((2*a*c + b^2)/e^2 + (2*d*((2*c^2*d)/e^3 - (2*b*c)/e^2))/e - (c^2*d^2)/e^
4) - x^3*((2*c^2*d)/(3*e^3) - (2*b*c)/(3*e^2)) + (c^2*x^5)/(5*e^2) + (x*(a^
2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2))
/(2*d*(d*e^4 + e^5*x^2)) + (atan((e^(1/2)*x*(a*e^2 + c*d^2 - b*d*e)*(a*e^2
- 7*c*d^2 + 3*b*d*e))/(d^(1/2)*(a^2*e^4 - 7*c^2*d^4 - 3*b^2*d^2*e^2 + 2*a*b
```

```
*d*e^3 + 10*b*c*d^3*e - 6*a*c*d^2*e^2)))*(a*e^2 + c*d^2 - b*d*e)*(a*e^2 - 7
*c*d^2 + 3*b*d*e))/(2*d^(3/2)*e^(9/2))
```

sympy [B] time = 3.79, size = 484, normalized size = 2.92

$$\frac{c^2x^5}{5e^2}+x^3\left(\frac{2bc}{3e^2}-\frac{2c^2d}{3e^3}\right)+x\left(\frac{2ac}{e^2}+\frac{b^2}{e^2}-\frac{4bcd}{e^3}+\frac{3c^2d^2}{e^4}\right)+\frac{x\left(a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+c^2d^4\right)}{2d^2e^4+2de^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**2,x)
```

```
[Out] c**2*x**5/(5*e**2) + x**3*(2*b*c/(3*e**2) - 2*c**2*d/(3*e**3)) + x*(2*a*c/e
**2 + b**2/e**2 - 4*b*c*d/e**3 + 3*c**2*d**2/e**4) + x*(a**2*e**4 - 2*a*b*d
*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4)/(2*d**
2*e**4 + 2*d*e**5*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a
*e**2 + 3*b*d*e - 7*c*d**2)*log(-d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - b
*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)/(a**2*e**4 + 2*a*b*d*e**3 - 6*
a*c*d**2*e**2 - 3*b**2*d**2*e**2 + 10*b*c*d**3*e - 7*c**2*d**4) + x)/4 + sq
rt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)*
log(d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*
d*e - 7*c*d**2)/(a**2*e**4 + 2*a*b*d*e**3 - 6*a*c*d**2*e**2 - 3*b**2*d**2*e
**2 + 10*b*c*d**3*e - 7*c**2*d**4) + x)/4
```

3.258 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3} dx$

Optimal. Leaf size=201

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(3a^2e^2+2abde+3b^2d^2\right)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}}-\frac{x\left(-3ae^2-5bde+13cd^2\right)\left(ae^2-bde+cd^2\right.}{8d^2e^4\left(d+ex^2\right)}$$

```
[Out] -c*(-2*b*e+3*c*d)*x/e^4+1/3*c^2*x^3/e^3+1/4*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(
e*x^2+d)^2-1/8*(-3*a*e^2-5*b*d*e+13*c*d^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/(e
*x^2+d)+1/8*(35*c^2*d^4-6*c*d^2*e*(-a*e+5*b*d)+e^2*(3*a^2*e^2+2*a*b*d*e+3*b
^2*d^2))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(9/2)
```

Rubi [A] time = 0.42, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1157, 1814, 1153, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(3a^2e^2+2abde+3b^2d^2\right)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}}-\frac{x\left(-3ae^2-5bde+13cd^2\right)\left(ae^2-bde+cd^2\right.}{8d^2e^4\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] -((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^3)/(3*e^3) + ((c*d^2 - b*d*e + a*e^2)
^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 5*b*d*e - 3*a*e^2)*(c*d^2 - b*
d*e + a*e^2)*x)/(8*d^2*e^4*(d + e*x^2)) + ((35*c^2*d^4 - 6*c*d^2*e*(5*b*d -
 a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(8*d^(5/2)*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
```

```
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx = \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\int \frac{\frac{\left(cd^2-bde-ae^2\right)\left(cd^2-bde+3ae^2\right)}{e^4} - \frac{4d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3} + \frac{4cd(cd-2be)x^4}{e^2} - \frac{4c^2dx^6}{e}}{\left(d+ex^2\right)^2} \, d}{4d}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+ae^2\right)x}{8d^2e^4\left(d+ex^2\right)} + \frac{\int \frac{11c^2d^4-2cd^2e(7bd-3a}{}}{}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+ae^2\right)x}{8d^2e^4\left(d+ex^2\right)} + \frac{\int \left(-\frac{8cd^2(3cd-2be)}{e^4}\right.}{}$$

$$= -\frac{c(3cd-2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+\right.}{8d^2e^4\left(d+ex^2\right)}$$

$$= -\frac{c(3cd-2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+\right.}{8d^2e^4\left(d+ex^2\right)}$$

Mathematica [A] time = 0.11, size = 217, normalized size = 1.08

$$-\frac{x\left(e^2\left(-3a^2e^2-2abde+5b^2d^2\right)-2cd^2e(9bd-5ae)+13c^2d^4\right)}{8d^2e^4\left(d+ex^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(3a^2e^2+2abde+3b^2d^2\right)+6c\right.}{8d^{5/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (c*(-3*c*d + 2*b*e)*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + e*(-(b*d) + a*e)
)^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c^2*d^4 - 2*c*d^2*e*(9*b*d - 5*a*e) +
 e^2*(5*b^2*d^2 - 2*a*b*d*e - 3*a^2*e^2))*x)/(8*d^2*e^4*(d + e*x^2)) + ((35
*c^2*d^4 + 6*c*d^2*e*(-5*b*d + a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^
2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))
```

fricas [B] time = 0.65, size = 794, normalized size = 3.95

$$\left[\frac{16\,c^2d^3e^4x^7-16\left(7\,c^2d^4e^3-6\,bcd^3e^4\right)x^5-2\left(175\,c^2d^5e^2-150\,bcd^4e^3-6\,abd^2e^5-9\,a^2de^6+15\left(b^2+2\,ac\right)d^3e^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^2*d^3*e^4*x^7 - 16*(7*c^2*d^4*e^3 - 6*b*c*d^3*e^4)*x^5 - 2*(175
*c^2*d^5*e^2 - 150*b*c*d^4*e^3 - 6*a*b*d^2*e^5 - 9*a^2*d*e^6 + 15*(b^2 + 2*
a*c)*d^3*e^4)*x^3 - 3*(35*c^2*d^6 - 30*b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2*d^
2*e^4 + 3*(b^2 + 2*a*c)*d^4*e^2 + (35*c^2*d^4*e^2 - 30*b*c*d^3*e^3 + 2*a*b*
```

```
d*e^5 + 3*a^2*e^6 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^4 + 2*(35*c^2*d^5*e - 30*b*c
*d^4*e^2 + 2*a*b*d^2*e^4 + 3*a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*e^3)*x^2)*sqrt
(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(35*c^2*d^6*e - 30
*b*c*d^5*e^2 + 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 + 3*(b^2 + 2*a*c)*d^4*e^3)*x)/
(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 8*(7*c^2
*d^4*e^3 - 6*b*c*d^3*e^4)*x^5 - (175*c^2*d^5*e^2 - 150*b*c*d^4*e^3 - 6*a*b*
d^2*e^5 - 9*a^2*d*e^6 + 15*(b^2 + 2*a*c)*d^3*e^4)*x^3 + 3*(35*c^2*d^6 - 30*
b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2*d^2*e^4 + 3*(b^2 + 2*a*c)*d^4*e^2 + (35*c
^2*d^4*e^2 - 30*b*c*d^3*e^3 + 2*a*b*d*e^5 + 3*a^2*e^6 + 3*(b^2 + 2*a*c)*d^2
*e^4)*x^4 + 2*(35*c^2*d^5*e - 30*b*c*d^4*e^2 + 2*a*b*d^2*e^4 + 3*a^2*d*e^5
+ 3*(b^2 + 2*a*c)*d^3*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c^2
*d^6*e - 30*b*c*d^5*e^2 + 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 + 3*(b^2 + 2*a*c)*d
^4*e^3)*x)/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]
```

giac [**A**] time = 0.18, size = 244, normalized size = 1.21

$$\frac{1}{3}\left(c^2x^3e^6 - 9\,c^2dxe^5 + 6\,bcxe^6\right)e^{(-9)} + \frac{\left(35\,c^2d^4 - 30\,bcd^3e + 3\,b^2d^2e^2 + 6\,acd^2e^2 + 2\,abde^3 + 3\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{(}}{8\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^6 - 9*c^2*d*x*e^5 + 6*b*c*x*e^6)*e^(-9) + 1/8*(35*c^2*d^4 -
30*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 2*a*b*d*e^3 + 3*a^2*e^4)*arc
tan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(5/2) - 1/8*(13*c^2*d^4*x^3*e - 18*b*c*d^
3*x^3*e^2 + 11*c^2*d^5*x + 5*b^2*d^2*x^3*e^3 + 10*a*c*d^2*x^3*e^3 - 14*b*c*
d^4*x*e - 2*a*b*d*x^3*e^4 + 3*b^2*d^3*x*e^2 + 6*a*c*d^3*x*e^2 - 3*a^2*x^3*e
^5 + 2*a*b*d^2*x*e^3 - 5*a^2*d*x*e^4)*e^(-4)/((x^2*e + d)^2*d^2)
```

maple [**B**] time = 0.01, size = 402, normalized size = 2.00

$$\frac{3a^2e\,x^3}{8\left(e\,x^2+d\right)^2d^2} + \frac{ab\,x^3}{4\left(e\,x^2+d\right)^2d} - \frac{5ac\,x^3}{4\left(e\,x^2+d\right)^2e} - \frac{5b^2x^3}{8\left(e\,x^2+d\right)^2e} + \frac{9bcd\,x^3}{4\left(e\,x^2+d\right)^2e^2} - \frac{13c^2d^2x^3}{8\left(e\,x^2+d\right)^2e^3} + \frac{5a^2x}{8\left(e\,x^2+d\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x)
```

```
[Out] 1/3*c^2/e^3*x^3+2*c/e^3*b*x-3*c^2*d/e^4*x+3/8/(e*x^2+d)^2*a^2/d^2*e*x^3+1/4
/(e*x^2+d)^2/d*x^3*a*b-5/4/(e*x^2+d)^2*a*c/e*x^3-5/8/e/(e*x^2+d)^2*x^3*b^2+
9/4/e^2/(e*x^2+d)^2*x^3*b*c*d-13/8/(e*x^2+d)^2*c^2*d^2/e^3*x^3+5/8/(e*x^2+d
)^2*a^2/d*x-1/4/e/(e*x^2+d)^2*a*b*x-3/4/(e*x^2+d)^2*a*c*d/e^2*x-3/8/e^2/(e*
x^2+d)^2*b^2*d*x+7/4/e^3/(e*x^2+d)^2*b*c*d^2*x-11/8/(e*x^2+d)^2*c^2*d^3/e^4
*x+3/8/(d*e)^(1/2)*a^2/d^2*arctan(1/(d*e)^(1/2)*e*x)+1/4/e/d/(d*e)^(1/2)*ar
ctan(1/(d*e)^(1/2)*e*x)*a*b+3/4/(d*e)^(1/2)*a*c/e^2*arctan(1/(d*e)^(1/2)*e*
x)+3/8/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b^2-15/4/e^3*d/(d*e)^(1/2)
*arctan(1/(d*e)^(1/2)*e*x)*b*c+35/8/(d*e)^(1/2)*c^2*d^2/e^4*arctan(1/(d*e)^
(1/2)*e*x)
```

maxima [**A**] time = 2.36, size = 245, normalized size = 1.22

$$-\frac{\left(13\,c^2d^4e - 18\,bcd^3e^2 - 2\,abde^4 - 3\,a^2e^5 + 5\left(b^2 + 2\,ac\right)d^2e^3\right)x^3 + \left(11\,c^2d^5 - 14\,bcd^4e + 2\,abd^2e^3 - 5\,a^2de^4 + 3\left(b^2\right.\right.}{8\left(d^2e^6x^4 + 2\,d^3e^5x^2 + d^4e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*c^2*d^4*e - 18*b*c*d^3*e^2 - 2*a*b*d*e^4 - 3*a^2*e^5 + 5*(b^2 + 2
*a*c)*d^2*e^3)*x^3 + (11*c^2*d^5 - 14*b*c*d^4*e + 2*a*b*d^2*e^3 - 5*a^2*d*e
^4 + 3*(b^2 + 2*a*c)*d^3*e^2)*x)/(d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4) +
1/3*(c^2*e*x^3 - 3*(3*c^2*d - 2*b*c*e)*x)/e^4 + 1/8*(35*c^2*d^4 - 30*b*c*d^
3*e + 2*a*b*d*e^3 + 3*a^2*e^4 + 3*(b^2 + 2*a*c)*d^2*e^2)*arctan(e*x/sqrt(d*
e))/(sqrt(d*e)*d^2*e^4)
```

mupad [B] time = 0.12, size = 257, normalized size = 1.28

$$\frac{c^2\,x^3}{3\,e^3}-x\,\left(\frac{3\,c^2\,d}{e^4}-\frac{2\,b\,c}{e^3}\right)-\frac{\frac{x\,\left(-5\,a^2\,e^4+2\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2-14\,b\,c\,d^3\,e+11\,c^2\,d^4\right)}{8\,d}-\frac{x^3\,\left(3\,a^2\,e^5+2\,a\,b\,d\,e^4-10\,a\,c\,d^2\,e^3-5\,b^2\,d^2\,e^3+\dots\right)}{8\,d^2}}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x)
```

```
[Out] (c^2*x^3)/(3*e^3) - x*((3*c^2*d)/e^4 - (2*b*c)/e^3) - ((x*(11*c^2*d^4 - 5*a
^2*e^4 + 3*b^2*d^2*e^2 + 2*a*b*d*e^3 - 14*b*c*d^3*e + 6*a*c*d^2*e^2))/(8*d)
 - (x^3*(3*a^2*e^5 - 13*c^2*d^4*e - 5*b^2*d^2*e^3 + 2*a*b*d*e^4 - 10*a*c*d^
2*e^3 + 18*b*c*d^3*e^2))/(8*d^2))/(d^2*e^4 + e^6*x^4 + 2*d*e^5*x^2) + (atan
((e^(1/2)*x)/d^(1/2))*(3*a^2*e^4 + 35*c^2*d^4 + 3*b^2*d^2*e^2 + 2*a*b*d*e^3
 - 30*b*c*d^3*e + 6*a*c*d^2*e^2))/(8*d^(5/2)*e^(9/2))
```

sympy [A] time = 17.72, size = 398, normalized size = 1.98

$$\frac{c^2x^3}{3e^3}+x\left(\frac{2bc}{e^3}-\frac{3c^2d}{e^4}\right)-\frac{\sqrt{-\frac{1}{d^5e^9}}\,\left(3a^2e^4+2abde^3+6acd^2e^2+3b^2d^2e^2-30bcd^3e+35c^2d^4\right)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}}\dots\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**3,x)
```

```
[Out] c**2*x**3/(3*e**3) + x*(2*b*c/e**3 - 3*c**2*d/e**4) - sqrt(-1/(d**5*e**9))*
(3*a**2*e**4 + 2*a*b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e**2 - 30*b*c*d
**3*e + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + sqrt(-1
/(d**5*e**9))*(3*a**2*e**4 + 2*a*b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e
**2 - 30*b*c*d**3*e + 35*c**2*d**4)*log(d**3*e**4*sqrt(-1/(d**5*e**9)) + x)
/16 + (x**3*(3*a**2*e**5 + 2*a*b*d*e**4 - 10*a*c*d**2*e**3 - 5*b**2*d**2*e*
*3 + 18*b*c*d**3*e**2 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 2*a*b*d**2*e**
3 - 6*a*c*d**3*e**2 - 3*b**2*d**3*e**2 + 14*b*c*d**4*e - 11*c**2*d**5))/(8*
d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)
```

3.259 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx$

Optimal. Leaf size=250

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(ae+5bd)+35c^2d^4\right)}{16d^{7/2}e^{9/2}}+\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(11\right.}{16d^3e^4\left(d+ex^2\right)}$$

```
[Out] c^2*x/e^4+1/6*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)^3-1/24*(-5*a*e^2-7*b*
d*e+19*c*d^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/(e*x^2+d)^2+1/16*(29*c^2*d^4-2*
c*d^2*e*(-a*e+11*b*d)+e^2*(5*a^2*e^2+2*a*b*d*e+b^2*d^2))*x/d^3/e^4/(e*x^2+d
)-1/16*(35*c^2*d^4-2*c*d^2*e*(a*e+5*b*d)-e^2*(5*a^2*e^2+2*a*b*d*e+b^2*d^2))
*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(9/2)
```

Rubi [A] time = 0.54, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1157, 1814, 388, 205}

$$\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(11bd-ae)+29c^2d^4\right)}{16d^3e^4\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(ae\right.}{16d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 - b*d*e + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((19*
c*d^2 - 7*b*d*e - 5*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(24*d^2*e^4*(d + e*x^
2)^2) + ((29*c^2*d^4 - 2*c*d^2*e*(11*b*d - a*e) + e^2*(b^2*d^2 + 2*a*b*d*e
+ 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2)) - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d
 + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(16*d^(7/2)*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^4}\,dx = \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} - \frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-5a^2e^2\right)}{e^4}-\frac{6d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{6cd(cd-2be)x^4}{e^2}}{\left(d+ex^2\right)^3}}{6d}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} - \frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2} + \frac{\int \frac{3\left(5c^2d^4-2cd^2e(3bd-a\right.}{}}{}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} - \frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2} + \frac{\left(29c^2d^4-2cd^2e\right.}{}$$

$$= \frac{c^2x}{e^4} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} - \frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2} + \frac{\left(29c^2d^4-\right.}{}$$

$$= \frac{c^2x}{e^4} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} - \frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2} + \frac{\left(29c^2d^4-\right.}{}$$

Mathematica [A] time = 0.15, size = 267, normalized size = 1.07

$$-\frac{x\left(e^2\left(-5a^2e^2-2abde+7b^2d^2\right)+2cd^2e(7ae-13bd)+19c^2d^4\right)}{24d^2e^4\left(d+ex^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(5a^2e^2+2abde+b^2d^2\right)-2\right.}{16d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((
19*c^2*d^4 + 2*c*d^2*e*(-13*b*d + 7*a*e) + e^2*(7*b^2*d^2 - 2*a*b*d*e - 5*a
^2*e^2))*x)/(24*d^2*e^4*(d + e*x^2)^2) + ((29*c^2*d^4 + 2*c*d^2*e*(-11*b*d
+ a*e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2))
 - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^
2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

fricas [B] time = 0.58, size = 1016, normalized size = 4.06

$$\left[\frac{96\,c^2d^4e^4x^7+6\left(77\,c^2d^5e^3-22\,bcd^4e^4+2\,abd^2e^6+5\,a^2de^7+\left(b^2+2\,ac\right)d^3e^5\right)x^5+16\left(35\,c^2d^6e^2-10\,bcd^5e^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(96*c^2*d^4*e^4*x^7 + 6*(77*c^2*d^5*e^3 - 22*b*c*d^4*e^4 + 2*a*b*d^2*
e^6 + 5*a^2*d*e^7 + (b^2 + 2*a*c)*d^3*e^5)*x^5 + 16*(35*c^2*d^6*e^2 - 10*b*
```

```
c*d^5*e^3 + 2*a*b*d^3*e^5 + 5*a^2*d^2*e^6 - (b^2 + 2*a*c)*d^4*e^4)*x^3 + 3*
(35*c^2*d^7 - 10*b*c*d^6*e - 2*a*b*d^4*e^3 - 5*a^2*d^3*e^4 - (b^2 + 2*a*c)*
d^5*e^2 + (35*c^2*d^4*e^3 - 10*b*c*d^3*e^4 - 2*a*b*d*e^6 - 5*a^2*e^7 - (b^2
 + 2*a*c)*d^2*e^5)*x^6 + 3*(35*c^2*d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d^2*e^5
 - 5*a^2*d*e^6 - (b^2 + 2*a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6*e - 10*b*c*d^5*
e^2 - 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 - (b^2 + 2*a*c)*d^4*e^3)*x^2)*sqrt(-d*e
)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^7*e - 10*b*c*
d^6*e^2 - 2*a*b*d^4*e^4 + 11*a^2*d^3*e^5 - (b^2 + 2*a*c)*d^5*e^3)*x)/(d^4*e
^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5), 1/48*(48*c^2*d^4*e^4*x^7
 + 3*(77*c^2*d^5*e^3 - 22*b*c*d^4*e^4 + 2*a*b*d^2*e^6 + 5*a^2*d*e^7 + (b^2
+ 2*a*c)*d^3*e^5)*x^5 + 8*(35*c^2*d^6*e^2 - 10*b*c*d^5*e^3 + 2*a*b*d^3*e^5
+ 5*a^2*d^2*e^6 - (b^2 + 2*a*c)*d^4*e^4)*x^3 - 3*(35*c^2*d^7 - 10*b*c*d^6*e
 - 2*a*b*d^4*e^3 - 5*a^2*d^3*e^4 - (b^2 + 2*a*c)*d^5*e^2 + (35*c^2*d^4*e^3
- 10*b*c*d^3*e^4 - 2*a*b*d*e^6 - 5*a^2*e^7 - (b^2 + 2*a*c)*d^2*e^5)*x^6 + 3
*(35*c^2*d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d^2*e^5 - 5*a^2*d*e^6 - (b^2 + 2*
a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6*e - 10*b*c*d^5*e^2 - 2*a*b*d^3*e^4 - 5*a^
2*d^2*e^5 - (b^2 + 2*a*c)*d^4*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 3
*(35*c^2*d^7*e - 10*b*c*d^6*e^2 - 2*a*b*d^4*e^4 + 11*a^2*d^3*e^5 - (b^2 + 2
*a*c)*d^5*e^3)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5)]
```

giac [A] time = 0.18, size = 296, normalized size = 1.18

$$c^2xe^{(-4)} - \frac{\left(35\,c^2d^4 - 10\,bcd^3e - b^2d^2e^2 - 2\,acd^2e^2 - 2\,abde^3 - 5\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{16\,d^{\frac{7}{2}}} + \frac{\left(87\,c^2d^4x^5e^2 - 66\,bcd^3x^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] c^2*x*e^(-4) - 1/16*(35*c^2*d^4 - 10*b*c*d^3*e - b^2*d^2*e^2 - 2*a*c*d^2*e^
2 - 2*a*b*d*e^3 - 5*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(7/2) + 1
/48*(87*c^2*d^4*x^5*e^2 - 66*b*c*d^3*x^5*e^3 + 136*c^2*d^5*x^3*e + 3*b^2*d^
2*x^5*e^4 + 6*a*c*d^2*x^5*e^4 - 80*b*c*d^4*x^3*e^2 + 57*c^2*d^6*x + 6*a*b*d
*x^5*e^5 - 8*b^2*d^3*x^3*e^3 - 16*a*c*d^3*x^3*e^3 - 30*b*c*d^5*x*e + 15*a^2
*x^5*e^6 + 16*a*b*d^2*x^3*e^4 - 3*b^2*d^4*x*e^2 - 6*a*c*d^4*x*e^2 + 40*a^2*
d*x^3*e^5 - 6*a*b*d^3*x*e^3 + 33*a^2*d^2*x*e^4)*e^(-4)/((x^2*e + d)^3*d^3)
```

maple [B] time = 0.01, size = 506, normalized size = 2.02

$$\frac{5a^2e^2x^5}{16\left(e\,x^2+d\right)^3d^3} + \frac{abe\,x^5}{8\left(e\,x^2+d\right)^3d^2} + \frac{ac\,x^5}{8\left(e\,x^2+d\right)^3d} + \frac{b^2x^5}{16\left(e\,x^2+d\right)^3d} - \frac{11bc\,x^5}{8\left(e\,x^2+d\right)^3e} + \frac{29c^2d\,x^5}{16\left(e\,x^2+d\right)^3e^2} + \frac{5a^2e\,x}{6\left(e\,x^2+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x)
```

```
[Out] c^2/e^4*x-1/6/e/(e*x^2+d)^3*x^3*b^2+1/16/(e*x^2+d)^3/d*x^5*b^2+11/16/(e*x^2
+d)^3*a^2/d*x+5/16/(d*e)^(1/2)*a^2/d^3*arctan(1/(d*e)^(1/2)*e*x)-1/8/(e*x^2
+d)^3*a*c*d/e^2*x+1/8/(d*e)^(1/2)*a*c/d/e^2*arctan(1/(d*e)^(1/2)*e*x)+1/8*e
/(e*x^2+d)^3/d^2*x^5*a*b-5/3/e^2/(e*x^2+d)^3*x^3*b*c*d-5/8/e^3/(e*x^2+d)^3*
b*c*d^2*x+1/8/e/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a*b-11/8/e/(e*x^2
+d)^3*x^5*b*c-1/8/e/(e*x^2+d)^3*a*b*x-1/16/e^2/(e*x^2+d)^3*b^2*d*x+1/16/e^2
/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b^2+5/8/e^3/(d*e)^(1/2)*arctan(1/(
d*e)^(1/2)*e*x)*b*c+1/3/(e*x^2+d)^3/d*x^3*a*b+29/16/(e*x^2+d)^3*c^2*d/e^2*x
^5+5/6/(e*x^2+d)^3*a^2/d^2*e*x^3-1/3/(e*x^2+d)^3*a*c/e*x^3+17/6/(e*x^2+d)^3
*c^2*d^2/e^3*x^3+19/16/(e*x^2+d)^3*c^2*d^3/e^4*x-35/16/(d*e)^(1/2)*c^2*d/e^
4*arctan(1/(d*e)^(1/2)*e*x)+5/16/(e*x^2+d)^3*a^2/d^3*e^2*x^5+1/8/(e*x^2+d)^
3*a*c/d*x^5
```

maxima [A] time = 2.39, size = 300, normalized size = 1.20

$$\frac{3\left(29\,c^2d^4e^2 - 22\,bcd^3e^3 + 2\,abde^5 + 5\,a^2e^6 + \left(b^2 + 2\,ac\right)d^2e^4\right)x^5 + 8\left(17\,c^2d^5e - 10\,bcd^4e^2 + 2\,abd^2e^4 + 5\,a^2de^5\right.}{48\left(d^3e^7x^6 + 3\,d^4e^6x^4 + 3\,d^5e^5x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(29*c^2*d^4*e^2 - 22*b*c*d^3*e^3 + 2*a*b*d*e^5 + 5*a^2*e^6 + (b^2 +
 2*a*c)*d^2*e^4)*x^5 + 8*(17*c^2*d^5*e - 10*b*c*d^4*e^2 + 2*a*b*d^2*e^4 + 5
*a^2*d*e^5 - (b^2 + 2*a*c)*d^3*e^3)*x^3 + 3*(19*c^2*d^6 - 10*b*c*d^5*e - 2*
a*b*d^3*e^3 + 11*a^2*d^2*e^4 - (b^2 + 2*a*c)*d^4*e^2)*x)/(d^3*e^7*x^6 + 3*d
^4*e^6*x^4 + 3*d^5*e^5*x^2 + d^6*e^4) + c^2*x/e^4 - 1/16*(35*c^2*d^4 - 10*b
*c*d^3*e - 2*a*b*d*e^3 - 5*a^2*e^4 - (b^2 + 2*a*c)*d^2*e^2)*arctan(e*x/sqrt
(d*e))/(sqrt(d*e)*d^3*e^4)
```

mupad [B] time = 4.60, size = 308, normalized size = 1.23

$$\frac{\frac{x^5\left(5\,a^2\,e^6+2\,a\,b\,d\,e^5+2\,a\,c\,d^2\,e^4+b^2\,d^2\,e^4-22\,b\,c\,d^3\,e^3+29\,c^2\,d^4\,e^2\right)}{16\,d^3} - \frac{x\left(-11\,a^2\,e^4+2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-19\,c^2\,d^4\right)}{16\,d} + \frac{x^3\left(5\,a^2\,e^5\right.}{}}{d^3\,e^4 + 3\,d^2\,e^5\,x^2 + 3\,d\,e^6\,x^4 + e^7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x)
```

```
[Out] ((x^5*(5*a^2*e^6 + b^2*d^2*e^4 + 29*c^2*d^4*e^2 + 2*a*b*d*e^5 + 2*a*c*d^2*e
^4 - 22*b*c*d^3*e^3))/(16*d^3) - (x*(b^2*d^2*e^2 - 19*c^2*d^4 - 11*a^2*e^4
+ 2*a*b*d*e^3 + 10*b*c*d^3*e + 2*a*c*d^2*e^2))/(16*d) + (x^3*(5*a^2*e^5 + 1
7*c^2*d^4*e - b^2*d^2*e^3 + 2*a*b*d*e^4 - 2*a*c*d^2*e^3 - 10*b*c*d^3*e^2))/
(6*d^2))/(d^3*e^4 + e^7*x^6 + 3*d*e^6*x^4 + 3*d^2*e^5*x^2) + (c^2*x)/e^4 +
(atan((e^(1/2)*x)/d^(1/2))*(5*a^2*e^4 - 35*c^2*d^4 + b^2*d^2*e^2 + 2*a*b*d*
e^3 + 10*b*c*d^3*e + 2*a*c*d^2*e^2))/(16*d^(7/2)*e^(9/2))
```

sympy [A] time = 94.00, size = 457, normalized size = 1.83

$$\frac{c^2x}{e^4} - \frac{\sqrt{-\frac{1}{d^7e^9}}\left(5a^2e^4 + 2abde^3 + 2acd^2e^2 + b^2d^2e^2 + 10bcd^3e - 35c^2d^4\right)\log\left(-d^4e^4\sqrt{-\frac{1}{d^7e^9}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7e^9}}\left(5a^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**4,x)
```

```
[Out] c**2*x/e**4 - sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*b*d*e**3 + 2*a*c*d**2
*e**2 + b**2*d**2*e**2 + 10*b*c*d**3*e - 35*c**2*d**4)*log(-d**4*e**4*sqrt(
-1/(d**7*e**9)) + x)/32 + sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*b*d*e**3
+ 2*a*c*d**2*e**2 + b**2*d**2*e**2 + 10*b*c*d**3*e - 35*c**2*d**4)*log(d**4
*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + (x**5*(15*a**2*e**6 + 6*a*b*d*e**5 + 6
*a*c*d**2*e**4 + 3*b**2*d**2*e**4 - 66*b*c*d**3*e**3 + 87*c**2*d**4*e**2) +
 x**3*(40*a**2*d*e**5 + 16*a*b*d**2*e**4 - 16*a*c*d**3*e**3 - 8*b**2*d**3*e
**3 - 80*b*c*d**4*e**2 + 136*c**2*d**5*e) + x*(33*a**2*d**2*e**4 - 6*a*b*d*
*3*e**3 - 6*a*c*d**4*e**2 - 3*b**2*d**4*e**2 - 30*b*c*d**5*e + 57*c**2*d**6
))/(48*d**6*e**4 + 144*d**5*e**5*x**2 + 144*d**4*e**6*x**4 + 48*d**3*e**7*x
**6)
```

3.260 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$

Optimal. Leaf size=317

$$-\frac{x\left(-e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(3ae+5bd)+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)+\right.}{128d^{9/2}e^{9/2}}$$

```
[Out] 1/8*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)^4-1/48*(-7*a*e^2-9*b*d*e+25*c*d
^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/(e*x^2+d)^3+1/192*(163*c^2*d^4-2*c*d^2*e*
(-3*a*e+59*b*d)+e^2*(35*a^2*e^2+10*a*b*d*e+3*b^2*d^2))*x/d^3/e^4/(e*x^2+d)^
2-1/128*(93*c^2*d^4-2*c*d^2*e*(3*a*e+5*b*d)-e^2*(35*a^2*e^2+10*a*b*d*e+3*b^
2*d^2))*x/d^4/e^4/(e*x^2+d)+1/128*(35*c^2*d^4+2*c*d^2*e*(3*a*e+5*b*d)+e^2*(
35*a^2*e^2+10*a*b*d*e+3*b^2*d^2))*arctan(x*e^(1/2)/d^(1/2))/d^(9/2)/e^(9/2)
```

Rubi [A] time = 0.65, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1157, 1814, 385, 205}

$$-\frac{x\left(-e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(3ae+5bd)+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{x\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(59b\right.}{192d^3e^4\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - ((25*c*d^2 - 9*b*d*
e - 7*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(48*d^2*e^4*(d + e*x^2)^3) + ((163*
c^2*d^4 - 2*c*d^2*e*(59*b*d - 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2
*e^2))*x)/(192*d^3*e^4*(d + e*x^2)^2) - ((93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3
*a*e) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(128*d^4*e^4*(d + e*x
^2)) + ((35*c^2*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d
*e + 35*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^5}\,dx = \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-7a^2e^2\right)}{e^4}-\frac{8d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{8cd(cd-2be)x^4}{e^2}}{\left(d+ex^2\right)^4}}{8d}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3} + \frac{\int \frac{19c^2d^4-2cd^2e(11bd-}{}}{}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3} + \frac{\left(163c^2d^4-2cd^2\right.}{}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3} + \frac{\left(163c^2d^4-2cd^2\right.}{}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3} + \frac{\left(163c^2d^4-2cd^2\right.}{}$$

Mathematica [A] time = 0.23, size = 345, normalized size = 1.09

$$-\frac{3\sqrt{d}\sqrt{e}x\left(-e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(3ae+5bd)+93c^2d^4\right)}{d+ex^2}+3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)+2cd^2e(3a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((48*d^(7/2)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))^2*x)/(d + e*x^2)^4 - (8*d^(
5/2)*Sqrt[e]*(25*c^2*d^4 + 2*c*d^2*e*(-17*b*d + 9*a*e) + e^2*(9*b^2*d^2 - 2
*a*b*d*e - 7*a^2*e^2))*x)/(d + e*x^2)^3 + (2*d^(3/2)*Sqrt[e]*(163*c^2*d^4 +
 2*c*d^2*e*(-59*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x
)/(d + e*x^2)^2 - (3*Sqrt[d]*Sqrt[e]*(93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3*a*e
) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(d + e*x^2) + 3*(35*c^2*d
^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(384*d^(9/2)*e^(9/2))
```

fricas [B] time = 0.74, size = 1266, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="fricas")
```

```
[Out] [-1/768*(6*(93*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e^8
 - 3*(b^2 + 2*a*c)*d^3*e^6)*x^7 + 2*(511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 - 11
0*a*b*d^3*e^6 - 385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + 2*(385*c^
2*d^7*e^2 + 110*b*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b^2 +
 2*a*c)*d^5*e^4)*x^3 + 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a
^2*d^4*e^4 + 3*(b^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e^5 + 1
0*a*b*d*e^7 + 35*a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*(35*c^2*d^5*e^3
 + 10*b*c*d^4*e^4 + 10*a*b*d^2*e^6 + 35*a^2*d*e^7 + 3*(b^2 + 2*a*c)*d^3*e^5
)*x^6 + 6*(35*c^2*d^6*e^2 + 10*b*c*d^5*e^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^
6 + 3*(b^2 + 2*a*c)*d^4*e^4)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*
b*d^4*e^4 + 35*a^2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*sqrt(-d*e)*log((
e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^8*e + 10*b*c*d^7*e^2
 + 10*a*b*d^5*e^4 - 93*a^2*d^4*e^5 + 3*(b^2 + 2*a*c)*d^6*e^3)*x)/(d^5*e^9*x
^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), -1/384*(3*(9
3*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e^8 - 3*(b^2 + 2
*a*c)*d^3*e^6)*x^7 + (511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 - 110*a*b*d^3*e^6 -
 385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + (385*c^2*d^7*e^2 + 110*b
*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b^2 + 2*a*c)*d^5*e^4)*
x^3 - 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a^2*d^4*e^4 + 3*(b
^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e^5 + 10*a*b*d*e^7 + 35*
a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*(35*c^2*d^5*e^3 + 10*b*c*d^4*e^4
 + 10*a*b*d^2*e^6 + 35*a^2*d*e^7 + 3*(b^2 + 2*a*c)*d^3*e^5)*x^6 + 6*(35*c^2
*d^6*e^2 + 10*b*c*d^5*e^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^6 + 3*(b^2 + 2*a*
c)*d^4*e^4)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*b*d^4*e^4 + 35*a^
2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) +
 3*(35*c^2*d^8*e + 10*b*c*d^7*e^2 + 10*a*b*d^5*e^4 - 93*a^2*d^4*e^5 + 3*(b^
2 + 2*a*c)*d^6*e^3)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8
*e^6*x^2 + d^9*e^5)]
```

giac [A] time = 0.19, size = 364, normalized size = 1.15

$$\frac{\left(35\,c^2d^4+10\,bcd^3e+3\,b^2d^2e^2+6\,acd^2e^2+10\,abde^3+35\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{128\,d^{\frac{9}{2}}}-\frac{\left(279\,c^2d^4x^7e^3-30\,bcd^3x^7e^4-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="giac")
```

```
[Out] 1/128*(35*c^2*d^4 + 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 10*a*b*d
*e^3 + 35*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(9/2) - 1/384*(279*
c^2*d^4*x^7*e^3 - 30*b*c*d^3*x^7*e^4 + 511*c^2*d^5*x^5*e^2 - 9*b^2*d^2*x^7*
e^5 - 18*a*c*d^2*x^7*e^5 + 146*b*c*d^4*x^5*e^3 + 385*c^2*d^6*x^3*e - 30*a*b
*d*x^7*e^6 - 33*b^2*d^3*x^5*e^4 - 66*a*c*d^3*x^5*e^4 + 110*b*c*d^5*x^3*e^2
+ 105*c^2*d^7*x - 105*a^2*x^7*e^7 - 110*a*b*d^2*x^5*e^5 + 33*b^2*d^4*x^3*e^
3 + 66*a*c*d^4*x^3*e^3 + 30*b*c*d^6*x*e - 385*a^2*d*x^5*e^6 - 146*a*b*d^3*x
^3*e^4 + 9*b^2*d^5*x*e^2 + 18*a*c*d^5*x*e^2 - 511*a^2*d^2*x^3*e^5 + 30*a*b*
d^4*x*e^3 - 279*a^2*d^3*x*e^4)*e^(-4)/((x^2*e + d)^4*d^4)
```

maple [A] time = 0.01, size = 412, normalized size = 1.30

$$\frac{35a^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\sqrt{de}\,d^4}+\frac{5ab\arctan\left(\frac{ex}{\sqrt{de}}\right)}{64\sqrt{de}\,d^3e}+\frac{3ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{64\sqrt{de}\,d^2e^2}+\frac{3b^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\sqrt{de}\,d^2e^2}+\frac{5bc\arctan\left(\frac{ex}{\sqrt{de}}\right)}{64\sqrt{de}\,d\,e^3}+\frac{35c^2\arctan\left(\frac{}{\sqrt{}}\right.}{128\sqrt{de}\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x)
```

```
[Out] (1/128*(35*a^2*e^4+10*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2+10*b*c*d^3*e-93
*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+110*a*b*d*e^3+66*a*c*d^2*e^2+33*b^2*
d^2*e^2-146*b*c*d^3*e-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4+146*a*b*d
*e^3-66*a*c*d^2*e^2-33*b^2*d^2*e^2-110*b*c*d^3*e-385*c^2*d^4)/d^2/e^3*x^3+1
/128*(93*a^2*e^4-10*a*b*d*e^3-6*a*c*d^2*e^2-3*b^2*d^2*e^2-10*b*c*d^3*e-35*c
^2*d^4)/d/e^4*x)/(e*x^2+d)^4+35/128/(d*e)^(1/2)*a^2/d^4*arctan(1/(d*e)^(1/2
)*e*x)+5/64/d^3/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a*b+3/64/(d*e)^(1/2
)*a*c/d^2/e^2*arctan(1/(d*e)^(1/2)*e*x)+3/128/d^2/e^2/(d*e)^(1/2)*arctan(1/
(d*e)^(1/2)*e*x)*b^2+5/64/d/e^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b*c+3
5/128/(d*e)^(1/2)*c^2/e^4*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.52, size = 366, normalized size = 1.15

$$-\frac{3\left(93\,c^2d^4e^3-10\,bcd^3e^4-10\,abde^6-35\,a^2e^7-3\left(b^2+2\,ac\right)d^2e^5\right)x^7+\left(511\,c^2d^5e^2+146\,bcd^4e^3-110\,abd^2e^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="maxima")
```

```
[Out] -1/384*(3*(93*c^2*d^4*e^3 - 10*b*c*d^3*e^4 - 10*a*b*d*e^6 - 35*a^2*e^7 - 3*
(b^2 + 2*a*c)*d^2*e^5)*x^7 + (511*c^2*d^5*e^2 + 146*b*c*d^4*e^3 - 110*a*b*d
^2*e^5 - 385*a^2*d*e^6 - 33*(b^2 + 2*a*c)*d^3*e^4)*x^5 + (385*c^2*d^6*e + 1
10*b*c*d^5*e^2 - 146*a*b*d^3*e^4 - 511*a^2*d^2*e^5 + 33*(b^2 + 2*a*c)*d^4*e
^3)*x^3 + 3*(35*c^2*d^7 + 10*b*c*d^6*e + 10*a*b*d^4*e^3 - 93*a^2*d^3*e^4 +
3*(b^2 + 2*a*c)*d^5*e^2)*x)/(d^4*e^8*x^8 + 4*d^5*e^7*x^6 + 6*d^6*e^6*x^4 +
4*d^7*e^5*x^2 + d^8*e^4) + 1/128*(35*c^2*d^4 + 10*b*c*d^3*e + 10*a*b*d*e^3
+ 35*a^2*e^4 + 3*(b^2 + 2*a*c)*d^2*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^
4*e^4)
```

mupad [B] time = 4.57, size = 375, normalized size = 1.18

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(35\,a^2\,e^4+10\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e+35\,c^2\,d^4\right)}{128\,d^{9/2}\,e^{9/2}}-\frac{x\left(-93\,a^2\,e^4+10\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+\right.}{128\,d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(35*a^2*e^4 + 35*c^2*d^4 + 3*b^2*d^2*e^2 + 10*a*
b*d*e^3 + 10*b*c*d^3*e + 6*a*c*d^2*e^2))/(128*d^(9/2)*e^(9/2)) - ((x*(35*c^
2*d^4 - 93*a^2*e^4 + 3*b^2*d^2*e^2 + 10*a*b*d*e^3 + 10*b*c*d^3*e + 6*a*c*d^
2*e^2))/(128*d*e^4) - (x^7*(35*a^2*e^4 - 93*c^2*d^4 + 3*b^2*d^2*e^2 + 10*a*
b*d*e^3 + 10*b*c*d^3*e + 6*a*c*d^2*e^2))/(128*d^4*e) + (x^3*(385*c^2*d^4 -
511*a^2*e^4 + 33*b^2*d^2*e^2 - 146*a*b*d*e^3 + 110*b*c*d^3*e + 66*a*c*d^2*e
^2))/(384*d^2*e^3) - (x^5*(385*a^2*e^4 - 511*c^2*d^4 + 33*b^2*d^2*e^2 + 110
*a*b*d*e^3 - 146*b*c*d^3*e + 66*a*c*d^2*e^2))/(384*d^3*e^2))/(d^4 + e^4*x^8
 + 4*d^3*e*x^2 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**5,x)
```

```
[Out] Timed out
```

3.261 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2} dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}$$
$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\left(3cd^2-e(bd+ae)\right)\int \frac{1}{d+ex^2}\,dx}{2de^2}$$
$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\left(3cd^2-e(bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

fricas [A] time = 0.68, size = 268, normalized size = 3.23

$$\left[\frac{4\,cd^2e^2x^3 + \left(3\,cd^3 - bd^2e - ade^2 + \left(3\,cd^2e - bde^2 - ae^3\right)x^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right) + 2\left(3\,cd^3e - bd^2e^2 + ad\right.}{4\left(d^2e^4x^2 + d^3e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

giac [A] time = 0.16, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-2)}}{2\left(x^2e + d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

maple [A] time = 0.00, size = 118, normalized size = 1.42

$$\frac{ax}{2\left(e\,x^2+d\right)d} + \frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d} - \frac{bx}{2\left(e\,x^2+d\right)e} + \frac{b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e} + \frac{cdx}{2\left(e\,x^2+d\right)e^2} - \frac{3cd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^2} + \frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/2/(e*x^2+d)*a/d*x+1/2/(d*e)^(1/2)*a/d*arctan(1/(d*e)^(1/2)*e*x)-1/2/(e*x^
2+d)*b/e*x+1/2/(d*e)^(1/2)*b/e*arctan(1/(d*e)^(1/2)*e*x)+1/2/(e*x^2+d)*c*d/
e^2*x-3/2/(d*e)^(1/2)*c*d/e^2*arctan(1/(d*e)^(1/2)*e*x)+c/e^2*x
```

maxima [A] time = 2.34, size = 84, normalized size = 1.01

$$\frac{\left(cd^2 - bde + ae^2\right)x}{2\left(de^3x^2 + d^2e^2\right)} + \frac{cx}{e^2} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*d^2 - b*d*e + a*e^2)*x/(d*e^3*x^2 + d^2*e^2) + c*x/e^2 - 1/2*(3*c*d^
2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^2)
```

mupad [B] time = 0.00, size = 77, normalized size = 0.93

$$\frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2 + b\,d\,e + a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{2\,d\,\left(e^3\,x^2 + d\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

sympy [B] time = 0.82, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x\left(ae^2 - bde + cd^2\right)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\,\left(ae^2 + bde - 3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\,\left(ae^2 + bde - 3cd^2\right)\log\left(d^2\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

3.262 $\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2} dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1814, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}$$

$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{\left(3cd^2-e(bd+ae)\right)\int \frac{1}{d+ex^2}\,dx}{2de^2}$$

$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{\left(3cd^2-e(bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

fricas [A] time = 0.61, size = 268, normalized size = 3.23

$$\left[\frac{4\,cd^2e^2x^3 + \left(3\,cd^3 - bd^2e - ade^2 + \left(3\,cd^2e - bde^2 - ae^3\right)x^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right) + 2\left(3\,cd^3e - bd^2e^2 + ade^3\right)}{4\left(d^2e^4x^2 + d^3e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

giac [A] time = 0.15, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-2)}}{2\left(x^2e + d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

maple [A] time = 0.01, size = 118, normalized size = 1.42

$$\frac{ax}{2\left(e\,x^2+d\right)d} + \frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,d} - \frac{bx}{2\left(e\,x^2+d\right)e} + \frac{b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e} + \frac{cdx}{2\left(e\,x^2+d\right)e^2} - \frac{3cd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^2} + \frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x)
```

```
[Out] 1/2/(e*x^2+d)*a/d*x+1/2/(d*e)^(1/2)*a/d*arctan(1/(d*e)^(1/2)*e*x)-1/2/(e*x^
2+d)*b/e*x+1/2/(d*e)^(1/2)*b/e*arctan(1/(d*e)^(1/2)*e*x)+1/2/(e*x^2+d)*c*d/
e^2*x-3/2/(d*e)^(1/2)*c*d/e^2*arctan(1/(d*e)^(1/2)*e*x)+c/e^2*x
```

maxima [A] time = 2.37, size = 84, normalized size = 1.01

$$\frac{\left(cd^2 - bde + ae^2\right)x}{2\left(de^3x^2 + d^2e^2\right)} + \frac{cx}{e^2} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*d^2 - b*d*e + a*e^2)*x/(d*e^3*x^2 + d^2*e^2) + c*x/e^2 - 1/2*(3*c*d^
2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^2)
```

mupad [B] time = 0.11, size = 77, normalized size = 0.93

$$\frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(-3\,c\,d^2 + b\,d\,e + a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{2\,d\,\left(e^3\,x^2 + d\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2*(b + c*x^2))/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

sympy [B] time = 0.86, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x\left(ae^2 - bde + cd^2\right)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\,\left(ae^2 + bde - 3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\,\left(ae^2 + bde - 3cd^2\right)\log\left(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x**2*(c*x**2+b))/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

3.263 $\int \frac{\left(d+ex^2\right)^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=459

$$\frac{\left(\frac{2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}+e(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b-}}\right.}{\sqrt{2}\,c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] e^2*(6*c^2*d^2+b^2*e^2-c*e*(a*e+4*b*d))*x/c^3+1/3*e^3*(-b*e+4*c*d)*x^3/c^2+
1/5*e^4*x^5/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e
*(-b*e+2*c*d)*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))+(2*c^4*d^4+b^4*e^4-4*b^2*
c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*
d^2))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e*(-b*e+2*c*d)*(2*c
^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))+(-2*c^4*d^4-b^4*e^4+4*b^2*c*e^3*(a*e+b*d)+4
*c^3*d^2*e*(3*a*e+b*d)-2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*d^2))/(-4*a*c+b^2
)^(1/2))/c^(7/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.54, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1170, 1166, 205}

$$\frac{\left(\frac{2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}+e(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b-}}\right.}{\sqrt{2}\,c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*
x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2
*c*e*(b*d + a*e)) + (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*
d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2
*e^2 - 2*c*e*(b*d + a*e)) - (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e)
- 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^4}{a+bx^2+cx^4}\,dx=\int \left(\frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)}{c^3}+\frac{e^3(4cd-be)x^2}{c^2}+\frac{e^4x^4}{c}+\frac{c^3d^4-6ac^2d^2e^2-ab^2e^4\cdots}{\cdots}\right.$$

$$=\frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3}+\frac{e^3(4cd-be)x^3}{3c^2}+\frac{e^4x^5}{5c}+\frac{\int \frac{c^3d^4-6ac^2d^2e^2-ab^2e^4+ace^3(4b\cdots}{\cdots}}{\cdots}$$

$$=\frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3}+\frac{e^3(4cd-be)x^3}{3c^2}+\frac{e^4x^5}{5c}+\frac{\left(e(2cd-be)\left(2c^2d^2+b^2e^{\cdots}\right.\right.}{\cdots}$$

$$=\frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3}+\frac{e^3(4cd-be)x^3}{3c^2}+\frac{e^4x^5}{5c}+\frac{\left(e(2cd-be)\left(2c^2d^2+b^2e^{\cdots}\right.\right.}{\cdots}$$

Mathematica [A] time = 0.69, size = 570, normalized size = 1.24

$$\frac{e^2x\left(-ce(ae+4bd)+b^2e^2+6c^2d^2\right)}{c^3}+\frac{\left(4c^3d^2e\left(d\sqrt{b^2-4ac}-3ae-bd\right)+2c^2e^2\left(-3bd\left(d\sqrt{b^2-4ac}-2ae\right)+ae\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*
x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((2*c^4*d^4 + b^3*(b - Sqrt[b^2 - 4*a*c])*
e^4 + 4*c^3*d^2*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*b*c*e^3*(-2*b^
2*d + 2*b*Sqrt[b^2 - 4*a*c]*d - 2*a*b*e + a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^
2*(3*b^2*d^2 - 3*b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + a*e*(-2*Sqrt[b^2 - 4*a
*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c^4*d^4
+ b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 4*c^3*d^2*e*(b*d + Sqrt[b^2 - 4*a*c]*d
+ 3*a*e) - 2*b*c*e^3*(2*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 2*b*(Sqrt[b^2 - 4*a
*c]*d + a*e)) + 2*c^2*e^2*(3*b^2*d^2 + a*e*(2*Sqrt[b^2 - 4*a*c]*d + a*e) +
3*b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.63, size = 9285, normalized size = 20.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/8*(4*(2*b^4*c^5 - 16*a*b^2*c^6 + 32*a^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*c^6 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6)*c^2*d^3*e + 2*(s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^6 + 2*b^4*c^6 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^7 + 8*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^2*c^7 - 16*a*b^2*c^7 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*c^8 + 32*a^2*c^8 - 2*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7)*d
^4*abs(c) - 6*(2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)
*c^2*d^2*e^2 + 2*(2*b^3*c^8 - 8*a*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b*c^8 - 2*(b^2 - 4*a*c)*b*c^8)*d^4 + 4*(2*b^6*c^3 - 18*a*b^4*c^4 +
48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^6*c + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^
3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 5*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*
a*c)*b^4*c^3 + 10*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*d*
e^3 - 12*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^3*c^5 + 2*a*b^4*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^3*c^6 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 16*a^2*b^2*c^6 - 4*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^7 + 32*a^3*c^7 - 2*(b^2 - 4*a*c)*a*b^2*c
^5 + 8*(b^2 - 4*a*c)*a^2*c^6)*d^2*abs(c)*e^2 - 4*(2*b^4*c^7 - 8*a*b^2*c^8 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^7 - 2*(b^2 - 4*a*c)*b^2*c
^7)*d^3*e - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(

b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)
*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*c^2*e^4 + 8*(sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^5*c
^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c)*a^
2*b*c^5)*d*abs(c)*e^3 + 6*(2*b^5*c^6 - 12*a*b^3*c^7 + 16*a^2*b*c^8 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^7 - 2*(b^2 - 4*a*c)*b^3*c^6 + 4*(b^2 - 4*a*
c)*a*b*c^7)*d^2*e^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2
 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*a*b^6*c^3 + 24*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 18*a^2*b
^4*c^4 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^5 + 48*a^3*b^2*c^5 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*c^6 - 32*a^4*c^6 - 2*(b^2 - 4*a*c)*a*b^4*c^3 + 10*(b^2 - 4*a*c)*a
^2*b^2*c^4 - 8*(b^2 - 4*a*c)*a^3*c^5)*abs(c)*e^4 - 4*(2*b^6*c^5 - 14*a*b^4*
c^6 + 24*a^2*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^6*c^3 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
5*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^
2*c^5 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 3*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*(b^
2 - 4*a*c)*b^4*c^5 + 6*(b^2 - 4*a*c)*a*b^2*c^6)*d*e^3 + (2*b^7*c^4 - 16*a*b
^5*c^5 + 36*a^2*b^3*c^6 - 16*a^3*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^7*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^6*c^3 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^4*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^5*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^6 - 2*(b^2 - 4*a*c)*b^5*c^4 + 8*(b^2 - 4*a*c)*a*b^3*c^5 - 4*(b^2 - 4*a*c)
*a^2*b*c^6)*e^4)*arctan(2*sqrt(1/2)*x/sqrt((b*c^5 + sqrt(b^2*c^10 - 4*a*c^1
1))/c^6))/((a*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^3*c^6 + 16*a^3*c^7 + 8*a^2*b*
c^7 + a*b^2*c^7 - 4*a^2*c^8)*c^2) - 1/8*(4*(2*b^4*c^5 - 16*a*b^2*c^6 + 32*a
^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3
+ 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^6 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(
b^2 - 4*a*c)*a*c^6)*c^2*d^3*e - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*

```
b^4*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^6 - 2*b^4*c^6 + 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*c^7 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c^7 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^7 + 16*a*b^2*c^7 - 4*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^8 - 32*a^2*c^8 + 2*(b^2 - 4*a*c
)*b^2*c^6 - 8*(b^2 - 4*a*c)*a*c^7)*d^4*abs(c) - 6*(2*b^5*c^4 - 16*a*b^3*c^5
 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3
 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 -
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c
)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*d^2*e^2 + 2*(2*b^3*c^8 - 8*a*b*c^9
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^7 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^7 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^8 - 2*(b^2 - 4*a*c)*b*c^8)*
d^4 + 4*(2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 9*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^3 + 10*(b^2 - 4*a*c)*a*b^2*c
^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*d*e^3 + 12*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^4*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^
5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 2*a*b^4*c^5 + 16*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^6 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^6 + 16*a^2*b^2*c^6 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^7 -
32*a^3*c^7 + 2*(b^2 - 4*a*c)*a*b^2*c^5 - 8*(b^2 - 4*a*c)*a^2*c^6)*d^2*abs(c
)*e^2 - 4*(2*b^4*c^7 - 8*a*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^7 - 2*(b^2 - 4*a*c)*b^2*c^7)*d^3*e - (2*b^7*c^2 - 20*a*b^5*c^3 +
 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2
+ 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 +
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 +
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2
 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4
)*c^2*e^4 - 8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^4*c^4 - 2*a*b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5
+ sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 16*a^2*b^3*c^5 - 4*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 32*a^3*b*c^6 + 2*(b^2 - 4
*a*c)*a*b^3*c^4 - 8*(b^2 - 4*a*c)*a^2*b*c^5)*d*abs(c)*e^3 + 6*(2*b^5*c^6 -
```

```
12*a*b^3*c^7 + 16*a^2*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^5*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^6
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^7 - 2*(
b^2 - 4*a*c)*b^3*c^6 + 4*(b^2 - 4*a*c)*a*b*c^7)*d^2*e^2 + 2*(sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 2*
a*b^6*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 10*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^4*c^4 + 18*a^2*b^4*c^4 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5
- 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 48*a^3*b^2*c^5 +
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 32*a^4*c^6 + 2*(b^2 - 4
*a*c)*a*b^4*c^3 - 10*(b^2 - 4*a*c)*a^2*b^2*c^4 + 8*(b^2 - 4*a*c)*a^3*c^5)*a
bs(c)*e^4 - 4*(2*b^6*c^5 - 14*a*b^4*c^6 + 24*a^2*b^2*c^7 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^3 + 7*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*(b^2 - 4*a*c)*b^4*c^5 + 6*(b^2 - 4*a*c)*a*
b^2*c^6)*d*e^3 + (2*b^7*c^4 - 16*a*b^5*c^5 + 36*a^2*b^3*c^6 - 16*a^3*b*c^7
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^2 + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^3 - 18*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 2*(b^2 - 4*a*c)*b^5*c^4 + 8*(b^2
- 4*a*c)*a*b^3*c^5 - 4*(b^2 - 4*a*c)*a^2*b*c^6)*e^4)*arctan(2*sqrt(1/2)*x/s
qrt((b*c^5 - sqrt(b^2*c^10 - 4*a*c^11))/c^6))/((a*b^4*c^5 - 8*a^2*b^2*c^6 -
 2*a*b^3*c^6 + 16*a^3*c^7 + 8*a^2*b*c^7 + a*b^2*c^7 - 4*a^2*c^8)*c^2) + 1/1
5*(3*c^4*x^5*e^4 + 20*c^4*d*x^3*e^3 - 5*b*c^3*x^3*e^4 + 90*c^4*d^2*x*e^2 -
60*b*c^3*d*x*e^3 + 15*b^2*c^2*x*e^4 - 15*a*c^3*x*e^4)/c^5
```

maple [B] time = 0.05, size = 1888, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 4/3/c*d*e^3*x^3-1/3*e^4/c^2*x^3*b+e^4/c^3*b^2*x-a/c^2*e^4*x+6/c*d^2*e^2*x-4
*e^3/c^2*b*d*x-2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^3*e-6/c/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)*a*b*d*e^3-6/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*d*e^
3+1/2/c^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e^4-1/2/c^3*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e^4-
c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^4-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
```

```
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*d^4-3/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d^2*e^2+2/c^2
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*e^4+6/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*a*d^2*e^2+2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^
3*e*b+6/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*d^2*e^2+2/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*d^3*e*b-1/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*e^4-1/c/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*e^4-1/2/c^3/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*b^4*e^4-1/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*e^4-
1/2/c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*e^4+2/c*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
c*x)*a*d*e^3-2/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d*e^3+3/c*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b*d^2*e^2+1/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*e^4-2/c*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*d*e^3
+2/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d*e^3-3/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d^2*e^2+2/c^2/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d*e^3-3/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*b^2*d^2*e^2+2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*a*b^2*e^4+2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d*e^3+2*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*d^3*e+1/5/c*e^4*x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*e^4*x^5 + 5*(4*c^2*d*e^3 - b*c*e^4)*x^3 + 15*(6*c^2*d^2*e^2 - 4
*b*c*d*e^3 + (b^2 - a*c)*e^4)*x)/c^3 + integrate((c^3*d^4 - 6*a*c^2*d^2*e^2
 + 4*a*b*c*d*e^3 - (a*b^2 - a^2*c)*e^4 + (4*c^3*d^3*e - 6*b*c^2*d^2*e^2 + 4
*(b^2*c - a*c^2)*d*e^3 - (b^3 - 2*a*b*c)*e^4)*x^2)/(c*x^4 + b*x^2 + a), x)/
c^3
```

mupad [B] time = 9.31, size = 29551, normalized size = 64.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4/(a + b*x^2 + c*x^4),x)
```

```
[Out] x*((b*((b*e^4)/c^2 - (4*d*e^3)/c))/c - (a*e^4)/c^2 + (6*d^2*e^2)/c) - x^3*(
(b*e^4)/(3*c^2) - (4*d*e^3)/(3*c)) + atan(((((16*a*c^8*d^4 + 16*a^3*c^6*e^4
 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^
2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 - (
2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c
- b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 2
8*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8
- 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d
^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*
c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^
5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^
2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2
)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7
*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a
*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c
^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d
^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*
e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2
*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2
)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*
d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^
(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(
-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2
))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b
^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*
a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8
+ a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*
d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^
2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b
^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a
^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c
^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*
(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70
*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*
b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*
c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) +
24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c
 - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4
*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^
2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e
^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) - (2*x*(b^8*e^8 + 2
*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b
^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2
- 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2
*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*
d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 2
80*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*
a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8
 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) -
11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 +
 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(
1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*
c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*
e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d
^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^
4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2
```

) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2)*1i - (((16*a*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) + (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4

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*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^
5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*
a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8
 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*
e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c
^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*
b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^
3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^
3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c -
 b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7
*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 -
56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b
^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c
^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^
4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a
*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c -
 b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*
c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)
^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e
^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^
(1/2)*1i)/((((16*a*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e
^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*
b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(
a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4
*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7
*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e
^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4
*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2)
 + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*
e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*
d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*
(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c -
b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*
d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e
^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 -
 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^
3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^
(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^
6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2)
 - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c
 - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d
^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2)
- 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7
 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)
^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^
8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^
5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3
*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*
d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1
/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*
b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^
3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d
*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^
6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*
a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^
2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*
```

d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^
(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e
^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84
*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 -
 8*a^2*b^2*c^8)))^(1/2) - (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*
c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4
 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^
4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c
^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 +
56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3
*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*
c^3*d*e^7))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1
/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*
e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3
*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448
*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4
*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 +
448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^
5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) -
 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b
^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5
*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 -
 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264
*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c
^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4
*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) -
 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4
*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*
b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) - (2*(a^4*
b^3*e^12 - 4*c^7*d^11*e + b^7*d^4*e^8 - 4*a*b^6*d^3*e^9 - 4*a^3*b^4*d*e^11
 - 12*a*c^6*d^9*e^3 + 4*a^5*c^2*d*e^11 + 22*b*c^6*d^10*e^2 - 8*b^6*c*d^5*e^7
 + 6*a^2*b^5*d^2*e^10 - 8*a^2*c^5*d^7*e^5 + 8*a^3*c^4*d^5*e^7 + 12*a^4*c^3*
d^3*e^9 - 52*b^2*c^5*d^9*e^3 + 69*b^3*c^4*d^8*e^4 - 56*b^4*c^3*d^7*e^5 + 28
*b^5*c^2*d^6*e^6 - 2*a^5*b*c*e^12 - 48*a^2*b^2*c^3*d^5*e^7 + 50*a^2*b^3*c^2
*d^4*e^8 + 8*a^3*b^2*c^2*d^3*e^9 + 54*a*b*c^5*d^8*e^4 + 26*a*b^5*c*d^4*e^8
+ 4*a^4*b^2*c*d*e^11 - 104*a*b^2*c^4*d^7*e^5 + 112*a*b^3*c^3*d^6*e^6 - 72*a
*b^4*c^2*d^5*e^7 + 28*a^2*b*c^4*d^6*e^6 - 28*a^2*b^4*c*d^3*e^9 - 20*a^3*b*c
^3*d^4*e^8 + 8*a^3*b^3*c*d^2*e^10 - 18*a^4*b*c^2*d^2*e^10))/c^5 + (((16*a*c
^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*
e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b
^2*c^6*d^2*e^2)/c^5 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(a*b^9*e^8 + b^3*c^7*
d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2)
 - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^
7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3
)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b
^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d
^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^
3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5
*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(
1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a
*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c
^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*
d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e
^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56
*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b
^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3
*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)

^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) + (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2)))*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3* c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2)*2i + atan(((((16*a*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) - (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*

```
e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3
*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^
8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2
*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*
c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3
*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b
^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a
^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c
- b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^
7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 +
 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*
b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*
c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c
^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*
a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c
- b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b
*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2
)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*
e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))
^(1/2)*1i - (((16*a*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*
e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2
*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*((
c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4
*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7
*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e
^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4
*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2)
 - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*
e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*
d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*
(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c -
b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*
d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e
^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 +
 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^
3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^
(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^
6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2)
 - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c
 - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*((c^7*d^8*(-(4*a*c - b^2
)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) +
 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7
- 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^
(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8
*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5
*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*
d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d
^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b
^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3
*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*
e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6
 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a
*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2
)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d
^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(
```

1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^
7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*
a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 -
8*a^2*b^2*c^8)))^(1/2) + (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c
^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4
- 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4
*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^
7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 5
6*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3*
c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c
^3*d*e^7))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^
8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^
8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c
^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a
^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a
*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 44
8*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5
- 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 2
8*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4
*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e
^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 1
12*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a
^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6
*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(
-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 2
8*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a
*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^
5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2)*1i)/((((16*a
*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^
5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a
*b^2*c^6*d^2*e^2)/c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*((c^7*d^8*(-(4*a*c -
b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2
) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e
^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^
3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*
b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*
d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c
^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^
5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^
(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*
a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*
c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2
*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*
e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5
6*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c -
b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^
3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3
)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d
e^7(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) +
84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7
 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^
7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 -
 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^
8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7
*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^
2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*
c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*

b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b
^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d
^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56
*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7
*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6
*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*
d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b
^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b
^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^
4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3
)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1
/2) - (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2
*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^
6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c
^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d
*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 +
56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a
*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*((c
^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*
a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*
e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^
8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*
a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2)
- 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e
^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d
^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(
-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b
^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d
^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^
2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 +
560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3
*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(
1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6
*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2)
- 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c
- b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) - (2*(a^4*b^3*e^12 - 4*c^7*d^11
*e + b^7*d^4*e^8 - 4*a*b^6*d^3*e^9 - 4*a^3*b^4*d*e^11 - 12*a*c^6*d^9*e^3 +
4*a^5*c^2*d*e^11 + 22*b*c^6*d^10*e^2 - 8*b^6*c*d^5*e^7 + 6*a^2*b^5*d^2*e^10
- 8*a^2*c^5*d^7*e^5 + 8*a^3*c^4*d^5*e^7 + 12*a^4*c^3*d^3*e^9 - 52*b^2*c^5*
d^9*e^3 + 69*b^3*c^4*d^8*e^4 - 56*b^4*c^3*d^7*e^5 + 28*b^5*c^2*d^6*e^6 - 2*
a^5*b*c*e^12 - 48*a^2*b^2*c^3*d^5*e^7 + 50*a^2*b^3*c^2*d^4*e^8 + 8*a^3*b^2*
c^2*d^3*e^9 + 54*a*b*c^5*d^8*e^4 + 26*a*b^5*c*d^4*e^8 + 4*a^4*b^2*c*d*e^11
- 104*a*b^2*c^4*d^7*e^5 + 112*a*b^3*c^3*d^6*e^6 - 72*a*b^4*c^2*d^5*e^7 + 28
*a^2*b*c^4*d^6*e^6 - 28*a^2*b^4*c*d^3*e^9 - 20*a^3*b*c^3*d^4*e^8 + 8*a^3*b^
3*c*d^2*e^10 - 18*a^4*b*c^2*d^2*e^10))/c^5 + (((16*a*c^8*d^4 + 16*a^3*c^6*e
^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*
e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 +
(2*x*(4*b^3*c^7 - 16*a*b*c^8)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7
*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 -
28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8
+ 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*
d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2
*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c
^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b
^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^

2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) + (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2)))*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a

```
*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b
*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^
4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3
- 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8
*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e
^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d
^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1
/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*
a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^
5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*
e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4
+ 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 2
8*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^
2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^
4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1
/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5
*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) +
 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2)*2i +
(e^4*x^5)/(5*c)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.264 $\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=316

$$\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+\right.\right.}{}$$

```
[Out] e^2*(-b*e+3*c*d)*x/c^2+1/3*e^3*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*(e*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))+(-b*e+2*c*d)*(c
^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*(e*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))-(-b*e+2*c*d)*(c^2*d^2+b^2*e^2-
c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))
^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1170, 1166, 205}

$$\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^3)/(3*c) + ((e*(3*c^2*d^2 + b^2*e^2 - c*
e*(3*b*d + a*e)) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(3*c^2*d^2 + b^2*e^2 -
 c*e*(3*b*d + a*e)) - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)
))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{a+bx^2+cx^4}\,dx = \int \left(\frac{e^2(3cd-be)}{c^2}+\frac{e^3x^2}{c}+\frac{c^2d^3-3acde^2+abe^3+e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)x^2}{c^2\left(a+bx^2+cx^4\right)}\right) d$$

$$= \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}+\frac{\int \frac{c^2d^3-3acde^2+abe^3+e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)x^2}{a+bx^2+cx^4}\,dx}{c^2}$$

$$= \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}+\frac{\left(e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)-\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\int}{2c^2}$$

$$= \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}+\frac{\left(e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\text{ta}}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.55, size = 402, normalized size = 1.27

$$\frac{\frac{3\sqrt{2}\left(3c^2de\left(d\sqrt{b^2-4ac}-2ae-bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)+b^2e^3\left(\sqrt{b^2-4ac}-b\right)+2c^3d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\left(3c^2de\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*Sqrt[c]*e^2*(3*c*d - b*e)*x + 2*c^(3/2)*e^3*x^3 + (3*Sqrt[2]*(2*c^3*d^3
+ b^2*(-b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*
d - 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e - a*Sqrt[b^
2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(-2*c^3*d^3 + b^2
*(b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e
) - c*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e
)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

fricas [B] time = 29.06, size = 9584, normalized size = 30.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*e^3*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a
*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2
*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 -
5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 -
30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*
e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^
6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*
(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3
 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*
c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2
+ 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4
*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*
```

```
c^5 - 4*a^2*c^6))*log(-2*(c^8*d^12 - 3*b*c^7*d^11*e + 3*(b^2*c^6 - 4*a*c^7)
*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2*c^6)*d
^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c^3 + 3*a^2*b
^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*e^7 - 9*
(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^7 + 21
*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^6 + 4*a^3*b^
4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^3*c - 3*a^5*
b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x + sqrt(1/2)*((b^2
*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(a*b^3*c^5 -
4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^4
+ 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a*b^6*c^2 - 7
2*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*b^5*c^2
- 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*a^3*b^4*c^2
+ 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c + 13*a^4*b^3*c
^2 - 4*a^5*b*c^3)*e^9 - ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2*b^2*c^7 - 4
*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^4*c^5 - 6*a^
3*b^2*c^6 + 8*a^4*c^7)*e^3)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^
8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*
b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60
*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*
c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*
c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*
c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4
)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^
4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e +
15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3
*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^
5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^1
2 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*
d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^
2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 +
 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5
*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*
b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*
c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4
*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*
b^2*c^5 - 4*a^2*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e
+ 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 -
 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*
b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d
^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8
)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*
b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7
 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b
^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^
3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^
5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a
^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(
a*b^2*c^5 - 4*a^2*c^6))*log(-2*(c^8*d^12 - 3*b*c^7*d^11*e + 3*(b^2*c^6 - 4*
a*c^7)*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2*
c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c^3 + 3
*a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*e^
7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^
7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^6 + 4*
a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^3*c -
3*a^5*b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x - sqrt(1/2)
*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(a*b^3*
c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^
5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a*b^6*c
```

^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*b^
5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*a^3*b^
4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c + 13*a^4
*b^3*c^2 - 4*a^5*b*c^3)*e^9 - ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2*b^2*c
^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^4*c^5
- 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*
a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 5
2*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^
6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^
3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*
a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 -
5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5
*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 +
a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^
5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c
^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 +
(a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^
10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2
*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*
a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5
e^7 + 15(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a
^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 4
4*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^
3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c +
11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)
))/(a*b^2*c^5 - 4*a^2*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*
d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3
*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5
+ (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*sqrt((
c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a
^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 42
8*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d
^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11
*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 -
44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*
a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c
+ 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^1
1)))/(a*b^2*c^5 - 4*a^2*c^6))*log(-2*(c^8*d^12 - 3*b*c^7*d^11*e + 3*(b^2*c^
6 - 4*a*c^7)*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 +
3*a^2*c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c
^3 + 3*a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*
d^5*e^7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 +
(a*b^7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^
6 + 4*a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^
3*c - 3*a^5*b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x + sqr
t(1/2)*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(
a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c
^6)*d^5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a
*b^6*c^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*
a^2*b^5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*
a^3*b^4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c +
13*a^4*b^3*c^2 - 4*a^5*b*c^3)*e^9 + ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2
*b^2*c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^
4*c^5 - 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2
+ 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c
^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*
d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 -
68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4
+ 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*

c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))*log(-2*(c^8*d^12 - 3*b*c^7*d^11*e + 3*(b^2*c^6 - 4*a*c^7)*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2*c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c^3 + 3*a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*e^7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^6 + 4*a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^3*c - 3*a^5*b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x - sqrt(1/2)*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a*b^6*c^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*b^5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*a^3*b^4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c + 13*a^4*b^3*c^2 - 4*a^5*b*c^3)*e^9 + ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2*b^2*c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^4*c^5 - 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2

```
*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*
b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*
a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 -
 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))) + 6*(3*c*d*e^2 - b*e^3)*x)/c^2
```

giac [B] time = 1.35, size = 6407, normalized size = 20.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(3*(2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d^2*e + 2*(s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^5 + 2*b^4*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 8*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^2*c^6 - 16*a*b^2*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*c^7 + 32*a^2*c^7 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6)*d
^3*abs(c) - 3*(2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c
^2*d*e^2 + 2*(2*b^3*c^7 - 8*a*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b*c^7 - 2*(b^2 - 4*a*c)*b*c^7)*d^3 + (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2
*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2
 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*c^2*e^3 - 6*(sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4
*a*c)*a^2*c^5)*d*abs(c)*e^2 - 3*(2*b^4*c^6 - 8*a*b^2*c^7 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
```

b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^6)*d^2*e + 2*(sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 +
 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c)*e^3 + 3*(2*b^5*c^5 - 12*a*b^3*c^6 + 16*a
^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^
3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^
5 + 4*(b^2 - 4*a*c)*a*b*c^6)*d*e^2 - (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2
*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 +
7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c
^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*e^3)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqr
t(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*
a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) - 1/8*(3*(2*b^4*c^4 - 1
6*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*
a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d^2*e - 2*(sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2
*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 2*b^4*c^5 + 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b*c^6 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^6 +
 16*a*b^2*c^6 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^7 - 32*a^2*c^
7 + 2*(b^2 - 4*a*c)*b^2*c^5 - 8*(b^2 - 4*a*c)*a*c^6)*d^3*abs(c) - 3*(2*b^5*
c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4
- 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2*d*e^2 + 2*(2*b^3*c
^7 - 8*a*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^7 - 2*(b^2 -
4*a*c)*b*c^7)*d^3 + (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*

```
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a
*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*c^2*e^3 + 6*(sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 +
16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6
 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*d*abs(
c)*e^2 - 3*(2*b^4*c^6 - 8*a*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^6)*d^2*e - 2*(sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3
 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*
b*c^4)*abs(c)*e^3 + 3*(2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a
*b*c^6)*d*e^2 - (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*
a*b^2*c^5)*e^3)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))
/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6
 + a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/3*(c^2*x^3*e^3 + 9*c^2*d*x*e^2 - 3*b*c*x
*e^3)/c^3
```

maple [B] time = 0.04, size = 1211, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3/c*e^3*x^3-e^3/c^2*b*x+3/c*d*e^2*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e^3-1/2
/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e^3+3/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d*e^2-3/2*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*d^2*e-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
a*b*e^3+3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*d*e^2+1/2/c^2/(-4*a*c+
```

```
b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e^3-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*b^2*d*e^2+3/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^2*e*b
-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^3-1/2/c*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*
e^3+1/2/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e^3-3/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d*e^2+3/2
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*d^2*e-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*e
^3+3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*d*e^2+1/2/c^2/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b^3*e^3-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b
^2*d*e^2+3/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^2*e*b-c/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*d^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ce^3x^3+3\left(3\,cde^2-be^3\right)x}{3\,c^2}-\frac{-\int\frac{c^2d^3-3\,acde^2+abe^3+\left(3\,c^2d^2e-3\,bcde^2+\left(b^2-ac\right)e^3\right)x^2}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*e^3*x^3 + 3*(3*c*d*e^2 - b*e^3)*x)/c^2 - integrate(-(c^2*d^3 - 3*a*c
*d*e^2 + a*b*e^3 + (3*c^2*d^2*e - 3*b*c*d*e^2 + (b^2 - a*c)*e^3)*x^2)/(c*x^
4 + b*x^2 + a), x)/c^2
```

mupad [B] time = 7.29, size = 17954, normalized size = 56.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3
- 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^6
*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6
*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2
)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^
2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c -
 b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1
/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 -
60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a
^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*d
^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/
2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-(4*a
*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2))/c^
3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^
6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^
```

```
6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^
2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a
^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c
- b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(
1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 -
 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*
a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*
d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1
/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2) -
(2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^
2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b
^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4
*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4))
/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4
*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2
*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c -
 b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 12
0*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a
*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3
)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^
4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 1
08*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c
^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)
^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2)
*1i - (((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3
- 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^6
*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6
*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2
)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^
2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c -
 b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1
/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 -
60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a
^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*d
^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/
2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-(4*a
*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2))/c^
3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^
6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^
6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^
2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a
^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c
- b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(
1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 -
 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*
a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*
d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1
/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2) +
(2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^
2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b
^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4
*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4))
/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4
*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2
*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c -
```

b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 12
0*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a
*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3
)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^
4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 1
08*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c
^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)
^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2)
1i)/((2(3*c^5*d^8*e - a^4*c*e^9 + a^3*b^2*e^9 - b^5*d^3*e^6 + 3*a*b^4*d^2
*e^7 - 3*a^2*b^3*d*e^8 + 8*a*c^4*d^6*e^3 - 12*b*c^4*d^7*e^2 + 6*b^4*c*d^4*e
^5 + 6*a^2*c^3*d^4*e^5 + 19*b^2*c^3*d^6*e^3 - 15*b^3*c^2*d^5*e^4 - 24*a*b*c
^3*d^5*e^4 - 14*a*b^3*c*d^3*e^6 + 27*a*b^2*c^2*d^4*e^5 - 12*a^2*b*c^2*d^3*e
^6 + 9*a^2*b^2*c*d^2*e^7))/c^3 + (((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*
c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 - (
2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c
- b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20
*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 +
a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6
- 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15
*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*
c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^
3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 18
0*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d
e^5(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 -
8*a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c
- b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 2
0*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6
+ a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^
6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 1
5*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2
*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e
^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 1
80*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*
d*e^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1
/2) - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5
- 8*a^2*b^2*c^6)))^(1/2) - (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a
*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2
- 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e -
6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e
^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*
a*c - b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6
- 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e
^6 + a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6
*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4
- 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*
b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^
3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5
+ 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)
^(1/2) - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c
^5 - 8*a^2*b^2*c^6)))^(1/2) + (((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3
*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 + (2*x
*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b
^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^

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4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^
3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 -
6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^
2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e
^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 +
 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a
^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^
5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2)
- 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*
a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c -
b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a
^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a
^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 -
 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a
^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*
e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3
+ 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*
a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2
)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e
^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8
*a^2*b^2*c^6)))^(1/2) + (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a*c^
5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 - 2
0*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e - 6*
b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e^5
- 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 2
0*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6
+ a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^
6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 1
5*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2
*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e
^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 1
80*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*
d*e^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1
/2) - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5
- 8*a^2*b^2*c^6)))^(1/2)))*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c -
b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a
^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a
^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 -
 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a
^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*
e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3
+ 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*
a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2
)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e
^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8
*a^2*b^2*c^6)))^(1/2)*2i + atan(((((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*
c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 - (
2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c
- b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20
*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 -
 a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6
 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 15
*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2*
c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^
```

3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 18
0*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c^2*d
e^5(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/
2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 -
 8*a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c
 - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 2
0*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6
- a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^
6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 1
5*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2
*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e
^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 1
80*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c^2*
d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1
/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5
- 8*a^2*b^2*c^6)))^(1/2) - (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a
*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2
- 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e -
 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e
^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*
a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6
- 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e
^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6
*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4
+ 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*
b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^
3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5
+ 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c
^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)
^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c
^5 - 8*a^2*b^2*c^6)))^(1/2)*1i - (((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*
c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 + (
2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c
- b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20
*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 -
 a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6
 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 15
*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2*
c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^
3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 18
0*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c^2*d
e^5(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/
2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 -
 8*a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c
 - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 2
0*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6
- a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^
6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 1
5*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2
*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e
^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 1
80*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c^2*
d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1
/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5

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- 8*a^2*b^2*c^6)))^(1/2) + (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a
*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2
- 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e -
 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e
^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*
a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6
- 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e
^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6
*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4
+ 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*
b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^
3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5
+ 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c
^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)
^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c
^5 - 8*a^2*b^2*c^6)))^(1/2)*1i)/((2*(3*c^5*d^8*e - a^4*c*e^9 + a^3*b^2*e^9
- b^5*d^3*e^6 + 3*a*b^4*d^2*e^7 - 3*a^2*b^3*d*e^8 + 8*a*c^4*d^6*e^3 - 12*b*
c^4*d^7*e^2 + 6*b^4*c*d^4*e^5 + 6*a^2*c^3*d^4*e^5 + 19*b^2*c^3*d^6*e^3 - 15
*b^3*c^2*d^5*e^4 - 24*a*b*c^3*d^5*e^4 - 14*a*b^3*c*d^3*e^6 + 27*a*b^2*c^2*d
^4*e^5 - 12*a^2*b*c^2*d^3*e^6 + 9*a^2*b^2*c*d^2*e^7))/c^3 + (((16*a*c^6*d^3
 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 +
12*a*b^2*c^4*d*e^2)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*
c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(
1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d
*e^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*
c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 1
05*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a
*b^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^
4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2
+ 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15
*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2
*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3
*c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^
(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*
d*e^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3
*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 -
105*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*
a*b^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d
^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2
 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 1
5*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^
2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2) - (2*x*(b^6*e^6 + 2*c^6
*d^6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^
2*e^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*
b^4*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c
^2*d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 +
b^3*c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^
3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c
^4*d*e^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*
a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3
 - 105*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) -
12*a*b^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^
4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*
e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5
```

```
- 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c -
 b^2)^3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2
*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2) + (((16*a*c^6*d^3 -
4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*
a*b^2*c^4*d*e^2)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5
*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2
) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^
5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5
*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*
a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^
2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e
^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 4
8*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*
c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^
4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3*c^
5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/
2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e
^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^
5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105
*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b
^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*
e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 +
48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a
*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e
^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2) + (2*x*(b^6*e^6 + 2*c^6*d^
6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e
^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4
*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*
d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + b^3
*c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^
(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*
d*e^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3
*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 -
105*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*
a*b^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d
^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2
 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 1
5*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^
2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2)))*(-(a*b^7*e^6 + b^3*c^
5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/
2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e
^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^
5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105
*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b
^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*
e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 +
48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a
*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e
^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2)*2i - x*((b*e^3)/c^2 - (3*d
*e^2)/c) + (e^3*x^3)/(3*c)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.265 $\int \frac{\left(d+ex^2\right)^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=238

$$\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] e^2*x/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e*(-b*e
+2*c*d)+(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*(e*(-b*e+2*c*d)+(-2*c^2*d^2-b^2*e^2+2*c*e*(a*e+b*d))/(-4*
a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1170, 1166, 205}

$$\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*x)/c + ((e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e) - (2*c^2*
d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{a+bx^2+cx^4}\,dx = \int \left(\frac{e^2}{c} + \frac{cd^2-ae^2+e(2cd-be)x^2}{c\left(a+bx^2+cx^4\right)}\right) dx$$

$$= \frac{e^2x}{c} + \frac{\int \frac{cd^2-ae^2+e(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{c}$$

$$= \frac{e^2x}{c} + \frac{\left(e(2cd-be) - \frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} + \frac{\left(e(2cd-be) + \frac{2c^2d^2+b^2e}{\sqrt{b}}\right.}{}$$

$$= \frac{e^2x}{c} + \frac{\left(e(2cd-be) + \frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e(2cd-be) - \frac{2c^2d^2+b^2e}{\sqrt{b}}\right.}{\sqrt{2}\,c^{3/2}}$$

Mathematica [A] time = 0.32, size = 269, normalized size = 1.13

$$\frac{\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*e^2*x + (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*
c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4
*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

fricas [B] time = 3.33, size = 4690, normalized size = 19.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^2*x - sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2
*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3
 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2
*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^
2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5
 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7
- (a^3*b^2 - a^4*c)*e^8)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*
c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c -
 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 +
 (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 - ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 -
 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c
^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a
*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*
(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2
```

```
*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^
2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 -
4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*
c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3
*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c +
a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) + sqrt(
1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c -
2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((
c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(
a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8
*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^
2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^8 - 2*b*c^4*
d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 +
 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^
2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*
e^8)*x - sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4
*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28
*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^
2*c + 4*a^4*c^2)*e^6 - ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*
a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^
6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^
4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c
^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7
)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a
^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6
*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b
^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a
^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b
^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^3*
d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (
a*b^3 - 3*a^2*b*c)*e^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d
^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c
^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*
c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^
7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d
^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4
+ 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2
*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x + sqrt(1/2)*(
(b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2
- 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 -
4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6
 + ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*
b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^
5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^
2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4
 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4
 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b
^3 - 3*a^2*b*c)*e^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*
e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)
*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2
)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7))
)/(a*b^2*c^3 - 4*a^2*c^4))) + sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e
+ 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e
^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d
^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a
^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^
4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*
a^2*c^4))*log(2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 -
4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2
```

```
*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 +
a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x - sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*
d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e
^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3
*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 + ((a*b^3*c^4 - 4*a
^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^
4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3
*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^
3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4
*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6
*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4
- (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*
e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*
b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 -
 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2
*c^4))))/c
```

giac [B] time = 1.14, size = 4107, normalized size = 17.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] x*e^2/c + 1/8*(2*(2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*d*e
+ 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^3*c^4 + 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 +
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^2*c^5 - 16*a*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*c^6 + 32*a^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*
c^5)*d^2*abs(c) - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2
*e^2 + 2*(2*b^3*c^6 - 8*a*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b*c^6 - 2*(b^2 - 4*a*c)*b*c^6)*d^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 -
16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3
*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c)*e^2 - 2*
(2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
```

)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*d*e + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*e^2)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*(2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*d*e - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 16*a*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^6 - 32*a^2*c^6 + 2*(b^2 - 4*a*c)*b^2*c^4 - 8*(b^2 - 4*a*c)*a*c^5)*d^2*abs(c) - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*e^2 + 2*(2*b^3*c^6 - 8*a*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b*c^6)*d^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c)*e^2 - 2*(2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*d*e + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*e^2)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)

maple [B] time = 0.03, size = 695, normalized size = 2.92

$$\frac{\sqrt{2}\, a\, e^{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{\sqrt{-4 a c+b^{2}}\, \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}+\frac{\sqrt{2}\, a\, e^{2} \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{\sqrt{-4 a c+b^{2}}\, \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}-\frac{\sqrt{2}\, b^{2} e^{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b\right.}}}\right.}{2 \sqrt{-4 a c+b^{2}}\, \sqrt{\left(-b+\sqrt{-4 a c+b\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/c*e^2*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e^2-2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d*e+1/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e^2-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*b^2*e^2+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d*e-c/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^2-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e^2
+2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*d*e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e^2-1/2/c
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e^2+1/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*b*d*e-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e^{2} x}{c} - \frac{-\int \frac{c d^{2} - a e^{2} + \left(2 \, c d e - b e^{2}\right) x^{2}}{c x^{4} + b x^{2} + a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] e^2*x/c - integrate(-(c*d^2 - a*e^2 + (2*c*d*e - b*e^2)*x^2)/(c*x^4 + b*x^2
 + a), x)/c
```

mupad [B] time = 6.48, size = 9600, normalized size = 40.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/c
 - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*
a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4
+ 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e
- 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e +
6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2
*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*e^4 + b^3
```

```
*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^
(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(
1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^
3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2
*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(
-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2
) - (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*c^
2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^
3))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^
2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c
*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b
*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a
^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^
3 - 8*a^2*b^2*c^4)))^(1/2)*1i - (((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^
3*d^2 + 4*a*b^2*c^2*e^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*e^4 + b
^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3
)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)
^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*
e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a
^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1
/2))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b
^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*
c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*
b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*
a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c
^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 1
2*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3
*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*
a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4
 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e
 - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e +
6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2
*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i)/((2*(2*c^3*d^5*e -
 a^2*b*e^6 - b^3*d^2*e^4 + 4*a*c^2*d^3*e^3 - 5*b*c^2*d^4*e^2 + 4*b^2*c*d^3*
e^3 + 2*a*b^2*d*e^5 + 2*a^2*c*d*e^5 - 6*a*b*c*d^2*e^4))/c + (((16*a*c^4*d^2
 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/c - (2*x*(4*b^3*c^3 -
16*a*b*c^4)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) -
 a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 +
a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 -
4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 -
 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b
^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-
(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e
^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3
*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e
 + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*
c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(b^4*e^4 + 2
*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c
*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*(-(a*b^5*e^4 +
 b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)
^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^
3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*
d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24
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*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e
^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^
(1/2) + (((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)
/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(
4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^
4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*
e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e
+ 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c
^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*e^4 + b
^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3
)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)
^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*
e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a
^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1
/2) + (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*
c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*
e^3))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*
b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2
*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a
*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24
*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*
c^3 - 8*a^2*b^2*c^4)))^(1/2)))*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*
c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 +
12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e -
32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*
a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*2i + atan(((((16*a*c^4*d
^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/c - (2*x*(4*b^3*c^3
- 16*a*b*c^4)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4
- a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 +
 a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 +
 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2
+ 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*
b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b
^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e
^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3
*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e
 - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*
c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(b^4*e^4 + 2
*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c
*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*((c^3*d^4*(-(4
*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^
3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3
)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d
*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*
a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^
3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(
1/2)*1i - (((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^
2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b
^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e
^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3
*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e
 - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*
```

c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*((c^3*d^4*(-(4*
a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3
)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)
^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*
e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a
^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1
/2) + (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*
c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*
e^3))/c)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b
^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*
c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*
b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*
a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c
^3 - 8*a^2*b^2*c^4)))^(1/2)*1i)/((2*(2*c^3*d^5*e - a^2*b*e^6 - b^3*d^2*e^4
+ 4*a*c^2*d^3*e^3 - 5*b*c^2*d^4*e^2 + 4*b^2*c*d^3*e^3 + 2*a*b^2*d*e^5 + 2*a
^2*c*d*e^5 - 6*a*b*c*d^2*e^4))/c + (((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2
*c^3*d^2 + 4*a*b^2*c^2*e^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((c^3*d^4*(-(
4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)
^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^
3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*
d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24
*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e
^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^
(1/2))/c)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*
b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2
*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a
*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24
*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*
c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 -
12*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^
3*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*
c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4
- 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e
+ 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e -
6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2
*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (((16*a*c^4*d^2 - 16
*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/c + (2*x*(4*b^3*c^3 - 16*a*
b*c^4)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2
e^4(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*
e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*
c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^
2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3
 - 8*a^2*b^2*c^4)))^(1/2))/c)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*
d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12
*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32
*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*
b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2
e^2(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(b^4*e^4 + 2*c^4*d^
4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 -
4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*((c^3*d^4*(-(4*a*c -
b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2
) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2)

```
 - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 +
8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2
*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)))*
((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-
(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(
4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4
 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3
*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^
2*b^2*c^4)))^(1/2)*2i + (e^2*x)/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.266 $\int \frac{d+ex^2}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=174

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] 1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*d)/
(-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a*c+b^2
)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1166, 205}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e -
(2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}} \\ &= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 172, normalized size = 0.99

$$\frac{\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b^2
 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt
[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

fricas [B] time = 0.91, size = 1525, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*
sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x + sqr
t(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2
*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (
a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2
- 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a
*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 -
 b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a
*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c
^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))
*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4
 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^
2))) + 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^
2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))
/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*
x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c
- 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e
^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*
e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b
^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(b*c*d^
2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e
^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^
2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d
^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c -
4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3
*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4
*a^2*c^2)))
```

giac [B] time = 0.87, size = 1402, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2
- 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2
+ a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*
c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b
^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arcta
n(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*
a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

maple [B] time = 0.02, size = 328, normalized size = 1.89

$$-\frac{\sqrt{2}\, be \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, be \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, cd \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b
*e-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/2*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/
2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 5.38, size = 4109, normalized size = 23.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^
2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2
+ a*b^4*c)))^(1/2) - 4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*
c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*
d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^
2*b^2*c^2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4
*b*c^2*d*e))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d
*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i +
((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^
2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*
c)))^(1/2) + 4*b^2*c^2*d - 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)
^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*
a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^
2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d
*e))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a
*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((x*(8*b
^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*
d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a
^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/
2) - 4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2
) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*
e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^
4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(
a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d
*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((x*(8*b^3*c^2 - 32
*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e
- 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 4*b^2*
c^2*d - 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d
^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^
2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2
) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(a*b^3*e^2 +
 a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*
a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*c^2*d^2*e + 2*a*c*e^3 - 2*b*
c*d*e^2))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e
```

```
- 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i - ata
n((((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/
2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c
*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b
^4*c)))^(1/2) - 4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b
^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 -
 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2
*c^2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^
2*d*e))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e -
4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i + ((x*(
8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3
*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 1
6*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^
(1/2) + 4*b^2*c^2*d - 16*a*c^3*d)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(
1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b
*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a
*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*
(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*
c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((x*(8*b^3*c^
2 - 32*a*b*c^3)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 +
 c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^
2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) -
4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b
^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 +
 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c))
)^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(a*b^3
*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(
8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((x*(8*b^3*c^2 - 32*a*b*
c^3)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a
*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 4*b^2*c^2*d
 - 16*a*c^3*d)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 +
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2
*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + x
*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(a*b^3*e^2 - a*e^
2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4
*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c
^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*c^2*d^2*e + 2*a*c*e^3 - 2*b*c*d*e
^2))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a
*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i
```

sympy [A] time = 20.95, size = 314, normalized size = 1.80

$$\operatorname{RootSum}\left(t^4\left(256a^3c^3-128a^2b^2c^2+16ab^4c\right)+t^2\left(-16a^2bce^2+64a^2c^2de+4ab^3e^2-16ab^2cde-16abc^2d^2+4b^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**3 - 128*a**2*b**2*c**2 + 16*a*b**4*c) + _t**2*(-
16*a**2*b*c*e**2 + 64*a**2*c**2*d*e + 4*a*b**3*e**2 - 16*a*b**2*c*d*e - 16*
a*b*c**2*d**2 + 4*b**3*c*d**2) + a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2
 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4, Lambda(_t, _t*log(x + (64*_t*
*3*a**3*c**2*e - 16*_t**3*a**2*b**2*c*e - 32*_t**3*a**2*b*c**2*d + 8*_t**3*
a*b**3*c*d - 2*_t*a**2*b*e**3 + 12*_t*a**2*c*d*e**2 - 6*_t*a*b*c*d**2*e - 4
```

```
*_t*a*c**2*d**3 + 2*_t*b**2*c*d**3)/(a**2*e**4 - a*b*d*e**3 + b*c*d**3*e -
c**2*d**4))))
```

3.267 $\int \frac{1}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*
a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*2^(1/2)*c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1093, 205}

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+bx^2+cx^4} \, dx &= \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} \, dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2} \, dx}{\sqrt{b^2-4ac}}\\ &= \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 0.74, size = 613, normalized size = 4.09

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\cdots}{ab^2\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

giac [B] time = 0.60, size = 1024, normalized size = 6.83

$$\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^4-8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ab^2c-2\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^3c-2b^4c+16\sqrt{2}\sqrt{bc\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
```

```
- 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^
2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs
(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 -
4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4
 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c
^3)*abs(c))
```

maple [A] time = 0.01, size = 116, normalized size = 0.77

$$-\frac{\sqrt{2}\,c\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 0.51, size = 763, normalized size = 5.09

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,x\,16\mathrm{i}-}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*
a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
```

```
- 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2
) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(b^3 + (
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 +
 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i
)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(((b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

sympy [**A**] time = 1.27, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac}{c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

3.268 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=254

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

```
[Out] e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/d^(1/2)-1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*a*
c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e+2*c
*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1170, 205, 1166}

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{cd-be-cex^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{cd-be-cex^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)} - \frac{\left(c\left(e+\frac{2c\ldots}{\sqrt{b^2\ldots}}\right.\right.}{2\ldots}$$

$$= -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\ldots}}{\sqrt{b+\sqrt{b^2\ldots}}}\right.}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde\ldots}$$

Mathematica [A] time = 0.27, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(e\sqrt{b^2-4ac}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-ae^2+bde-cd^2)} + \frac{\sqrt{c}\left(e\sqrt{b^2-4ac}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(-ae^2+bde-cd^2)} + \frac{e^{3/\ldots}}{\sqrt{d}\,(\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) +
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 2.53, size = 7650, normalized size = 30.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*
c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
```

sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e +
2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^3*c^3 - 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^2*c^4 + 16*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*c^5 - 32*a^2*c^5 + 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^
4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2
- 4*a*c)*a*b*c^4)*d^3*e^2 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a
*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (
2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 + 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 2*
b^6*c + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 6*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^4*c^2 + 14*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 3*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*b^4*c -
6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d*abs(c*d^2 - b*d*e +
a*e^2)*e^2 - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a
*e^2)^2*e + 2*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2
*b*c^3)*d*e^4 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt

(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(c*d^2 - b*d*e + a*e^2)*e^3 - (2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 + sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4) - 1/8*(2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 16*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 + 32*a^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^3*e^2 + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sq

```
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c + 8*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 14*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*b^4*c + 6*(b^2 - 4*a*c)*a*b^2*c^2 +
 8*(b^2 - 4*a*c)*a^2*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 - (2*b^4*c^2 - 1
6*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*
c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 -
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2
*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^2*e + 2*(2*a*b^5*c^2
- 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*
c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 -
 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3)*d*e^4 + 2*(sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*
b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a
^2*b*c^2)*abs(c*d^2 - b*d*e + a*e^2)*e^3 - (2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)
*a^2*b^2*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 -
 sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^
2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 -
8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*
c^5)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2
*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*ab
s(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^
2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c
^4)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c -
2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*a
bs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c
 + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c*d^2 - b*d*e +
a*e^2)*abs(c)*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*d^2 - b*d*e + a*
e^2)*sqrt(d))
```

maple [B] time = 0.02, size = 480, normalized size = 1.89

$$\frac{\sqrt{2}\, bce \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2 - bde + c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} + \frac{\sqrt{2}\, bce \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2 - bde + c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] e^2/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)+1/2/(a*e^2-b*
d*e+c*d^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)*d-1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1
/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/(a*e^2
-b*d*e+c*d^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] e^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) - integrate((
c*e*x^2 - c*d + b*e)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

mupad [B] time = 9.45, size = 23640, normalized size = 93.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2
 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3
*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2
 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3
*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*
b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d
^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2
*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*
b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 +
b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^
2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 -
 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^
3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^
2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*
c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) -
16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*
c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^
2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*
b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*
b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 +
512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^
```

5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6
+ 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e
^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 -
640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96
*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2
*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c
^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*
a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5
*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a
^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c
^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2
*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2
*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 +
a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*
b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 1
6*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c
*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2
*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b
^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*
c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*
d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2
*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 -
8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3
*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2
*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^
4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^
2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*
d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^
4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16
*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*
a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 1
6*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4
*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2
d^2(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*
d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 1
2*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*
a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*
e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*
d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a
^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5
*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*
e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e
- 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16
*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*
d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b
*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e
- 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 -
512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*
d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 +
128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^
2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128

*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3
*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*
d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3
*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128
*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d
^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*
b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*
c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3
*e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7
))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7
*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2
*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4
*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2
*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e -
32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*
c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 -
4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^
5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d
^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12
*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a
^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e
^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d
^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^
2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^5*e^2 + b^3*c^2*d
^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a
*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^
2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*
a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d
^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 +
192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3
- 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*
d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2
*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4
*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c
^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32
*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(
-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b
^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2
*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4
+ 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2
*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*
a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*
d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*
e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4
*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e
^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 2
88*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7
+ 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*
a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6
*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^

5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 1
6*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5
*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 6
4*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*
c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*
e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5
- 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*
d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16
*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*
e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*
e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*
c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3
*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 -
20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*
b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*
b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^
3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*
a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - ((-
(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^
3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*
d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4
+ 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*
b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a
^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d
^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 -
112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^
6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d
^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d
^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a
*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^
2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*
a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d
^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2
- 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8
*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a
^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
- 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*
(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d
^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 1
28*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2
*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*
c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4
*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4
*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4
+ 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) -
64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d
^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96

*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*2i + atan((((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d

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^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*
e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8
- 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c
^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4
 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c
^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*
e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*
a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*
b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*
(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*
e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4
- 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 3
2*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e
^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/
2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16
*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^
2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d
^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3
*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)
))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b
*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*
e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4
+ 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6
*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b
^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 +
 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 +
16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 -
 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e
^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^
5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c
*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e
 + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 1
6*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5
*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*
b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*
e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*
d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 +
a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b
^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16
*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*
d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^
9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^
3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4
*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e
^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^
2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 -
672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*
a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*
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b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^
8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3
*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6
 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384
*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4
*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 +
 b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 +
a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c
^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3
- 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d
^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a
^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4
 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5
*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*
e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e
+ 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16
*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*
d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b
*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e
 - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^
2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 -
8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*
a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)
((x(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e
^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d
*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a
*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*
a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3
*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e
^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*
b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*
a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^
3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3
*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e
^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*
a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e
 + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b
^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c
^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 -
32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^
3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*
d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d
^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e
^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 -
1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288
*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e
^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b
^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e

^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 3
2*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3
*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c
^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3
*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e +
12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16
*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2
*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c
*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*
a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^
3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5
) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a
*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*
d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b
^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a
*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3
 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - ((-(b^5*e^2 + b^3*
c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^
4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a
^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e
+ 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^
4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e
^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^
4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b
*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^
2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 -
8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*
a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)
*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e
- 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2
*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b
^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4
+ a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2
- 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*
d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^
3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^
3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*
e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*
a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 3
84*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*
b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5
*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c
^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^
2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^
3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^
4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256
*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5
*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3

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*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b
^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b
^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3
*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a
^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^
3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*
e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2
 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8
*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a
^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
- 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2))
)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*
a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*
c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*
d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*
a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e -
32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c
^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*2i - (log(b^5*d*(-d*e^3)^(5/2) - b^
5*d^3*e^8*x + c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^(3/2) - 16*a^3*c^2*e*(-d
*e^3)^(5/2) - c^5*d^8*e*(-d*e^3)^(1/2) + b^2*c^3*d^5*(-d*e^3)^(3/2) - a*b^4
*e*(-d*e^3)^(5/2) - 7*a*b^3*c*d*(-d*e^3)^(5/2) + 17*a^2*c^3*d^3*e^2*(-d*e^3
)^(3/2) + a*b^4*d^2*e^9*x + 2*a*c^4*d^6*e^5*x - 2*b*c^4*d^7*e^4*x + 2*b^4*c
*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^(5/2) + 8*a^2*b^2*c*e*(-d*e^3)^(5/2) +
 17*a^2*c^3*d^4*e^7*x + 16*a^3*c^2*d^2*e^9*x + b^2*c^3*d^6*e^5*x - b^3*c^2*
d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^(3/2) + 2*b^4*c*d^3*e^2*(-d*e^3)^(3/2) +
 2*b*c^4*d^7*e^2*(-d*e^3)^(1/2) - 12*a*b^2*c^2*d^4*e^7*x - 12*a^2*b*c^2*d^3
*e^8*x - 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^(3/2) + 2*a*
b*c^3*d^5*e^6*x + 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^(3/2))*(-d
*e^3)^(1/2))/(2*(c*d^3 + a*d*e^2 - b*d^2*e)) + (log(b^5*d*(-d*e^3)^(5/2) +
b^5*d^3*e^8*x - c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^(3/2) - 16*a^3*c^2*e*(
-d*e^3)^(5/2) - c^5*d^8*e*(-d*e^3)^(1/2) + b^2*c^3*d^5*(-d*e^3)^(3/2) - a*b
^4*e*(-d*e^3)^(5/2) - 7*a*b^3*c*d*(-d*e^3)^(5/2) + 17*a^2*c^3*d^3*e^2*(-d*e
^3)^(3/2) - a*b^4*d^2*e^9*x - 2*a*c^4*d^6*e^5*x + 2*b*c^4*d^7*e^4*x - 2*b^4
*c*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^(5/2) + 8*a^2*b^2*c*e*(-d*e^3)^(5/2)
 - 17*a^2*c^3*d^4*e^7*x - 16*a^3*c^2*d^2*e^9*x - b^2*c^3*d^6*e^5*x + b^3*c^
2*d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^(3/2) + 2*b^4*c*d^3*e^2*(-d*e^3)^(3/2)
 + 2*b*c^4*d^7*e^2*(-d*e^3)^(1/2) + 12*a*b^2*c^2*d^4*e^7*x + 12*a^2*b*c^2*d
^3*e^8*x + 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^(3/2) - 2*
a*b*c^3*d^5*e^6*x - 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^(3/2))*(
-d*e^3)^(1/2))/(2*c*d^3 + 2*a*d*e^2 - 2*b*d^2*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.269 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=429

$$\frac{\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)^2}-\frac{\sqrt{c}\left(-2ce\left(-d\sqrt{b^2-4ac}\cdots\right.\right.}{\sqrt{2}\cdots}$$

```
[Out] 1/2*e^2*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)+1/2*e^(3/2)*arctan(x*e^(1/2)/d^(1
/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)+e^(3/2)*(-b*e+2*c*d)*arctan(x*e^(1/2)/d^(1
/2))/(a*e^2-b*d*e+c*d^2)^2/d^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-2*c*e*(b
*d+a*e+d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(-4*a*c+b^2)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2))*c^(1/2)*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(b*d
+a*e-d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(-4*a*c+b^2)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.41, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1170, 199, 205, 1166}

$$\frac{\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)^2}-\frac{\sqrt{c}\left(-2ce\left(-d\sqrt{b^2-4ac}\cdots\right.\right.}{\sqrt{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(2*d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + (Sqrt[c]*(2*c^2*d^2 + b
*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4
*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (Sqrt[c]*(2*
c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d
+ a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (
e^(3/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e
+ a*e^2)^2) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 - b*d
*e + a*e^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}\,dx = \int\left(\frac{e^2}{\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)^2} - \frac{e^2(-2cd+be)}{\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} + \frac{c^2d^2+b \cdots}{\left(cd^{\cdots}\right.}\right.$$

$$= \frac{\int \frac{c^2d^2+b^2e^2-ce(2bd+ae)-ce(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{\left(cd^2-bde+ae^2\right)^2} + \frac{\left(e^2(2cd-be)\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2-bde+ae^2\right)^2} + \frac{e^2\int \frac{1}{\left(d+ex^{2}\cdots\right.}}{cd^2-bde \cdots}$$

$$= \frac{e^2x}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{e^{3/2}(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)^2} + \frac{e^2\int \frac{1}{d+ex^{\cdots}}}{2d\left(cd^2-bd\cdots\right.}$$

$$= \frac{e^2x}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{\sqrt{c}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(\cdots\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^{\cdots}}}}$$

Mathematica [A] time = 0.75, size = 354, normalized size = 0.83

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\cdots\right.}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\cdots}{2\left(e(ae-bd)+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((e^2*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^2)) + (Sqrt[2]*Sqrt[c]*(2*c
^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d +
 a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-2*c^2*d^2 + b*(-b
 + Sqrt[b^2 - 4*a*c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]) + (e^(3/2)*(5*c*d^2 + e*(-3*b*d + a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/d^(3/2))/(2*(c*d^2 + e*(-(b*d) + a*e))^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 2.51, size = 13225, normalized size = 30.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 - 3*b*d*e^3 + a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((
c^2*d^5 - 2*b*c*d^4*e + b^2*d^3*e^2 + 2*a*c*d^3*e^2 - 2*a*b*d^2*e^3 + a^2*d
*e^4)*sqrt(d)) - 2*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^3 - b^7
*c^3 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 11*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 12*a*b^5*c^4 + 3*b^6*c^4 + 96*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 88*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^5 - 48*a^2*b^3*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c^5 - 28*a*b^4*c^5 + 5*b^5*c^5 - 128*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*c^6 - 176*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^6 + 64*a^3*b*c^6 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^6 + 80*a^2*b^2*c^6 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^6 -
24*a*b^3*c^6 - 11*b^4*c^6 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
c^7 - 64*a^3*c^7 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^7 + 16*
a^2*b*c^7 - 8*a*b^2*c^7 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^8 +
 80*a^2*c^8 + 16*a*b*c^8 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^5*c^3 + sqrt(b^2 - 4*a*c)*b^6*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 11*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 12*sqrt(b^2 - 4*a*c)*a*b^4*c^4
- 5*sqrt(b^2 - 4*a*c)*b^5*c^4 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^5 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 + 48*sqrt(b^2 - 4*a*c)*a^2*b^2*c^5 - 16*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 40*sqrt(b^2 - 4*
a*c)*a*b^3*c^5 + 7*sqrt(b^2 - 4*a*c)*b^4*c^5 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 64*sqrt(b^2 - 4*a*c)*a^3*c^6 + 3
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 80*sq
rt(b^2 - 4*a*c)*a^2*b*c^6 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^6 - 56*sqrt(b^2 - 4*a*c)*a*b^2*c^6 - 3*sqrt(b^2 - 4*a*c)
*b^3*c^6 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c
^7 + 112*sqrt(b^2 - 4*a*c)*a^2*c^7 + 60*sqrt(b^2 - 4*a*c)*a*b*c^7 - 24*sqrt
(b^2 - 4*a*c)*a*c^8 + 2*(b^2 - 4*a*c)*b^4*c^4 - 16*(b^2 - 4*a*c)*a*b^2*c^5
- 12*(b^2 - 4*a*c)*b^3*c^5 + 32*(b^2 - 4*a*c)*a^2*c^6 + 48*(b^2 - 4*a*c)*a*
b*c^6 + 14*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7)*arctan(2*sqrt(1/2
)*x/sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b
^2*d*e^3 + a^2*b*e^4 + sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*
b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^4)^2 - 4*(a*c^2*d^4 - 2*a*b*c*d^3*e +
 a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a^2*b*d*e^3 + a^3*e^4)*(c^3*d^4 - 2*b*
c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/
(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3
+ a^2*c*e^4)))/((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c - 16*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^7*c^2 - 2*b^8*c^2 + 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c^3 + 60*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 7*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^3 + 16*a*b^6*c^3 + 8*b^7*c^3 -
256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 240*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 84*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c^4 - 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 - 32
*a*b^5*c^4 - 6*b^6*c^4 + 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^
5 + 320*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 336*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 256*a^3*b^2*c^5 + 72*sqrt(2)*sqr
```

t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 128*a^2*b^3*c^5 - 24*a*b^4*c^5 - 4
48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 512*a^4*c^6 - 240*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + 512*a^3*b*c^6 - 24*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 + 224*a^2*b^2*c^6 + 96*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^7 - 128*a^3*c^7 - 16*a*b^2*c^7 + 64*a^2
*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c + 12
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 5*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 48*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 52*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 7*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 64*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 176*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 72*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 3*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 24*sqrt(b^2 - 4*a*c)*a*b
^4*c^4 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
c^5 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c
^5 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5
 + 192*sqrt(b^2 - 4*a*c)*a^2*b^2*c^5 + 32*sqrt(b^2 - 4*a*c)*a*b^3*c^5 + 48*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 384*sqr
t(b^2 - 4*a*c)*a^3*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c^6 - 128*sqrt(b^2 - 4*a*c)*a^2*b*c^6 + 8*sqrt(b^2 - 4*a*c)*
a*b^2*c^6 + 96*sqrt(b^2 - 4*a*c)*a^2*c^7 - 16*sqrt(b^2 - 4*a*c)*a*b*c^7 + 2
*(b^2 - 4*a*c)*b^6*c^2 - 24*(b^2 - 4*a*c)*a*b^4*c^3 - 8*(b^2 - 4*a*c)*b^5*c
^3 + 96*(b^2 - 4*a*c)*a^2*b^2*c^4 + 64*(b^2 - 4*a*c)*a*b^3*c^4 + 6*(b^2 - 4
*a*c)*b^4*c^4 - 128*(b^2 - 4*a*c)*a^3*c^5 - 128*(b^2 - 4*a*c)*a^2*b*c^5 - 4
8*(b^2 - 4*a*c)*a*b^2*c^5 + 96*(b^2 - 4*a*c)*a^2*c^6 + 64*(b^2 - 4*a*c)*a*b
*c^6)*d^2*abs(c) + 4*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 +
 4*a*b^7*c^2 - 36*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 4*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 48*a^2*b^5*c^3 - 10*a*b^
6*c^3 + 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 32*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + 192*a^3*b^3*c^4 - sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 56*a^2*b^4*c^4 + 8*a*b^5*c^4 - 1
92*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 64*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 256*a^4*b*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 32*a^3*b^2*c^5 + 2*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^5 - 6*a*b^4*c^5 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*c^6 - 384*a^4*c^6 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^6 - 128*a^3*b*c^6 + 16*a^2*b^2*c^6 + 8*a*b^3*c^6 + 32*a^3*c^7 - 32*a
^2*b*c^7 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*
c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^
2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 +
 4*sqrt(b^2 - 4*a*c)*a*b^6*c^2 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^4*c^3 - 48*sqrt(b^2 - 4*a*c)*a^2*b^4*c^3 - 10*sqrt(b^
2 - 4*a*c)*a*b^5*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*c^4 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b*c^4 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^4 + 192*sqrt(b^2 - 4*a*c)*a^3*b^2*c^4 - 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 80*sqrt(b^2 - 4*a*c)*a^2*
b^3*c^4 + 16*sqrt(b^2 - 4*a*c)*a*b^4*c^4 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 256*sqrt(b^2 - 4*a*c)*a^4*c^5 + 48*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 160*s
qrt(b^2 - 4*a*c)*a^3*b*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^5 - 96*sqrt(b^2 - 4*a*c)*a^2*b^2*c^5 - 18*sqrt(b^2 -
4*a*c)*a*b^3*c^5 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*c^6 + 128*sqrt(b^2 - 4*a*c)*a^3*c^6 + 40*sqrt(b^2 - 4*a*c)*a^2*b*c
^6 + 8*sqrt(b^2 - 4*a*c)*a*b^2*c^6 - 16*sqrt(b^2 - 4*a*c)*a^2*c^7 + 8*(b^2

- 4*a*c)*a*b^3*c^4 - 32*(b^2 - 4*a*c)*a^2*b*c^5 - 12*(b^2 - 4*a*c)*a*b^2*c^
5 - 16*(b^2 - 4*a*c)*a^2*c^6)*d*abs(c)*e - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^8 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c + 6*a*b^8*c + 96*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^5*c^2 - sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 -
80*a^2*b^6*c^2 - 12*a*b^7*c^2 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^4*b^2*c^3 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 20*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 384*a^3*b^4*c^3 - 5*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 80*a^2*b^5*c^3 + 10*a*b^
6*c^3 + 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^4 - 64*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 208*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^2*c^4 - 768*a^4*b^2*c^4 + 56*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^4 - 64*a^3*b^3*c^4 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c^4 - 24*a^2*b^4*c^4 - 12*a*b^5*c^4 - 448*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*c^5 + 512*a^5*c^5 - 144*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b*c^5 - 256*a^4*b*c^5 - 40*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^5 - 32*a^3*b^2*c^5 + 32*a^2*b^3*c^5 + 16*a*b^4*c^5 + 9
6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^6 - 128*a^4*c^6 + 64*a^3*b*
c^6 - 80*a^2*b^2*c^6 + 64*a^3*c^7 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^7 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^5*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^6*c + 8*sqrt(b^2 - 4*a*c)*a*b^7*c + 48*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 96*sqrt(b^2 - 4*a*c)*a^2*b^5*c^2
- 20*sqrt(b^2 - 4*a*c)*a*b^6*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 384*sqrt(b^2 - 4*a*c)*a^3*b^3*c^3 - 5*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 136*sqrt
(b^2 - 4*a*c)*a^2*b^4*c^3 + 32*sqrt(b^2 - 4*a*c)*a*b^5*c^3 - 192*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 48*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 512*sqrt(b^2 - 4*a*
c)*a^4*b*c^4 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^2*c^4 - 128*sqrt(b^2 - 4*a*c)*a^3*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 160*sqrt(b^2 - 4*a*c)*a^2*b
^3*c^4 - 36*sqrt(b^2 - 4*a*c)*a*b^4*c^4 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 384*sqrt(b^2 - 4*a*c)*a^4*c^5 - 32*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 128*sqr
t(b^2 - 4*a*c)*a^3*b*c^5 + 88*sqrt(b^2 - 4*a*c)*a^2*b^2*c^5 + 16*sqrt(b^2 -
4*a*c)*a*b^3*c^5 + 96*sqrt(b^2 - 4*a*c)*a^3*c^6 - 48*sqrt(b^2 - 4*a*c)*a^2
*b*c^6 + 2*(b^2 - 4*a*c)*a*b^6*c - 24*(b^2 - 4*a*c)*a^2*b^4*c^2 - 8*(b^2 -
4*a*c)*a*b^5*c^2 + 96*(b^2 - 4*a*c)*a^3*b^2*c^3 + 64*(b^2 - 4*a*c)*a^2*b^3*
c^3 + 22*(b^2 - 4*a*c)*a*b^4*c^3 - 128*(b^2 - 4*a*c)*a^4*c^4 - 128*(b^2 - 4
*a*c)*a^3*b*c^4 - 112*(b^2 - 4*a*c)*a^2*b^2*c^4 - 24*(b^2 - 4*a*c)*a*b^3*c^
4 + 96*(b^2 - 4*a*c)*a^3*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^5)*abs(c)*e^2) + 2*
(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^3 + b^7*c^3 - 24*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 11*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^5*c^4 - 12*a*b^5*c^4 - 3*b^6*c^4 + 96*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 88*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^5 + 48*a^2*b^3*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c
^5 + 28*a*b^4*c^5 - 5*b^5*c^5 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*c^6 - 176*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 64*a^3*b
*c^6 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 80*a^2*b^2*c^
6 - 7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 24*a*b^3*c^6 + 11*b
^4*c^6 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^7 + 64*a^3*c^7 +
44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^7 - 16*a^2*b*c^7 + 8*a*b^2
*c^7 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^8 - 80*a^2*c^8 - 16*a*

b*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + sqrt(b^2 - 4*a*c)*b^6*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 12*sqrt(b^2 - 4*a*c)*a*b^4*c^4 - 5*sqrt(b^2 - 4*a*c)*b^5*c^4 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 48*sqrt(b^2 - 4*a*c)*a^2*b^2*c^5 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 40*sqrt(b^2 - 4*a*c)*a*b^3*c^5 + 7*sqrt(b^2 - 4*a*c)*b^4*c^5 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 64*sqrt(b^2 - 4*a*c)*a^3*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 80*sqrt(b^2 - 4*a*c)*a^2*b*c^6 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - 56*sqrt(b^2 - 4*a*c)*a*b^2*c^6 - 3*sqrt(b^2 - 4*a*c)*b^3*c^6 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^7 + 112*sqrt(b^2 - 4*a*c)*a^2*c^7 + 60*sqrt(b^2 - 4*a*c)*a*b*c^7 - 24*sqrt(b^2 - 4*a*c)*a*c^8 - 2*(b^2 - 4*a*c)*b^4*c^4 + 16*(b^2 - 4*a*c)*a*b^2*c^5 + 12*(b^2 - 4*a*c)*b^3*c^5 - 32*(b^2 - 4*a*c)*a^2*c^6 - 48*(b^2 - 4*a*c)*a*b*c^6 - 14*(b^2 - 4*a*c)*b^2*c^6 - 8*(b^2 - 4*a*c)*a*c^7)*arctan(2*sqrt(1/2)*x/sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^4 - sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^4)^2 - 4*(a*c^2*d^4 - 2*a*b*c*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a^2*b*d*e^3 + a^3*e^4)*(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^2 + 2*b^8*c^2 + 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 60*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^3 - 16*a*b^6*c^3 - 8*b^7*c^3 - 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 240*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 84*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 32*a*b^5*c^4 + 6*b^6*c^4 + 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^5 + 320*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 336*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 256*a^3*b^2*c^5 + 72*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 128*a^2*b^3*c^5 + 24*a*b^4*c^5 - 448*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 - 512*a^4*c^6 - 240*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 512*a^3*b*c^6 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 224*a^2*b^2*c^6 + 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^7 + 128*a^3*c^7 + 16*a*b^2*c^7 - 64*a^2*c^8 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 72*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 24*sqrt(b^2 - 4*a*c)*a*b^4*c^4 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 192*sqrt(b^2 - 4*a*c)*a^2*b^2*c^5 + 32*sqrt(b^2 - 4*a*c)*a*b^3*c^5 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 384*sqrt(b^2 - 4*a*c)*a^3*c^6 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 128*sqrt(b^2 - 4*a*c)*a^2*b*c^6 + 8*sqrt(b^2 - 4*a*c)*a*b^2*c^6 + 96*sqrt(b^2 - 4*a*c)*a^2*c^7 - 16*sqrt(b^2 - 4*a*c)*a*b*c^7 - 2*(b^2 - 4*a*c)*b^6*c^2 + 24*(b^2 - 4*a*c)*a*b^4*c^3 + 8*(b^2 - 4*a*c)*b^5*c^3 - 96*(b^2 - 4*a*

```
c)*a^2*b^2*c^4 - 64*(b^2 - 4*a*c)*a*b^3*c^4 - 6*(b^2 - 4*a*c)*b^4*c^4 + 128
*(b^2 - 4*a*c)*a^3*c^5 + 128*(b^2 - 4*a*c)*a^2*b*c^5 + 48*(b^2 - 4*a*c)*a*b
^2*c^5 - 96*(b^2 - 4*a*c)*a^2*c^6 - 64*(b^2 - 4*a*c)*a*b*c^6)*d^2*abs(c) +
4*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 4*a*b^7*c^2 - 36*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 4*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 48*a^2*b^5*c^3 + 10*a*b^6*c^3 + 144*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 32*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b^3*c^4 - 192*a^3*b^3*c^4 - sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^4*c^4 - 56*a^2*b^4*c^4 - 8*a*b^5*c^4 - 192*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^5 + 256*a^4*b*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^5 - 32*a^3*b^2*c^5 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^5 + 6*a*b^4*c^5 + 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 3
84*a^4*c^6 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + 128*a^3*
b*c^6 - 16*a^2*b^2*c^6 - 8*a*b^3*c^6 - 32*a^3*c^7 + 32*a^2*b*c^7 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c + 12*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 4*sqrt(b^2 - 4*a*c
)*a*b^6*c^2 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^2*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^3 - 48*sqrt(b^2 - 4*a*c)*a^2*b^4*c^3 - 10*sqrt(b^2 - 4*a*c)*a*b^5*c^
3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 +
32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 40
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 19
2*sqrt(b^2 - 4*a*c)*a^3*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 80*sqrt(b^2 - 4*a*c)*a^2*b^3*c^4 + 16*sqrt(b
^2 - 4*a*c)*a*b^4*c^4 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*c^5 - 256*sqrt(b^2 - 4*a*c)*a^4*c^5 - 48*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 160*sqrt(b^2 - 4*a*c)*a^
3*b*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2
*c^5 - 96*sqrt(b^2 - 4*a*c)*a^2*b^2*c^5 - 18*sqrt(b^2 - 4*a*c)*a*b^3*c^5 +
24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 128*
sqrt(b^2 - 4*a*c)*a^3*c^6 + 40*sqrt(b^2 - 4*a*c)*a^2*b*c^6 + 8*sqrt(b^2 - 4
*a*c)*a*b^2*c^6 - 16*sqrt(b^2 - 4*a*c)*a^2*c^7 - 8*(b^2 - 4*a*c)*a*b^3*c^4
+ 32*(b^2 - 4*a*c)*a^2*b*c^5 + 12*(b^2 - 4*a*c)*a*b^2*c^5 + 16*(b^2 - 4*a*c
)*a^2*c^6)*d*abs(c)*e - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8 - 16
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b^7*c - 6*a*b^8*c + 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*b^4*c^2 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^
2 - sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 + 80*a^2*b^6*c^2 + 12
*a*b^7*c^2 - 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 48*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 20*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 384*a^3*b^4*c^3 - 5*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 80*a^2*b^5*c^3 - 10*a*b^6*c^3 + 256*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^4 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^4*b*c^4 + 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*
c^4 + 768*a^4*b^2*c^4 + 56*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c^4 + 64*a^3*b^3*c^4 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4
+ 24*a^2*b^4*c^4 + 12*a*b^5*c^4 - 448*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*c^5 - 512*a^5*c^5 - 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b*c^5 + 256*a^4*b*c^5 - 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^5 + 32*a^3*b^2*c^5 - 32*a^2*b^3*c^5 - 16*a*b^4*c^5 + 96*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 128*a^4*c^6 - 64*a^3*b*c^6 + 80*a^2*b^2*c^
6 - 64*a^3*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^7 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^
5*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c + 8
*sqrt(b^2 - 4*a*c)*a*b^7*c - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^3*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
```

```
^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^5*c^2 - 96*sqrt(b^2 - 4*a*c)*a^2*b^5*c^2 - 20*sqrt(b^2 - 4*
a*c)*a*b^6*c^2 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b*c^3 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^2*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^3 + 384*sqrt(b^2 - 4*a*c)*a^3*b^3*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 136*sqrt(b^2 - 4*a*c)*a^2*b
^4*c^3 + 32*sqrt(b^2 - 4*a*c)*a*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 512*sqrt(b^2 - 4*a*c)*a^4*b*c^4 - 40*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 128*
sqrt(b^2 - 4*a*c)*a^3*b^2*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^4 - 160*sqrt(b^2 - 4*a*c)*a^2*b^3*c^4 - 36*sqrt(b^
2 - 4*a*c)*a*b^4*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*c^5 - 384*sqrt(b^2 - 4*a*c)*a^4*c^5 + 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 128*sqrt(b^2 - 4*a*c)*a^3*
b*c^5 + 88*sqrt(b^2 - 4*a*c)*a^2*b^2*c^5 + 16*sqrt(b^2 - 4*a*c)*a*b^3*c^5 +
 96*sqrt(b^2 - 4*a*c)*a^3*c^6 - 48*sqrt(b^2 - 4*a*c)*a^2*b*c^6 - 2*(b^2 - 4
*a*c)*a*b^6*c + 24*(b^2 - 4*a*c)*a^2*b^4*c^2 + 8*(b^2 - 4*a*c)*a*b^5*c^2 -
96*(b^2 - 4*a*c)*a^3*b^2*c^3 - 64*(b^2 - 4*a*c)*a^2*b^3*c^3 - 22*(b^2 - 4*a
*c)*a*b^4*c^3 + 128*(b^2 - 4*a*c)*a^4*c^4 + 128*(b^2 - 4*a*c)*a^3*b*c^4 + 1
12*(b^2 - 4*a*c)*a^2*b^2*c^4 + 24*(b^2 - 4*a*c)*a*b^3*c^4 - 96*(b^2 - 4*a*c
)*a^3*c^5 - 32*(b^2 - 4*a*c)*a^2*b*c^5)*abs(c)*e^2) + 1/2*x*e^2/((c*d^3 - b
*d^2*e + a*d*e^2)*(x^2*e + d))
```

maple [B] time = 0.03, size = 1141, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*e^4/(a*e^2-b*d*e+c*d^2)^2/d*x/(e*x^2+d)*a-1/2*e^3/(a*e^2-b*d*e+c*d^2)^2
*x/(e*x^2+d)*b+1/2*e^2/(a*e^2-b*d*e+c*d^2)^2*d*x/(e*x^2+d)*c+1/2*e^4/(a*e^2
-b*d*e+c*d^2)^2/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a-3/2*e^3/(a*e^2-b*
d*e+c*d^2)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b+5/2*e^2/(a*e^2-b*d*e+c
*d^2)^2*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c-1/2/(a*e^2-b*d*e+c*d^2)^2
*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)*b*e^2+1/(a*e^2-b*d*e+c*d^2)^2*c^2*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*d*e+1/(a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^2*e^2+1/(a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*b*d*e-1/(a*e^2-b*d*e+c*d^2)^2*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*d^2+1/2/(a*e^2-b*d*e+c*d^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e^2-1/(a*e^2-b*d
*e+c*d^2)^2*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d*e+1/(a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2*c/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e^2+1/(a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)*b*d*e-1/(a*e^2-b*d*e+c*d^2)^2*c^3/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*e^2*x/(c*d^4 - b*d^3*e + a*d^2*e^2 + (c*d^3*e - b*d^2*e^2 + a*d*e^3)*x^
2) + 1/2*(5*c*d^2*e^2 - 3*b*d*e^3 + a*e^4)*arctan(e*x/sqrt(d*e))/((c^2*d^5
- 2*b*c*d^4*e - 2*a*b*d^2*e^3 + a^2*d*e^4 + (b^2 + 2*a*c)*d^3*e^2)*sqrt(d*e
)) + integrate((c^2*d^2 - 2*b*c*d*e + (b^2 - a*c)*e^2 - (2*c^2*d*e - b*c*e^
2)*x^2)/(c*x^4 + b*x^2 + a), x)/(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*
e^4 + (b^2 + 2*a*c)*d^2*e^2)
```

mupad [B] time = 10.28, size = 91169, normalized size = 212.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan(((((x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8
*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48
*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e
^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^
8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a
^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*
b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 -
 12*a^2*b*c*d^5*e^5)) - (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c
^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 +
 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c
^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^
9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*
a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11
+ 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b
^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^
5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5
*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10
+ a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d
^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*
d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2
*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - ((-d^3*e^3)^(1/2)*((x*(32*c^11*d^13*e^2
 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*
d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6
- 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336
*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c
^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*
e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6
*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*
c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720
*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^1
3 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10
*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7
*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*
e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^
6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d
*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^
4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e
^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*
```

```
e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d
^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16
 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 1
0880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*
b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*
c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*
d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^1
1*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*
b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 40
0*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e
^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*
c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680
*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^
13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^
4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^
6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 36
48*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 -
 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 -
 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 +
 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e
^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2
*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 -
4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b
^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 -
4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^
5)) - (x*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e)*(1024*a^2*c^11*d^16*e
^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9
 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 -
1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*
b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8
*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2
*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a
^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e
^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^
2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 1376
0*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8
*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^
2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4
224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*
e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4
*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 2150
4*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^
15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^
3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c
^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a
*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 57
6*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 +
320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e
^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*
d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(8*(c^2*d^7
 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)
*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7
+ 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 +
 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3
 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c
*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*
a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a
^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(
-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^
```

```
2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)
*(a*e^2 + 5*c*d^2 - 3*b*d*e)*1i)/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 -
2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)) + (((x*(54*c^9*d^6*e^5 - 2*a^
3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a
^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e
^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^
2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5
- 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6
*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e -
12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (((2*a^2*b
^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26
*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*
d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 -
 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^
2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4
*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^
2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^
5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*
e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e
^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*
b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c
*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*
c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5))
+ ((-d^3*e^3)^(1/2)*((x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d
^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28
*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*
c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^
10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8
+ 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*
a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 -
 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2
*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b
^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 74
0*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 78
52*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 81
6*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 72
16*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 56
96*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 3
36*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*
d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7
*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*
d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - ((
(128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^
3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c
^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^
14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e
^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32
*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7
 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4
*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3
*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 -
9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4
*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b
^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20
672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*
e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c
^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3
*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b
^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^
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9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*
a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 1
6*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2
*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7
+ 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 +
6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3
+ 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*(-d^3*e^3)^(1/2)*(a*e^2 +
5*c*d^2 - 3*b*d*e)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216
*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^
7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^1
0*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^
14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*
e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*
e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^
5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1
664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^1
2*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3
*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12
+ 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6
*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^
4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 -
2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d
^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b
^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 84
48*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^
17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^1
6*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^
7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*
c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2
*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11
520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^1
4 + 5376*a^8*b*c^4*d^3*e^16))/(8*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b
*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6
*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^
6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8
*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c
*d^5*e^5)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2
*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(a*
e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d
^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 -
3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^
4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e)*1i)/(
4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*
c*d^5*e^2)))/((5*c^8*d^3*e^6 - 3*b*c^7*d^2*e^7 + a*c^7*d*e^8)/(c^4*d^10 + a
^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*
e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6
*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*
d^6*e^4 - 12*a^2*b*c*d^5*e^5) - (((x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*
a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 +
107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*
d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/
(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^
7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6
+ 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e
^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((2*a^2*b^6*c^2*e^13 - 20
0*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^1
3 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^
2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5
*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a

^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - ((-d^3*e^3)^(1/2)*((x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (x*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 -

22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*
d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^
3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^1
0 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^
2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280
*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^
13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^
5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^
5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14
+ 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d
^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2
*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b
^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 953
6*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1
856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4
 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^
9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c
^4*d^3*e^16))/(8*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b
*d^4*e^3 + 2*a*c*d^5*e^2)*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d
^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*
e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d
^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-d^
3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d
^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2 -
3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4
*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c
^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^
5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^
3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)) + (((x*
(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a
^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3
*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2
*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*
e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6
 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*
e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*
d^5*e^5)) + (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14
*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7
*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 +
 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c
^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*
d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^
8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e
^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^
10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12
- 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^
8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a
^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*
b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 -
 12*a^2*b*c*d^5*e^5)) + ((-d^3*e^3)^(1/2)*((x*(32*c^11*d^13*e^2 + 48*a^6*b*
c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4
*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c
^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^1
1*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 -
 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^
9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*
a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12
 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5
*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4

*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*
a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*
a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a
*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a
^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*
a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 +
b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c
*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*
c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*
a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c
^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7
*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^1
5*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5
 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 25
6*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 1216
0*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*
e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^
2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a
^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12
 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5
*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4
*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 +
 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^
3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9
*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*
c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c
^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b
*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*
a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*
a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*
e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3
 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*
e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*(-d
^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e)*(1024*a^2*c^11*d^16*e^3 + 5120*a^
3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*
c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4
*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15
*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^
7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 +
8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d
^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^
2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 +
 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^
6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088
*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e
^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*
c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*
a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^1
3 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^
5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6
*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 -
 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2
 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^1
4*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*
d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c
^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a
^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 2
0224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(8*(c^2*d^7 + a^2*d^3*e
^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)*(c^4*d^10 +

a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^
8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d
^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*
c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d
e))/(4(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3
+ 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 +
b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1
/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 -
2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*
c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2
*a*b*d^4*e^3 + 2*a*c*d^5*e^2))))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*
e)*1i)/(2*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^
3 + 2*a*c*d^5*e^2)) - atan(((((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a
^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e
^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b
^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^
4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 +
672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e
^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984
*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*
a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2
*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d
^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c
^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*
c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a
*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^
4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9
*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^
14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1
792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*
b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2
*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12
864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e
^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c
^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*
a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^1
0 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^
3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^
5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 -
3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*
e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^
11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d
^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^
12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c
^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b
*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^
6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e
^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^
8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*
c*d^5*e^5)) - (x*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4
+ c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e -
32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4
*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 +
4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*
(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^
3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) +
42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*

a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2
*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4
+ 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64
*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 14
4*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4
*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*
e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a
^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^
4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120
*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a
^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*
c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d
^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12
*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10
 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^
7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648
*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^1
2 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4
*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1
088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^1
0*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b
^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 64
00*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*
e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2
*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*
a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^1
6 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*
e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*
d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c
^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^1
0*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 3712
0*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12
+ 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d
^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2
+ 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4
 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*
e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^
4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*
c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c
^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b
^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a
*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e
^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*
e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c
^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*
d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c
^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d
^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d
^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*
d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a
^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*
d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3
- 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(32*c^11*d^13*
e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^
10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e
^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 +
336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^

5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d
^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*
d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b
^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3
720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*
e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d
^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*
d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d
^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7
*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^
3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 +
 a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^
8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d
^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*
c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*
c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 3
2*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 +
 a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 +
 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c
^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c
- b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*
c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a
*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^
2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96
*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5
*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4
*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^
2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e
+ 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^
3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^
5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2))*((b^4*e^4*(-
(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)
^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3
*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*
c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2
e^2(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24
*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1
/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^
2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5
*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4
 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*
e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2
*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3
*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^
4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*
d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*
a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5
*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d
^3*e^5)))^(1/2) - (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 -
118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4
*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^
3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a
^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*
e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6
*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*
d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^

4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*1i - (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4

*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^
4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b
^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d
^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8
+ 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*
b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*
a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*
d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*
d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2
*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 -
 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2
*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^
5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))
^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12
*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^1
3 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 +
512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480
*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^1
0*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^
2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^
8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c
^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840
*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*
e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8
*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1
280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6
*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3
*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576
*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^
14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^
4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*
b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*
a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 -
9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9
- 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*
e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7
*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*
b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5
 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 +
6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e -
 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*
(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^
3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c
^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^
5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) -
24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^
(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*
a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a
^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e
^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^
2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a
^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a
^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*
a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*
c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 +
6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b

^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3
*d^3*e^5)))^(1/2) - (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^
11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*
a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c
^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^1
0*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 +
 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a
^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 -
3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*
e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^
4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740
*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 785
2*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816
*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 721
6*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 569
6*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 33
6*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d
^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*
e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d
^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^
4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c -
 b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4
*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2)
- 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2
*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1
/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^
2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2
 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d
^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8
*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*
b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6
- 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2
+ 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4
 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*
a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*
e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20
*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5
*b*c^3*d^3*e^5)))^(1/2))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 -
 b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5
*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*
e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*
c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3
- 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3
 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*
d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8
 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*
d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*
e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*
e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2
*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*
b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7
*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 4
4*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(54*c^9*d^6*e^5 -
 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 -
 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*

d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4
*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5
*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^
3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9
*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*
e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b
^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b
^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) -
6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2
) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)
^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 -
 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8
 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d
^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^
6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 -
44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 +
32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 +
 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*
b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^
7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a
^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b
*c^3*d^3*e^5)))^(1/2)*1i)/((5*c^8*d^3*e^6 - 3*b*c^7*d^2*e^7 + a*c^7*d*e^8)/
(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 +
 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 +
6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3
+ 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5) + (((2*a^2*b^6*c^2*e^13 - 200*a*
c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 +
480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^
8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8
 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b
^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*
a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*
c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c
^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7
*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*
d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*
b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2
*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c
^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15
*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 -
 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 21
76*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4
*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^
5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^
10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4
*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*
a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8
 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4
*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^
4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12
- 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*
d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b
^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 10
24*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 +
8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 +
 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 +
 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^

9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (x*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 +

72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2
*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*
c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7
 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d
^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 +
 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 -
 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5
+ 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b
*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2
+ 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a
^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) +
(x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*
d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2
048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^
5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*
d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^
9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 -
 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4
*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2
*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648
*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^1
3 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7
+ 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10
+ 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7
+ 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11
 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14
))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3
*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*
e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^
7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2
)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*
a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*
a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2
+ 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d
^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*
c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*
e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a
*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 1
6*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5
*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4
*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6
*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5
*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^
3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64
*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^
6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 -
 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1
/2))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(
-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*
d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^
3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^
3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b
^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4
*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^
3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^
8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5

+ 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3
*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4
*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^
3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d
^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6
*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4
*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6
- 192*a^5*b*c^3*d^3*e^5)))^(1/2) - (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22
*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9
+ 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7
*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))
/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e
^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^
6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*
e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^
3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^
3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^
2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 +
4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3
*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*
d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^
2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b
*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*
a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d
*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c
^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e
^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e
^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*
e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a
^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*
e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 1
92*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2
) + (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4
*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9
+ 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4
*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e
^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10
- 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^
6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 207
2*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*
a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a
^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*
d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4
*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*
d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*
b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*
d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7
*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^
2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2
240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^
9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^
2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9
+ 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^
5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b
^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 4
80*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6
*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6
*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 238

4*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d

*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3
)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3
 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^
2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*
a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3
*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8
 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5
+ 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*
d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*
d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3
*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^
7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*
b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*
e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 -
 192*a^5*b*c^3*d^3*e^5)))^(1/2) - (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15
+ 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*
c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^
8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 2
68*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6
*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4
*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c
^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^
2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11
 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*
d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8
*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5
*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*
d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^
2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5
*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e
^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6
- 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e
^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d
^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*
d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3
*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c -
b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c
*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*
c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e
*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b
^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*
e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2
*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^
3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^
6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^
2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b
^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*
c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 6
4*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*
c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2
*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) -
 b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^
4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*
e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*
d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*

b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*2i - atan(((((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 +

96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c
^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^1
0 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 -
354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6
- 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*
a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*
b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3
*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^
6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e
^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^
8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*
c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^
13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e
^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2
+ 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 224
0*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*
c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*
b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 +
3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*
e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4
*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480
*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e
^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c
^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*
a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14
- 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*
e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^
10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7
*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d
^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*
c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^
2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 -
4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a
^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12
*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (x*(-(b^7*e^
4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2
)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4
*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*
b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2
*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2)
+ 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3
)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 7
2*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 +
a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4
*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*
d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44
*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32
*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 3
2*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^
7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7
+ 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3
*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c
^3*d^3*e^5)))^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216
*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^
7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^1
0*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^
14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*
e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*

e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d

^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4
d^4(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^
3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c -
 b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*
c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a
*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*
e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*
b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2
e^2(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^
2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d
^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a
^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b
^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*
b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5
*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 -
64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6
*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^
2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*
(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*
e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^
2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^
5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*
a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a
^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e
^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*
e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8
*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e
^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*
a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*
a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64
*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5
*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32
*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b
*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) - (x*(
54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^
2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*
e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*
b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e
^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6
- 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e
^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d
^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4
d^4(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^
3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c -
 b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*
c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a
*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*
e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*
b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2
e^2(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^
2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d
^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a
^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b
^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*
b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5

*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*1i - (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*(1024*a^2*c^1

1*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*
d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4
*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3
 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 +
3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*
b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 -
 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^
5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^
2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^
8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6
*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 5376
0*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*
e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*
c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952
*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^1
5 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^
3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^
7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 2
56*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 -
 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*
e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9
*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^
9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^
6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(2*
(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 +
 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 +
6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3
+ 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4
e^4(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b
^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a
*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e
+ 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 -
 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^
2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7
*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^
7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*
d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2
+ 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3
- 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5
 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*
b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2
 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*
a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) -
 (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5
*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 -
2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a
^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7
*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e
^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8
- 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^
4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^
2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 64
8*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^
13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7
 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10
 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7

+ 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^1
1 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^1
4))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^
3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4
*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d
^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^
4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 2
0*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 2
5*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^
2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4
*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^
3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^
2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8
*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 +
16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b
^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a
^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d
^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d
^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*
d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 -
64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*
d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5
- 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^
(1/2))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^
4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c
^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^
2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d
*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c
- b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(
-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3
*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c
*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*
e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*
c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*
c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2
*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^
3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*
a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*
d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e
^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 -
22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e
^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*
c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^
9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^
3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4
*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d
^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^
4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 2
0*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 2
5*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^
2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4
*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^
3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^
2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8
*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 +

16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*1i)/((5*c^8*d^3*e^6 - 3*b*c^7*d^2*e^7 + a*c^7*d*e^8)/(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5) + (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (x*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^

(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12

+ 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260
*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^
3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c
^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^
2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10
+ 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d
^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^
9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c
^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^
3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*
c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4
*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^1
0 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3
*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^
2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b
^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(
4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4
 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e
^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d
^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b
^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*
b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8
 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^
2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2
+ 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2
*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3
*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^
5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^
7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^
2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^
4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2))*(-(b^7*e
^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^
2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^
4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6
*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^
2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2)
 + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^
3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 +
72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8
+ a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^
4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6
*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 4
4*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 3
2*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 +
32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b
^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7
 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^
3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*
c^3*d^3*e^5)))^(1/2) - (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e
^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^
7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6
*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^1
0 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3
*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^
2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b

^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d

^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^
2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) -
 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c
^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*
e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*
a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 +
 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*
c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*
c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3
*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e
^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6
*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^
3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^
5)))^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*
d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6
*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^
2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 -
4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512
*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 505
6*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^1
1*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b
^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 2
7840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*
d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3
*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10
 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3
*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5
*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 -
 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^
5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^
2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*
a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8
512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^
6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*
e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d
^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b
*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*
a^8*b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5
*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^
3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9
*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7
*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c -
b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*
b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) +
 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*
c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/
2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2
)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2
+ 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^
8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*
d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b
^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 -
 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 +
 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4
+ 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a
*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e

^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*
a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*
b*c^3*d^3*e^5)))^(1/2) - (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^
10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15
- 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*
a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^
8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*
e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7
584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^
10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2
*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a
^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12
+ 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6
- 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9
+ 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12
+ 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14
+ 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13
- 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*
b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c
*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*
c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))
*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4
*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e
^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^
(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 +
6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)
^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*
c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d
^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3
*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 +
a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 +
6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^
2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^
6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d
^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*
e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*
c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^
4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 1
92*a^5*b*c^3*d^3*e^5)))^(1/2))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c
- b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*
a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a
^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9
*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2
*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c -
b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^
3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a
c - b^2)^3)^(1/2))/(8(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b
^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*
c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a
^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c
^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c
^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*
c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e +
32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*
c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*
e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(54*c^9*d^
6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5

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*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b
^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e
^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*
b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c
*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*
c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))
*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4
*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e
^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^
(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 +
 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)
^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*
c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d
^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3
*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 +
 a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 +
6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^
2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^
6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d
^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*
e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*
c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^
4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 1
92*a^5*b*c^3*d^3*e^5)))^(1/2)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c
 - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32
*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 +
a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 -
9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^
2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c -
 b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c
^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*
b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2
*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*
a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*
c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*
c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2
*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e +
 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3
*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5
*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*2i + (e^2*x)/(
2*d*(d + e*x^2)*(a*e^2 + c*d^2 - b*d*e))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.270 $\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=563

$$\frac{x\left(c\left(-\frac{abe(ae^2+3cd^2)}{c}-2ad\left(cd^2-3ae^2\right)+b^2d^3\right)-x^2\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(ab^3e^3-b\right.}{}$$

```
[Out] 1/2*x*(c*(b^2*d^3-2*a*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2)/c)-(a*b^2*e^
3+2*a*c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*x^2)/a/c/(-4*a*c+b^2)/(c*
x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(a*
b^3*e^3+6*a*c*(a*e^2+c*d^2)*(2*c*d+e*(-4*a*c+b^2)^(1/2))-b^2*(c^2*d^3-3*a*c
*d*e^2+a*e^3*(-4*a*c+b^2)^(1/2))-b*c*(c*d^2*(12*a*e+d*(-4*a*c+b^2)^(1/2))+a
*e^2*(8*a*e+3*d*(-4*a*c+b^2)^(1/2))))/a/c^(3/2)/(-4*a*c+b^2)^(3/2)*2^(1/2)/
(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2))*(a*b^3*e^3+6*a*c*(a*e^2+c*d^2)*(2*c*d-e*(-4*a*c+b^2)^(1/2))-b^
2*(c^2*d^3-3*a*c*d*e^2-a*e^3*(-4*a*c+b^2)^(1/2))+b*c*(c*d^2*(-12*a*e+d*(-4*
a*c+b^2)^(1/2))+a*e^2*(-8*a*e+3*d*(-4*a*c+b^2)^(1/2))))/a/c^(3/2)/(-4*a*c+b
^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 3.52, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1205, 1166, 205}

$$-\frac{\left(-b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac\left(ae^2+cd^2\right)\left(e\sqrt{b^2-4ac}+2cd\right)-bc\left(cd^2\left(d\sqrt{b^2-4ac}+12ae\right)\right.\right.}{2\sqrt{2}\,ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(c*(b^2*d^3 - 2*a*d*(c*d^2 - 3*a*e^2) - (a*b*e*(3*c*d^2 + a*e^2))/c) - (
a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^2))/(2*a
*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((a*b^3*e^3 + 6*a*c*(2*c*d + Sqrt[b
^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 -
4*a*c]*e^3) - b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2
- 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(
c^2*d^3 - 3*a*c*d*e^2 - a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4
*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(c\left(b^2d^3-2ad\left(cd^2-3ae^2\right)-\frac{abe\left(3cd^2+ae^2\right)}{c}\right)-\left(ab^2e^3+2ace\left(3cd^2-ae^2\right)-bcd\left(cd^2+3\right.\right.\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(c\left(b^2d^3-2ad\left(cd^2-3ae^2\right)-\frac{abe\left(3cd^2+ae^2\right)}{c}\right)-\left(ab^2e^3+2ace\left(3cd^2-ae^2\right)-bcd\left(cd^2+3\right.\right.\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(c\left(b^2d^3-2ad\left(cd^2-3ae^2\right)-\frac{abe\left(3cd^2+ae^2\right)}{c}\right)-\left(ab^2e^3+2ace\left(3cd^2-ae^2\right)-bcd\left(cd^2+3\right.\right.\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 1.63, size = 540, normalized size = 0.96

$$\frac{\frac{2\sqrt{c}\,x\left(b\left(-a^2e^3-3acde\left(d-ex^2\right)+c^2d^3x^2\right)+b^2\left(cd^3-ae^3x^2\right)+2ac\left(ae^2\left(3d+ex^2\right)-cd^2\left(d+3ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(-ab^3e^3+b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)-6ac\left(ae^2+c\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*x*(b^2*(c*d^3 - a*e^3*x^2) + b*(-(a^2*e^3) + c^2*d^3*x^2 - 3*a*
c*d*e*(d - e*x^2)) + 2*a*c*(a*e^2*(3*d + e*x^2) - c*d^2*(d + 3*e*x^2))))/((
b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*e^3) - 6*a*c*(2*c*d +
 Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt
[b^2 - 4*a*c]*e^3) + b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sq
rt[b^2 - 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a
*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(-(c^2
*d^3) + 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4*a
*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[
b^2 - 4*a*c]]))/(4*a*c^(3/2))
```

fricas [B] time = 84.43, size = 12117, normalized size = 21.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*
x^3 - sqrt(1/2)*(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b
^3*c - 4*a^2*b*c^2)*x^2)*sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6
 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92
*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^
3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*
d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 + (a^3*b^6*c^3 - 12*a^4
*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*
a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*
c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^
3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 +
2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 -
18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2
*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4
 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*
b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64
*a^6*c^6))*log(-((5*b^4*c^6 - 81*a*b^2*c^7 + 324*a^2*c^8)*d^12 - 3*(3*b^5*c
^5 - 65*a*b^3*c^6 + 324*a^2*b*c^7)*d^11*e + 3*(b^6*c^4 - 42*a*b^4*c^5 + 252
*a^2*b^2*c^6 + 432*a^3*c^7)*d^10*e^2 + (b^7*c^3 + 3*a*b^5*c^4 + 33*a^2*b^3*
c^5 - 2916*a^3*b*c^6)*d^9*e^3 + 9*(a*b^6*c^3 - 15*a^2*b^4*c^4 + 195*a^3*b^2
*c^5 + 180*a^4*c^6)*d^8*e^4 - 162*(a^3*b^3*c^4 + 12*a^4*b*c^5)*d^7*e^5 + 16
2*(a^4*b^3*c^3 + 12*a^5*b*c^4)*d^5*e^7 - 9*(a^3*b^6*c - 15*a^4*b^4*c^2 + 19
5*a^5*b^2*c^3 + 180*a^6*c^4)*d^4*e^8 - (a^3*b^7 + 3*a^4*b^5*c + 33*a^5*b^3*
c^2 - 2916*a^6*b*c^3)*d^3*e^9 - 3*(a^4*b^6 - 42*a^5*b^4*c + 252*a^6*b^2*c^2
 + 432*a^7*c^3)*d^2*e^10 + 3*(3*a^5*b^5 - 65*a^6*b^3*c + 324*a^7*b*c^2)*d*e
^11 - (5*a^6*b^4 - 81*a^7*b^2*c + 324*a^8*c^2)*e^12)*x + 1/2*sqrt(1/2)*((b^
8*c^4 - 23*a*b^6*c^5 + 190*a^2*b^4*c^6 - 672*a^3*b^2*c^7 + 864*a^4*c^8)*d^9
 + 9*(a*b^7*c^4 - 15*a^2*b^5*c^5 + 72*a^3*b^3*c^6 - 112*a^4*b*c^7)*d^8*e +
3*(a^2*b^6*c^4 + 28*a^3*b^4*c^5 - 272*a^4*b^2*c^6 + 576*a^5*c^7)*d^7*e^2 +
(a^2*b^7*c^3 - 80*a^3*b^5*c^4 + 592*a^4*b^3*c^5 - 1152*a^5*b*c^6)*d^6*e^3 +
 15*(a^3*b^6*c^3 - 8*a^4*b^4*c^4 + 16*a^5*b^2*c^5)*d^5*e^4 - 6*(a^3*b^7*c^2
 - 17*a^4*b^5*c^3 + 88*a^5*b^3*c^4 - 144*a^6*b*c^5)*d^4*e^5 - (a^3*b^8*c -
5*a^4*b^6*c^2 + 100*a^5*b^4*c^3 - 816*a^6*b^2*c^4 + 1728*a^7*c^5)*d^3*e^6 -
 3*(a^4*b^7*c - 32*a^5*b^5*c^2 + 208*a^6*b^3*c^3 - 384*a^7*b*c^4)*d^2*e^7 -
 54*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)*d*e^8 - (a^5*b^7 - 17*a^6*b^
5*c + 88*a^7*b^3*c^2 - 144*a^8*b*c^3)*e^9 - ((a^3*b^9*c^4 - 20*a^4*b^7*c^5
+ 144*a^5*b^5*c^6 - 448*a^6*b^3*c^7 + 512*a^7*b*c^8)*d^3 + 3*(a^4*b^8*c^4 -
 8*a^5*b^6*c^5 + 128*a^7*b^2*c^7 - 256*a^8*c^8)*d^2*e - 12*(a^5*b^7*c^4 - 1
2*a^6*b^5*c^5 + 48*a^7*b^3*c^6 - 64*a^8*b*c^7)*d*e^2 - (a^5*b^8*c^3 - 24*a^
6*b^6*c^4 + 192*a^7*b^4*c^5 - 640*a^8*b^2*c^6 + 768*a^9*c^7)*e^3)*sqrt(-(10
8*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*
a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a
^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4
- 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^
6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*
c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^
7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6
- 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))*sqrt(-((b^5*c^3 - 15*a*b^
3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*
e - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c
^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a
^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^
6 + (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108
*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a
^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^
3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 -
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18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6
e^6 + 12(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c
^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7
*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 -
12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*
c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))) + sqrt(1/2)*(a^2*b^2*c - 4*a^3*c^2 + (
a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2)*sqrt(-((b^5*c^3 -
15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c
^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 +
36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4
*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b
*c^2)*e^6 + (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*sq
rt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^
7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^
6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b
^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*
c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 -
9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c
- 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b
^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*
a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))*log(-((5*b^4*c^6 - 81*a*b^2*c^7
+ 324*a^2*c^8)*d^12 - 3*(3*b^5*c^5 - 65*a*b^3*c^6 + 324*a^2*b*c^7)*d^11*e
+ 3*(b^6*c^4 - 42*a*b^4*c^5 + 252*a^2*b^2*c^6 + 432*a^3*c^7)*d^10*e^2 + (b^
7*c^3 + 3*a*b^5*c^4 + 33*a^2*b^3*c^5 - 2916*a^3*b*c^6)*d^9*e^3 + 9*(a*b^6*c
^3 - 15*a^2*b^4*c^4 + 195*a^3*b^2*c^5 + 180*a^4*c^6)*d^8*e^4 - 162*(a^3*b^3
*c^4 + 12*a^4*b*c^5)*d^7*e^5 + 162*(a^4*b^3*c^3 + 12*a^5*b*c^4)*d^5*e^7 - 9
*(a^3*b^6*c - 15*a^4*b^4*c^2 + 195*a^5*b^2*c^3 + 180*a^6*c^4)*d^4*e^8 - (a^
3*b^7 + 3*a^4*b^5*c + 33*a^5*b^3*c^2 - 2916*a^6*b*c^3)*d^3*e^9 - 3*(a^4*b^6
- 42*a^5*b^4*c + 252*a^6*b^2*c^2 + 432*a^7*c^3)*d^2*e^10 + 3*(3*a^5*b^5 -
65*a^6*b^3*c + 324*a^7*b*c^2)*d*e^11 - (5*a^6*b^4 - 81*a^7*b^2*c + 324*a^8*
c^2)*e^12)*x - 1/2*sqrt(1/2)*((b^8*c^4 - 23*a*b^6*c^5 + 190*a^2*b^4*c^6 - 6
72*a^3*b^2*c^7 + 864*a^4*c^8)*d^9 + 9*(a*b^7*c^4 - 15*a^2*b^5*c^5 + 72*a^3*
b^3*c^6 - 112*a^4*b*c^7)*d^8*e + 3*(a^2*b^6*c^4 + 28*a^3*b^4*c^5 - 272*a^4*
b^2*c^6 + 576*a^5*c^7)*d^7*e^2 + (a^2*b^7*c^3 - 80*a^3*b^5*c^4 + 592*a^4*b^
3*c^5 - 1152*a^5*b*c^6)*d^6*e^3 + 15*(a^3*b^6*c^3 - 8*a^4*b^4*c^4 + 16*a^5*
b^2*c^5)*d^5*e^4 - 6*(a^3*b^7*c^2 - 17*a^4*b^5*c^3 + 88*a^5*b^3*c^4 - 144*a
^6*b*c^5)*d^4*e^5 - (a^3*b^8*c - 5*a^4*b^6*c^2 + 100*a^5*b^4*c^3 - 816*a^6*
b^2*c^4 + 1728*a^7*c^5)*d^3*e^6 - 3*(a^4*b^7*c - 32*a^5*b^5*c^2 + 208*a^6*b
^3*c^3 - 384*a^7*b*c^4)*d^2*e^7 - 54*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*
c^4)*d*e^8 - (a^5*b^7 - 17*a^6*b^5*c + 88*a^7*b^3*c^2 - 144*a^8*b*c^3)*e^9
- ((a^3*b^9*c^4 - 20*a^4*b^7*c^5 + 144*a^5*b^5*c^6 - 448*a^6*b^3*c^7 + 512*
a^7*b*c^8)*d^3 + 3*(a^4*b^8*c^4 - 8*a^5*b^6*c^5 + 128*a^7*b^2*c^7 - 256*a^8
*c^8)*d^2*e - 12*(a^5*b^7*c^4 - 12*a^6*b^5*c^5 + 48*a^7*b^3*c^6 - 64*a^8*b*
c^7)*d*e^2 - (a^5*b^8*c^3 - 24*a^6*b^6*c^4 + 192*a^7*b^4*c^5 - 640*a^8*b^2*
c^6 + 768*a^9*c^7)*e^3)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^
9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^
7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^
4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 +
18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^
5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3
)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c
+ 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9
*c^9)))*sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 -
6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e
^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*
c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5
- 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 + (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5
*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9
- (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7

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)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4
*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 +
18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5
*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)
*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c +
 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*
c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))) - sqr
t(1/2)*(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*
a^2*b*c^2)*x^2)*sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b
^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^
4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 9
2*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (
a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 - (a^3*b^6*c^3 - 12*a^4*b^4*c^4
+ 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3
*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a
^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5
 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^
4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*
c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*
a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7
*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 -
 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)
)*log(-((5*b^4*c^6 - 81*a*b^2*c^7 + 324*a^2*c^8)*d^12 - 3*(3*b^5*c^5 - 65*a
*b^3*c^6 + 324*a^2*b*c^7)*d^11*e + 3*(b^6*c^4 - 42*a*b^4*c^5 + 252*a^2*b^2*
c^6 + 432*a^3*c^7)*d^10*e^2 + (b^7*c^3 + 3*a*b^5*c^4 + 33*a^2*b^3*c^5 - 291
6*a^3*b*c^6)*d^9*e^3 + 9*(a*b^6*c^3 - 15*a^2*b^4*c^4 + 195*a^3*b^2*c^5 + 18
0*a^4*c^6)*d^8*e^4 - 162*(a^3*b^3*c^4 + 12*a^4*b*c^5)*d^7*e^5 + 162*(a^4*b^
3*c^3 + 12*a^5*b*c^4)*d^5*e^7 - 9*(a^3*b^6*c - 15*a^4*b^4*c^2 + 195*a^5*b^2
*c^3 + 180*a^6*c^4)*d^4*e^8 - (a^3*b^7 + 3*a^4*b^5*c + 33*a^5*b^3*c^2 - 291
6*a^6*b*c^3)*d^3*e^9 - 3*(a^4*b^6 - 42*a^5*b^4*c + 252*a^6*b^2*c^2 + 432*a^
7*c^3)*d^2*e^10 + 3*(3*a^5*b^5 - 65*a^6*b^3*c + 324*a^7*b*c^2)*d*e^11 - (5*
a^6*b^4 - 81*a^7*b^2*c + 324*a^8*c^2)*e^12)*x + 1/2*sqrt(1/2)*((b^8*c^4 - 2
3*a*b^6*c^5 + 190*a^2*b^4*c^6 - 672*a^3*b^2*c^7 + 864*a^4*c^8)*d^9 + 9*(a*b
^7*c^4 - 15*a^2*b^5*c^5 + 72*a^3*b^3*c^6 - 112*a^4*b*c^7)*d^8*e + 3*(a^2*b^
6*c^4 + 28*a^3*b^4*c^5 - 272*a^4*b^2*c^6 + 576*a^5*c^7)*d^7*e^2 + (a^2*b^7*
c^3 - 80*a^3*b^5*c^4 + 592*a^4*b^3*c^5 - 1152*a^5*b*c^6)*d^6*e^3 + 15*(a^3*
b^6*c^3 - 8*a^4*b^4*c^4 + 16*a^5*b^2*c^5)*d^5*e^4 - 6*(a^3*b^7*c^2 - 17*a^4
*b^5*c^3 + 88*a^5*b^3*c^4 - 144*a^6*b*c^5)*d^4*e^5 - (a^3*b^8*c - 5*a^4*b^6
*c^2 + 100*a^5*b^4*c^3 - 816*a^6*b^2*c^4 + 1728*a^7*c^5)*d^3*e^6 - 3*(a^4*b
^7*c - 32*a^5*b^5*c^2 + 208*a^6*b^3*c^3 - 384*a^7*b*c^4)*d^2*e^7 - 54*(a^6*
b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)*d*e^8 - (a^5*b^7 - 17*a^6*b^5*c + 88*
a^7*b^3*c^2 - 144*a^8*b*c^3)*e^9 + ((a^3*b^9*c^4 - 20*a^4*b^7*c^5 + 144*a^5
*b^5*c^6 - 448*a^6*b^3*c^7 + 512*a^7*b*c^8)*d^3 + 3*(a^4*b^8*c^4 - 8*a^5*b^
6*c^5 + 128*a^7*b^2*c^7 - 256*a^8*c^8)*d^2*e - 12*(a^5*b^7*c^4 - 12*a^6*b^5
*c^5 + 48*a^7*b^3*c^6 - 64*a^8*b*c^7)*d*e^2 - (a^5*b^8*c^3 - 24*a^6*b^6*c^4
 + 192*a^7*b^4*c^5 - 640*a^8*b^2*c^6 + 768*a^9*c^7)*e^3)*sqrt(-(108*a^3*b*c
^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*
d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d
^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*
b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 1
2*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*
e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*
d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*
b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))*sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 6
0*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*
a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e
^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^
2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 - (a^3*
b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108*a^3*b*c^
```

```
6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d
^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^
10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b
*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12
*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e
^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d
*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b
^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*
a^5*b^2*c^5 - 64*a^6*c^6))) + sqrt(1/2)*(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2
 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2)*sqrt(-((b^5*c^3 - 15*a*b^3
*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e
 - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^
4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^
4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6
 - (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108*
a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^
2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3
*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 -
18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*
e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^
4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*
b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 -
12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c
^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))*log(-((5*b^4*c^6 - 81*a*b^2*c^7 + 324*a^
2*c^8)*d^12 - 3*(3*b^5*c^5 - 65*a*b^3*c^6 + 324*a^2*b*c^7)*d^11*e + 3*(b^6*
c^4 - 42*a*b^4*c^5 + 252*a^2*b^2*c^6 + 432*a^3*c^7)*d^10*e^2 + (b^7*c^3 + 3
*a*b^5*c^4 + 33*a^2*b^3*c^5 - 2916*a^3*b*c^6)*d^9*e^3 + 9*(a*b^6*c^3 - 15*a
^2*b^4*c^4 + 195*a^3*b^2*c^5 + 180*a^4*c^6)*d^8*e^4 - 162*(a^3*b^3*c^4 + 12
*a^4*b*c^5)*d^7*e^5 + 162*(a^4*b^3*c^3 + 12*a^5*b*c^4)*d^5*e^7 - 9*(a^3*b^6
*c - 15*a^4*b^4*c^2 + 195*a^5*b^2*c^3 + 180*a^6*c^4)*d^4*e^8 - (a^3*b^7 + 3
*a^4*b^5*c + 33*a^5*b^3*c^2 - 2916*a^6*b*c^3)*d^3*e^9 - 3*(a^4*b^6 - 42*a^5
*b^4*c + 252*a^6*b^2*c^2 + 432*a^7*c^3)*d^2*e^10 + 3*(3*a^5*b^5 - 65*a^6*b^
3*c + 324*a^7*b*c^2)*d*e^11 - (5*a^6*b^4 - 81*a^7*b^2*c + 324*a^8*c^2)*e^12
)*x - 1/2*sqrt(1/2)*((b^8*c^4 - 23*a*b^6*c^5 + 190*a^2*b^4*c^6 - 672*a^3*b^
2*c^7 + 864*a^4*c^8)*d^9 + 9*(a*b^7*c^4 - 15*a^2*b^5*c^5 + 72*a^3*b^3*c^6 -
 112*a^4*b*c^7)*d^8*e + 3*(a^2*b^6*c^4 + 28*a^3*b^4*c^5 - 272*a^4*b^2*c^6 +
 576*a^5*c^7)*d^7*e^2 + (a^2*b^7*c^3 - 80*a^3*b^5*c^4 + 592*a^4*b^3*c^5 - 1
152*a^5*b*c^6)*d^6*e^3 + 15*(a^3*b^6*c^3 - 8*a^4*b^4*c^4 + 16*a^5*b^2*c^5)*
d^5*e^4 - 6*(a^3*b^7*c^2 - 17*a^4*b^5*c^3 + 88*a^5*b^3*c^4 - 144*a^6*b*c^5)
*d^4*e^5 - (a^3*b^8*c - 5*a^4*b^6*c^2 + 100*a^5*b^4*c^3 - 816*a^6*b^2*c^4 +
 1728*a^7*c^5)*d^3*e^6 - 3*(a^4*b^7*c - 32*a^5*b^5*c^2 + 208*a^6*b^3*c^3 -
384*a^7*b*c^4)*d^2*e^7 - 54*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)*d*e^
8 - (a^5*b^7 - 17*a^6*b^5*c + 88*a^7*b^3*c^2 - 144*a^8*b*c^3)*e^9 + ((a^3*b
^9*c^4 - 20*a^4*b^7*c^5 + 144*a^5*b^5*c^6 - 448*a^6*b^3*c^7 + 512*a^7*b*c^8
)*d^3 + 3*(a^4*b^8*c^4 - 8*a^5*b^6*c^5 + 128*a^7*b^2*c^7 - 256*a^8*c^8)*d^2
*e - 12*(a^5*b^7*c^4 - 12*a^6*b^5*c^5 + 48*a^7*b^3*c^6 - 64*a^8*b*c^7)*d*e^
2 - (a^5*b^8*c^3 - 24*a^6*b^6*c^4 + 192*a^7*b^4*c^5 - 640*a^8*b^2*c^6 + 768
*a^9*c^7)*e^3)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*
c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e
 - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^
8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b
^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9
*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^1
0 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*
c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))*s
qrt(-((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^
2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(1
1*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*
e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*
```

```
b^3*c + 60*a^5*b*c^2)*e^6 - (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5
- 64*a^6*c^6)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c
^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e
- 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8
*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^
2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*
(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10
 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c
^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a
^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))) - 2*(3*a*b*c*d
^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3)*x)/(a^2*b^2*c - 4
*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2)
```

giac [B] time = 2.46, size = 8983, normalized size = 15.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c^2*d^3*x^3 - 6*a*c^2*d^2*x^3*e + 3*a*b*c*d*x^3*e^2 + b^2*c*d^3*x -
2*a*c^2*d^3*x - a*b^2*x^3*e^3 + 2*a^2*c*x^3*e^3 - 3*a*b*c*d^2*x*e + 6*a^2*c
*d*x*e^2 - a^2*b*x*e^3)/((c*x^4 + b*x^2 + a)*(a*b^2*c - 4*a^2*c^2)) + 1/16*
((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 -
2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^3 - 6*(2*a*b^2*c^4 - 8*a^2
*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a
*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^6*c^3 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 2*a*b^6*c^4 + 64*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 20*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^5 + 28*a^2*b^4*c^5 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^
6 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 10*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 128*a^3*b^2*c^6 + 24*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 192*a^4*c^7 + 2*(b^2 - 4*a*c)*a*b^4*c^
4 - 20*(b^2 - 4*a*c)*a^2*b^2*c^5 + 48*(b^2 - 4*a*c)*a^3*c^6)*d^3*abs(a*b^2*
c - 4*a^2*c^2) + 3*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^
2*d*e^2 + 6*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*a^2*b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*
c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 16*a^3*b^3*c^5
- 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*a^4*b*c^6 + 2*(b
^2 - 4*a*c)*a^2*b^3*c^4 - 8*(b^2 - 4*a*c)*a^3*b*c^5)*d^2*abs(a*b^2*c - 4*a^
2*c^2)*e + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 224*a^4*b^3*c^8 - 384*a^5*b*c^
9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4 +
 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 -
112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 -
 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 -
```

```
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 + 19
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 + 96*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 + 16*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 48*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 2*(b^
2 - 4*a*c)*a^2*b^5*c^6 + 32*(b^2 - 4*a*c)*a^3*b^3*c^7 - 96*(b^2 - 4*a*c)*a^
4*b*c^8)*d^3 + (2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^
2*c^3)*(a*b^2*c - 4*a^2*c^2)^2*e^3 - 12*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 -
 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 2*a^3*b^4*c^4 + 16
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^2*c^5 + 16*a^4*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6
 - 32*a^5*c^6 + 2*(b^2 - 4*a*c)*a^3*b^2*c^4 - 8*(b^2 - 4*a*c)*a^4*c^5)*d*ab
s(a*b^2*c - 4*a^2*c^2)*e^2 + 12*(2*a^3*b^6*c^6 - 16*a^4*b^4*c^7 + 32*a^5*b^
2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c
^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^
5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5
 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6
 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 2*
(b^2 - 4*a*c)*a^3*b^4*c^6 + 8*(b^2 - 4*a*c)*a^4*b^2*c^7)*d^2*e + 2*(sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^4*c^3 - 2*a^3*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c
^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 16*a^4*b^3*c^4 - 4*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 32*a^5*b*c^5 + 2*(b^2 - 4*a*c)*a^3*b^3*
c^3 - 8*(b^2 - 4*a*c)*a^4*b*c^4)*abs(a*b^2*c - 4*a^2*c^2)*e^3 - 3*(2*a^3*b^
7*c^5 - 8*a^4*b^5*c^6 - 32*a^5*b^3*c^7 + 128*a^6*b*c^8 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 - 2*(b^2 - 4*a*c)*a^3*b^5*c^5 +
32*(b^2 - 4*a*c)*a^5*b*c^7)*d*e^2 - (2*a^3*b^8*c^4 - 32*a^4*b^6*c^5 + 160*a
^5*b^4*c^6 - 256*a^6*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^6*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^5*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^4*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^5*b^2*c^6 - 2*(b^2 - 4*a*c)*a^3*b^6*c^4 + 24*(b^2 - 4*a*c)*a^4
*b^4*c^5 - 64*(b^2 - 4*a*c)*a^5*b^2*c^6)*e^3)*arctan(2*sqrt(1/2)*x/sqrt((a*
```

```
b^3*c - 4*a^2*b*c^2 + sqrt((a*b^3*c - 4*a^2*b*c^2)^2 - 4*(a^2*b^2*c - 4*a^3
*c^2)*(a*b^2*c^2 - 4*a^2*c^3)))/(a*b^2*c^2 - 4*a^2*c^3)))/((a^3*b^6*c^3 - 1
2*a^4*b^4*c^4 - 2*a^3*b^5*c^4 + 48*a^5*b^2*c^5 + 16*a^4*b^3*c^5 + a^3*b^4*c
^5 - 64*a^6*c^6 - 32*a^5*b*c^6 - 8*a^4*b^2*c^6 + 16*a^5*c^7)*abs(a*b^2*c -
4*a^2*c^2)*abs(c)) - 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^
3 - 6*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^2*e - 2*(sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^4*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c
^4 + 2*a*b^6*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 +
 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 28*a^2*b^4*c^5 - 96*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*c^6 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b*c^6 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 128*a^3*
b^2*c^6 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 - 192*a^4*c^7
- 2*(b^2 - 4*a*c)*a*b^4*c^4 + 20*(b^2 - 4*a*c)*a^2*b^2*c^5 - 48*(b^2 - 4*a*
c)*a^3*c^6)*d^3*abs(a*b^2*c - 4*a^2*c^2) + 3*(2*a*b^3*c^3 - 8*a^2*b*c^4 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c
^3)*(a*b^2*c - 4*a^2*c^2)^2*d*e^2 - 6*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 2*a^2*b^5*c^4 + 16*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 8*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^3*c^5 - 16*a^3*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
*c^6 + 32*a^4*b*c^6 - 2*(b^2 - 4*a*c)*a^2*b^3*c^4 + 8*(b^2 - 4*a*c)*a^3*b*c
^5)*d^2*abs(a*b^2*c - 4*a^2*c^2)*e + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 224*
a^4*b^3*c^8 - 384*a^5*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^7*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b^5*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^6*c^5 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^4*b^3*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b^4*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^5*c^6 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^5*b*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b^3*c^7 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b*c^8 - 2*(b^2 - 4*a*c)*a^2*b^5*c^6 + 32*(b^2 - 4*a*c)*a^3*b^3
*c^7 - 96*(b^2 - 4*a*c)*a^4*b*c^8)*d^3 + (2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48
*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4
+ 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 24*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 12*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2
*c^2 + 12*(b^2 - 4*a*c)*a^2*c^3)*(a*b^2*c - 4*a^2*c^2)^2*e^3 + 12*(sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^4*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
3*c^4 + 2*a^3*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5
```

```
+ 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 - 16*a^4*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^4*c^6 + 32*a^5*c^6 - 2*(b^2 - 4*a*c)*a^3*b^2*c^4 + 8*(
b^2 - 4*a*c)*a^4*c^5)*d*abs(a*b^2*c - 4*a^2*c^2)*e^2 + 12*(2*a^3*b^6*c^6 -
16*a^4*b^4*c^7 + 32*a^5*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^5*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^5*b^2*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^4*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b^4*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^4*b^2*c^7 - 2*(b^2 - 4*a*c)*a^3*b^4*c^6 + 8*(b^2 - 4*a*c)*a^4*b
^2*c^7)*d^2*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 2*a^3*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
4*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 16*a^4*b^
3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 32*a^5*b*c^5
- 2*(b^2 - 4*a*c)*a^3*b^3*c^3 + 8*(b^2 - 4*a*c)*a^4*b*c^4)*abs(a*b^2*c - 4*
a^2*c^2)*e^3 - 3*(2*a^3*b^7*c^5 - 8*a^4*b^5*c^6 - 32*a^5*b^3*c^7 + 128*a^6*
b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^
4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4
 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 -
64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^6 - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 + 16
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 - 2*(b
^2 - 4*a*c)*a^3*b^5*c^5 + 32*(b^2 - 4*a*c)*a^5*b*c^7)*d*e^2 - (2*a^3*b^8*c^
4 - 32*a^4*b^6*c^5 + 160*a^5*b^4*c^6 - 256*a^6*b^2*c^7 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 2*(b^2 - 4*a*c)*a^3*b^6*
c^4 + 24*(b^2 - 4*a*c)*a^4*b^4*c^5 - 64*(b^2 - 4*a*c)*a^5*b^2*c^6)*e^3)*arc
tan(2*sqrt(1/2)*x/sqrt((a*b^3*c - 4*a^2*b*c^2 - sqrt((a*b^3*c - 4*a^2*b*c^2
)^2 - 4*(a^2*b^2*c - 4*a^3*c^2)*(a*b^2*c^2 - 4*a^2*c^3)))/(a*b^2*c^2 - 4*a^
2*c^3)))/((a^3*b^6*c^3 - 12*a^4*b^4*c^4 - 2*a^3*b^5*c^4 + 48*a^5*b^2*c^5 +
16*a^4*b^3*c^5 + a^3*b^4*c^5 - 64*a^6*c^6 - 32*a^5*b*c^6 - 8*a^4*b^2*c^6 +
16*a^5*c^7)*abs(a*b^2*c - 4*a^2*c^2)*abs(c))
```

maple [B] time = 0.05, size = 1846, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4/a/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d^3+2*a/(4*a*c-b^2)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)*b*e^3-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*b^3*e^3-3/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
```

```
b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b
^2*d*e^2+1/4/a/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d^3+2*a/(4*a*c-b^2)/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e^3-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e^3-3/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*b^2*d*e^2-3*a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*d*e^2+3/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*b*d^2*e+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d
^3-3*a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d*e^2+3/(4*a*c-b
^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d^2*e+1/4/a/(4*a*c-b^2)*c/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d^3+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*b^2*e^3+3/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d*e^2-3/2/(4*a*c-b^
2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*d^2*e-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*d^3-1/4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e^3-3/4/(4*a*
c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)*b*d*e^2+3/2/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^2*
e-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^3-3/2*a/(4*a*c-
b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*e^3+3/2*a/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e^3+(-1
/2*(2*a^2*c*e^3-a*b^2*e^3+3*a*b*c*d*e^2-6*a*c^2*d^2*e+b*c^2*d^3)/a/c/(4*a*c
-b^2)*x^3+1/2/c*(a^2*b*e^3-6*a^2*c*d*e^2+3*a*b*c*d^2*e+2*a*c^2*d^3-b^2*c*d^
3)/(4*a*c-b^2)/a*x)/(c*x^4+b*x^2+a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(bc^2d^3 - 6\,ac^2d^2e + 3\,abcde^2 - \left(ab^2 - 2\,a^2c\right)e^3\right)x^3 - \left(3\,abcd^2e - 6\,a^2cde^2 + a^2be^3 - \left(b^2c - 2\,ac^2\right)d^3\right)x}{2\left(a^2b^2c - 4\,a^3c^2 + \left(ab^2c^2 - 4\,a^2c^3\right)x^4 + \left(ab^3c - 4\,a^2bc^2\right)x^2\right)} - \frac{-\int \frac{3\,abc}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*x^
3 - (3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3)*x)/
(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c
^2)*x^2) - 1/2*integrate(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 + (b^2
*c - 6*a*c^2)*d^3 + (b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 + (a*b^2 - 6
*a^2*c)*e^3)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a^2*c^2)
```

mupad [B] time = 8.79, size = 29030, normalized size = 51.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x)

[Out] - ((x^3*(b*c^2*d^3 - a*b^2*e^3 + 2*a^2*c*e^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2))/(2*a*c*(4*a*c - b^2)) - (x*(2*a*c^2*d^3 + a^2*b*e^3 - b^2*c*d^3 - 6*a^2*c*d*e^2 + 3*a*b*c*d^2*e))/(2*a*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^5*c^3*e^6 - 72*a^2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a

^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*1i - (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^5*c^3*e^6 - 72*a^2*c^6*d^6 - a^2*b^6*e^

6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 - 74*a^4*b^2*c^2*e^6
- 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*d^4*e^2 + 44*a^
2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*d^2*e^4 - 6*a*b
^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*a^4*b*c^3*d*e^5
 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4*c^3 + a^2*b^4*
c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 384
0*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 +
 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d
^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 38
40*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2
*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c -
 b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^
7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b
^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*
a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 +
8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b
^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^
5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*
d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d
*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(
-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(40
96*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^
6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*1i)/((5*a^4*b^4*e^9 +
216*a^6*c^2*e^9 + 5*b^3*c^5*d^9 - 66*a^5*b^2*c*e^9 + a*b^7*d^3*e^6 - 9*a^3*
b^5*d*e^8 + 216*a^2*c^6*d^8*e - 9*b^4*c^4*d^8*e + 3*a^2*b^6*d^2*e^7 + 864*a
^3*c^5*d^6*e^3 + 1296*a^4*c^4*d^4*e^5 + 864*a^5*c^3*d^2*e^7 + 3*b^5*c^3*d^7
*e^2 + b^6*c^2*d^6*e^3 - 36*a*b*c^6*d^9 + 624*a^2*b^2*c^4*d^6*e^3 - 6*a^2*b
^3*c^3*d^5*e^4 - 108*a^2*b^4*c^2*d^4*e^5 + 1020*a^3*b^2*c^3*d^4*e^5 + 128*a
^3*b^3*c^2*d^3*e^6 + 384*a^4*b^2*c^2*d^2*e^7 + 54*a*b^2*c^5*d^8*e + 6*a*b^6
*c*d^4*e^5 + 153*a^4*b^3*c*d*e^8 - 612*a^5*b*c^2*d*e^8 + 24*a*b^3*c^4*d^7*e
^2 - 46*a*b^4*c^3*d^6*e^3 - 3*a*b^5*c^2*d^5*e^4 - 720*a^2*b*c^5*d^7*e^2 - 3
*a^2*b^5*c*d^3*e^6 - 1944*a^3*b*c^4*d^5*e^4 - 90*a^3*b^4*c*d^2*e^7 - 1872*a
^4*b*c^3*d^3*e^6)/(4*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*
c^3)) + (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*
a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c
^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3
072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3
*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5
*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^
3)) - (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*
d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*
c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*
a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c
^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*
a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/
2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^
2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e
^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*
d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2
*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2
)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*
e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 38
4*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*
a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664
*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^
2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 +
 a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840
*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 +

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 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2
*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d
^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c
^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a
^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^
7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a
^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2
) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2
 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^
2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d
^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*
c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)
^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e
 - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384
*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a
^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*
a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2
)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 +
a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*
a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^5*c^3*e^6 - 72*a^2*c^6*d
^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 - 74*a
^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*
d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*
d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*
a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4
*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3
*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*
a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) -
 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b
^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) -
288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3
*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e
^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e
^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d
^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5
*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2
*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9
)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e -
 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 46
08*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 38
4*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*
a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)
^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^
5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (((61
44*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*a^6*c^6*d*e^2
 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c^6*d^3 + 16*a
^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3072*a^5*b*c^6
*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3*b^6*c^3*d*e^
2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5*b^2*c^5*d*e^
2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x*((27
*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4
*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a
^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5
 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*
b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e
^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^
7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^
8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*
b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768
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*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3
- 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) +
9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^1
0*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6
*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d
*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^
2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) -
 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3
 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7
- 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*
c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*
a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*
d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^
4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5
- 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b
^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^
6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7
*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8
*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b
^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*
a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 -
 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9
*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10
*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*
d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*
e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2
*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) -
6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3
- 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 -
 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^5*c^3*e^6 - 72*a^2*c^6*d^6 - a^2*b^6*
e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 - 74*a^4*b^2*c^2*e^
6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*d^4*e^2 + 44*
a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*d^2*e^4 - 6*a
*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*a^4*b*c^3*d*e
^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4*c^3 + a^2*b^
4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3
840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6
 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8
*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 -
3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c
^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c
 - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*
b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4
*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 1382
4*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4
+ 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a
*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*
c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^
7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4
*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e
*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(
4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*
b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)))*((27*a*b^9*c^4*d^6
 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c
- b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*
a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7
*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a
*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*
```

b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3
+ 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3
+ 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^
3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d
^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^
3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*
e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 10
8*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a
^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a
^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a
^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e
^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10
*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2
*c^8)))^(1/2)*2i - atan((((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*
b*c^5*e^3 + 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3
 - 5632*a^4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^
5*b^3*c^4*e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c
^4*d^2*e - 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4
*d*e^2 - 4608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^
2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6
+ 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*
e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*
c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6
 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^
7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*
a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a
^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*
a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 1
3824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e
^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) -
6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b
^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2
*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*
c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^
5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a
^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 -
 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*
b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 +
 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e
^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c
^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6
- 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7
*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a
*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^
3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a
^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13
824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^
4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^
2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6
*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^
6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*
c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c
^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5
e(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32
*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^
6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^5*c^3*e
^6 - 72*a^2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3

*b^4*c*e^6 - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*1i - (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a

^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^5*c^3*e^6 - 72*a^2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*1i)/((5*a^4*b^4*e^9 + 216*a^6*c^2*e^9 + 5*b^3*c^5*d^9 - 66*a^5*b^2*c*e^9 + a*b^7*d^3*e^6 - 9*a^3*b^5*d*e^8 + 216*a^2*c^6*d^8*e - 9*b^4*c^4*d^8*e + 3*a^2*b^6*d^2*e^7 + 864*a^3*c^5*d^6*e^3 + 1296*a^4*c^4*d^4*e^5 + 864*a^5*c^3*d^2*e^7 + 3*b^5*c^3*d^7*e^2 + b^6*c^2*d^6*e^3 - 36*a*b*c^6*d^9 + 624*a^2*b^2*c^4*d^6*e^3 - 6*a^2*b^3*c^3*d^5*e^4 - 108*a^2*b^4*c^2*d^4*e^5 + 1020*a^3*b^2*c^3*d^4*e^5 + 128*a^3*b^3*c^2*d^3*e^6 + 384*a^4*b^2*c^2*d^2*e^7 + 54*a*b^2*c^5*d^8*e + 6*a*b^6*c*d^4*e^5 + 153*a^4*b^3*c*d*e^8 - 612*a^5*b*c^2*d*e^8 + 24*a*b^3*c^4*d^7*e^2 - 46*a*b^4*c^3*d^6*e^3 - 3*a*b^5*c^2*d^5*e^4 - 720*a^2*b*c^5*d^7*e^2 - 3*a^2*b^5*c*d^3*e^6 - 1944*a^3*b*c^4*d^5*e^4 - 90*a^3*b^4*c*d^2*e^7 - 1872*a^4*b*c^3*d^3*e^6)/(4*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*

d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*
d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/
2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*
a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^
5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6
*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c
^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1
280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*
c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 +
 a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*
e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^
9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*
a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6
*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^
5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(
-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 3
84*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3
744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4
 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d
^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d
^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2
) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a
^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5
*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*
b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^
3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 12
80*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^5*
c^3*e^6 - 72*a^2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 1
6*a^3*b^4*c*e^6 - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*
e^4 - 102*a^2*b^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*
e^4 - 174*a^3*b^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6
*a^2*b^5*c*d*e^5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*
c^2*d*e^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^
6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c
 - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4
*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^
7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*
a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7
*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3
 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3
 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e
^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*
d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b
^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2
e^4(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 1
08*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*
a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*
a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*
a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*
e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^1
0*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^
2*c^8)))^(1/2) + (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^
3 + 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*
a^4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^
4*e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e
 - 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 -
 4608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a

^4*b^2*c^3)) + (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a
^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 38
40*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*
e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*
a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^
6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^
2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c
^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*
c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5
*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 819
2*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*
a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10
*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d
^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5
*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^
5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4
*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*
a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c
^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*
b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c -
8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^
5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 384
0*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e
 - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a
^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6
 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2
)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^
4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c
^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*
b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192
*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a
*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*
c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^
5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*
e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5
 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*
a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a
^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^
6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^5*c^3*e^6 - 72*
a^2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e
^6 - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2
*b^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3
*b^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*
e^5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(
2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*
d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1
/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9
)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1
504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)
^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6
- b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*
c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*
c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*
b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 374
4*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^
4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c
 - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^
4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d
*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4

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*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d
*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a
^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2
)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 +
 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e
^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c
^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^
6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b
^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) -
18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 8
8*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 -
768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e
^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*
d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^
(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6
*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4
*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b
^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*
b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)
^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*
b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*
b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.271 $\int \frac{\left(d+ex^2\right)^2}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=386

$$\frac{x\left(x^2\left(abe^2-4acde+bcd^2\right)-2abde-2a\left(cd^2-ae^2\right)+b^2d^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(\frac{b^2\left(cd^2-ae^2\right)+8abcde-4ac\left(ae^2+3cd^2\right)}{\sqrt{b^2-4ac}}+abe^2-4acde+\right.}{2\sqrt{2}\,a\sqrt{c}\,\left(b^2-4ac\right)\sqrt{b-\sqrt{b}}}$$

```
[Out] 1/2*x*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+c*d^2)+(a*b*e^2-4*a*c*d*e+b*c*d^2)*x^2
)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2))*(b*c*d^2-4*a*c*d*e+a*b*e^2+(8*a*b*c*d*e+b^2*(-a*e^2+c*d^2)
-4*a*c*(a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/
(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2))*(b*c*d^2-4*a*c*d*e+a*b*e^2+(-8*a*b*c*d*e-b^2*(-a*e^2+c*d^2)+4*
a*c*(a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 2.08, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1205, 1166, 205}

$$\frac{x\left(x^2\left(abe^2-4acde+bcd^2\right)-2abde-2a\left(cd^2-ae^2\right)+b^2d^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(\frac{b^2\left(cd^2-ae^2\right)+8abcde-4ac\left(ae^2+3cd^2\right)}{\sqrt{b^2-4ac}}+abe^2-4acde+\right.}{2\sqrt{2}\,a\sqrt{c}\,\left(b^2-4ac\right)\sqrt{b-\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d^2 - 4*a*c*d*e
+ a*b*e^2 + (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a*e^2))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d^2
 - 4*a*c*d*e + a*b*e^2 - (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^
2 + a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
```

```
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2d^2-2abde-2a\left(cd^2-ae^2\right)+\left(bcd^2-4acde+abe^2\right)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b^2d^2-2abde+2a\left(3cd^2+ae^2\right) \ldots}{a+bx^2 \ldots}}{2a\left(b^2 \ldots\right.}$$

$$= \frac{x\left(b^2d^2-2abde-2a\left(cd^2-ae^2\right)+\left(bcd^2-4acde+abe^2\right)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd^2-4acde+abe^2- \ldots\right.}{\ldots}$$

$$= \frac{x\left(b^2d^2-2abde-2a\left(cd^2-ae^2\right)+\left(bcd^2-4acde+abe^2\right)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd^2-4acde+abe^2+ \ldots\right.}{2\sqrt{2}\,\ldots}$$

Mathematica [A] time = 1.11, size = 415, normalized size = 1.08

$$\frac{\frac{2x\left(2a^2e^2+abe\left(ex^2-2d\right)-2acd\left(d+2ex^2\right)+b^2d^2+bcd^2x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b^2\left(cd^2-ae^2\right)-4ac\left(e\left(d\sqrt{b^2-4ac}+ae\right)+3cd^2\right)+b\left(cd\left(d\sqrt{b^2-4ac}+8ae\right)+ae^2\sqrt{b^2-4ac}\right)\right)\tan \ldots}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \ldots}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^2 + a*b*e*(-2*d + e*x^2) - 2*a*c*d*(
d + 2*e*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d^2 -
 a*e^2) - 4*a*c*(3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b*(a*Sqrt[b^2 -
 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-(c*d^2) + a*e^2) + b*(a*Sqrt[b^2 - 4*a*c]*
e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e)) + 4*a*c*(3*c*d^2 + e*(-(Sqrt[b^2 -
 4*a*c]*d) + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

fricas [B] time = 11.08, size = 7338, normalized size = 19.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^
2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*
b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*
e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^
```

3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c
^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*
d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^
2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*
c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 6
4*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*lo
g(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c
^3 + 324*a^2*b*c^4)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a
^3*c^4)*d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3 +
(3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^3*b^3*c + 12*
a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d^2*e^6 - 2*(a^4
*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6*c)*e^8)*x + 1/2*sqrt(1/2)*((b
^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6
+ 6*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(
2*a^2*b^6*c - a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3
*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b^4*c +
96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b
*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 - ((a^3*b^9*c - 20
*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(
a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7
*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c
^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*
a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b
^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*
c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*
b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*
e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^
3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c
^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*
d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^
2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*
c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 6
4*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*
b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6
*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8
*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c
- 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3
+ 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3
+ 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3
*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a
^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 4
8*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*
d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^7*e + (b^6*c - 51*a*b^
4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*
c^2 - 244*a^3*b*c^3)*d^5*e^3 + (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3
)*d^4*e^4 - 10*(a^3*b^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c
- 48*a^5*c^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^
6*c)*e^8)*x - 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*
a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c
^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c - a^3*b^4*c^2 - 88*a^4*b^2*c^3 +
240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e
^3 - (a^3*b^6 - 18*a^4*b^4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a
^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)*e^6 - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3
*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4
- 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^
8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2
*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 -

```
9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c -
 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a
^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6
*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8
*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c
- 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3
 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3
+ 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3
*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a
^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 4
8*a^5*b^2*c^3 - 64*a^6*c^4))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*
a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^
3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3
+ 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c
^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6
*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c
^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^
2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/
(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3
 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*
b*c^4)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*d^6*e
^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3 + (3*a^2*b^4*c
+ 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^3*b^3*c + 12*a^4*b*c^2)*d^
3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5
*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6*c)*e^8)*x + 1/2*sqrt(1/2)*((b^8*c - 23*a*b
^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c
- 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c -
 a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^5*c - 8*a^
4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b^4*c + 96*a^5*b^2*c^
2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d*e^5 +
 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 + ((a^3*b^9*c - 20*a^4*b^7*c^2
+ 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8
*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b
^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c^2*d^5*e^3 +
8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 8
1*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^
3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*
b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*
a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^
3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3
+ 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c
^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6
*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c
^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^
2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/
(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) - sqrt(1/2)*((
a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqr
t(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2
- 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c
 + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*
c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*
d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)
*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6
*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 +
48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3
 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^8 - 2*(3*b^
5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a
^2*b^2*c^3 + 432*a^3*c^4)*d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3
```

```
*b*c^3)*d^5*e^3 + (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 1
0*(a^3*b^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2
)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6*c)*e^8)*x -
 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 +
 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*
b*c^4)*d^5*e + 2*(2*a^2*b^6*c - a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)
*d^4*e^2 - 12*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6
 - 18*a^4*b^4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^
5*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6
+ ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^
7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5
)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)
*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^
8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)
*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*
d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqr
t(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2
- 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c
 + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*
c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*
d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)
*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6
*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 +
48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3
 - 64*a^6*c^4))) - 2*(2*a*b*d*e - 2*a^2*e^2 - (b^2 - 2*a*c)*d^2)*x)/((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

giac [B] time = 1.85, size = 6390, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d^2*x^3 - 4*a*c*d*x^3*e + a*b*x^3*e^2 + b^2*d^2*x - 2*a*c*d^2*x -
2*a*b*d*x*e + 2*a^2*x*e^2)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*(
(2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(
b^2 - 4*a*c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d^2 - 4*(2*a*b^2*c^3 - 8*a^2*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*
b^2 - 4*a^2*c)^2*d*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c -
 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 2*a*b^6*c^2 + 64*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 28*a^2*b
^4*c^3 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 48*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^4 - 128*a^3*b^2*c^4 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^5 + 192*a^4*c^5 + 2*(b^2 - 4*a*c)*a*b^4*c^2 - 20*(b^2 - 4*a
*c)*a^2*b^2*c^3 + 48*(b^2 - 4*a*c)*a^3*c^4)*d^2*abs(a*b^2 - 4*a^2*c) + (2*a
*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2
*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2*e^2 + 4*(sqrt(2)*sqrt(b*c + sqr
```

t(b^2 - 4*a*c)*c)*a^2*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*a^2*b^
5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + 8*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^3*c^3 + 16*a^3*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^4 - 32*a^4*b*c^4 + 2*(b^2 - 4*a*c)*a^2*b^3*c^2 - 8*(b^2 - 4*
a*c)*a^3*b*c^3)*d*abs(a*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^3 - 40*a^3*b^5*c^4
+ 224*a^4*b^3*c^5 - 384*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^7*c + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^5*b*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^2*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^3*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*b^5*c^3 + 32*(b^2 - 4*a*c)*a^3
*b^3*c^4 - 96*(b^2 - 4*a*c)*a^4*b*c^5)*d^2 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*
c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*a^3*b^4*c^2
 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^3 + 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^2*c^3 + 16*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*c^4 - 32*a^5*c^4 + 2*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a*c)*a^4*c^3)
*abs(a*b^2 - 4*a^2*c)*e^2 + 8*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b^2*
c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c +
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 1
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*(b^2
 - 4*a*c)*a^3*b^4*c^3 + 8*(b^2 - 4*a*c)*a^4*b^2*c^4)*d*e - (2*a^3*b^7*c^2 -
 8*a^4*b^5*c^3 - 32*a^5*b^3*c^4 + 128*a^6*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^6*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*b*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c^2 + 32*(b^2 - 4*a*c
)*a^5*b*c^4)*e^2)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^
3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c -
 4*a^2*c^2)))/((a^3*b^6*c - 12*a^4*b^4*c^2 - 2*a^3*b^5*c^2 + 48*a^5*b^2*c^3
 + 16*a^4*b^3*c^3 + a^3*b^4*c^3 - 64*a^6*c^4 - 32*a^5*b*c^4 - 8*a^4*b^2*c^4
 + 16*a^5*c^5)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^3 - 8*a*b*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(a*
b^2 - 4*a^2*c)^2*d^2 - 4*(2*a*b^2*c^3 - 8*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4*a^2*c)^2*d*e -
2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 14*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c

)*a*b^5*c^2 + 2*a*b^6*c^2 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^2*c^3 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 28*a^2*b^4*c^3 - 96*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^4 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 +
 128*a^3*b^2*c^4 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 192
*a^4*c^5 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 20*(b^2 - 4*a*c)*a^2*b^2*c^3 - 48*(b
^2 - 4*a*c)*a^3*c^4)*d^2*abs(a*b^2 - 4*a^2*c) + (2*a*b^3*c^2 - 8*a^2*b*c^3
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)
*(a*b^2 - 4*a^2*c)^2*e^2 - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 2*a^2*b^5*c^2 + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 -
16*a^3*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 32*a
^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^3*c^2 + 8*(b^2 - 4*a*c)*a^3*b*c^3)*d*abs(a
*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 224*a^4*b^3*c^5 - 384
*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^7*c + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
5*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6
*c^2 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
3*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c
^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^
4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^
4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^
4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5
- 2*(b^2 - 4*a*c)*a^2*b^5*c^3 + 32*(b^2 - 4*a*c)*a^3*b^3*c^4 - 96*(b^2 - 4*
a*c)*a^4*b*c^5)*d^2 + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 2*a^3*b^4*c^2 + 16*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^5*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 16*a^4*b^
2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 32*a^5*c^4 - 2*
(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 - 4*a*c)*a^4*c^3)*abs(a*b^2 - 4*a^2*c)*e
^2 + 8*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b^2*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*(b^2 - 4*a*c)*a^3*b^4*c^3 +
 8*(b^2 - 4*a*c)*a^4*b^2*c^4)*d*e - (2*a^3*b^7*c^2 - 8*a^4*b^5*c^3 - 32*a^5
*b^3*c^4 + 128*a^6*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^5*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6
*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b
^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b
c^4 - 2(b^2 - 4*a*c)*a^3*b^5*c^2 + 32*(b^2 - 4*a*c)*a^5*b*c^4)*e^2)*arcta
n(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a
^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6
*c - 12*a^4*b^4*c^2 - 2*a^3*b^5*c^2 + 48*a^5*b^2*c^3 + 16*a^4*b^3*c^3 + a^3

```
*b^4*c^3 - 64*a^6*c^4 - 32*a^5*b*c^4 - 8*a^4*b^2*c^4 + 16*a^5*c^5)*abs(a*b^
2 - 4*a^2*c)*abs(c))
```

maple [B] time = 0.04, size = 1223, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2/a*(a*b*e^2-4*a*c*d*e+b*c*d^2)/(4*a*c-b^2)*x^3-1/2*(2*a^2*e^2-2*a*b*d*
e-2*a*c*d^2+b^2*d^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*b*e^2-1/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d*e+1/4/a/(4*a
*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d^2-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*e^2-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*b^2*e^2+2/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d*
e-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^2+1/4/a/(4*a
*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d^2-1/4/(4*a*c-b^2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*b*e^2+1/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d*e-1/4/a/(4*a*
c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d^2-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*e^2-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*
e^2+2/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d*e-3/(4*a*c-b^
2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^2+1/4/a/(4*a*c-b^2)*c/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^3 - (2*a*b*d*e - 2*a^2*e^2 - (b^2 -
2*a*c)*d^2)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*
a^2*b*c)*x^2) + 1/2*integrate((2*a*b*d*e - 2*a^2*e^2 + (b^2 - 6*a*c)*d^2 +
(b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2
*c)
```

mupad [B] time = 9.84, size = 18785, normalized size = 48.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b
^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e
^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a
^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 -
64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11*c*d^4 + a^3*b^9*e
^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d
^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(
-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2
*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2
*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1
344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 7
2*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4
*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b
^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^
(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^1
2*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c
^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b
^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^1
1*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4
 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^
4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*
c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d
^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*
b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*
a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*
c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^
5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c
^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(72*a^2*c^5*d^4 + 8*
a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^
2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 8
0*a^2*b*c^4*d^3*e - 16*a^3*b*c^3*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a
*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-
(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(
1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504
*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*
c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2
*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e
 - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 25
6*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656
*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3
*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c
^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c
^6)))^(1/2)*1i - (((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2
- 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^
3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5
*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*
(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11*c*d^4
+ a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*
a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 -
b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^
3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*
a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*
d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*
c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*
e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e
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- 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 16*a^3*b*c^3*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i)/((5*b^3*c^4*d^6 - 3*a^3*b^3*c*e^6 - 4*a^4*b*c^2*e^6 + 144*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 - 6*b^4*c^3*d^5*e + 160*a^3*c^4*d^3*e^3 + b^5*c^2*d^4*e^2 - 36*a*b*c^5*d^6 + 152*a^2*b^2*c^3*d^3*e^3 - 29*a^2*b^3*c^2*d^2*e^4 + 36*a*b^2*c^4*d^5*e + a*b^5*c*d^2*e^4 + 2*a^2*b^4*c*d*e^5 + 11*a*b^3*c^3*d^4*e^2 - 8*a*b^4*c^2*d^3*e^3 - 300*a^2*b*c^4*d^4*e^2 - 140*a^3*b*c^3*d^2*e^4 + 36*a^3*b^2*c^2*d*e^5)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*

a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4
*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*
a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*
a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9
*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^
2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d
^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5
*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)
+ (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a*b^2*c^4*d^4 + a^2
*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*a^2*b^2*c^3*d^2*e^
2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 16*a^3*b*c^3*d*e^3 - 12*a^2*b^
3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3
*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b
*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c
d^4(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 2
88*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b
^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e
^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e
^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 2
56*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768
*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^
2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a
^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7
*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e
^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a
^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^
4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 153
6*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
)) + (x*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27
*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2)
- 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3
*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840
*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^
3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4
*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2
+ 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3
072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*
a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 -
1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b
*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 1
6*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c -
b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a
*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2)
+ 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3
*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*
e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2
+ 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2
*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^
4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6
*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(
-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 +
240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6))
)^(1/2) - (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a*b^2*c^4*d
^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*a^2*b^2*c^
3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 16*a^3*b*c^3*d*e^3 - 1
2*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d
^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 38

40*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*2i - ((x^3*(a*b*e^2 + b*c*d^2 - 4*a*c*d*e))/(2*a*(4*a*c - b^2)) + (x*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + atan(((((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 16*a^3*b*c^3*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3

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840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^
4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*
d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 +
 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*
c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*
b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*
d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^
3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^
9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 +
 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i - (((6144*a^5*c^6*d^2 + 20
48*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*
d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 153
6*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c
^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2)) + (x*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11
*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2
)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*
a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4
*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*
a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*
a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9
*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^
2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d
^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5
*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*
(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(
a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) -
a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*
d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2
)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4
+ 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^
6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c
^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2
*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*
e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3
+ 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b
*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*
b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8
*b^2*c^6)))^(1/2) - (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a
*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*
a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 16*a^3*b*c^3
*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(
(a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2
*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*
b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*
a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c
^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*
a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 +
 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*
b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^
2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*
(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b
^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i)/((5*b^3*c^4*d^6 -
 3*a^3*b^3*c*e^6 - 4*a^4*b*c^2*e^6 + 144*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 -
 6*b^4*c^3*d^5*e + 160*a^3*c^4*d^3*e^3 + b^5*c^2*d^4*e^2 - 36*a*b*c^5*d^6 +
 152*a^2*b^2*c^3*d^3*e^3 - 29*a^2*b^3*c^2*d^2*e^4 + 36*a*b^2*c^4*d^5*e + a*
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b^5*c*d^2*e^4 + 2*a^2*b^4*c*d*e^5 + 11*a*b^3*c^3*d^4*e^2 - 8*a*b^4*c^2*d^3*
e^3 - 300*a^2*b*c^4*d^4*e^2 - 140*a^3*b*c^3*d^2*e^4 + 36*a^3*b^2*c^2*d*e^5)
/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (((6144*a^5*c
^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a
^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b^4*c^
3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 38
4*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3
*b^4*c + 48*a^4*b^2*c^2)) - (x*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9
*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(
4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1
/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*
a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c
^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*
e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e
 + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256
*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*
a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*
e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^
2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^
6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3
*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^
9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4
 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(
4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b
^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e
^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 134
4*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*
a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b
^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4
*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1
/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*
c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5
 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3
*d^4 - 14*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^
2*e^2 + 44*a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 1
6*a^3*b*c^3*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3
*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 2
7*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2)
 + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^
3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 384
0*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d
^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^
4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2
 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 +
3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2
*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^
(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 -
 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (((6144*
a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1
920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b
^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e
 - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^
3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^
4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^
9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 +
1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*
b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3
*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d
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^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e
- 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 +
6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c
*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^
10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b
^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^
4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c -
b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^
6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^
4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*
a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*
c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2
+ 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3
+ 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*
a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^
5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^
9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*
b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^
4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^
4*c^3*d^4 - 14*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c
^4*d^2*e^2 + 44*a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*
e - 16*a^3*b*c^3*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 -
8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^
4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^
(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c
^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4
- 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^1
0*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b
^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^
2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e
^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^
2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2
)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*
c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)))*((a
^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d
^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*
c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^
7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5
*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^
3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 2
4*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^
6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*
c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^
2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4
096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6
*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

3.272 $\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=293

$$\frac{x\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-\right.}{2\sqrt{2}\,a\left(b^2-\right.}$$

```
[Out] 1/2*x*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*d-
2*a*e+(4*a*b*e-12*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*c^(1/2)*(b*d-2*a*e+(-4*a*b*e+12*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2)
)/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1178, 1166, 205}

$$\frac{x\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-\right.}{2\sqrt{2}\,a\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*e + (b^2*d - 12*a*c*d + 4*a*b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a
*e - (b^2*d - 12*a*c*d + 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b^2d+6acd-abe-c(bd-2ae)x^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(bd-2ae-\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+}}{4a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(bd-2ae+\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{}}{\sqrt{b-\sqrt{b}}}\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.75, size = 310, normalized size = 1.06

$$\frac{\frac{2x\left(b\left(cdx^2-ae\right)-2ac\left(d+ex^2\right)+b^2d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b\left(d\sqrt{b^2-4ac}+4ae\right)-2a\left(e\sqrt{b^2-4ac}+6cd\right)+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2-4ac}-2ae\sqrt{b}\right.}{(b}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d + b*(-(a*e) + c*d*x^2) - 2*a*c*(d + e*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 - 4*a*c]*d + 4*a
*e) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - 4*a*b*e - 2
*a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

fricas [B] time = 2.83, size = 4573, normalized size = 15.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c*d - 2*a*c*e)*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b
^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b
*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*
c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3
*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2
*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4 - (3*b^5*c - 65*
a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 28*a^2*b^2*c^2)*d^2*e^2 -
 (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*c^2)*e^4)*x + 1/
2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^
4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112*a^4*b*c^3)*d^2*
e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3)*d*e^2 + (a^3*b
^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 - ((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b
```

```
^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*a^5*b^6*c + 128*
a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b
^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c
)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(
(b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2
)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2
*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^
4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*b^3*c +
60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 1
2*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3
 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2
 - 64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4 - (3*b^5
*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 28*a^2*b^2*c^2)*d
^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*c^2)*e^4
)*x - 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3
+ 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112*a^4*b*c
^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3)*d*e^2
+ (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 - ((a^3*b^9 - 20*a^4*b^7*c + 1
44*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*a^5*b^6*
c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4
- 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 -
 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24
*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a
^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c
+ 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2
*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 1
2*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*
c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*
b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2
*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 +
4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5
*b^2*c^2 - 64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4
- (3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 28*a^2*b^
2*c^2)*d^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*
c^2)*e^4)*x + 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*
b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112
*a^4*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3
)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 + ((a^3*b^9 - 20*a^4*b
^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*
a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4
 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a
^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)))*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^
2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*
a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^
3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3
*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c
- 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5
 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*
e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
```

```
 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2
)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c
^4)*d^4 - (3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 2
8*a^2*b^2*c^2)*d^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c
+ 4*a^4*c^2)*e^4)*x - 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 -
672*a^3*b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c
^2 - 112*a^4*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32
*a^5*c^3)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 + ((a^3*b^9 -
20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*
b^8 - 8*a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 +
 a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*
e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 -
6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*
a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b
^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^
2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - 2*(a*b*e - (
b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3
- 4*a^2*b*c)*x^2)
```

giac [B] time = 1.76, size = 4433, normalized size = 15.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*x^3*e + b^2*d*x - 2*a*c*d*x - a*b*x*e)/((c*x^4 + b*x
^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d - 2*
(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*
c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^
2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2
 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(
b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*d*abs(a*b^2 - 4*a^2*c)
 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3 + 2*(b^2 - 4*a*c)*a^2*b
^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*abs(a*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^2 -
 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
```

```
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 -
4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*d + 4*(2*a^3*b^6*c^2 - 16*
a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*e)*
arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 -
 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^
3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*
b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2
 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^
2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a
*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*a
^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*(b^2 - 4
*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*d*abs(a*b^2 - 4*a^2*c) - 2*(s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
^4*c + 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 +
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 32*a^4*b*c^3 - 2*(b^2 - 4*a*c)*a^2*b^3*c +
8*(b^2 - 4*a*c)*a^3*b*c^2)*abs(a*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^2 - 40*a^3
*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*
a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*d + 4*(2*a^3*b^6*c^2 - 16*a^4*b^4
```

```
*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5
*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4
*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*
c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*e)*arctan(
2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2
*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 -
 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2
 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^
2*c)*abs(c))
```

maple [B] time = 0.08, size = 1761, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)
/c)*d+1/2/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*e-1/4/(4*a*c
-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b*d-12*c^3/(-4*a*c+b^2)^(1
/2)/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d*a-8*c^2/(-4*a*c+b^2)^
(1/2)/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+3/4*c/(-4*a*c+b
^2)^(1/2)/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*d-2*c^2/(4*
a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-3/2*c/(4*a*c-b^2)/(4*a*c+3
*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e+c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*b*d+3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*
d+4*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b*e+3*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b^3*e+1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/
c+1/2*b/c)*d+1/2/(4*a*c-b^2)*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*e-1/4
/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*b*d-12*c^3/(-4*a*c+
b^2)^(1/2)/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d*a-8*c^2/(-4*a*c+b
^2)^(1/2)/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+3/4*c/(-4*a*c+
b^2)^(1/2)/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*d+2*c^2/(4*a*
c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+3/2*c/(4*a*c-b^2)/(4*a*c+3*b^2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^2*e-c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
b*d-3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d+4*c^2/(-4*a*
c+b^2)^(1/2)/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e+3*c/(-4*a*c
```

```
+b^2)^(1/2)/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bcd-2\,ace)x^3-\left(abe-\left(b^2-2\,ac\right)d\right)x}{2\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}-\frac{-\int\frac{abe+(bcd-2\,ace)x^2+\left(b^2-6\,ac\right)d}{cx^4+bx^2+a}\,dx}{2\left(ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d - 2*a*c*e)*x^3 - (a*b*e - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2
*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - 1/2*integrate(-(
a*b*e + (b*c*d - 2*a*c*e)*x^2 + (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a
*b^2 - 4*a^2*c)
```

mupad [B] time = 9.39, size = 12350, normalized size = 42.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4
*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*
a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2)) - (x*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)
^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*
c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*
a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2
 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e
 + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b
*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
 + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5
)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*
c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 +
 a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840
*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1
504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b
^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6
*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1
536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4
096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7
*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 +
 b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40
*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^
2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(
1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*
c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2
+ 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2)
 + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*
c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3
 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6*d - 28
8*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*
e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c^5
*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11
*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c -
```

b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288
*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5
*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)
^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128
*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2
))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 -
16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c
^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^
(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c
^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a
^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2
- 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e
+ 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*
d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)
))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*
d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4
+ 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c
- b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 -
768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d
^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a
*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2
*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d
*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*
a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a
^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b
^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*
a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3
- 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c
^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*
b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e
^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e
- 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*
b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*
c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4
- 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5
*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*
d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b
^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*
a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*
c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^
9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*
a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6
*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5*d^
2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2
*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10
*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 -
96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*
a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2
*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2
- 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (((614
4*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d +
16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*a*b^8*c^2*d

- 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (8*a^3*c^4*e^3 + 5*b^3*c^4*d^3 + 72*a^2*c^5*d^2*e - 3*b^4*c^3*d^2*e + 6*a^2*b^2*c^3*e^3 - 36*a*b*c^5*d^3 + 18*a*b^2*c^4*d^2*e + 3*a*b^3*c^3*d*e^2 - 60*a^2*b*c^4*d*e^2)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i + atan(((((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8

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*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x
*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*
b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d
^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*
e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*
b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9
*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 1
92*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2)*1i - (((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 563
2*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e
+ 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2)) + (x*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2
- b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^
6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3
840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c
*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c
*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2
*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^
10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2
*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*
b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^
2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3
840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2
+ 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^
5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*
a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e
+ 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^
2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e -
 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)
^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^
7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^
2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/
2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^
4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^5*c^6*d -
288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^
2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c
^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((a^2
*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c -
 b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 28
8*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^
5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2
)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 12
8*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/
2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a
^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 -
 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*
c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^1
1*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c
^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a
^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2
- 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e
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- 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*
d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)
))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*
d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9
*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 7
68*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^
2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*
b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*
b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*
e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a
^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^
8*b^2*c^5)))^(1/2) + (((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c
^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*
b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^
2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^
2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c
^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 +
27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36
*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c
^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3
- 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)
^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^
7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^
2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/
2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^
4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*
a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3
*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a
^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c
 - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e -
288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*
b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b
^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e +
128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (8*a^3*c^4*e^
3 + 5*b^3*c^4*d^3 + 72*a^2*c^5*d^2*e - 3*b^4*c^3*d^2*e + 6*a^2*b^2*c^3*e^3
- 36*a*b*c^5*d^3 + 18*a*b^2*c^4*d^2*e + 3*a*b^3*c^3*d*e^2 - 60*a^2*b*c^4*d*
e^2)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*((a^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2
)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^
2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^
2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)
^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^
4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/
(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b
^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i + ((x*(a*b*e - b^2
*d + 2*a*c*d))/(2*a*(4*a*c - b^2)) + (c*x^3*(2*a*e - b*d))/(2*a*(4*a*c - b^
2)))/(a + b*x^2 + c*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.273 $\int \frac{1}{\left(a+bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=252

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{}}$$

```
[Out] 1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/
2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)
^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1092, 1166, 205}

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{b^2-2ac-2\left(b^2-4ac\right)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}} +$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} -$$

Mathematica [A] time = 0.42, size = 243, normalized size = 0.96

$$\frac{\frac{2x\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

fricas [B] time = 1.10, size = 2309, normalized size = 9.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6
- 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 67
2*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 4
48*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*
x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8
```

```
64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 +
512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 +
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)
*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sq
rt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(
(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

giac [B] time = 0.60, size = 2682, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1
/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*
b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^
3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c
)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
```

```
*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b
^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c
)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sq
rt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2
 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*
a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 6
4*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)
*abs(c)) - 1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5
*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7
+ 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 -
 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b
*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*
(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 4
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*
(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))
*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2
- 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a
^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3
*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^
2 - 4*a^2*c)*abs(c))
```

maple [B] time = 0.06, size = 733, normalized size = 2.91

$$\frac{\sqrt{2}\, b^{2} c \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4 \sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, a}+\frac{\sqrt{2}\, b^{2} c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4 \sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, a}-\frac{\cdots}{\sqrt{-4a \cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b+c/(-4*a*c+b^2
)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)-1/4/(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b^2+1/4*c/(
4*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
```

```
(1/2))*c)^(1/2)*c*x)+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b^2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*b-c/(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)+1/4/
(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*b
^2-1/4*c/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c
-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

mupad [B] time = 6.26, size = 6404, normalized size = 25.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 +
 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4
*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
 - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 38
40*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840
*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*
b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
- b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 384
0*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2
 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2
)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4
*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 409
6*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b
^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a
^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-
(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1
504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c
^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
```

```
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^
5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^
5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11
+ b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*
b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16
*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 24
0*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(
1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4
- 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^
8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(10
24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2
*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*
b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27
*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 2
4*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*a^5*c^3 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5)))^(1/2)*2i + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*
b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3
- 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2
*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5
*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^
3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*
(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*
c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^
3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*
a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*
```

```
b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*
a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 +
 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b
^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3
840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^
2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^
2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^
7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*
c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*
(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7
*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c
^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((
((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a
^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x
*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2
- 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^
9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 -
1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b
*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 1
6*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5
*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c
 + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^
10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2
*c^5)))^(1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3
*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a
^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4)
)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^
3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a
^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))
/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)
^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3
*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^
9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c
^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 - b^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^
3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a
^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i
```

sympy [A] time = 170.28, size = 394, normalized size = 1.56

$$\frac{-bcx^3 + x\left(2ac - b^2\right)}{8a^3c - 2a^2b^2 + x^4\left(8a^2c^2 - 2ab^2c\right) + x^2\left(8a^2bc - 2ab^3\right)} + \operatorname{RootSum}\left(t^4\left(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-b*c*x**3 + x*(2*a*c - b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2
- 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSum(_t**4*(1048576*a**9
*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*c
**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**2*(-
61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2
*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 +
25*b**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672*_t**3*
a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**4*b**7*c + 64*_t
**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*b**2*c**3 + 740*_t*a**2*b*
*4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(324*a**2*c**4 - 81*a*b**2*c**3 + 5*b
**4*c**2))))
```

3.274 $\int \frac{1}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=660

$$-\frac{\sqrt{c}\,e^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)^2}-\frac{\sqrt{c}\,e^2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)^2}+\frac{x\left(cx^2\left(2ace+b^2(-e)+bcd\right)\right.}{2a\left(b^2-4ac\right)\left(a+bx\right.}$$

```
[Out] 1/2*x*(b^2*c*d-2*a*c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^2)/a/(-4
*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)+e^(7/2)*arctan(x*e^(1/2)/d^(1
/2))/(a*e^2-b*d*e+c*d^2)^2/d^(1/2)-1/2*e^2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*
d*e+c*d^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*d-b^2*e+2*a*c*e+(8*a*b*c*e-1
2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*
d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*e^2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(
a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1
/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*d-b^2*e+2*a*c*e+(-8*
a*b*c*e+12*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/(a*e^2
-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 2.87, antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1238, 205, 1178, 1166}

$$\frac{x\left(cx^2\left(2ace+b^2(-e)+bcd\right)+3abce-2ac^2d+b^2cd+b^3(-e)\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(\frac{8abce-12ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x
^2))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x^2 + c*x^4)) - (Sqr
t[c]*e^2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 -
b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e + (b^2*c*d - 12*a*c^2
*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*e^2*(e + (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b
^2*e + 2*a*c*e - (b^2*c*d - 12*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(
b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2}\,dx = \int \left(\frac{e^4}{\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} + \frac{cd-be-cex^2}{\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)^2} - \cdots\right.$$

$$= -\frac{e^2\int \frac{-cd+be+cex^2}{a+bx^2+cx^4}\,dx}{\left(cd^2-bde+ae^2\right)^2} + \frac{e^4\int \frac{1}{d+ex^2}\,dx}{\left(cd^2-bde+ae^2\right)^2} + \frac{\int \frac{cd-be-cex^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{x\left(b^2cd-2ac^2d-b^3e+3abce+c\left(bcd-b^2e+2ace\right)x^2\right)}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\cdots\right.}{\sqrt{d}\,\left(cd^2-bde\cdots\right.}$$

$$= \frac{x\left(b^2cd-2ac^2d-b^3e+3abce+c\left(bcd-b^2e+2ace\right)x^2\right)}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\,e^2\left(e-\frac{2cd-\cdots}{\sqrt{b^2-\cdots}}\right.}{\sqrt{2}\sqrt{b-\sqrt{b^2-\cdots}}}$$

$$= \frac{x\left(b^2cd-2ac^2d-b^3e+3abce+c\left(bcd-b^2e+2ace\right)x^2\right)}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\,e^2\left(e-\frac{2cd-\cdots}{\sqrt{b^2-\cdots}}\right.}{\sqrt{2}\sqrt{b-\sqrt{b^2-\cdots}}}$$

Mathematica [A] time = 2.79, size = 708, normalized size = 1.07

$$\frac{\frac{2x\left(e(ae-bd)+cd^2\right)\left(-bc\left(3ae+cdx^2\right)+2ac^2\left(d-ex^2\right)+b^3e+b^2c\left(ex^2-d\right)\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b^2\left(-cde\left(2d\sqrt{b^2-4ac}+3ae\right)-3ae^3\sqrt{b^2-4ac}+c^2d^3\right)+2ac\left(cde\left(d\sqrt{b^2-4\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*(c*d^2 + e*(-(b*d) + a*e))*x*(b^3*e - b*c*(3*a*e + c*d*x^2) + 2*a*c^2*(
d - e*x^2) + b^2*c*(-d + e*x^2)))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) +
(Sqrt[2]*Sqrt[c]*(b^4*d*e^2 + 2*a*c*(-6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c]*e^3
 + c*d*e*(Sqrt[b^2 - 4*a*c]*d - 14*a*e)) + b^3*e*(-2*c*d^2 + e*(Sqrt[b^2 -
4*a*c]*d - 3*a*e)) + b^2*(c^2*d^3 - 3*a*Sqrt[b^2 - 4*a*c]*e^3 - c*d*e*(2*Sq
rt[b^2 - 4*a*c]*d + 3*a*e)) + b*c*(a*e^2*(-(Sqrt[b^2 - 4*a*c]*d) + 16*a*e)
+ c*d^2*(Sqrt[b^2 - 4*a*c]*d + 20*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*Sqrt[c]*(-(b^4*d*e^2) - b^2*(c^2*d^3 + 3*a*Sqrt[b^2 - 4*a*c]*e^3
 + c*d*e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e)) + b^3*e*(2*c*d^2 + e*(Sqrt[b^2 -
4*a*c]*d + 3*a*e)) + 2*a*c*(6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c]*e^3 + c*d*e*(
Sqrt[b^2 - 4*a*c]*d + 14*a*e)) + b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 20*a*e)
- a*e^2*(Sqrt[b^2 - 4*a*c]*d + 16*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
+ (4*e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d])/(4*(c*d^2 + e*(-(b*d) +
a*e))^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.06, size = 3841, normalized size = 5.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] e^4/(a*e^2-b*d*e+c*d^2)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)+1/2/(a*e^2-
b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^3*e^3+1/(a*e^2-b*d*e+c*d^2)^
2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c^3*d^3+1/4/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-
b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*d*e^2+1/4/(a*e^2-b*d*e+c*d
^2)^2/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*d*e^2-1/
2/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*b^3*d^2*e+1/4/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c^3*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*b*d^3-1/2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)*c/a/(4*a*c-b^2)*x^
3*b^3*d*e^2+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)*c^2/a/(4*a*c-b^2)*x^3*b
^2*d^2*e+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)/a/(4*a*c-b^2)*x*b^3*c*d^2*
e-1/4/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c^3*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d^3-3/4/
```

(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b
^3*e^3+1/4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d*e^2
-1/4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d*e^2-1/
2/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
c*x)*b^3*d^2*e+1/4/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^3*d*e^2+1/4/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c^3/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*b^2*d^3+4/(a*e^2-b*d*e+c*d^2)^2*a/(4*a*c-b^2)*c^2/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e^3-3/4/(a*e^2-b*d*e+c*d^2)^2/(4*
a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d*e^2+5/(a*e^2-b*
d*e+c*d^2)^2/(4*a*c-b^2)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d^2*e+
1/2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b^2*e^3-1/(a*e^
2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)*c^3/(4*a*c-b^2)*x^3*d^2*e-1/(a*e^2-b*d*e+c
*d^2)^2/(c*x^4+b*x^2+a)*c^2*a/(4*a*c-b^2)*x^3*e^3-3/4/(a*e^2-b*d*e+c*d^2)^2
/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e^3-7/(a*e^2-b*
d*e+c*d^2)^2*a/(4*a*c-b^2)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d*e
^2+1/2/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d^2*e
+4/(a*e^2-b*d*e+c*d^2)^2*a/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*b*e^3-7/(a*e^2-b*d*e+c*d^2)^2*a/(4*a*c-b^2)*c^3/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*d*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c^2*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*b^2*d^2*e-1/4/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*b^3*d*e^2+1/4/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c^3/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d^3+5/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^3/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d^2*e-3/4/(a*e^2-b*d*e+c*d^2)^2/
(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d*e^2-3/(a*e^2-b*
d*e+c*d^2)^2/(4*a*c-b^2)*c^4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^3+5/2/
(a*e^2-b*d*e+c*d^2)^2*a/(4*a*c-b^2)*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e^3-5/2/(a*e^
2-b*d*e+c*d^2)^2*a/(4*a*c-b^2)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e^3-3/2/(a*e^2-b*d*e+
c*d^2)^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b*e^3*c+1/(a*e^2-b*d*e+c*d^2)^2/(c
*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*c^2*d*e^2+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^
2+a)/(4*a*c-b^2)*x*b^2*c*d*e^2-5/2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)/(4
*a*c-b^2)*x*b*c^2*d^2*e-1/2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)/a/(4*a*c-
b^2)*x*b^4*d*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)/a/(4*a*c-b^2)*x*
b^2*c^2*d^3+1/2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^3*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*d^2*e-3/4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e^
3-3/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(

```
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*d^3-1/2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^3*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d^
2*e+3/4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e^3+1/2/
(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*b*d*e^2-1/2/(a*e^
2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)*c^3/a/(4*a*c-b^2)*x^3*b*d^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 +
 (b^2 + 2*a*c)*d^2*e^2)*sqrt(d*e)) + 1/2*((b*c^2*d - (b^2*c - 2*a*c^2)*e)*x
^3 + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*x)/(((a*b^2*c^2 - 4*a^2*c^3)
*d^2 - (a*b^3*c - 4*a^2*b*c^2)*d*e + (a^2*b^2*c - 4*a^3*c^2)*e^2)*x^4 + (a^
2*b^2*c - 4*a^3*c^2)*d^2 - (a^2*b^3 - 4*a^3*b*c)*d*e + (a^3*b^2 - 4*a^4*c)*
e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d^2 - (a*b^4 - 4*a^2*b^2*c)*d*e + (a^2*b^3 -
 4*a^3*b*c)*e^2)*x^2) + 1/2*integrate(((b^2*c^2 - 6*a*c^3)*d^3 - (2*b^3*c -
 11*a*b*c^2)*d^2*e + (b^4 - 2*a*b^2*c - 14*a^2*c^2)*d*e^2 - (3*a*b^3 - 13*a
^2*b*c)*e^3 + (b*c^3*d^3 - 2*(b^2*c^2 - a*c^3)*d^2*e + (b^3*c - a*b*c^2)*d*
e^2 - (3*a*b^2*c - 10*a^2*c^2)*e^3)*x^2)/(c*x^4 + b*x^2 + a), x)/((a*b^2*c^
2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c
 - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)
*e^4)
```

mupad [B] time = 16.46, size = 237586, normalized size = 359.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - atan((((((((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*
c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*
b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 91750
4*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 +
 10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c
^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78
848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*
c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4
608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^
5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 68454
4*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c
^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 15
5136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3
*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864
256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7
*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7
 + 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4
*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*
e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*
a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^1
1*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 -
12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^1
0*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^
12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6
```

*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*
d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2
088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^
9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^1
3 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920
*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9
*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7
609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^1
0*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*
e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600
576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c
^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^1
1 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*
a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^
8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 -
 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^1
3*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c
^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^
17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 14
08*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d
*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^
9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^
11 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a
^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 -
 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8
+ 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b
^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1
024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b
^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4
*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*
a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 1
92*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3
 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2
*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c
^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*
d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 +
 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a
^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7
*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*
b^3*c^3*d*e^7)) - (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a
^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 2
13*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^
7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*
a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(
1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6
 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^
4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a
*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4
*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b
^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 1545
6*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^
4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c
^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2
*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^
9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*
a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b
^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^
4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d
^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3

*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 37683

20*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^

2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10
*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b
^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*
b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c
^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2
) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a
*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2
*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2
)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^
5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^
10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*
a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^
2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4
*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^
5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^
12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4
*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 -
42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e
^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c
^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^
7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3
+ 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*
d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9
*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5
- 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e
- 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96
*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^
5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*
a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*
a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 4
9152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(626688*a^
10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b
^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^
8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 12697
6*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1
067008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2
 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 1
44*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^1
5*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*
c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 -
 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^
8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*
a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*
e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*
b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*
e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3
*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 +
 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c
^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 -
 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^
6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 -
175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11
*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7
- 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*
b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^
12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a
^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*
e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496

*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8
*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4
739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^1
2*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8
*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*
c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^
16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11
680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*
d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^
7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11
 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*
c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*
b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4
*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e
^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*
c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*
d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4
*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*
c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b
^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*
a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1
536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e
^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 -
1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*
b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c
*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d
^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*
d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6
 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*
c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*
b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*
d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*
a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7
*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a
*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)
^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 6
00*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e
^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^
6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^
5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 -
 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2
*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6
*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9
)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5
 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a
*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*
a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 168
96*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22
400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 -
 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)
^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^
5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) -
42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a
*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*
b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 2
4*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^
8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 614
4*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11

*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^
5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 +
16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2
+ 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^
5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c
^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a
^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 +
1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*
d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*
b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 1
7920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*
d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^
9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*
e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e
+ 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6
- 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7
+ 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d
*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (3269
12*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3
*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a
^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080
*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532
736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*
b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7
*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10
+ 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3
+ 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5
*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 234
88*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^
3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^
3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e
^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b
^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7
- 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*
c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 2
36800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c
^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1
106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^
2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^
9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6
*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4
*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b
^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 34
2272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*
e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b
^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*
(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^
5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 2
56*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^
4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 15
36*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3
*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a
^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 12
8*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 +
512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*
e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*
d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2
*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d
*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e -

4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*
a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7
*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*
b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9
*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^
8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e +
4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b
^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6
+ 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 2
5*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/
2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*
c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*
d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^
8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168
*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^
2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c
^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)
^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2
*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*
b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e
^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^
3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e
^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d
^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*
d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e
*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a
*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2
)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4
e^2(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/
2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4
*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8
- 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*
c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^
8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6
144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d
^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d
^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^
2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b
^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 145
6*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e
^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c
^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*
a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3
- 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4
*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*
a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*
a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^1
1*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c
*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*
c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*
c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^1
1*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(22800*a^6*c^9*e^1
3 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15
592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 3030
4*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^
5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4
*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7
+ 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*

d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a
^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^1
1 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 +
 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8
 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 -
 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^
8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8
*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6
*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^
4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8
 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7
*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6
*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^
6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5
*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*
c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6
*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536
*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6
 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 102
4*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7
*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^
3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*
e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e
^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3
840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e
^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^1
2*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6
 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4
*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^
3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c
 - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1
/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*
a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2
 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d
^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b
^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59
392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-
(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^
4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(
1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 5
1*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^
12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3
*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*
a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400
*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*
b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)
^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(
-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*
a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c
 - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c
^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)
^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a
^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 +
 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a
^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^
4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 +
 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 163
84*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 8
4*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e

^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*
d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*
b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 13
44*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5
*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3
*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 1792
0*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3
*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d
^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3
 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e +
84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 1
5360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 +
5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^
7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*1i - (((((
1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61
440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16
- 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^
12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^1
0*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 -
 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*
c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6
+ 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*
c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 2457
6*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^1
0*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 -
 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13
*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2
560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c
^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^
5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^
4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6
*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a
^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*
d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 +
4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*
b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^1
0 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^
5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^1
2*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31
076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^
7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^1
1 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6
*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11
*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37
101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^
8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^1
4 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 3436544
0*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c
^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 -
 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*
a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d
^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 -
9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12
*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*
a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 -
 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^
9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c
^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 70451
20*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*

e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*
a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d
e^15)/(16(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c
*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*
c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 +
a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d
^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e
^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*
e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d
^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^
2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b
^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a
^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 +
512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*
a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c
^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*
e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^
3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7
)) + (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 +
3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c
*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b
^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d
^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a
^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*
b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*
c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^
(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 60
0*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^
2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6
*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5
*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 -
59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*
b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)
^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 -
51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*
b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a
^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 1689
6*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 224
00*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 -
4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^
9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5
*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 4
2*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*
c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b
*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24
*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8
+ 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144
*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*
b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5
+ 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 1
6384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 +
84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5
*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^
4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^
7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 +
1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d
^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b

^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17
920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d
^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9
*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e
^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e
+ 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 -
 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7
+ 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*
e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576
*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^
11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 157
2864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*
e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12
*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 2
56*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*
d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 1433
6*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8
*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b
^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12
*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384
*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*
e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^
4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*
d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 +
490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15
*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 -
163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*
b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d
^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 22
79680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*
c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 3
27680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*
b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*
d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10
864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b
^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^
13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*
a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^
11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8
 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^
7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^
4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728
*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*
c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^
9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*
a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*
d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9
502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*
b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*
e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080
*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3
*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*
e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 292
8640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^1
1*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*
d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 +
 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*
a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*
e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9

```
*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8
 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^1
1*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e
^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*
b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9
*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^
5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4
 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^
8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*
c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^
7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5
*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*
a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 -
 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e
^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*
d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7
 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3
*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b
^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*
c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c
^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6
 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c
^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^
14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^
8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 106
56*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4
*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) +
22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d
^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e
^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4
*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*
b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 3
7632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^
3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(
1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*
c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-
(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1
116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 +
15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e
+ 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5
 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4
*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*
c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^
(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) +
120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c
 - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*
a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d
^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 2
40*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a
^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^
6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^
6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5
*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c
^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6
*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 -
10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^
```

2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b
^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 465
92*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*
e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*
b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*
b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4
*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*
e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d
^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d
*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c
^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(626688*a^10*b*c^8*e^15
 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15
+ 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^1
5 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^
11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^
11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^1
0*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d
^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9
 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2
 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^
7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 7
8496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*
d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^
2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12
*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*
a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*
e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^
11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 +
 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b
^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8
 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^
9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13
 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5
*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e
^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a
^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d
^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 305964
8*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10
*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2
370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^
3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*
a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 60
00*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 +
 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^
12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*
c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601
536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*
e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^
8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/
(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4
*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8
- 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12
*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 +
 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*
a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 19
2*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 -
 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^
5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d
^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c

^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9

*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*
d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10
*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16
*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^
3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*
b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6
 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^
7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*
a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^
8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a
^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^
11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*
b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10
 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2
*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 -
 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^
3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^
4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*
a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a
*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 2
40*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^1
3 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^
12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*
a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13
 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8
 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 +
 a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^
7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3
*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^
4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*
e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^
5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4
*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*
c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6
*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 204
8*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6
 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*
b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*
d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d
^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^
7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 -
 b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^
5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 +
 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3
*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 +
9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b
^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6
*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^
7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 +
 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180
*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3
 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*
d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5
*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 +
60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 3
9*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*
e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c -
 b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*

b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^
4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a
^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296
*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b
^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^
4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) +
86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c
 - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*
b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9
)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e
^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13
*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 128
0*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b
^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*
c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 1638
4*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^
3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1
344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e
^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^
3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^
7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 -
21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d
^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b
^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12
288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*
d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*
e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e -
 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 51
20*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 2
4576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 4
9152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e
^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(22800*a^6*c^9*e^13 + 36*a^2*b^
8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c
^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^
4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^
6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*
c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b
^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 12
54*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^
3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*
c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c
^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*
c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c
^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*
b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*
(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^
5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 2
56*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^
4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 15
36*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3
*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a
^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 12
8*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 +
 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*
e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*
d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2
*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d
*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e -
4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*

a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7
*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*
b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9
*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^
8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e +
4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b
^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6
+ 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 2
5*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/
2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*
c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*
d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^
8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168
*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^
2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c
^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)
^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2
*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*
b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e
^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^
3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e
^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d
^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*
d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e
*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a
*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2
)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4
e^2(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/
2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4
*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8
- 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*
c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^
8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6
144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d
^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d
^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^
2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b
^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 145
6*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e
^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c
^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*
a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3
- 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4
*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*
a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*
a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^1
1*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c
*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*
c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*
c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^1
1*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*1i)/((2000*a^4*c^9*e^12
+ 21*a^2*b^4*c^7*e^12 - 520*a^3*b^2*c^8*e^12 + 1296*a^2*c^11*d^4*e^8 + 432
0*a^3*c^10*d^2*e^10 + 25*b^4*c^9*d^4*e^8 - 60*b^5*c^8*d^3*e^9 + 35*b^6*c^7*
d^2*e^10 + 192*a^2*b^2*c^9*d^2*e^10 - 112*a*b^5*c^7*d*e^11 - 4480*a^3*b*c^9
*d*e^11 - 360*a*b^2*c^10*d^4*e^8 + 832*a*b^3*c^9*d^3*e^9 - 362*a*b^4*c^8*d^
2*e^10 - 2880*a^2*b*c^10*d^3*e^9 + 1440*a^2*b^3*c^8*d*e^11)/(8*(a^6*b^8*e^8
+ 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7
+ a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c

^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^
3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d
^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6
*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d
^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^
4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2
*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^
6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 20
48*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6
 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2
*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c
*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*
d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e
^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (((((1048576*a^13*
c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c
^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^1
2*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 5898
24*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^
10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8
*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4
 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*
b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 +
 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^1
2*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 -
1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b
^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9
 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*
c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3
 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a
^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^
8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 11200
0*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d
^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 60
78464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b
^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e
^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a
^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^
3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 30801
92*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^
5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9
 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^
6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2
*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 80
28160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^
6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^1
2 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*
a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9
*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 -
 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9
*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d
^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 432
5376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*
b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15
 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^
12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*
c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^1
8*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7
012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12
*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600

*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*
d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a
^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*
b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256
*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*
e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536
*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b
^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4
*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*
a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 5
12*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^
4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^
4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c
^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e
^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*
a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^
5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b
^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - (x*((27*
a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c
^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*
a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e
+ 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4
*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^
6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 +
25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(
1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^1
3*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^
3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*
b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 71
68*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*
e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2
*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*
a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c
e^6(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*
e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d
*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8
*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^
5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5
e(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11
*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b
^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d
^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(
1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-
(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/
(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e
^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^
8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*
d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 -
6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14
*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7
*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*
c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5
*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1
456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6
*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10
*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 3225
6*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^

3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c
^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 2150
4*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 1638
4*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b
^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13
*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^
5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^
7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a
^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576*a^15*c^8*e^1
7 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^
17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2
*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184
*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^1
1 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c
^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14
336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^
5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^
2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14
*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680
*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5
*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a
^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^
14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 28
67456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^
13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10
 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^
5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13
*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 515
9936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^1
2*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11
 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3
*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13
*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 13
72160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^
10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e
^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6
*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*
d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 -
 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a
^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^
6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 3148
8*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13
*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7
 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*
a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d
^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55
808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^
3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e
^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 526336
0*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4
*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^1
0 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 150732
80*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8
*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5
*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 83
55840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^1
2*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*
d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024
*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*

e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^1
0*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e
^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 2070937
6*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*b^8*e^8 + 256
*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*
b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8
 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11
*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4
 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 +
 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3
 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*
e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d
^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*
c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7
*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 102
4*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c
^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e
^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e
- 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3
072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11
*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6
*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072
*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3
- 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2
*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^
3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4
*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8
*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*
b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*
b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 28
71*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e
^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*
c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928
*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3
*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9
)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7
*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)
^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c
^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4
*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^
6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*
b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9
)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(
4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^
3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3
*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/
2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 +
4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^
7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6
*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^
2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e
^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^1
0*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^1
4*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a
^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 -
6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4
*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8
*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504

*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 +

2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e
^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a
^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10
*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^
6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d
*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^
6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a
*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^
6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*
d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6
 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^
5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*
e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528
*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^
2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 +
180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*
e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c
^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*
a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3
 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
- 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c
- b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d
^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*
c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a
^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232
*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 134
4*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23
296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c
- b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2
e^4(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2)
 + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a
^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2
)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^1
2*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b
^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 -
1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^
9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b
^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 1
6384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6
*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3
+ 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^
4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10
*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080
*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6
 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^
5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^
9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 +
 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c
^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6
*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*
e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 +
 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e
+ 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7
- 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^
3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (326912*a^8*c^9*d*e^13 - 241664
*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4
*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a
^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120

*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40
*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^
8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6
*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 -
16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10
*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*
b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 2
6384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*
d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6 - 12643
2*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^
5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^
3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*
e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b
^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9
 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b
^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^1
2 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e
^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8
*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5
*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3
*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3
*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 -
76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d
*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7
*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8
*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*
d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^
8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5
 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*
a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*
b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^
4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 80
0*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 +
 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*
e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4
*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*
c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e
 - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384
*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5
*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*
b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6
 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*
c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9
*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^
2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6
*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 -
30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b
^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3
 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e
^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2
*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b
^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 168
96*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*
e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3
*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2
e^4(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
 - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*
c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2
) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e

- 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5
*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*
e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^
2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2)
- 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c -
b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*
d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1
/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3
*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108
*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9
*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*
b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7
*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 -
1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3
*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^
6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^
6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*
c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6
*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 6
72*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4
*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4
*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720
*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^
4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*
c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*
a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b
^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*
c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^
6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*
c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c
^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^
11*b^3*c^5*d*e^7)))^(1/2) - (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 -
600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 129
6*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512
*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c
^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10
+ 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e
^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^
7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516
*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 -
41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 -
2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 -
232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 +
416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^1
2 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8
+ 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 +
a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^
7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3
*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^
4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*
e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^
5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4
*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*
c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6
*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 204
8*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6
- 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*
b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*
d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d

^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^
7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 -
 b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^
5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 +
 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3
*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 +
9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b
^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6
*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^
7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 +
 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180
*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3
 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*
d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5
*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 +
60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 3
9*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*
e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c -
 b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*
b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^
4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a
^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296
*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b
^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^
4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) +
86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c
 - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*
b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9
)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e
^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13
*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 128
0*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b
^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*
c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 1638
4*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^
3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1
344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e
^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^
3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^
7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 -
21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d
^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b
^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12
288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*
d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*
e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e -
 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 51
20*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 2
4576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 4
9152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e
^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (((((1048576*a^13*c^8*e^16 + 256*a
^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680
*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 -
 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10
*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 + 10616832*a^
11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 +
 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^1
1*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7

- 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*

e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9
*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^
7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*
(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7
*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 +
6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*
a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3
- 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d
^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*
b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 6
0928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39
*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e
 + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c -
b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b
^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4
*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^
5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*
a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^
2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4
*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 8
6*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c
- b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b
*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)
^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^
8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*
d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280
*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^
8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c
^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384
*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3
*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 13
44*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^
4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3
*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7
*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 2
1504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^
4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^
4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 122
88*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d
^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e
^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e -
12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 512
0*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24
576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49
152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^
5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12
*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12
*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048
576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*
e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13
*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2
048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7
*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 716
8*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e
^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^
3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^
11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176
*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*

e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400 *a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c ^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^1 6*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160* a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^ 8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15 *c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4 - 23592960* a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^ 9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6 *b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^ 14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527 424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^ 6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e ^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^11 - 726784* a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d ^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 3112 9600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8* b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7* e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^12 + 128614 4*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3 *d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a ^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^ 4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 4980 7360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10 *b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7* d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12 - 24576000 *a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c ^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^ 14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e^15 - 55050 24*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13 *e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9* b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a ^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3 *e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 2 56*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16 *a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^ 2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4 *b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5* d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2* d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^ 2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^ 6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^ 6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 51 2*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7* e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2 *b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5 *c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d ^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^ 3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15* d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)

^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 3

2064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2)

- 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4

+ 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 2

1504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e

- 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*

a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24
*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b
^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3
*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4
*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^1
1*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b
^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672
*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2
+ 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d
^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b
^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21
504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*
e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5
*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 573
44*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7
- 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4
*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a
^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6
*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a
^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 1536
0*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)
))^(1/2)*2i - atan((((((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 61
44*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 +
983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14
*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^
12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 53
08416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d
^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 1827
84*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5
*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2
*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13
*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 140377
6*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*
d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*
a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^
14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 518
1440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9
*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9
+ 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b
^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^
2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920
*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^
9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 73
4080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^1
2*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 -
4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a
^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9
*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 60
43520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11
*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^
6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096
*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*
d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 66
1632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b
^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*
e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 31516
16*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3
*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^1
3 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504

*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 2

4576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 1
40*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e
^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*
d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^
11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 22
40*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*
e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*
c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504
*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e
^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*
c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d
^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4
 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e -
 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e
 - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^
3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^
5*d*e^7)))^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10
*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 98304
0*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3
 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^
12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242
880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14
*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2
*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^
8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^
11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 +
 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b
^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9
 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*
c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^
4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144
*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*
d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304
*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^1
2*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5
 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*
a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d
^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*
a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13
*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5
 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400
*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6
*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 +
25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*
c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12
*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 501
26848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*
c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12
 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^
14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^1
1*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11
075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b
^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^
13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*
a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^1
0*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 +
 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*
b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^
8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 1

4974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^
10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^1
0*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^1
3 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 1258291
2*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^
4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16
 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*
c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 380
10880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^
11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 -
 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*
b^3*c^7*d*e^16))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*
a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96
*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c
^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*
a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*
c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6
*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*
b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a
^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2
240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6
 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d
^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^
7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*
a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b
^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c
^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*
c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*
e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b
^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5
 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*
c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2
077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800
*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-
(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2
)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2
 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d
^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^
5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 1024
0*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e
^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4
e^2(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2)
 - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^
2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*
e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 -
84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1
030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 -
 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2
+ 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e
^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c -
b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*
d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2
) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-
(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 3
4*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8
 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^
5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 -
 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*

a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a

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^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6
 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a
^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512
*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1
152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4
 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5
*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2
*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b
^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^
7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e
^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e +
52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072
*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*
a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a
^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 2
13*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^
7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*
a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(
1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6
 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^
4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a
*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4
*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b
^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 1545
6*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^
4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c
^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2
*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^
9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*
a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b
^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^
4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d
^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3
*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*
d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2
*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(
4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^
2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)
^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^
3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(
1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13
*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4
*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c
^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8
 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4
*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^
8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*
c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5
*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 22
40*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e
^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c
^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*
a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6
+ 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5
*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a
^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16
384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3
*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b
```

^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^1
2*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b
^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^
10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(
1/2) - (326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^1
4 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^1
4 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11
*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d
^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^1
1*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 +
 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*
c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c
^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 5
2144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d
^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2
*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^
5 - 36224*a^3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b
^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 -
 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c
^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277
000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*
d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 1596
32*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*
d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*
c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a
*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a
*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 153
6*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2
 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4
*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5
*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 -
113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*
d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*
c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4
*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 +
 a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*
d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*
e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6
*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*
d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c
^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*
b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*
a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 +
 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024
*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*
c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3
*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e
^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^
7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 38
40*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^
6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12
*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6
 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*
b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3
*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c -
 b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/
2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a
^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 -
 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^

4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152 *a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2

)*1i - (((((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c
^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b
^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504
*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 +
10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^
9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 788
48*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c
^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 46
08*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5
*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544
*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^
8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155
136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*
d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 8642
56*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*
c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7
+ 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*
b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e
^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a
^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11
*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 1
2657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10
*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^1
2 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*
b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d
^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 20
88960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9
*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13
- 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*
a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*
d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 76
09856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10
*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e
^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 246005
76*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^
5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11
- 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a
^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8
*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 -
4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13
*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^
6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^1
7*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 140
8*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*
e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9
*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^1
1 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^
11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 -
16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 +
96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^
2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 10
24*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^
10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*
b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a
^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 19
2*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3
- 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*
e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^
3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d

*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9* d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b

^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*
a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*
b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d
^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5
+ 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 -
384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*
((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^
5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 2
6880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*
d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 384
0*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c
^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*
e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)
^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) -
6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^
10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032
*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2
- 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7
*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^
6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c
- b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*
d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) -
106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*
b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2
*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*
c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^
2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b
^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^
3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2)
- 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*
c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*
c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)
^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5
e(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1
/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^1
0*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a
^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2
*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*
e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5
*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^1
2*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*
b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 4
2*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^
3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^
6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7
*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 +
32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d
^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*
b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 -
21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e -
16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*
a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5
*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a
^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a
^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49
152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(626688*a^1
0*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^
11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8
*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976

*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 10
67008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2
- 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 14
4*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15
*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c
^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 -
36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8
*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a
^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e
^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b
^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e
^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*
b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 +
77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^
3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 -
916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6
*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 1
75296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*
c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 -
1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b
^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^1
2 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^
6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e
^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*
a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*
d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 47
39072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12
*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*
d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c
^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^1
6*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 116
80*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d
*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7
*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11
+ 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c
^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b
^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*
b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^
8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c
^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d
^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*
d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c
^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^
7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a
^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 15
36*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^
6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1
024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b
^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*
d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^
5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d
e^7)))((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 +
3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c
*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b
^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d
^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a
^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*
b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*
c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^

(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 60
0*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^
2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6
*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5
*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 -
59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*
b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)
^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 +
51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*
b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a
^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 1689
6*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 224
00*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 +
4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^
9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5
*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 4
2*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*
c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b
*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24
*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8
+ 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144
*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*
b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5
+ 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 1
6384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 +
84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5
*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^
4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^
7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 +
1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d
^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b
^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17
920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d
^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9
*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e
^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e
+ 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 -
15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7
+ 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*
e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (32691
2*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*
b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^
6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*
a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 5327
36*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b
^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*
d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10
+ 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3
+ 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*
c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 2348
8*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3
*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3
*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^
8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^
8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 -
191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c
^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 23

6800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^
8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 11
06496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2
 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9
*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*
e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*
d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^
12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342
272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e
^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^
4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(
a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5
*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 25
6*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4
*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 153
6*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*
b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^
4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128
*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 +
512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e
^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d
^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*
c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*
e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4
*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a
^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*
b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b
^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*
d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8
*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e +
4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^
3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 +
 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25
*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2
) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c
^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d
^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8
*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*
a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2
 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^
7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^
9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b
^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^
6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3
 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^
5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^
5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d
*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*
(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*
b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)
^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2
) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*
a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32
*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8
- 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c
^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8
 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 61

44*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^

3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 593
92*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4
*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1
/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51
*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^1
2*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*
b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a
^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*
a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b
^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^
(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-
(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a
*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c -
 b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^
4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^
9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^
8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 +
240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^
8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4
*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 +
6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 1638
4*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84
*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^
3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d
^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b
^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 134
4*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*
e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*
c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920
*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*
e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^
7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3
+ 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 8
4*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15
360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5
120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7
 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*1i)/((2000*
a^4*c^9*e^12 + 21*a^2*b^4*c^7*e^12 - 520*a^3*b^2*c^8*e^12 + 1296*a^2*c^11*d
^4*e^8 + 4320*a^3*c^10*d^2*e^10 + 25*b^4*c^9*d^4*e^8 - 60*b^5*c^8*d^3*e^9 +
 35*b^6*c^7*d^2*e^10 + 192*a^2*b^2*c^9*d^2*e^10 - 112*a*b^5*c^7*d*e^11 - 44
80*a^3*b*c^9*d*e^11 - 360*a*b^2*c^10*d^4*e^8 + 832*a*b^3*c^9*d^3*e^9 - 362*
a*b^4*c^8*d^2*e^10 - 2880*a^2*b*c^10*d^3*e^9 + 1440*a^2*b^3*c^8*d*e^11)/(8*
(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^
5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 2
56*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^
4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 15
36*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3
*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a
^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 12
8*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 +
 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*
e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*
d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2
*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d
*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e -
4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*
a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7
*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (((((1

048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 614
40*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 -
1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^1
2*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10
*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 -
2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c
^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 +
130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c
^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576
*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10
*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 -
798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*
c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 25
60*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^
12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5
- 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4
*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*
e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^
4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d
^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4
055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b
^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10
- 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5
*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12
*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 310
76352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7
*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11
+ 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*
b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*
d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 371
01568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8
*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14
- 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440
*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^
7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 -
10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a
^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^
2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9
117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*
b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a
*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 -
7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9
+ 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^
2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 704512
0*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e
^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a
^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*
e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*
e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c
^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a
^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^
6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^
2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e
^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^
6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2
*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^
4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^
7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 5
12*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a

^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 25

6*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^

4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d
^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 -
 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*
b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^
3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c
^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^
5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 +
 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^
6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^1
4*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8
*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 1065
6*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*
c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 2
2528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^
4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^
3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*
d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b
^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37
632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3
*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1
/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c
^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(
4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 11
16*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 1
5232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e +
 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5
- 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*
a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c
*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(
1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(
-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 1
20*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c -
 b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7
*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a
^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^
8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 24
0*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^
12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6
 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6
 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*
e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^
2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*
b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 1
0080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2
*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^
6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 4659
2*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e
^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b
^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b
*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*
d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e
^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^
7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*
e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^
7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(626688*a^10*b*c^8*e^15
- 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 +
 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15
 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^1

1*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^1
1*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10
*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^
9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9
- 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2
- 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7
*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78
496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d
^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2
*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*
e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a
^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e
^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^1
1*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 +
162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^
4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8
- 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9
*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13
+ 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*
b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^
10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^
5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^
6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648
*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*
d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 23
70048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3
*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a
*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 600
0*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 +
1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^1
2 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c
^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 16015
36*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e
^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8
*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(
8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*
a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 -
256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*
d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 +
1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a
^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192
*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 -
128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5
+ 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^
4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^
5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b
^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4
*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e
- 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 102
4*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a
^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*
a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c
^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*
a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e
+ 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4
*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^
6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 -
25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(
1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^1

3*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^
3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*
b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 71
68*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*
e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2
*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e
^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*
a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c
e^6(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*
e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d
*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8
*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^
5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5
e(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11
*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b
^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d
^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(
1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-
(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/
(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e
^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^
8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*
d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 -
 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14
*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7
*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*
c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5
*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1
456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6
*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10
*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 3225
6*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^
3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c
^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 2150
4*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 1638
4*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b
^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13
*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^
5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^
7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a
^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (326912*a^8*c^9*d*e
^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14
 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^1
4 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*
e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d
^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*
e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*
b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3
*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b
^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6
 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7
*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a
^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8
*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^
3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^1
1 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b
^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 +
 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*

c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3 *c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4 *c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a *b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a* b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 24 0*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^1 2*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a ^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7 *d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3* b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4 *e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e ^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5 *e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4* d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c ^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6* b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048 *a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b ^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d ^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^ 7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5 *d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3* e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9 *a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^ 7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6* (-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7 *c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180* a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d ^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5* b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 6 0928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39 *a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^ 2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b ^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4 *b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^ 5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296* a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^ 2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4 *(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 8 6*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b *c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9) ^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^ 8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13* d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280 *a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^ 8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c

^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384
*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3
*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 13
44*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^
4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3
*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7
*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 2
1504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^
4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^
4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 122
88*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d
^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e
^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e -
12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 512
0*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24
576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49
152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^
5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8
*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^
8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4
*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6
 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c
^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^
3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 125
4*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3
*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c
^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^
11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c
^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^
12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b
^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(
a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5
*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 25
6*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4
*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 153
6*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*
b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^
4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128
*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 +
512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e
^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d
^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*
c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*
e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4
*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a
^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*
b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b
^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*
d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8
*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e +
4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^
3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 +
 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25
*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2
) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c
^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d
^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8
*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*
a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2

+ 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^
7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^
9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b
^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^
6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3
+ 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^
5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^
5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d
*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*
(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*
b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)
^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2
) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*
a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32
*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8
- 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c
^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8
+ 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 61
44*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^
2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^
2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2
*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^
12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456
*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^
2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^
2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a
^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 -
46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*
d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a
^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a
^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11
*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*
d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c
^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c
^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11
*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (((((1048576*a^13*c^8*
e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e
^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^
2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a
^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 +
10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^1
1*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 15
4112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13
*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 332
8*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^
14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291
520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*
c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77
440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*
d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 29
24544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b
^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^
8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^
4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e
^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 607846
4*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c
^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 +

6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6

*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3
+ 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5
+ 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5
*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*
e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(
-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b
^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^
9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2)
- 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a
*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*
(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 -
4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^
6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8
+ 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 614
4*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2
*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2
*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*
d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^1
2*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*
a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2
- 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2
*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^
8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 -
46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d
^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^
10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^
12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*
c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d
^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^
7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^
3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*
b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576*a^15*c^8*e^17 +
256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 -
327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7
*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^1
0*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 +
9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*
d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*
a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^
10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^
19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3
- 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3
*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9
*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b
^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e
^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 286745
6*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c
^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 1
3824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^
13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4
+ 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936
*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^
6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41
216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^1
4*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4
- 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 137216
0*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c
^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11

+ 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^1
5*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13
*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 388
95616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b
^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^
11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^
7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^1
2*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 5
5476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*
b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e
^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*
a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^
11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10
+ 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^
9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2
*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 +
7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a
^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5
*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^1
2 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 835584
0*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^
3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*
e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8
*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6
+ 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^
9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16
- 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^
13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*b^8*e^8 + 256*a^6
*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*
c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 9
6*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3
*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1
024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*
a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 9
0*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3
+ 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e
^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*
d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3
*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^
6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d
^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 -
384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92
*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*
a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4
*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(
4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6
*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 28
8*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4
e^6(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^
6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c
- b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3
*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14
*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11
*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a
^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 +
16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*
d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6
*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3
*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1

/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b
^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^
(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d
^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7
*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^
4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^
7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1
/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*
c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*
c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9
)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3
e^3(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) +
108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096
*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 +
a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6
*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^
8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 +
a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^
9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^
2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b
^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720
*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4
- 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4
*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8
*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 -
6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^
4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*
b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 +
96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a
^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b
^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^
7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*
b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10
*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 2457
6*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^
9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c
^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b
^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a
^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 177
3568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*
b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13
*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^
5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*
c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 -
16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7
*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a
^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^
13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3
*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*
e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9
*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 +
6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c
^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 +
1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7
*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 -
189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^
2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8
+ 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*
b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^

13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^
6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3
*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 68730
88*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^
7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 -
352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e
^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8
*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*
d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14
*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 6758
40*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^
8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7
*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14
+ 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 +
256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 +
a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7
*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*
b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4
*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e
^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5
*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*
d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c
^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*
b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048
*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 -
1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b
^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d
^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^
7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7
- 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 -
b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5
d^6(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 +
3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*
e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9
*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^
7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*
(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7
*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 +
6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*
a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3
- 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d
^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*
b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 6
0928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39
*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e
+ 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c -
b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b
^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4
*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^
5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*
a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^
2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4
*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 8
6*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c
- b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b
*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)
^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^
8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*

d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280
*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^
8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c
^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384
*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3
*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 13
44*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^
4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3
*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7
*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 2
1504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^
4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^
4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 122
88*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d
^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e
^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e -
12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 512
0*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24
576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49
152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^
5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (326912*a^8*c^9*d*e^13 - 241664*a^8
*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9
*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b
^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120*a^5
*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40*b^5
*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^8*c^
9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8
+ 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 - 16*b
^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10*e^4
- 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*
c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384
*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*
e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6 - 126432*a^
3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^
9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^
9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*e^8
+ 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c
^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9 + 3
88032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c
^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^12 +
64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e^3 +
2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6
+ 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9
+ 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2
*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c
^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 - 7692
8*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^1
3 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c
^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8
+ 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8
- 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^
4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6
*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*
c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*
c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^
8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^
5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 204
8*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5
+ 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3

```
*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*
d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4
*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4
*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8
*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^
5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b
^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c -
b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5
*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^
7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(
4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 3024
0*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^
9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b
^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 -
 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2
*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c
^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a
^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3
- 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6
*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4
*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*
a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^
2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) +
150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5
824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e -
 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5
+ 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^
4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*
b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)
^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^
5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
- 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(
4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2
*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^1
0*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12
*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8
 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280
*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^1
6*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^
2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^
2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*
d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8
*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a
^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4
 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7
*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8
*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 -
21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*
d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*
b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*
c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^
4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^
7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*
d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d
^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b
^3*c^5*d*e^7)))^(1/2) + (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*
a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^
2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5
*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d
```

^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^

10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^
14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*
a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 -
6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^
4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^
8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 2150
4*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e
^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c
^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*
a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*
e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7
- 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*
a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a
^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576
*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152
*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 +
24576*a^11*b^3*c^5*d*e^7)))^(1/2)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^
15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b
^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*
e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7
*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4
*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240
*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9
)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^
6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 -
1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*
e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^
3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^
4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 -
47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*
d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*
(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a
*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2
*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 1
50*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 58
24*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e -
3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 +
7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4
- 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b
^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^
9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5
*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) -
12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4
*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*
b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10
*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*
c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8
+ 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*
a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16
*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2
+ 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2
- 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d
^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*
c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^
6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4
+ 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*
d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*
b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 2
1504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d

^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b
^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c
^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4
*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7
*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d
^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^
5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^
3*c^5*d*e^7)))^(1/2)*2i - ((x*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(2
a(a*b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d
*e)) - (c*x^3*(2*a*c*e - b^2*e + b*c*d))/(2*a*(a*b^2*e^2 - 4*a*c^2*d^2 - 4*
a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e)))/(a + b*x^2 + c*x^4) - (ata
n((((((-d*e^7)^(1/2)*((326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a
^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*
a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 1152
0*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 33
6384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 2
16*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^
9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5
*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 +
1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c
^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 1304
0*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e
^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*
b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7
+ 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7
*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 4
74112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^
9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 1955
84*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*
d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 4889
60*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^
13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d
^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*
d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5
*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b
*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 158
08*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^
13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c
^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 2440
96*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^
8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d
^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^
9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6
+ 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^
2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*
a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 11
52*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4
- 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*
e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*
d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^
4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7
*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^
3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 5
2*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*
a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a
^8*b^3*c^3*d*e^7)) + (((x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14
+ 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 -
242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^
15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d

^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a^9*
c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*
d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*
e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 +
16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12
*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^
2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8
+ 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c
^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 1280
0*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12
*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436
736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*
e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3
*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*
e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*
a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*
e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^
4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^
9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 324096
0*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d
^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442
688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9
*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 2
47296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^
3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^1
3 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6
*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168
*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 81
92*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10
+ 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^
14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*
c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 5
14768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*
d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792
*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*
c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c
^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96
*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*
e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 10
24*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a
^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90
*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 +
800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^
2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d
^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*
c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6
*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^
7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 -
384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*
a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a
^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - (((1048576*a^13*c^8*e^16 + 25
6*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327
680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^1
6 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d
^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 + 10616832
*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^
2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*
b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*
e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^1

6*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - (x*(-d*e^7)^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880

*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^1
5 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13
*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 -
 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^
3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*
a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8
*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 1
12384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3
*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 3481
60*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10
*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e
^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4
*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^
12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680
*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*
c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8
 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*
b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^1
3 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680
*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*
c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8
 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*
a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d
^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 1422
1312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b
^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^
9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 62003
20*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c
^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 31
45728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8
*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d
^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 16
8960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11
*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^
7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400
*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7
*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 5
58080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^1
0*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8
*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14
 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 2392064
0*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5
*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e
^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 340
7872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*
d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 2595225
6*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^
9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 167936
0*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^
6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(16
*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d
^3*e^2)*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^
8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6
*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2
*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*
e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2
- 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2
 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*
e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d

^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*
c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*
b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512
*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9
*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*
d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5
 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 -
 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))
*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*
b*d^2*e^3 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*
d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2))))/(2*(c^2*d^5 + a^2
*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)) + (x*(
22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*
b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^
12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11
*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8
 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2
*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 +
1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*
d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^
4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b
^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*
a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b
^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*
a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 +
21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4
*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^
5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256
*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e
^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6
*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 5
12*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 -
 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e
^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d
^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c
^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7
*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*
b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5
*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e
+ 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 30
72*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 102
4*a^8*b^3*c^3*d*e^7)))*(-d*e^7)^(1/2)*1i)/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3
*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)) - ((((-d*e^7)^(1/2)*((
326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264
*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 2329
60*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 7
8080*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 +
 532736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 -
464*b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10
*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*
e^10 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11
*e^3 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2
*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 +
 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^
5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 3622
4*a^3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d
^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a
^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*
e^7 - 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*

b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12
 + 236800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b
^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11
 + 1106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^1
2*e^2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^1
0*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7
*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12
*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a
^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13
- 342272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*
d^6*e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a
^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/
(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 -
4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8
 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^1
2*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2
+ 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92
*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 1
92*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2
- 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e
^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*
d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*
c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8
*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c
^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*
e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1
024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384
*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - ((
(x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15
 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^1
5 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^1
3*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^1
2*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*
c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8
*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*
e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 +
3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^
11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 745
76*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^
6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*
b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2
 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b
^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7
 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c
^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 84
16*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^
12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 -
314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^
7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 6
2064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*
c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 -
 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b
^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^
8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6
*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e
^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 282240
0*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9
*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a
*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 4

80*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^

3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (x*(-d*e^7)^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^1

1 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7
*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12
*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55
476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b
^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^
12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a
^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^1
1*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 +
 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9
*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*
e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7
208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^
10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*
d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12
 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840
*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3
*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e
^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*
b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 +
 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9
*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 -
 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^1
3*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(16*(c^2*d^5 + a^2*d*e^4 +
 b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)*(a^6*b^8*e^8 +
256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a
^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*
d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b
^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*
e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^
2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*
e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d
^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^
6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b
^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*
a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 -
1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^
9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^
4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7
*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7
 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*(-d*e^7)^(1/2))/(2*(c^
2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e
^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e -
 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2))))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 -
2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)) - (x*(22800*a^6*c^9*e^13 + 36
*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^
5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*
c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10
*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 -
 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 384
0*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^
10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3
*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24
*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*
a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332
*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*
a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 41
20*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^1
2))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8
 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*

d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*
b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e
^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 -
92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2
- 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e
^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^
3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c
^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b
^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*
a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*
b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d
^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5
+ 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 -
384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*
(-d*e^7)^(1/2)*1i)/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*
a*b*d^2*e^3 + 2*a*c*d^3*e^2)))/(((((-d*e^7)^(1/2)*((326912*a^8*c^9*d*e^13 -
241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13
552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 3
72736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 +
197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^
11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 +
496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*
c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*
e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^
12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 162
72*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*
e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^
11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6
- 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6
*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 2
5264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^
10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 5605
6*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*
d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 67048
8*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*
d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12
*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*
c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*
c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b
^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67
968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8
*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^
6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731
008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*
a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b
^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8
+ 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*
d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4
+ 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 +
52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3
- 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e
^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^
6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c
^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*
b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024
*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^
3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^
4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e -
92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 30

72*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (((x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - (((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*

a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^1
4 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^
10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e
^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^
15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 2
4576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*
c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^
6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b
^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11
 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^
5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11
*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544
*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*
d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 384
0*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^
13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5
 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a
^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*
e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080
*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*
d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 -
 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6
*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*
e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*
a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c
^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 +
 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7
*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*
e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 3436
5440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^
6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^1
4 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 306216
96*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^
6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13
 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a
^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 35
84*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^
6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7
*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^1
3*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 70
45120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d
^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 49356
80*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^
7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^
6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b
^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8
 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^
7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^
6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d
^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^
5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7
*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^
6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 153
6*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6
 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 10
24*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^
7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d
^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5
*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*

e^7)) - (x*(-d*e^7)^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(16*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^1

0*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*
e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*
e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^
4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^
3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^
3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^
6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1
152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e +
64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^1
1*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5
*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e
^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^
5 + 1024*a^8*b^3*c^3*d*e^7)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2
*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/(
2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*
d^3*e^2))))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2
*e^3 + 2*a*c*d^3*e^2)) + (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600
*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a
^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^
5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*
d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4
*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8
- 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d
^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^
3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 410
88*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 26
04*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 23
2*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 41
6*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 -
63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 2
56*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^
2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d
^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^
11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e
^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2
+ 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e
^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^
5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6
*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^
5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a
^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1
024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9
*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4
*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*
e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 -
3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*(-d*e^7)^(1/2))/(2*(c^2
*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^
2)) - (2000*a^4*c^9*e^12 + 21*a^2*b^4*c^7*e^12 - 520*a^3*b^2*c^8*e^12 + 129
6*a^2*c^11*d^4*e^8 + 4320*a^3*c^10*d^2*e^10 + 25*b^4*c^9*d^4*e^8 - 60*b^5*c
^8*d^3*e^9 + 35*b^6*c^7*d^2*e^10 + 192*a^2*b^2*c^9*d^2*e^10 - 112*a*b^5*c^7
*d*e^11 - 4480*a^3*b*c^9*d*e^11 - 360*a*b^2*c^10*d^4*e^8 + 832*a*b^3*c^9*d^
3*e^9 - 362*a*b^4*c^8*d^2*e^10 - 2880*a^2*b*c^10*d^3*e^9 + 1440*a^2*b^3*c^8
*d*e^11)/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*
c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4
*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 +
a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*
d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*
e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6
*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*

d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c
^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*
b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*
a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 +
512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024
*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*
c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3
*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e
^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^
7)) + ((((-d*e^7)^(1/2)*((326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 4
8*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 757
76*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 1
1520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 +
336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2
+ 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264
*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*
d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^1
2 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^
4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 1
3040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^
4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a
^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*
e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*
b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12
+ 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4
*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 1
95584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c
^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 4
88960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d
*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^1
1*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c
^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*
c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^
2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 +
15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d
*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*
b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 2
44096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4
*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^
5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256
*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e
^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6
*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 5
12*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 -
1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e
^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d
^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c
^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7
*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*
b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5
*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e
+ 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 30
72*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 102
4*a^8*b^3*c^3*d*e^7)) - (((x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^
14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^1
5 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7
*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^1
3*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a
^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c

^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^

3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (x*(-d*e^7)^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b

^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(16*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a

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^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 +
512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*
a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c
^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*
e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^
3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7
)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2
*a*b*d^2*e^3 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b
^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2))))/(2*(c^2*d^5 +
a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)) - (
x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a
^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3
*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c
^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*
e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*
a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9
 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^
10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712
*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*
a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 12
72*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*
a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 321
28*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10
 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*
c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6
*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 -
256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^
2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6
+ 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3
+ 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^
4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^
4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^
5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^
6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*
a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a
^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*
d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7
*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 -
 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 +
1024*a^8*b^3*c^3*d*e^7)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3
*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2))))*(-d*e^7)^(1/2)*1i)/(
c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3
*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.275 $\int \frac{1}{(d+ex^2)^2 (a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=1077

$$\frac{xe^4}{2d\left(cd^2-bed+ae^2\right)^2\left(ex^2+d\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}\left(cd^2-bed+ae^2\right)^2}+\frac{2(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\sqrt{d}\left(cd^2-bed+ae^2\right)^3}+\frac{\sqrt{2}\sqrt{c}\left(3c^2d^2+b\left(b\right.\right.}{}$$

```
[Out] 1/2*e^4*x/d/(a*e^2-b*d*e+c*d^2)^2/(e*x^2+d)+1/2*x*(a*b*c*e*(-b*e+2*c*d)+(-2
*a*c+b^2)*(c^2*d^2+b^2*e^2-c*e*(a*e+2*b*d))-c*(2*b^2*c*d*e-4*a*c^2*d*e-b^3*
e^2-b*c*(-3*a*e^2+c*d^2))*x^2)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+
b*x^2+a)+1/2*e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)^
2+2*e^(7/2)*(-b*e+2*c*d)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)^3/d^
(1/2)+e^2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^
(1/2)*(3*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-c*e*(3*b*d+a*e+2*d*(-4*a*c+b^
2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b
^4*e^2-b^3*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-4*a*c^2*(3*c*d^2-e*(3*a*e+d*(-4*a
*c+b^2)^(1/2)))-b*c*(3*a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(16*a*e+d*(-4*a*c+b^2)^
(1/2)))+b^2*c*(c*d^2-e*(9*a*e+2*d*(-4*a*c+b^2)^(1/2)))))/a/(-4*a*c+b^2)^(3/2
)/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-e^2*arctan(x*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)*(3*c^2*d^2+b*e
^2*(b-(-4*a*c+b^2)^(1/2))-c*e*(3*b*d+a*e-2*d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*
d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^4*e^2-b^3*e*(2*c*d+
e*(-4*a*c+b^2)^(1/2))+b*c*(3*a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-16*a*e+d*(-4*a*
c+b^2)^(1/2)))-4*a*c^2*(3*c*d^2+e*(-3*a*e+d*(-4*a*c+b^2)^(1/2)))+b^2*c*(c*d
^2+e*(-9*a*e+2*d*(-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*
d^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 12.64, antiderivative size = 1077, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1238, 199, 205, 1178, 1166}

$$\frac{xe^4}{2d\left(cd^2-bed+ae^2\right)^2\left(ex^2+d\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}\left(cd^2-bed+ae^2\right)^2}+\frac{2(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\sqrt{d}\left(cd^2-bed+ae^2\right)^3}+\frac{\sqrt{2}\sqrt{c}\left(3c^2d^2+b\left(b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) + (x*(a*b*c*e*(2*c*d -
b*e) + (b^2 - 2*a*c)*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e)) - c*(2*b^2*c*d
*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^2))/(2*a*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*e^2*(3*
c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*c]*
d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^3) + (Sqrt[c
]*(b^4*e^2 - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) - 4*a*c^2*(3*c*d^2 - e*(Sq
rt[b^2 - 4*a*c]*d + 3*a*e)) + b^2*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 9*a
*e)) - b*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 16*a*e))
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^
2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (
Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d
 - 2*Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
```

```
^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)^3) - (Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + b
*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 16*a*e)) + b^2*c
*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 9*a*e)) - 4*a*c^2*(3*c*d^2 + e*(Sqrt[b
^2 - 4*a*c]*d - 3*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)^2) + (2*e^(7/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^3) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2}\,dx = \int\left(\frac{e^4}{\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)^2} - \frac{2e^4(-2cd+be)}{\left(cd^2-bde+ae^2\right)^3\left(d+ex^2\right)} + \frac{c^2d^2+}{(c}\right.$$

$$= \frac{e^2\int\frac{3c^2d^2+2b^2e^2-ce(5bd+ae)-2ce(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{\left(cd^2-bde+ae^2\right)^3} + \frac{\left(2e^4(2cd-be)\right)\int\frac{1}{d+ex^2}\,dx}{\left(cd^2-bde+ae^2\right)^3} + \frac{\int}{}$$

$$= \frac{e^4x}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} + \frac{x\left(abce(2cd-be)+\left(b^2-2ac\right)\left(c^2d^2+b\right.\right.}{2a\left(b^2-\right.}$$

$$= \frac{e^4x}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} + \frac{x\left(abce(2cd-be)+\left(b^2-2ac\right)\left(c^2d^2+b\right.\right.}{2a\left(b^2-\right.}$$

$$= \frac{e^4x}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} + \frac{x\left(abce(2cd-be)+\left(b^2-2ac\right)\left(c^2d^2+b\right.\right.}{2a\left(b^2-\right.}$$

Mathematica [A] time = 5.84, size = 1020, normalized size = 0.95

$$\frac{1}{4}\left(\frac{2xe^4}{d\left(cd^2+e(ae-bd)\right)^2\left(ex^2+d\right)} + \frac{2\left(9cd^2+e(ae-5bd)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{d^{3/2}\left(cd^2+e(ae-bd)\right)^3} - \frac{2x\left(e^2b^4+ce\left(ex^2-2d\right)b^3+c\left(\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*e^4*x)/(d*(c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x^2)) - (2*x*(b^4*e^2 + b
^3*c*e*(-2*d + e*x^2) + 2*a*c^2*(a*e^2 - c*d*(d - 2*e*x^2)) + b^2*c*(-4*a*e
^2 + c*d*(d - 2*e*x^2)) + b*c^2*(c*d^2*x^2 - 3*a*e*(-2*d + e*x^2))))/(a*(-b
^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sq
rt[c]*(b^5*d*e^3 + b^3*e*(c*d - Sqrt[b^2 - 4*a*c]*e)*(3*c*d^2 + 5*a*e^2) +
b^4*e^2*(-3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - 5*a*e)) - 4*a*c^2*(-3*c^2*d^4
+ c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^3*(9*Sqrt[b^2 - 4*a*c]*d + 7
*a*e)) - b*c*(-19*a^2*Sqrt[b^2 - 4*a*c]*e^4 + 2*a*c*d*e^2*(-3*Sqrt[b^2 - 4*
a*c]*d + 26*a*e) + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d + 28*a*e)) + b^2*c*(-(c^2*d
^4) + 3*c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*e^3*(7*Sqrt[b^2 - 4*a*c]*
d + 29*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))^3
) - (Sqrt[2]*Sqrt[c]*(b^5*d*e^3 + b^3*e*(c*d + Sqrt[b^2 - 4*a*c]*e)*(3*c*d^
2 + 5*a*e^2) - b^2*c*(c^2*d^4 + a*e^3*(7*Sqrt[b^2 - 4*a*c]*d - 29*a*e) + 3*
c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - b^4*e^2*(3*c*d^2 + e*(Sqrt[b^2 - 4
*a*c]*d + 5*a*e)) + 4*a*c^2*(3*c^2*d^4 + a*e^3*(9*Sqrt[b^2 - 4*a*c]*d - 7*a
*e) + c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)) + b*c*(-19*a^2*Sqrt[b^2 - 4*a
*c]*e^4 + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d - 28*a*e) - 2*a*c*d*e^2*(3*Sqrt[b^2
- 4*a*c]*d + 26*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d
- a*e))^3) + (2*e^(7/2)*(9*c*d^2 + e*(-5*b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(d^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^3))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.08, size = 5709, normalized size = 5.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(9*c*d^2*e^4 - 5*b*d*e^5 + a*e^6)*arctan(e*x/sqrt(d*e))/((c^3*d^7 - 3*b
*c^2*d^6*e - 3*a^2*b*d^2*e^5 + a^3*d*e^6 + 3*(b^2*c + a*c^2)*d^5*e^2 - (b^3
 + 6*a*b*c)*d^4*e^3 + 3*(a*b^2 + a^2*c)*d^3*e^4)*sqrt(d*e)) + 1/2*((b*c^3*d
^3*e - 2*(b^2*c^2 - 2*a*c^3)*d^2*e^2 + (b^3*c - 3*a*b*c^2)*d*e^3 + (a*b^2*c
 - 4*a^2*c^2)*e^4)*x^5 + (b*c^3*d^4 - (b^2*c^2 - 2*a*c^3)*d^3*e - (b^3*c -
3*a*b*c^2)*d^2*e^2 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^3 + (a*b^3 - 4*a^2*b
*c)*e^4)*x^3 + ((b^2*c^2 - 2*a*c^3)*d^4 - 2*(b^3*c - 3*a*b*c^2)*d^3*e + (b^
4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^2 + (a^2*b^2 - 4*a^3*c)*e^4)*x)/((a^2*b^2*
c^2 - 4*a^3*c^3)*d^6 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^5*e + (a^2*b^4 - 2*a^3
*b^2*c - 8*a^4*c^2)*d^4*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d^3*e^3 + (a^4*b^2 -
4*a^5*c)*d^2*e^4 + ((a*b^2*c^3 - 4*a^2*c^4)*d^5*e - 2*(a*b^3*c^2 - 4*a^2*b*
c^3)*d^4*e^2 + (a*b^4*c - 2*a^2*b^2*c^2 - 8*a^3*c^3)*d^3*e^3 - 2*(a^2*b^3*c
 - 4*a^3*b*c^2)*d^2*e^4 + (a^3*b^2*c - 4*a^4*c^2)*d*e^5)*x^6 + ((a*b^2*c^3
- 4*a^2*c^4)*d^6 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e - (a*b^4*c - 6*a^2*b^2*c
^2 + 8*a^3*c^3)*d^4*e^2 + (a*b^5 - 4*a^2*b^3*c)*d^3*e^3 - (2*a^2*b^4 - 9*a^
3*b^2*c + 4*a^4*c^2)*d^2*e^4 + (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x^4 + ((a*b^3*c
^2 - 4*a^2*b*c^3)*d^6 - (2*a*b^4*c - 9*a^2*b^2*c^2 + 4*a^3*c^3)*d^5*e + (a*
b^5 - 4*a^2*b^3*c)*d^4*e^2 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^3*e^3 -
(a^3*b^3 - 4*a^4*b*c)*d^2*e^4 + (a^4*b^2 - 4*a^5*c)*d*e^5)*x^2) - 1/2*integ
rate(-((b^2*c^3 - 6*a*c^4)*d^4 - (3*b^3*c^2 - 16*a*b*c^3)*d^3*e + 3*(b^4*c
- 3*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - (b^5 + 6*a*b^3*c - 44*a^2*b*c^2)*d*e^3
 + (5*a*b^4 - 24*a^2*b^2*c + 14*a^3*c^2)*e^4 + (b*c^4*d^4 - (3*b^2*c^3 - 4*
a*c^4)*d^3*e + 3*(b^3*c^2 - 2*a*b*c^3)*d^2*e^2 - (b^4*c + 7*a*b^2*c^2 - 36*
a^2*c^3)*d*e^3 + (5*a*b^3*c - 19*a^2*b*c^2)*e^4)*x^2)/(c*x^4 + b*x^2 + a),
x)/((a*b^2*c^3 - 4*a^2*c^4)*d^6 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 3*(a*
b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3*
b*c^2)*d^3*e^3 + 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*d^2*e^4 - 3*(a^3*b^3
 - 4*a^4*b*c)*d*e^5 + (a^4*b^2 - 4*a^5*c)*e^6)
```

mupad [B] time = 17.81, size = 97073, normalized size = 90.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log(root(128723189760*a^14*b^4*c^9*d^13*e^14*z^6 + 128723189760*a^12
*b^4*c^11*d^17*e^10*z^6 - 8432455680*a^11*b^12*c^4*d^11*e^16*z^6 - 84324556
80*a^7*b^12*c^8*d^19*e^8*z^6 + 12673351680*a^11*b^11*c^5*d^12*e^15*z^6 + 12
673351680*a^8*b^11*c^8*d^18*e^9*z^6 - 72637480960*a^12*b^9*c^6*d^12*e^15*z^
6 - 72637480960*a^9*b^9*c^9*d^18*e^9*z^6 - 21048344576*a^9*b^12*c^6*d^15*e^
12*z^6 - 16609443840*a^17*b^3*c^7*d^8*e^19*z^6 - 16609443840*a^10*b^3*c^14*
d^22*e^5*z^6 + 145332633600*a^13*b^5*c^9*d^14*e^13*z^6 + 145332633600*a^12*
b^5*c^10*d^16*e^11*z^6 + 123740356608*a^14*b^5*c^8*d^12*e^15*z^6 + 12374035
6608*a^11*b^5*c^11*d^18*e^9*z^6 + 3460300800*a^17*b^5*c^5*d^6*e^21*z^6 + 34
60300800*a^8*b^5*c^14*d^24*e^3*z^6 - 7751073792*a^15*b^7*c^5*d^8*e^19*z^6 -
 7751073792*a^8*b^7*c^12*d^22*e^5*z^6 + 12041846784*a^14*b^7*c^6*d^10*e^17*
z^6 + 12041846784*a^9*b^7*c^11*d^20*e^7*z^6 - 325545099264*a^14*b^3*c^10*d^
14*e^13*z^6 - 325545099264*a^13*b^3*c^11*d^16*e^11*z^6 - 3330539520*a^13*b^
10*c^4*d^9*e^18*z^6 - 3330539520*a^7*b^10*c^10*d^21*e^6*z^6 + 157789716480*
a^12*b^7*c^8*d^14*e^13*z^6 + 157789716480*a^11*b^7*c^9*d^16*e^11*z^6 + 3749
2359168*a^11*b^10*c^6*d^13*e^14*z^6 + 37492359168*a^9*b^10*c^8*d^17*e^10*z^
6 + 301989888*a^8*b^3*c^16*d^26*e*z^6 - 7266631680*a^17*b^4*c^6*d^7*e^20*z^
6 - 7266631680*a^9*b^4*c^14*d^23*e^4*z^6 - 201326592*a^20*b*c^6*d^4*e^23*z^
6 - 188743680*a^7*b^5*c^15*d^26*e*z^6 + 45747339264*a^13*b^8*c^6*d^11*e^16*
z^6 + 45747339264*a^9*b^8*c^10*d^19*e^8*z^6 - 74612736*a^10*b^16*c*d^9*e^18
*z^6 - 2768240640*a^16*b^7*c^4*d^6*e^21*z^6 - 2768240640*a^7*b^7*c^13*d^24*
e^3*z^6 + 69746688*a^11*b^15*c*d^8*e^19*z^6 + 62914560*a^6*b^7*c^14*d^26*e*
z^6 + 2752020480*a^10*b^13*c^4*d^12*e^15*z^6 + 2752020480*a^7*b^13*c^7*d^18
*e^9*z^6 + 55148544*a^9*b^17*c*d^10*e^17*z^6 - 45957120*a^12*b^14*c*d^7*e^2
0*z^6 - 2724986880*a^14*b^9*c^4*d^8*e^19*z^6 - 2724986880*a^7*b^9*c^11*d^22
*e^5*z^6 - 25952256*a^8*b^18*c*d^11*e^16*z^6 + 21086208*a^13*b^13*c*d^6*e^2
1*z^6 - 11796480*a^5*b^9*c^13*d^26*e*z^6 - 6438912*a^14*b^12*c*d^5*e^22*z^6
 + 5406720*a^7*b^19*c*d^12*e^15*z^6 + 1622016*a^6*b^20*c*d^13*e^14*z^6 - 15
23712*a^5*b^21*c*d^14*e^13*z^6 + 1179648*a^15*b^11*c*d^4*e^23*z^6 + 1179648
*a^4*b^11*c^12*d^26*e*z^6 + 442368*a^4*b^22*c*d^15*e^12*z^6 - 98304*a^16*b^
10*c*d^3*e^24*z^6 - 49152*a^3*b^23*c*d^16*e^11*z^6 - 49152*a^3*b^13*c^11*d^
26*e*z^6 + 6897106944*a^9*b^13*c^5*d^14*e^13*z^6 + 6897106944*a^8*b^13*c^6*
d^16*e^11*z^6 - 2422210560*a^16*b^6*c^5*d^7*e^20*z^6 - 2422210560*a^8*b^6*c
^13*d^23*e^4*z^6 + 255785435136*a^14*b^2*c^11*d^15*e^12*z^6 + 41004564480*a
^15*b^4*c^8*d^11*e^16*z^6 + 41004564480*a^11*b^4*c^12*d^19*e^8*z^6 + 227082
2400*a^13*b^11*c^3*d^8*e^19*z^6 + 2270822400*a^6*b^11*c^10*d^22*e^5*z^6 + 2
3677108224*a^14*b^8*c^5*d^9*e^18*z^6 + 23677108224*a^8*b^8*c^11*d^21*e^6*z^
6 + 212600881152*a^15*b^2*c^10*d^13*e^14*z^6 + 212600881152*a^13*b^2*c^12*d
^17*e^10*z^6 + 75157733376*a^15*b^5*c^7*d^10*e^17*z^6 + 75157733376*a^10*b^
5*c^12*d^20*e^7*z^6 - 251217838080*a^13*b^6*c^8*d^13*e^14*z^6 - 25121783808
0*a^11*b^6*c^10*d^17*e^10*z^6 - 1952907264*a^14*b^10*c^3*d^7*e^20*z^6 - 195
2907264*a^6*b^10*c^11*d^23*e^4*z^6 - 27691057152*a^13*b^9*c^5*d^10*e^17*z^6
 - 27691057152*a^8*b^9*c^10*d^20*e^7*z^6 - 1902673920*a^8*b^15*c^4*d^14*e^1
3*z^6 - 1902673920*a^7*b^15*c^5*d^16*e^11*z^6 + 10465050624*a^10*b^11*c^6*d
^14*e^13*z^6 + 10465050624*a^9*b^11*c^7*d^16*e^11*z^6 + 1613905920*a^9*b^14
*c^4*d^13*e^14*z^6 + 1613905920*a^7*b^14*c^6*d^17*e^10*z^6 - 33218887680*a^
17*b*c^9*d^10*e^17*z^6 - 33218887680*a^12*b*c^14*d^20*e^7*z^6 + 1524695040*
a^10*b^14*c^3*d^11*e^16*z^6 + 1524695040*a^6*b^14*c^7*d^19*e^8*z^6 - 147220
0704*a^18*b^4*c^5*d^5*e^22*z^6 - 1472200704*a^8*b^4*c^15*d^25*e^2*z^6 - 830
47219200*a^16*b^3*c^8*d^10*e^17*z^6 - 83047219200*a^11*b^3*c^13*d^20*e^7*z^
6 + 44291850240*a^17*b^2*c^8*d^9*e^18*z^6 + 44291850240*a^11*b^2*c^14*d^21*
e^6*z^6 + 1308131328*a^8*b^14*c^5*d^15*e^12*z^6 - 201326592*a^9*b*c^17*d^26
```

*e*z^6 + 48530718720*a^12*b^8*c^7*d^13*e^14*z^6 + 48530718720*a^10*b^8*c^9*
d^17*e^10*z^6 - 1242644480*a^12*b^12*c^3*d^9*e^18*z^6 - 1242644480*a^6*b^12
*c^9*d^21*e^6*z^6 + 9813196800*a^12*b^10*c^5*d^11*e^16*z^6 + 9813196800*a^8
*b^10*c^9*d^19*e^8*z^6 - 93012885504*a^15*b*c^11*d^14*e^13*z^6 - 9301288550
4*a^14*b*c^12*d^16*e^11*z^6 + 177305812992*a^13*b^4*c^10*d^15*e^12*z^6 + 52
730658816*a^10*b^10*c^7*d^15*e^12*z^6 - 1180106752*a^9*b^15*c^3*d^12*e^15*z
^6 - 1180106752*a^6*b^15*c^6*d^18*e^9*z^6 + 1023672320*a^15*b^9*c^3*d^6*e^2
1*z^6 + 1023672320*a^6*b^9*c^12*d^24*e^3*z^6 + 975175680*a^17*b^6*c^4*d^5*e
^22*z^6 + 975175680*a^7*b^6*c^14*d^25*e^2*z^6 - 11072962560*a^18*b*c^8*d^8*
e^19*z^6 - 11072962560*a^11*b*c^15*d^22*e^5*z^6 + 9412018176*a^18*b^2*c^7*d
^7*e^20*z^6 + 9412018176*a^10*b^2*c^15*d^23*e^4*z^6 + 805306368*a^19*b^2*c^
6*d^5*e^22*z^6 + 805306368*a^9*b^2*c^16*d^25*e^2*z^6 - 133809831936*a^14*b^
6*c^7*d^11*e^16*z^6 - 133809831936*a^10*b^6*c^11*d^19*e^8*z^6 - 2214592512*
a^19*b*c^7*d^6*e^21*z^6 - 2214592512*a^10*b*c^16*d^24*e^3*z^6 + 82216747008
*a^13*b^7*c^7*d^12*e^15*z^6 + 82216747008*a^10*b^7*c^10*d^18*e^9*z^6 - 5866
29120*a^12*b^13*c^2*d^8*e^19*z^6 - 586629120*a^5*b^13*c^9*d^22*e^5*z^6 + 56
8565760*a^7*b^16*c^4*d^15*e^12*z^6 - 4844421120*a^16*b^4*c^7*d^9*e^18*z^6 -
4844421120*a^10*b^4*c^13*d^21*e^6*z^6 + 531210240*a^11*b^14*c^2*d^9*e^18*z
^6 + 531210240*a^5*b^14*c^8*d^21*e^6*z^6 - 527155200*a^11*b^13*c^3*d^10*e^1
7*z^6 - 527155200*a^6*b^13*c^8*d^20*e^7*z^6 + 43470028800*a^11*b^8*c^8*d^15
*e^12*z^6 - 107874877440*a^11*b^9*c^7*d^14*e^13*z^6 - 107874877440*a^10*b^9
*c^8*d^16*e^11*z^6 + 9018408960*a^12*b^11*c^4*d^10*e^17*z^6 + 9018408960*a^
7*b^11*c^9*d^20*e^7*z^6 + 421994496*a^13*b^12*c^2*d^7*e^20*z^6 + 421994496*
a^5*b^12*c^10*d^23*e^4*z^6 - 66437775360*a^16*b*c^10*d^12*e^15*z^6 - 664377
75360*a^13*b*c^13*d^18*e^9*z^6 + 26159874048*a^16*b^5*c^6*d^8*e^19*z^6 + 26
159874048*a^9*b^5*c^13*d^22*e^5*z^6 - 369098752*a^18*b^3*c^6*d^6*e^21*z^6 -
369098752*a^9*b^3*c^15*d^24*e^3*z^6 + 351436800*a^8*b^16*c^3*d^13*e^14*z^6
+ 351436800*a^6*b^16*c^5*d^17*e^10*z^6 - 334233600*a^16*b^8*c^3*d^5*e^22*z
^6 - 334233600*a^6*b^8*c^13*d^25*e^2*z^6 + 301989888*a^19*b^3*c^5*d^4*e^23*
z^6 - 266010624*a^10*b^15*c^2*d^10*e^17*z^6 - 266010624*a^5*b^15*c^7*d^20*e
^7*z^6 - 305198530560*a^12*b^6*c^9*d^15*e^12*z^6 - 203292672*a^14*b^11*c^2*
d^6*e^21*z^6 - 203292672*a^5*b^11*c^11*d^24*e^3*z^6 - 188743680*a^18*b^5*c^
4*d^4*e^23*z^6 + 120418467840*a^16*b^2*c^9*d^11*e^16*z^6 + 120418467840*a^1
2*b^2*c^13*d^19*e^8*z^6 - 17293934592*a^10*b^12*c^5*d^13*e^14*z^6 - 1729393
4592*a^8*b^12*c^7*d^17*e^10*z^6 + 104890368*a^8*b^17*c^2*d^12*e^15*z^6 + 10
4890368*a^5*b^17*c^5*d^18*e^9*z^6 + 4390256640*a^15*b^8*c^4*d^7*e^20*z^6 +
4390256640*a^7*b^8*c^12*d^23*e^4*z^6 - 91750400*a^6*b^18*c^3*d^15*e^12*z^6
+ 79134720*a^7*b^17*c^3*d^14*e^13*z^6 + 79134720*a^6*b^17*c^4*d^16*e^11*z^6
- 74612736*a^4*b^16*c^7*d^21*e^6*z^6 - 72990720*a^7*b^18*c^2*d^13*e^14*z^6
- 72990720*a^5*b^18*c^4*d^17*e^10*z^6 + 69746688*a^4*b^15*c^8*d^22*e^5*z^6
+ 63700992*a^15*b^10*c^2*d^5*e^22*z^6 + 63700992*a^5*b^10*c^12*d^25*e^2*z^
6 + 62914560*a^17*b^7*c^3*d^4*e^23*z^6 + 55148544*a^4*b^17*c^6*d^20*e^7*z^6
- 45957120*a^4*b^14*c^9*d^23*e^4*z^6 - 25952256*a^4*b^18*c^5*d^19*e^8*z^6
- 25165824*a^20*b^2*c^5*d^3*e^24*z^6 + 21086208*a^4*b^13*c^10*d^24*e^3*z^6
+ 20643840*a^6*b^19*c^2*d^14*e^13*z^6 + 20643840*a^5*b^19*c^3*d^16*e^11*z^6
+ 15728640*a^19*b^4*c^4*d^3*e^24*z^6 - 11796480*a^16*b^9*c^2*d^4*e^23*z^6
- 6438912*a^4*b^12*c^11*d^25*e^2*z^6 + 5406720*a^4*b^19*c^4*d^18*e^9*z^6 -
5242880*a^18*b^6*c^3*d^3*e^24*z^6 + 3784704*a^3*b^18*c^6*d^21*e^6*z^6 - 324
4032*a^3*b^19*c^5*d^20*e^7*z^6 - 3244032*a^3*b^17*c^7*d^22*e^5*z^6 + 202752
0*a^3*b^20*c^4*d^19*e^8*z^6 + 2027520*a^3*b^16*c^8*d^23*e^4*z^6 - 1622016*a
^9*b^16*c^2*d^11*e^16*z^6 - 1622016*a^5*b^16*c^6*d^19*e^8*z^6 + 1622016*a^4
*b^20*c^3*d^17*e^10*z^6 - 1523712*a^4*b^21*c^2*d^16*e^11*z^6 + 983040*a^17*
b^8*c^2*d^3*e^24*z^6 - 901120*a^3*b^21*c^3*d^18*e^9*z^6 - 901120*a^3*b^15*c
^9*d^24*e^3*z^6 + 270336*a^3*b^22*c^2*d^17*e^10*z^6 + 270336*a^3*b^14*c^10*
d^25*e^2*z^6 + 172032*a^5*b^20*c^2*d^15*e^12*z^6 - 38593888256*a^15*b^6*c^6
*d^9*e^18*z^6 - 38593888256*a^9*b^6*c^12*d^21*e^6*z^6 - 210386288640*a^15*b
^3*c^9*d^12*e^15*z^6 - 210386288640*a^12*b^3*c^12*d^18*e^9*z^6 + 1550214758
4*a^15*c^12*d^15*e^12*z^6 + 1107296256*a^19*c^8*d^7*e^20*z^6 + 1107296256*a
^11*c^16*d^23*e^4*z^6 + 13287555072*a^16*c^11*d^13*e^14*z^6 + 13287555072*a

^14*c^13*d^17*e^10*z^6 + 201326592*a^20*c^7*d^5*e^22*z^6 + 201326592*a^10*c
^17*d^25*e^2*z^6 + 16777216*a^21*c^6*d^3*e^24*z^6 + 3784704*a^9*b^18*d^9*e^
18*z^6 - 3244032*a^10*b^17*d^8*e^19*z^6 - 3244032*a^8*b^19*d^10*e^17*z^6 +
2027520*a^11*b^16*d^7*e^20*z^6 + 2027520*a^7*b^20*d^11*e^16*z^6 - 901120*a^
12*b^15*d^6*e^21*z^6 - 901120*a^6*b^21*d^12*e^15*z^6 + 270336*a^13*b^14*d^5
*e^22*z^6 + 270336*a^5*b^22*d^13*e^14*z^6 - 49152*a^14*b^13*d^4*e^23*z^6 -
49152*a^4*b^23*d^14*e^13*z^6 + 4096*a^15*b^12*d^3*e^24*z^6 + 4096*a^3*b^24*
d^15*e^12*z^6 - 25165824*a^8*b^2*c^17*d^27*z^6 + 15728640*a^7*b^4*c^16*d^27
*z^6 - 5242880*a^6*b^6*c^15*d^27*z^6 + 983040*a^5*b^8*c^14*d^27*z^6 - 98304
*a^4*b^10*c^13*d^27*z^6 + 4096*a^3*b^12*c^12*d^27*z^6 + 8304721920*a^17*c^1
0*d^11*e^16*z^6 + 8304721920*a^13*c^14*d^19*e^8*z^6 + 3690987520*a^18*c^9*d
^9*e^18*z^6 + 3690987520*a^12*c^15*d^21*e^6*z^6 + 16777216*a^9*c^18*d^27*z^
6 - 8493371392*a^6*b^8*c^9*d^14*e^9*z^4 + 1458044928*a^8*b*c^14*d^17*e^6*z^
4 - 12604538880*a^11*b^4*c^8*d^8*e^15*z^4 - 8303067136*a^9*b^5*c^9*d^11*e^1
2*z^4 - 5588058112*a^13*b*c^9*d^7*e^16*z^4 - 3892838400*a^8*b^2*c^13*d^16*e
^7*z^4 - 3611713536*a^8*b^8*c^7*d^10*e^13*z^4 + 7819006464*a^7*b^9*c^7*d^11
*e^12*z^4 - 7782137856*a^8*b^7*c^8*d^11*e^12*z^4 + 7780433920*a^12*b^2*c^9*
d^8*e^15*z^4 - 12020465664*a^7*b^5*c^11*d^15*e^8*z^4 + 3176792064*a^8*b^3*c
^12*d^15*e^8*z^4 - 322633728*a^15*b*c^7*d^3*e^20*z^4 + 210829312*a^7*b*c^15
*d^19*e^4*z^4 + 15623258112*a^9*b^6*c^8*d^10*e^13*z^4 + 25165824*a^15*b^3*c
^5*d*e^22*z^4 - 15728640*a^14*b^5*c^4*d*e^22*z^4 + 12582912*a^5*b^2*c^16*d^
22*e*z^4 - 11730944*a^4*b^4*c^15*d^22*e*z^4 + 5242880*a^13*b^7*c^3*d*e^22*z
^4 - 4561920*a*b^15*c^7*d^17*e^6*z^4 + 4521984*a^3*b^6*c^14*d^22*e*z^4 + 44
60544*a*b^14*c^8*d^18*e^5*z^4 + 3538944*a^6*b*c^16*d^21*e^2*z^4 + 3108864*a
*b^16*c^6*d^16*e^7*z^4 - 3027200*a*b^13*c^9*d^19*e^4*z^4 - 2345472*a^5*b^17
*c*d^7*e^16*z^4 - 2307072*a^8*b^14*c*d^4*e^19*z^4 + 1824768*a^6*b^16*c*d^6*
e^17*z^4 + 1734912*a^9*b^13*c*d^3*e^20*z^4 + 1419264*a*b^12*c^10*d^20*e^3*z
^4 - 1191168*a*b^17*c^5*d^15*e^8*z^4 - 983040*a^12*b^9*c^2*d*e^22*z^4 + 964
608*a^4*b^18*c*d^8*e^15*z^4 - 866304*a^2*b^8*c^13*d^22*e*z^4 + 703488*a^7*b
^15*c*d^5*e^18*z^4 - 608256*a^10*b^12*c*d^2*e^21*z^4 - 440832*a*b^11*c^11*d
^21*e^2*z^4 + 275968*a*b^19*c^3*d^13*e^10*z^4 - 159744*a^2*b^20*c*d^10*e^13
*z^4 - 153600*a*b^20*c^2*d^12*e^11*z^4 + 64512*a^3*b^19*c*d^9*e^14*z^4 + 19
746062336*a^8*b^6*c^9*d^12*e^11*z^4 - 15333588992*a^10*b^4*c^9*d^10*e^13*z^
4 + 6702170112*a^7*b^4*c^12*d^16*e^7*z^4 + 15167913984*a^10*b^3*c^10*d^11*e
^12*z^4 - 2256638976*a^5*b^11*c^7*d^13*e^10*z^4 + 2254307328*a^5*b^7*c^11*d
^17*e^6*z^4 - 2200633344*a^6*b^5*c^12*d^17*e^6*z^4 + 6457131008*a^11*b^3*c^
9*d^9*e^14*z^4 - 2128785408*a^5*b^8*c^10*d^16*e^7*z^4 - 2126057472*a^6*b^11
*c^6*d^11*e^12*z^4 + 2038349824*a^12*b^5*c^6*d^5*e^18*z^4 + 2037841920*a^5*
b^10*c^8*d^14*e^9*z^4 + 3615621120*a^9*b*c^13*d^15*e^8*z^4 + 1900019712*a^1
1*b^2*c^10*d^10*e^13*z^4 + 1867698432*a^9*b^9*c^5*d^7*e^16*z^4 - 6157369344
*a^9*b^4*c^10*d^12*e^11*z^4 - 1856913408*a^7*b^10*c^6*d^10*e^13*z^4 + 17891
32800*a^6*b^4*c^13*d^18*e^5*z^4 + 6082658304*a^8*b^4*c^11*d^14*e^9*z^4 + 60
29549568*a^11*b^5*c^7*d^7*e^16*z^4 + 6010159104*a^6*b^7*c^10*d^15*e^8*z^4 +
 1703182336*a^7*b^7*c^9*d^13*e^10*z^4 + 1658388480*a^11*b^6*c^6*d^6*e^17*z^
4 + 5917114368*a^10*b^6*c^7*d^8*e^15*z^4 - 1591197696*a^11*b^7*c^5*d^5*e^18
*z^4 - 1526464512*a^8*b^10*c^5*d^8*e^15*z^4 - 5772607488*a^12*b^4*c^7*d^6*e
^17*z^4 - 1423507456*a^13*b^4*c^6*d^4*e^19*z^4 - 1387266048*a^7*b^3*c^13*d^
17*e^6*z^4 + 2976120832*a^10*b*c^12*d^13*e^10*z^4 - 9906946048*a^9*b^2*c^12
*d^14*e^9*z^4 - 18421874688*a^8*b^5*c^10*d^13*e^10*z^4 + 1141217280*a^6*b^1
2*c^5*d^10*e^13*z^4 - 9714364416*a^7*b^8*c^8*d^12*e^11*z^4 - 16777216*a^16*
b*c^6*d*e^22*z^4 + 98304*a^11*b^11*c*d*e^22*z^4 + 81920*a*b^10*c^12*d^22*e*
z^4 + 39168*a*b^21*c*d^11*e^12*z^4 - 1091829760*a^5*b^6*c^12*d^18*e^5*z^4 +
 1046740992*a^14*b^2*c^7*d^4*e^19*z^4 - 6884425728*a^12*b*c^10*d^9*e^14*z^4
 + 987445248*a^4*b^10*c^9*d^16*e^7*z^4 + 984087552*a^5*b^12*c^6*d^12*e^11*z
^4 - 9564585984*a^9*b^7*c^7*d^9*e^14*z^4 - 5266857984*a^10*b^7*c^6*d^7*e^16
*z^4 - 892145664*a^7*b^11*c^5*d^9*e^14*z^4 - 2444623872*a^11*b*c^11*d^11*e^
12*z^4 + 768540672*a^12*b^3*c^8*d^7*e^16*z^4 + 5048322048*a^8*b^9*c^6*d^9*e
^14*z^4 + 5047612416*a^6*b^9*c^8*d^13*e^10*z^4 - 732492288*a^4*b^11*c^8*d^1
5*e^8*z^4 + 9266921472*a^7*b^6*c^10*d^14*e^9*z^4 - 645857280*a^6*b^6*c^11*d

^16*e^7*z^4 - 623867904*a^4*b^9*c^10*d^17*e^6*z^4 - 622067712*a^6*b^3*c^14*
d^19*e^4*z^4 + 582617088*a^10*b^8*c^5*d^6*e^17*z^4 + 577119744*a^7*b^12*c^4
*d^8*e^15*z^4 + 552566784*a^12*b^6*c^5*d^4*e^19*z^4 + 549224448*a^9*b^8*c^6
*d^8*e^15*z^4 - 526565376*a^9*b^10*c^4*d^6*e^17*z^4 + 511520256*a^10*b^9*c^
4*d^5*e^18*z^4 + 13393723392*a^9*b^3*c^11*d^13*e^10*z^4 - 2066350080*a^14*b
*c^8*d^5*e^18*z^4 + 4718592000*a^13*b^2*c^8*d^6*e^17*z^4 - 314572800*a^7*b^
2*c^14*d^18*e^5*z^4 + 287250432*a^4*b^13*c^6*d^13*e^10*z^4 + 4565827584*a^1
0*b^5*c^8*d^9*e^14*z^4 - 250785792*a^4*b^14*c^5*d^12*e^11*z^4 + 235536384*a
^13*b^3*c^7*d^5*e^18*z^4 - 232683264*a^8*b^11*c^4*d^7*e^16*z^4 - 199627776*
a^5*b^14*c^4*d^10*e^13*z^4 - 190267392*a^12*b^7*c^4*d^3*e^20*z^4 + 18489139
2*a^6*b^10*c^7*d^12*e^11*z^4 + 180502528*a^4*b^7*c^12*d^19*e^4*z^4 + 178877
952*a^3*b^13*c^7*d^15*e^8*z^4 + 172490752*a^14*b^3*c^6*d^3*e^20*z^4 + 16394
6496*a^13*b^5*c^5*d^3*e^20*z^4 + 155839488*a^8*b^12*c^3*d^6*e^17*z^4 + 1550
00832*a^5*b^5*c^13*d^19*e^4*z^4 - 152076288*a^4*b^6*c^13*d^20*e^3*z^4 - 137
592576*a^3*b^12*c^8*d^16*e^7*z^4 - 133693440*a^14*b^4*c^5*d^2*e^21*z^4 - 11
6767488*a^3*b^9*c^11*d^19*e^4*z^4 - 108985344*a^3*b^14*c^6*d^14*e^9*z^4 - 1
06223616*a^6*b^13*c^4*d^9*e^14*z^4 + 106119168*a^3*b^10*c^10*d^18*e^5*z^4 +
 102432768*a^5*b^4*c^14*d^20*e^3*z^4 + 102113280*a^4*b^12*c^7*d^14*e^9*z^4
+ 100674048*a^5*b^9*c^9*d^15*e^8*z^4 + 90439680*a^13*b^6*c^4*d^2*e^21*z^4 -
 86808576*a^6*b^14*c^3*d^8*e^15*z^4 + 86245376*a^6*b^2*c^15*d^20*e^3*z^4 +
79011840*a^4*b^8*c^11*d^18*e^5*z^4 + 78345216*a^4*b^15*c^4*d^11*e^12*z^4 +
78006528*a^11*b^9*c^3*d^3*e^20*z^4 - 73253376*a^9*b^11*c^3*d^5*e^18*z^4 + 6
7524608*a^3*b^8*c^12*d^20*e^3*z^4 + 67108864*a^15*b^2*c^6*d^2*e^21*z^4 - 61
590528*a^10*b^10*c^3*d^4*e^19*z^4 + 61559808*a^5*b^15*c^3*d^9*e^14*z^4 - 59
637760*a^5*b^3*c^15*d^21*e^2*z^4 + 58638336*a^4*b^5*c^14*d^21*e^2*z^4 - 408
28416*a^7*b^13*c^3*d^7*e^16*z^4 - 35639296*a^2*b^12*c^9*d^18*e^5*z^4 - 3129
3440*a^12*b^8*c^3*d^2*e^21*z^4 + 29933568*a^5*b^13*c^5*d^11*e^12*z^4 + 2779
3920*a^2*b^11*c^10*d^19*e^4*z^4 + 27168768*a^2*b^13*c^8*d^17*e^6*z^4 - 2360
2176*a^7*b^14*c^2*d^6*e^17*z^4 - 23248896*a^3*b^7*c^13*d^21*e^2*z^4 + 20929
536*a^3*b^15*c^5*d^13*e^10*z^4 + 18428928*a^9*b^12*c^2*d^4*e^19*z^4 + 18026
496*a^6*b^15*c^2*d^7*e^16*z^4 - 16261632*a^10*b^11*c^2*d^3*e^20*z^4 + 15128
064*a^3*b^16*c^4*d^12*e^11*z^4 - 14060544*a^2*b^10*c^11*d^20*e^3*z^4 + 1317
8880*a^2*b^16*c^5*d^14*e^9*z^4 - 11244288*a^3*b^17*c^3*d^11*e^12*z^4 - 1050
9312*a^2*b^15*c^6*d^15*e^8*z^4 - 7262208*a^4*b^17*c^2*d^9*e^14*z^4 - 704563
2*a^2*b^17*c^4*d^13*e^10*z^4 - 6285312*a^2*b^14*c^7*d^16*e^7*z^4 + 5996544*
a^11*b^10*c^2*d^2*e^21*z^4 + 4558336*a^2*b^9*c^12*d^21*e^2*z^4 + 4478976*a^
11*b^8*c^4*d^4*e^19*z^4 + 2850816*a^4*b^16*c^3*d^10*e^13*z^4 + 2629632*a^3*
b^11*c^9*d^17*e^6*z^4 + 2503680*a^3*b^18*c^2*d^10*e^13*z^4 + 1627136*a^2*b^
18*c^3*d^12*e^11*z^4 + 1605120*a^8*b^13*c^2*d^5*e^18*z^4 + 1483776*a^5*b^16
*c^2*d^8*e^15*z^4 + 139776*a^2*b^19*c^2*d^11*e^12*z^4 - 8542224384*a^10*b^2
*c^11*d^12*e^11*z^4 - 3072*b^22*c*d^12*e^11*z^4 - 3072*b^12*c^11*d^22*e*z^4
 - 1572864*a^6*c^17*d^22*e*z^4 - 4096*a^10*b^13*d*e^22*z^4 - 4096*a*b^22*d^
10*e^13*z^4 - 6144*a^12*b^10*c*e^23*z^4 - 983040*a^5*b*c^17*d^23*z^4 - 6912
*a*b^9*c^13*d^23*z^4 + 1824522240*a^13*c^10*d^8*e^15*z^4 + 1730150400*a^12*
c^11*d^10*e^13*z^4 + 958922752*a^14*c^9*d^6*e^17*z^4 - 537919488*a^9*c^14*d
^16*e^7*z^4 + 508559360*a^11*c^12*d^12*e^11*z^4 - 500170752*a^10*c^13*d^14*
e^9*z^4 + 246939648*a^15*c^8*d^4*e^19*z^4 - 199229440*a^8*c^15*d^18*e^5*z^4
 - 29884416*a^7*c^16*d^20*e^3*z^4 + 25165824*a^16*c^7*d^2*e^21*z^4 + 236544
*b^17*c^6*d^17*e^6*z^4 - 202752*b^18*c^5*d^16*e^7*z^4 - 202752*b^16*c^7*d^1
8*e^5*z^4 + 126720*b^19*c^4*d^15*e^8*z^4 + 126720*b^15*c^8*d^19*e^4*z^4 - 5
6320*b^20*c^3*d^14*e^9*z^4 - 56320*b^14*c^9*d^20*e^3*z^4 + 16896*b^21*c^2*d
^13*e^10*z^4 + 16896*b^13*c^10*d^21*e^2*z^4 + 110080*a^7*b^16*d^4*e^19*z^4
+ 110080*a^4*b^19*d^7*e^16*z^4 - 75520*a^8*b^15*d^3*e^20*z^4 - 75520*a^3*b^
20*d^8*e^15*z^4 - 56320*a^6*b^17*d^5*e^18*z^4 - 56320*a^5*b^18*d^6*e^17*z^4
 + 25600*a^9*b^14*d^2*e^21*z^4 + 25600*a^2*b^21*d^9*e^14*z^4 - 1572864*a^16
*b^2*c^5*e^23*z^4 + 983040*a^15*b^4*c^4*e^23*z^4 - 327680*a^14*b^6*c^3*e^23
*z^4 + 61440*a^13*b^8*c^2*e^23*z^4 + 983040*a^4*b^3*c^16*d^23*z^4 - 385024*
a^3*b^5*c^15*d^23*z^4 + 73728*a^2*b^7*c^14*d^23*z^4 + 256*b^23*d^11*e^12*z^
4 + 1048576*a^17*c^6*e^23*z^4 + 256*b^11*c^12*d^23*z^4 + 256*a^11*b^12*e^23

*z^4 + 948695040*a^8*b*c^10*d^6*e^13*z^2 + 348917760*a^7*b*c^11*d^8*e^11*z^
2 - 125030400*a^9*b*c^9*d^4*e^15*z^2 - 50728960*a^6*b*c^12*d^10*e^9*z^2 - 4
4298240*a^5*b*c^13*d^12*e^7*z^2 - 36495360*a^10*b*c^8*d^2*e^17*z^2 + 296755
20*a^8*b^6*c^5*d*e^18*z^2 - 24170496*a^9*b^4*c^6*d*e^18*z^2 - 17202816*a^7*
b^8*c^4*d*e^18*z^2 - 14561280*a^4*b*c^14*d^14*e^5*z^2 + 5532416*a^6*b^10*c^
3*d*e^18*z^2 + 4128768*a^10*b^2*c^7*d*e^18*z^2 - 2662400*a^3*b*c^15*d^16*e^
3*z^2 + 1184512*a*b^12*c^6*d^9*e^10*z^2 - 1136160*a*b^13*c^5*d^8*e^11*z^2 -
 1017600*a^5*b^12*c^2*d*e^18*z^2 - 744768*a*b^11*c^7*d^10*e^9*z^2 + 607872*
a*b^14*c^4*d^7*e^12*z^2 - 424064*a*b^6*c^12*d^15*e^4*z^2 + 408576*a*b^5*c^1
3*d^16*e^3*z^2 + 361152*a*b^10*c^8*d^11*e^8*z^2 - 287408*a*b^9*c^9*d^12*e^7
*z^2 - 260448*a^3*b^15*c*d^2*e^17*z^2 - 203904*a*b^4*c^14*d^17*e^2*z^2 + 20
0832*a*b^8*c^10*d^13*e^6*z^2 + 126720*a*b^7*c^11*d^14*e^5*z^2 - 123968*a*b^
15*c^3*d^6*e^13*z^2 - 39168*a*b^16*c^2*d^5*e^14*z^2 + 11904*a^2*b^16*c*d^3*
e^16*z^2 + 1824135552*a^7*b^4*c^8*d^5*e^14*z^2 - 1457252352*a^8*b^2*c^9*d^5
*e^14*z^2 - 1405209600*a^7*b^5*c^7*d^4*e^15*z^2 - 184320*a^2*b*c^16*d^18*e*
z^2 + 100608*a^4*b^14*c*d*e^18*z^2 + 53248*a*b^3*c^15*d^18*e*z^2 + 26448*a*
b^17*c*d^4*e^15*z^2 + 1067599872*a^8*b^3*c^8*d^4*e^15*z^2 - 930828288*a^7*b
^3*c^9*d^6*e^13*z^2 + 920760000*a^6*b^4*c^9*d^7*e^12*z^2 - 806639616*a^6*b^
3*c^10*d^8*e^11*z^2 - 791052480*a^6*b^6*c^7*d^5*e^14*z^2 + 772237824*a^6*b^
7*c^6*d^4*e^15*z^2 - 701025408*a^5*b^6*c^8*d^7*e^12*z^2 + 443340288*a^5*b^5
*c^9*d^8*e^11*z^2 + 433047552*a^7*b^6*c^6*d^3*e^16*z^2 + 405741312*a^5*b^7*
c^7*d^6*e^13*z^2 + 293652480*a^6*b^2*c^11*d^9*e^10*z^2 - 276962688*a^6*b^8*
c^5*d^3*e^16*z^2 - 247804272*a^8*b^4*c^7*d^3*e^16*z^2 + 213564384*a^4*b^8*c
^7*d^7*e^12*z^2 - 202596816*a^5*b^9*c^5*d^4*e^15*z^2 - 182520896*a^4*b^9*c^
6*d^6*e^13*z^2 - 153489408*a^5*b^3*c^11*d^10*e^9*z^2 - 152151552*a^7*b^2*c^
10*d^7*e^12*z^2 + 115859712*a^5*b^2*c^12*d^11*e^8*z^2 + 108085248*a^9*b^3*c
^7*d^2*e^17*z^2 + 105536256*a^4*b^5*c^10*d^10*e^9*z^2 - 98323200*a^6*b^5*c^
8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^9*d^9*e^10*z^2 + 89464512*a^5*b^10*c^4*
d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*d^2*e^17*z^2 + 68315904*a^5*b^8*c^6*d^5
*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8*e^11*z^2 - 62951040*a^9*b^2*c^8*d^3*e^
16*z^2 + 49056768*a^4*b^10*c^5*d^5*e^14*z^2 + 47614464*a^3*b^8*c^8*d^9*e^10
*z^2 + 35604480*a^4*b^2*c^13*d^13*e^6*z^2 + 33983040*a^3*b^11*c^5*d^6*e^13*
z^2 - 33515520*a^4*b^3*c^12*d^12*e^7*z^2 - 33463808*a^3*b^7*c^9*d^10*e^9*z^
2 - 25128864*a^4*b^4*c^11*d^11*e^8*z^2 - 23193728*a^3*b^10*c^6*d^7*e^12*z^2
 + 21015456*a^6*b^9*c^4*d^2*e^17*z^2 + 19924176*a^4*b^11*c^4*d^4*e^15*z^2 -
 19251216*a^3*b^9*c^7*d^8*e^11*z^2 - 16434048*a^5*b^4*c^10*d^9*e^10*z^2 - 1
6289664*a^3*b^12*c^4*d^5*e^14*z^2 - 15059328*a^4*b^12*c^3*d^3*e^16*z^2 - 10
766016*a^2*b^10*c^7*d^9*e^10*z^2 - 10453632*a^5*b^11*c^3*d^2*e^17*z^2 - 994
0992*a^3*b^3*c^13*d^14*e^5*z^2 + 8373696*a^2*b^11*c^6*d^8*e^11*z^2 + 777676
8*a^3*b^2*c^14*d^15*e^4*z^2 + 7077888*a^3*b^5*c^11*d^12*e^7*z^2 + 6798240*a
^2*b^9*c^8*d^10*e^9*z^2 - 3589440*a^2*b^6*c^11*d^13*e^6*z^2 + 3544320*a^3*b
^6*c^10*d^11*e^8*z^2 + 3128064*a^2*b^5*c^12*d^14*e^5*z^2 + 2346336*a^4*b^13
*c^2*d^2*e^17*z^2 - 2261568*a^2*b^8*c^9*d^11*e^8*z^2 - 2125824*a^2*b^13*c^4
*d^6*e^13*z^2 + 2002560*a^3*b^4*c^12*d^13*e^6*z^2 + 1927680*a^2*b^7*c^10*d^
12*e^7*z^2 + 1814784*a^2*b^14*c^3*d^5*e^14*z^2 - 1807104*a^2*b^12*c^5*d^7*e
^12*z^2 + 1637808*a^3*b^13*c^3*d^4*e^15*z^2 + 1083456*a^3*b^14*c^2*d^3*e^16
*z^2 - 792384*a^2*b^4*c^13*d^15*e^4*z^2 - 657408*a^2*b^3*c^14*d^16*e^3*z^2
+ 608256*a^7*b^7*c^5*d^2*e^17*z^2 + 595968*a^2*b^2*c^15*d^17*e^2*z^2 - 4986
24*a^2*b^15*c^2*d^4*e^15*z^2 - 3840*b^18*c*d^5*e^14*z^2 - 3840*b^5*c^14*d^1
8*e*z^2 + 2064384*a^11*c^8*d*e^18*z^2 - 4160*a^3*b^16*d*e^18*z^2 - 4160*a*b
^18*d^3*e^16*z^2 - 1290240*a^11*b*c^7*e^19*z^2 - 9840*a^5*b^13*c*e^19*z^2 -
 5760*a*b^2*c^16*d^19*z^2 - 280581120*a^8*c^11*d^7*e^12*z^2 + 110278656*a^9
*c^10*d^5*e^14*z^2 - 89479168*a^7*c^12*d^9*e^10*z^2 + 34464000*a^10*c^9*d^3
*e^16*z^2 + 54240*b^15*c^4*d^8*e^11*z^2 + 54240*b^8*c^11*d^15*e^4*z^2 - 499
20*b^14*c^5*d^9*e^10*z^2 - 49920*b^9*c^10*d^14*e^5*z^2 - 37376*b^16*c^3*d^7
*e^12*z^2 - 37376*b^7*c^12*d^16*e^3*z^2 + 28480*b^13*c^6*d^10*e^9*z^2 + 284
80*b^10*c^9*d^13*e^6*z^2 + 15936*b^17*c^2*d^6*e^13*z^2 + 15936*b^6*c^13*d^1
7*e^2*z^2 - 7920*b^12*c^7*d^11*e^8*z^2 - 7920*b^11*c^8*d^12*e^7*z^2 + 74895
36*a^5*c^14*d^13*e^6*z^2 + 6084096*a^6*c^13*d^11*e^8*z^2 + 2280448*a^4*c^15

*d^15*e^4*z^2 + 350208*a^3*c^16*d^17*e^2*z^2 + 11616*a^2*b^17*d^2*e^17*z^2
- 3515904*a^9*b^5*c^5*e^19*z^2 + 3440640*a^10*b^3*c^6*e^19*z^2 + 1870848*a^
8*b^7*c^4*e^19*z^2 - 572272*a^7*b^9*c^3*e^19*z^2 + 101856*a^6*b^11*c^2*e^19
*z^2 + 400*b^19*d^4*e^15*z^2 + 400*b^4*c^15*d^19*z^2 + 20736*a^2*c^17*d^19*
z^2 + 400*a^4*b^15*e^19*z^2 - 3969216*a^4*b*c^10*d^3*e^12 - 3001536*a^3*b*c
^11*d^5*e^10 - 419904*a^2*b*c^12*d^7*e^8 + 184608*a^4*b^3*c^8*d*e^14 - 1530
36*a*b^4*c^10*d^6*e^9 + 127008*a*b^3*c^11*d^7*e^8 + 63108*a*b^6*c^8*d^4*e^1
1 - 29160*a*b^2*c^12*d^8*e^7 - 21060*a^3*b^5*c^7*d*e^14 - 21060*a*b^7*c^7*d
^3*e^12 + 5460*a*b^5*c^9*d^5*e^10 - 404544*a^5*b*c^9*d*e^14 + 1251872*a^3*b
^3*c^9*d^3*e^12 + 844224*a^4*b^2*c^9*d^2*e^13 + 820512*a^2*b^3*c^10*d^5*e^1
0 + 750672*a^3*b^2*c^10*d^4*e^11 - 657498*a^2*b^4*c^9*d^4*e^11 - 487116*a^3
*b^4*c^8*d^2*e^13 + 160704*a^2*b^2*c^11*d^6*e^9 + 58806*a^2*b^6*c^7*d^2*e^1
3 + 13140*a^2*b^5*c^8*d^3*e^12 + 15286*b^6*c^9*d^6*e^9 - 9540*b^7*c^8*d^5*e
^10 - 9540*b^5*c^10*d^7*e^8 + 2025*b^8*c^7*d^4*e^11 + 2025*b^4*c^11*d^8*e^7
 + 3367008*a^4*c^11*d^4*e^11 + 1166400*a^3*c^12*d^6*e^9 + 705600*a^5*c^10*d
^2*e^13 + 104976*a^2*c^13*d^8*e^7 - 17640*a^5*b^2*c^8*e^15 + 2025*a^4*b^4*c
^7*e^15 + 38416*a^6*c^9*e^15, z, k)*(root(128723189760*a^14*b^4*c^9*d^13*e^
14*z^6 + 128723189760*a^12*b^4*c^11*d^17*e^10*z^6 - 8432455680*a^11*b^12*c^
4*d^11*e^16*z^6 - 8432455680*a^7*b^12*c^8*d^19*e^8*z^6 + 12673351680*a^11*b
^11*c^5*d^12*e^15*z^6 + 12673351680*a^8*b^11*c^8*d^18*e^9*z^6 - 72637480960
*a^12*b^9*c^6*d^12*e^15*z^6 - 72637480960*a^9*b^9*c^9*d^18*e^9*z^6 - 210483
44576*a^9*b^12*c^6*d^15*e^12*z^6 - 16609443840*a^17*b^3*c^7*d^8*e^19*z^6 -
16609443840*a^10*b^3*c^14*d^22*e^5*z^6 + 145332633600*a^13*b^5*c^9*d^14*e^1
3*z^6 + 145332633600*a^12*b^5*c^10*d^16*e^11*z^6 + 123740356608*a^14*b^5*c^
8*d^12*e^15*z^6 + 123740356608*a^11*b^5*c^11*d^18*e^9*z^6 + 3460300800*a^17
*b^5*c^5*d^6*e^21*z^6 + 3460300800*a^8*b^5*c^14*d^24*e^3*z^6 - 7751073792*a
^15*b^7*c^5*d^8*e^19*z^6 - 7751073792*a^8*b^7*c^12*d^22*e^5*z^6 + 120418467
84*a^14*b^7*c^6*d^10*e^17*z^6 + 12041846784*a^9*b^7*c^11*d^20*e^7*z^6 - 325
545099264*a^14*b^3*c^10*d^14*e^13*z^6 - 325545099264*a^13*b^3*c^11*d^16*e^1
1*z^6 - 3330539520*a^13*b^10*c^4*d^9*e^18*z^6 - 3330539520*a^7*b^10*c^10*d^
21*e^6*z^6 + 157789716480*a^12*b^7*c^8*d^14*e^13*z^6 + 157789716480*a^11*b^
7*c^9*d^16*e^11*z^6 + 37492359168*a^11*b^10*c^6*d^13*e^14*z^6 + 37492359168
*a^9*b^10*c^8*d^17*e^10*z^6 + 301989888*a^8*b^3*c^16*d^26*e*z^6 - 726663168
0*a^17*b^4*c^6*d^7*e^20*z^6 - 7266631680*a^9*b^4*c^14*d^23*e^4*z^6 - 201326
592*a^20*b*c^6*d^4*e^23*z^6 - 188743680*a^7*b^5*c^15*d^26*e*z^6 + 457473392
64*a^13*b^8*c^6*d^11*e^16*z^6 + 45747339264*a^9*b^8*c^10*d^19*e^8*z^6 - 746
12736*a^10*b^16*c*d^9*e^18*z^6 - 2768240640*a^16*b^7*c^4*d^6*e^21*z^6 - 276
8240640*a^7*b^7*c^13*d^24*e^3*z^6 + 69746688*a^11*b^15*c*d^8*e^19*z^6 + 629
14560*a^6*b^7*c^14*d^26*e*z^6 + 2752020480*a^10*b^13*c^4*d^12*e^15*z^6 + 27
52020480*a^7*b^13*c^7*d^18*e^9*z^6 + 55148544*a^9*b^17*c*d^10*e^17*z^6 - 45
957120*a^12*b^14*c*d^7*e^20*z^6 - 2724986880*a^14*b^9*c^4*d^8*e^19*z^6 - 27
24986880*a^7*b^9*c^11*d^22*e^5*z^6 - 25952256*a^8*b^18*c*d^11*e^16*z^6 + 21
086208*a^13*b^13*c*d^6*e^21*z^6 - 11796480*a^5*b^9*c^13*d^26*e*z^6 - 643891
2*a^14*b^12*c*d^5*e^22*z^6 + 5406720*a^7*b^19*c*d^12*e^15*z^6 + 1622016*a^6
*b^20*c*d^13*e^14*z^6 - 1523712*a^5*b^21*c*d^14*e^13*z^6 + 1179648*a^15*b^1
1*c*d^4*e^23*z^6 + 1179648*a^4*b^11*c^12*d^26*e*z^6 + 442368*a^4*b^22*c*d^1
5*e^12*z^6 - 98304*a^16*b^10*c*d^3*e^24*z^6 - 49152*a^3*b^23*c*d^16*e^11*z^
6 - 49152*a^3*b^13*c^11*d^26*e*z^6 + 6897106944*a^9*b^13*c^5*d^14*e^13*z^6
+ 6897106944*a^8*b^13*c^6*d^16*e^11*z^6 - 2422210560*a^16*b^6*c^5*d^7*e^20*
z^6 - 2422210560*a^8*b^6*c^13*d^23*e^4*z^6 + 255785435136*a^14*b^2*c^11*d^1
5*e^12*z^6 + 41004564480*a^15*b^4*c^8*d^11*e^16*z^6 + 41004564480*a^11*b^4*
c^12*d^19*e^8*z^6 + 2270822400*a^13*b^11*c^3*d^8*e^19*z^6 + 2270822400*a^6*
b^11*c^10*d^22*e^5*z^6 + 23677108224*a^14*b^8*c^5*d^9*e^18*z^6 + 2367710822
4*a^8*b^8*c^11*d^21*e^6*z^6 + 212600881152*a^15*b^2*c^10*d^13*e^14*z^6 + 21
2600881152*a^13*b^2*c^12*d^17*e^10*z^6 + 75157733376*a^15*b^5*c^7*d^10*e^17
*z^6 + 75157733376*a^10*b^5*c^12*d^20*e^7*z^6 - 251217838080*a^13*b^6*c^8*d
^13*e^14*z^6 - 251217838080*a^11*b^6*c^10*d^17*e^10*z^6 - 1952907264*a^14*b
^10*c^3*d^7*e^20*z^6 - 1952907264*a^6*b^10*c^11*d^23*e^4*z^6 - 27691057152*
a^13*b^9*c^5*d^10*e^17*z^6 - 27691057152*a^8*b^9*c^10*d^20*e^7*z^6 - 190267

3920*a^8*b^15*c^4*d^14*e^13*z^6 - 1902673920*a^7*b^15*c^5*d^16*e^11*z^6 + 1
0465050624*a^10*b^11*c^6*d^14*e^13*z^6 + 10465050624*a^9*b^11*c^7*d^16*e^11
*z^6 + 1613905920*a^9*b^14*c^4*d^13*e^14*z^6 + 1613905920*a^7*b^14*c^6*d^17
*e^10*z^6 - 33218887680*a^17*b*c^9*d^10*e^17*z^6 - 33218887680*a^12*b*c^14*
d^20*e^7*z^6 + 1524695040*a^10*b^14*c^3*d^11*e^16*z^6 + 1524695040*a^6*b^14
*c^7*d^19*e^8*z^6 - 1472200704*a^18*b^4*c^5*d^5*e^22*z^6 - 1472200704*a^8*b
^4*c^15*d^25*e^2*z^6 - 83047219200*a^16*b^3*c^8*d^10*e^17*z^6 - 83047219200
*a^11*b^3*c^13*d^20*e^7*z^6 + 44291850240*a^17*b^2*c^8*d^9*e^18*z^6 + 44291
850240*a^11*b^2*c^14*d^21*e^6*z^6 + 1308131328*a^8*b^14*c^5*d^15*e^12*z^6 -
201326592*a^9*b*c^17*d^26*e*z^6 + 48530718720*a^12*b^8*c^7*d^13*e^14*z^6 +
48530718720*a^10*b^8*c^9*d^17*e^10*z^6 - 1242644480*a^12*b^12*c^3*d^9*e^18
*z^6 - 1242644480*a^6*b^12*c^9*d^21*e^6*z^6 + 9813196800*a^12*b^10*c^5*d^11
*e^16*z^6 + 9813196800*a^8*b^10*c^9*d^19*e^8*z^6 - 93012885504*a^15*b*c^11*
d^14*e^13*z^6 - 93012885504*a^14*b*c^12*d^16*e^11*z^6 + 177305812992*a^13*b
^4*c^10*d^15*e^12*z^6 + 52730658816*a^10*b^10*c^7*d^15*e^12*z^6 - 118010675
2*a^9*b^15*c^3*d^12*e^15*z^6 - 1180106752*a^6*b^15*c^6*d^18*e^9*z^6 + 10236
72320*a^15*b^9*c^3*d^6*e^21*z^6 + 1023672320*a^6*b^9*c^12*d^24*e^3*z^6 + 97
5175680*a^17*b^6*c^4*d^5*e^22*z^6 + 975175680*a^7*b^6*c^14*d^25*e^2*z^6 - 1
1072962560*a^18*b*c^8*d^8*e^19*z^6 - 11072962560*a^11*b*c^15*d^22*e^5*z^6 +
9412018176*a^18*b^2*c^7*d^7*e^20*z^6 + 9412018176*a^10*b^2*c^15*d^23*e^4*z
^6 + 805306368*a^19*b^2*c^6*d^5*e^22*z^6 + 805306368*a^9*b^2*c^16*d^25*e^2*
z^6 - 133809831936*a^14*b^6*c^7*d^11*e^16*z^6 - 133809831936*a^10*b^6*c^11*
d^19*e^8*z^6 - 2214592512*a^19*b*c^7*d^6*e^21*z^6 - 2214592512*a^10*b*c^16*
d^24*e^3*z^6 + 82216747008*a^13*b^7*c^7*d^12*e^15*z^6 + 82216747008*a^10*b^
7*c^10*d^18*e^9*z^6 - 586629120*a^12*b^13*c^2*d^8*e^19*z^6 - 586629120*a^5*
b^13*c^9*d^22*e^5*z^6 + 568565760*a^7*b^16*c^4*d^15*e^12*z^6 - 4844421120*a
^16*b^4*c^7*d^9*e^18*z^6 - 4844421120*a^10*b^4*c^13*d^21*e^6*z^6 + 53121024
0*a^11*b^14*c^2*d^9*e^18*z^6 + 531210240*a^5*b^14*c^8*d^21*e^6*z^6 - 527155
200*a^11*b^13*c^3*d^10*e^17*z^6 - 527155200*a^6*b^13*c^8*d^20*e^7*z^6 + 434
70028800*a^11*b^8*c^8*d^15*e^12*z^6 - 107874877440*a^11*b^9*c^7*d^14*e^13*z
^6 - 107874877440*a^10*b^9*c^8*d^16*e^11*z^6 + 9018408960*a^12*b^11*c^4*d^1
0*e^17*z^6 + 9018408960*a^7*b^11*c^9*d^20*e^7*z^6 + 421994496*a^13*b^12*c^2
*d^7*e^20*z^6 + 421994496*a^5*b^12*c^10*d^23*e^4*z^6 - 66437775360*a^16*b*c
^10*d^12*e^15*z^6 - 66437775360*a^13*b*c^13*d^18*e^9*z^6 + 26159874048*a^16
*b^5*c^6*d^8*e^19*z^6 + 26159874048*a^9*b^5*c^13*d^22*e^5*z^6 - 369098752*a
^18*b^3*c^6*d^6*e^21*z^6 - 369098752*a^9*b^3*c^15*d^24*e^3*z^6 + 351436800*
a^8*b^16*c^3*d^13*e^14*z^6 + 351436800*a^6*b^16*c^5*d^17*e^10*z^6 - 3342336
00*a^16*b^8*c^3*d^5*e^22*z^6 - 334233600*a^6*b^8*c^13*d^25*e^2*z^6 + 301989
888*a^19*b^3*c^5*d^4*e^23*z^6 - 266010624*a^10*b^15*c^2*d^10*e^17*z^6 - 266
010624*a^5*b^15*c^7*d^20*e^7*z^6 - 305198530560*a^12*b^6*c^9*d^15*e^12*z^6
- 203292672*a^14*b^11*c^2*d^6*e^21*z^6 - 203292672*a^5*b^11*c^11*d^24*e^3*z
^6 - 188743680*a^18*b^5*c^4*d^4*e^23*z^6 + 120418467840*a^16*b^2*c^9*d^11*e
^16*z^6 + 120418467840*a^12*b^2*c^13*d^19*e^8*z^6 - 17293934592*a^10*b^12*c
^5*d^13*e^14*z^6 - 17293934592*a^8*b^12*c^7*d^17*e^10*z^6 + 104890368*a^8*b
^17*c^2*d^12*e^15*z^6 + 104890368*a^5*b^17*c^5*d^18*e^9*z^6 + 4390256640*a^
15*b^8*c^4*d^7*e^20*z^6 + 4390256640*a^7*b^8*c^12*d^23*e^4*z^6 - 91750400*a
^6*b^18*c^3*d^15*e^12*z^6 + 79134720*a^7*b^17*c^3*d^14*e^13*z^6 + 79134720*
a^6*b^17*c^4*d^16*e^11*z^6 - 74612736*a^4*b^16*c^7*d^21*e^6*z^6 - 72990720*
a^7*b^18*c^2*d^13*e^14*z^6 - 72990720*a^5*b^18*c^4*d^17*e^10*z^6 + 69746688
*a^4*b^15*c^8*d^22*e^5*z^6 + 63700992*a^15*b^10*c^2*d^5*e^22*z^6 + 63700992
*a^5*b^10*c^12*d^25*e^2*z^6 + 62914560*a^17*b^7*c^3*d^4*e^23*z^6 + 55148544
*a^4*b^17*c^6*d^20*e^7*z^6 - 45957120*a^4*b^14*c^9*d^23*e^4*z^6 - 25952256*
a^4*b^18*c^5*d^19*e^8*z^6 - 25165824*a^20*b^2*c^5*d^3*e^24*z^6 + 21086208*a
^4*b^13*c^10*d^24*e^3*z^6 + 20643840*a^6*b^19*c^2*d^14*e^13*z^6 + 20643840*
a^5*b^19*c^3*d^16*e^11*z^6 + 15728640*a^19*b^4*c^4*d^3*e^24*z^6 - 11796480*
a^16*b^9*c^2*d^4*e^23*z^6 - 6438912*a^4*b^12*c^11*d^25*e^2*z^6 + 5406720*a^
4*b^19*c^4*d^18*e^9*z^6 - 5242880*a^18*b^6*c^3*d^3*e^24*z^6 + 3784704*a^3*b
^18*c^6*d^21*e^6*z^6 - 3244032*a^3*b^19*c^5*d^20*e^7*z^6 - 3244032*a^3*b^17
*c^7*d^22*e^5*z^6 + 2027520*a^3*b^20*c^4*d^19*e^8*z^6 + 2027520*a^3*b^16*c^

8*d^23*e^4*z^6 - 1622016*a^9*b^16*c^2*d^11*e^16*z^6 - 1622016*a^5*b^16*c^6*
d^19*e^8*z^6 + 1622016*a^4*b^20*c^3*d^17*e^10*z^6 - 1523712*a^4*b^21*c^2*d^
16*e^11*z^6 + 983040*a^17*b^8*c^2*d^3*e^24*z^6 - 901120*a^3*b^21*c^3*d^18*e
^9*z^6 - 901120*a^3*b^15*c^9*d^24*e^3*z^6 + 270336*a^3*b^22*c^2*d^17*e^10*z
^6 + 270336*a^3*b^14*c^10*d^25*e^2*z^6 + 172032*a^5*b^20*c^2*d^15*e^12*z^6
- 38593888256*a^15*b^6*c^6*d^9*e^18*z^6 - 38593888256*a^9*b^6*c^12*d^21*e^6
*z^6 - 210386288640*a^15*b^3*c^9*d^12*e^15*z^6 - 210386288640*a^12*b^3*c^12
*d^18*e^9*z^6 + 15502147584*a^15*c^12*d^15*e^12*z^6 + 1107296256*a^19*c^8*d
^7*e^20*z^6 + 1107296256*a^11*c^16*d^23*e^4*z^6 + 13287555072*a^16*c^11*d^1
3*e^14*z^6 + 13287555072*a^14*c^13*d^17*e^10*z^6 + 201326592*a^20*c^7*d^5*e
^22*z^6 + 201326592*a^10*c^17*d^25*e^2*z^6 + 16777216*a^21*c^6*d^3*e^24*z^6
+ 3784704*a^9*b^18*d^9*e^18*z^6 - 3244032*a^10*b^17*d^8*e^19*z^6 - 3244032
*a^8*b^19*d^10*e^17*z^6 + 2027520*a^11*b^16*d^7*e^20*z^6 + 2027520*a^7*b^20
*d^11*e^16*z^6 - 901120*a^12*b^15*d^6*e^21*z^6 - 901120*a^6*b^21*d^12*e^15*
z^6 + 270336*a^13*b^14*d^5*e^22*z^6 + 270336*a^5*b^22*d^13*e^14*z^6 - 49152
*a^14*b^13*d^4*e^23*z^6 - 49152*a^4*b^23*d^14*e^13*z^6 + 4096*a^15*b^12*d^3
*e^24*z^6 + 4096*a^3*b^24*d^15*e^12*z^6 - 25165824*a^8*b^2*c^17*d^27*z^6 +
15728640*a^7*b^4*c^16*d^27*z^6 - 5242880*a^6*b^6*c^15*d^27*z^6 + 983040*a^5
*b^8*c^14*d^27*z^6 - 98304*a^4*b^10*c^13*d^27*z^6 + 4096*a^3*b^12*c^12*d^27
*z^6 + 8304721920*a^17*c^10*d^11*e^16*z^6 + 8304721920*a^13*c^14*d^19*e^8*z
^6 + 3690987520*a^18*c^9*d^9*e^18*z^6 + 3690987520*a^12*c^15*d^21*e^6*z^6 +
16777216*a^9*c^18*d^27*z^6 - 8493371392*a^6*b^8*c^9*d^14*e^9*z^4 + 1458044
928*a^8*b*c^14*d^17*e^6*z^4 - 12604538880*a^11*b^4*c^8*d^8*e^15*z^4 - 83030
67136*a^9*b^5*c^9*d^11*e^12*z^4 - 5588058112*a^13*b*c^9*d^7*e^16*z^4 - 3892
838400*a^8*b^2*c^13*d^16*e^7*z^4 - 3611713536*a^8*b^8*c^7*d^10*e^13*z^4 + 7
819006464*a^7*b^9*c^7*d^11*e^12*z^4 - 7782137856*a^8*b^7*c^8*d^11*e^12*z^4
+ 7780433920*a^12*b^2*c^9*d^8*e^15*z^4 - 12020465664*a^7*b^5*c^11*d^15*e^8*
z^4 + 3176792064*a^8*b^3*c^12*d^15*e^8*z^4 - 322633728*a^15*b*c^7*d^3*e^20*
z^4 + 210829312*a^7*b*c^15*d^19*e^4*z^4 + 15623258112*a^9*b^6*c^8*d^10*e^13
*z^4 + 25165824*a^15*b^3*c^5*d*e^22*z^4 - 15728640*a^14*b^5*c^4*d*e^22*z^4
+ 12582912*a^5*b^2*c^16*d^22*e*z^4 - 11730944*a^4*b^4*c^15*d^22*e*z^4 + 524
2880*a^13*b^7*c^3*d*e^22*z^4 - 4561920*a*b^15*c^7*d^17*e^6*z^4 + 4521984*a^
3*b^6*c^14*d^22*e*z^4 + 4460544*a*b^14*c^8*d^18*e^5*z^4 + 3538944*a^6*b*c^1
6*d^21*e^2*z^4 + 3108864*a*b^16*c^6*d^16*e^7*z^4 - 3027200*a*b^13*c^9*d^19*
e^4*z^4 - 2345472*a^5*b^17*c*d^7*e^16*z^4 - 2307072*a^8*b^14*c*d^4*e^19*z^4
+ 1824768*a^6*b^16*c*d^6*e^17*z^4 + 1734912*a^9*b^13*c*d^3*e^20*z^4 + 1419
264*a*b^12*c^10*d^20*e^3*z^4 - 1191168*a*b^17*c^5*d^15*e^8*z^4 - 983040*a^1
2*b^9*c^2*d*e^22*z^4 + 964608*a^4*b^18*c*d^8*e^15*z^4 - 866304*a^2*b^8*c^13
*d^22*e*z^4 + 703488*a^7*b^15*c*d^5*e^18*z^4 - 608256*a^10*b^12*c*d^2*e^21*
z^4 - 440832*a*b^11*c^11*d^21*e^2*z^4 + 275968*a*b^19*c^3*d^13*e^10*z^4 - 1
59744*a^2*b^20*c*d^10*e^13*z^4 - 153600*a*b^20*c^2*d^12*e^11*z^4 + 64512*a^
3*b^19*c*d^9*e^14*z^4 + 19746062336*a^8*b^6*c^9*d^12*e^11*z^4 - 15333588992
*a^10*b^4*c^9*d^10*e^13*z^4 + 6702170112*a^7*b^4*c^12*d^16*e^7*z^4 + 151679
13984*a^10*b^3*c^10*d^11*e^12*z^4 - 2256638976*a^5*b^11*c^7*d^13*e^10*z^4 +
2254307328*a^5*b^7*c^11*d^17*e^6*z^4 - 2200633344*a^6*b^5*c^12*d^17*e^6*z^
4 + 6457131008*a^11*b^3*c^9*d^9*e^14*z^4 - 2128785408*a^5*b^8*c^10*d^16*e^7
*z^4 - 2126057472*a^6*b^11*c^6*d^11*e^12*z^4 + 2038349824*a^12*b^5*c^6*d^5*
e^18*z^4 + 2037841920*a^5*b^10*c^8*d^14*e^9*z^4 + 3615621120*a^9*b*c^13*d^1
5*e^8*z^4 + 1900019712*a^11*b^2*c^10*d^10*e^13*z^4 + 1867698432*a^9*b^9*c^5
*d^7*e^16*z^4 - 6157369344*a^9*b^4*c^10*d^12*e^11*z^4 - 1856913408*a^7*b^10
*c^6*d^10*e^13*z^4 + 1789132800*a^6*b^4*c^13*d^18*e^5*z^4 + 6082658304*a^8*
b^4*c^11*d^14*e^9*z^4 + 6029549568*a^11*b^5*c^7*d^7*e^16*z^4 + 6010159104*a
^6*b^7*c^10*d^15*e^8*z^4 + 1703182336*a^7*b^7*c^9*d^13*e^10*z^4 + 165838848
0*a^11*b^6*c^6*d^6*e^17*z^4 + 5917114368*a^10*b^6*c^7*d^8*e^15*z^4 - 159119
7696*a^11*b^7*c^5*d^5*e^18*z^4 - 1526464512*a^8*b^10*c^5*d^8*e^15*z^4 - 577
2607488*a^12*b^4*c^7*d^6*e^17*z^4 - 1423507456*a^13*b^4*c^6*d^4*e^19*z^4 -
1387266048*a^7*b^3*c^13*d^17*e^6*z^4 + 2976120832*a^10*b*c^12*d^13*e^10*z^4
- 9906946048*a^9*b^2*c^12*d^14*e^9*z^4 - 18421874688*a^8*b^5*c^10*d^13*e^1
0*z^4 + 1141217280*a^6*b^12*c^5*d^10*e^13*z^4 - 9714364416*a^7*b^8*c^8*d^12

*e^11*z^4 - 16777216*a^16*b*c^6*d*e^22*z^4 + 98304*a^11*b^11*c*d*e^22*z^4 + 81920*a*b^10*c^12*d^22*e*z^4 + 39168*a*b^21*c*d^11*e^12*z^4 - 1091829760*a^5*b^6*c^12*d^18*e^5*z^4 + 1046740992*a^14*b^2*c^7*d^4*e^19*z^4 - 6884425728*a^12*b*c^10*d^9*e^14*z^4 + 987445248*a^4*b^10*c^9*d^16*e^7*z^4 + 984087552*a^5*b^12*c^6*d^12*e^11*z^4 - 9564585984*a^9*b^7*c^7*d^9*e^14*z^4 - 5266857984*a^10*b^7*c^6*d^7*e^16*z^4 - 892145664*a^7*b^11*c^5*d^9*e^14*z^4 - 2444623872*a^11*b*c^11*d^11*e^12*z^4 + 768540672*a^12*b^3*c^8*d^7*e^16*z^4 + 5048322048*a^8*b^9*c^6*d^9*e^14*z^4 + 5047612416*a^6*b^9*c^8*d^13*e^10*z^4 - 732492288*a^4*b^11*c^8*d^15*e^8*z^4 + 9266921472*a^7*b^6*c^10*d^14*e^9*z^4 - 645857280*a^6*b^6*c^11*d^16*e^7*z^4 - 623867904*a^4*b^9*c^10*d^17*e^6*z^4 - 622067712*a^6*b^3*c^14*d^19*e^4*z^4 + 582617088*a^10*b^8*c^5*d^6*e^17*z^4 + 577119744*a^7*b^12*c^4*d^8*e^15*z^4 + 552566784*a^12*b^6*c^5*d^4*e^19*z^4 + 549224448*a^9*b^8*c^6*d^8*e^15*z^4 - 526565376*a^9*b^10*c^4*d^6*e^17*z^4 + 511520256*a^10*b^9*c^4*d^5*e^18*z^4 + 13393723392*a^9*b^3*c^11*d^13*e^10*z^4 - 2066350080*a^14*b*c^8*d^5*e^18*z^4 + 4718592000*a^13*b^2*c^8*d^6*e^17*z^4 - 314572800*a^7*b^2*c^14*d^18*e^5*z^4 + 287250432*a^4*b^13*c^6*d^13*e^10*z^4 + 4565827584*a^10*b^5*c^8*d^9*e^14*z^4 - 250785792*a^4*b^14*c^5*d^12*e^11*z^4 + 235536384*a^13*b^3*c^7*d^5*e^18*z^4 - 232683264*a^8*b^11*c^4*d^7*e^16*z^4 - 199627776*a^5*b^14*c^4*d^10*e^13*z^4 - 190267392*a^12*b^7*c^4*d^3*e^20*z^4 + 184891392*a^6*b^10*c^7*d^12*e^11*z^4 + 180502528*a^4*b^7*c^12*d^19*e^4*z^4 + 178877952*a^3*b^13*c^7*d^15*e^8*z^4 + 172490752*a^14*b^3*c^6*d^3*e^20*z^4 + 163946496*a^13*b^5*c^5*d^3*e^20*z^4 + 155839488*a^8*b^12*c^3*d^6*e^17*z^4 + 155000832*a^5*b^5*c^13*d^19*e^4*z^4 - 152076288*a^4*b^6*c^13*d^20*e^3*z^4 - 137592576*a^3*b^12*c^8*d^16*e^7*z^4 - 133693440*a^14*b^4*c^5*d^2*e^21*z^4 - 116767488*a^3*b^9*c^11*d^19*e^4*z^4 - 108985344*a^3*b^14*c^6*d^14*e^9*z^4 - 106223616*a^6*b^13*c^4*d^9*e^14*z^4 + 106119168*a^3*b^10*c^10*d^18*e^5*z^4 + 102432768*a^5*b^4*c^14*d^20*e^3*z^4 + 102113280*a^4*b^12*c^7*d^14*e^9*z^4 + 100674048*a^5*b^9*c^9*d^15*e^8*z^4 + 90439680*a^13*b^6*c^4*d^2*e^21*z^4 - 86808576*a^6*b^14*c^3*d^8*e^15*z^4 + 86245376*a^6*b^2*c^15*d^20*e^3*z^4 + 79011840*a^4*b^8*c^11*d^18*e^5*z^4 + 78345216*a^4*b^15*c^4*d^11*e^12*z^4 + 78006528*a^11*b^9*c^3*d^3*e^20*z^4 - 73253376*a^9*b^11*c^3*d^5*e^18*z^4 + 67524608*a^3*b^8*c^12*d^20*e^3*z^4 + 67108864*a^15*b^2*c^6*d^2*e^21*z^4 - 61590528*a^10*b^10*c^3*d^4*e^19*z^4 + 61559808*a^5*b^15*c^3*d^9*e^14*z^4 - 59637760*a^5*b^3*c^15*d^21*e^2*z^4 + 58638336*a^4*b^5*c^14*d^21*e^2*z^4 - 40828416*a^7*b^13*c^3*d^7*e^16*z^4 - 35639296*a^2*b^12*c^9*d^18*e^5*z^4 - 31293440*a^12*b^8*c^3*d^2*e^21*z^4 + 29933568*a^5*b^13*c^5*d^11*e^12*z^4 + 27793920*a^2*b^11*c^10*d^19*e^4*z^4 + 27168768*a^2*b^13*c^8*d^17*e^6*z^4 - 23602176*a^7*b^14*c^2*d^6*e^17*z^4 - 23248896*a^3*b^7*c^13*d^21*e^2*z^4 + 20929536*a^3*b^15*c^5*d^13*e^10*z^4 + 18428928*a^9*b^12*c^2*d^4*e^19*z^4 + 18026496*a^6*b^15*c^2*d^7*e^16*z^4 - 16261632*a^10*b^11*c^2*d^3*e^20*z^4 + 15128064*a^3*b^16*c^4*d^12*e^11*z^4 - 14060544*a^2*b^10*c^11*d^20*e^3*z^4 + 13178880*a^2*b^16*c^5*d^14*e^9*z^4 - 11244288*a^3*b^17*c^3*d^11*e^12*z^4 - 10509312*a^2*b^15*c^6*d^15*e^8*z^4 - 7262208*a^4*b^17*c^2*d^9*e^14*z^4 - 7045632*a^2*b^17*c^4*d^13*e^10*z^4 - 6285312*a^2*b^14*c^7*d^16*e^7*z^4 + 5996544*a^11*b^10*c^2*d^2*e^21*z^4 + 4558336*a^2*b^9*c^12*d^21*e^2*z^4 + 4478976*a^11*b^8*c^4*d^4*e^19*z^4 + 2850816*a^4*b^16*c^3*d^10*e^13*z^4 + 2629632*a^3*b^11*c^9*d^17*e^6*z^4 + 2503680*a^3*b^18*c^2*d^10*e^13*z^4 + 1627136*a^2*b^18*c^3*d^12*e^11*z^4 + 1605120*a^8*b^13*c^2*d^5*e^18*z^4 + 1483776*a^5*b^16*c^2*d^8*e^15*z^4 + 139776*a^2*b^19*c^2*d^11*e^12*z^4 - 8542224384*a^10*b^2*c^11*d^12*e^11*z^4 - 3072*b^22*c*d^12*e^11*z^4 - 3072*b^12*c^11*d^22*e*z^4 - 1572864*a^6*c^17*d^22*e*z^4 - 4096*a^10*b^13*d*e^22*z^4 - 4096*a*b^22*d^10*e^13*z^4 - 6144*a^12*b^10*c*e^23*z^4 - 983040*a^5*b*c^17*d^23*z^4 - 6912*a*b^9*c^13*d^23*z^4 + 1824522240*a^13*c^10*d^8*e^15*z^4 + 1730150400*a^12*c^11*d^10*e^13*z^4 + 958922752*a^14*c^9*d^6*e^17*z^4 - 537919488*a^9*c^14*d^16*e^7*z^4 + 508559360*a^11*c^12*d^12*e^11*z^4 - 500170752*a^10*c^13*d^14*e^9*z^4 + 246939648*a^15*c^8*d^4*e^19*z^4 - 199229440*a^8*c^15*d^18*e^5*z^4 - 29884416*a^7*c^16*d^20*e^3*z^4 + 25165824*a^16*c^7*d^2*e^21*z^4 + 236544*b^17*c^6*d^17*e^6*z^4 - 202752*b^18*c^5*d^16*e^7*z^4 - 202752*b^16*c^7*d^18*e^5*z^4 + 126720*b^19*c^4*d^15*e^8*z^4 + 126720 *b^15*c^8*d^19*e^4*z^4 - 56320*b^20*c^3*d^14*e^9*z^4 - 56320*b^14*c^9*d^20*e^3*z^4 + 16896*b^21*c^2*d^13*e^10*z^4 + 16896*b^13*c^10*d^21*e^2*z^4 + 110080*a^7*b^16*d^4*e^19*z^4 + 110080*a^4*b^19*d^7*e^16*z^4 - 75520*a^8*b^15*d^3*e^20*z^4 - 75520*a^3*b^20*d^8*e^15*z^4 - 56320*a^6*b^17*d^5*e^18*z^4 - 56320*a^5*b^18*d^6*e^17*z^4 + 25600*a^9*b^14*d^2*e^21*z^4 + 25600*a^2*b^21*d^9*e^14*z^4 - 1572864*a^16*b^2*c^5*e^23*z^4 + 983040*a^15*b^4*c^4*e^23*z^4 - 327680*a^14*b^6*c^3*e^23*z^4 + 61440*a^13*b^8*c^2*e^23*z^4 + 983040*a^4*b^3*c^16*d^23*z^4 - 385024*a^3*b^5*c^15*d^23*z^4 + 73728*a^2*b^7*c^14*d^23*z^4 + 256*b^23*d^11*e^12*z^4 + 1048576*a^17*c^6*e^23*z^4 + 256*b^11*c^12*d^23*z^4 + 256*a^11*b^12*e^23*z^4 + 948695040*a^8*b*c^10*d^6*e^13*z^2 + 348917760*a^7*b*c^11*d^8*e^11*z^2 - 125030400*a^9*b*c^9*d^4*e^15*z^2 - 50728960*a^6*b*c^12*d^10*e^9*z^2 - 44298240*a^5*b*c^13*d^12*e^7*z^2 - 36495360*a^10*b*c^8*d^2*e^17*z^2 + 29675520*a^8*b^6*c^5*d*e^18*z^2 - 24170496*a^9*b^4*c^6*d*e^18*z^2 - 17202816*a^7*b^8*c^4*d*e^18*z^2 - 14561280*a^4*b*c^14*d^14*e^5*z^2 + 5532416*a^6*b^10*c^3*d*e^18*z^2 + 4128768*a^10*b^2*c^7*d*e^18*z^2 - 2662400*a^3*b*c^15*d^16*e^3*z^2 + 1184512*a*b^12*c^6*d^9*e^10*z^2 - 1136160*a*b^13*c^5*d^8*e^11*z^2 - 1017600*a^5*b^12*c^2*d*e^18*z^2 - 744768*a*b^11*c^7*d^10*e^9*z^2 + 607872*a*b^14*c^4*d^7*e^12*z^2 - 424064*a*b^6*c^12*d^15*e^4*z^2 + 408576*a*b^5*c^13*d^16*e^3*z^2 + 361152*a*b^10*c^8*d^11*e^8*z^2 - 287408*a*b^9*c^9*d^12*e^7*z^2 - 260448*a^3*b^15*c*d^2*e^17*z^2 - 203904*a*b^4*c^14*d^17*e^2*z^2 + 200832*a*b^8*c^10*d^13*e^6*z^2 + 126720*a*b^7*c^11*d^14*e^5*z^2 - 123968*a*b^15*c^3*d^6*e^13*z^2 - 39168*a*b^16*c^2*d^5*e^14*z^2 + 11904*a^2*b^16*c*d^3*e^16*z^2 + 1824135552*a^7*b^4*c^8*d^5*e^14*z^2 - 1457252352*a^8*b^2*c^9*d^5*e^14*z^2 - 1405209600*a^7*b^5*c^7*d^4*e^15*z^2 - 184320*a^2*b*c^16*d^18*e*z^2 + 100608*a^4*b^14*c*d*e^18*z^2 + 53248*a*b^3*c^15*d^18*e*z^2 + 26448*a*b^17*c*d^4*e^15*z^2 + 1067599872*a^8*b^3*c^8*d^4*e^15*z^2 - 930828288*a^7*b^3*c^9*d^6*e^13*z^2 + 920760000*a^6*b^4*c^9*d^7*e^12*z^2 - 806639616*a^6*b^3*c^10*d^8*e^11*z^2 - 791052480*a^6*b^6*c^7*d^5*e^14*z^2 + 772237824*a^6*b^7*c^6*d^4*e^15*z^2 - 701025408*a^5*b^6*c^8*d^7*e^12*z^2 + 443340288*a^5*b^5*c^9*d^8*e^11*z^2 + 433047552*a^7*b^6*c^6*d^3*e^16*z^2 + 405741312*a^5*b^7*c^7*d^6*e^13*z^2 + 293652480*a^6*b^2*c^11*d^9*e^10*z^2 - 276962688*a^6*b^8*c^5*d^3*e^16*z^2 - 247804272*a^8*b^4*c^7*d^3*e^16*z^2 + 213564384*a^4*b^8*c^7*d^7*e^12*z^2 - 202596816*a^5*b^9*c^5*d^4*e^15*z^2 - 182520896*a^4*b^9*c^6*d^6*e^13*z^2 - 153489408*a^5*b^3*c^11*d^10*e^9*z^2 - 152151552*a^7*b^2*c^10*d^7*e^12*z^2 + 115859712*a^5*b^2*c^12*d^11*e^8*z^2 + 108085248*a^9*b^3*c^7*d^2*e^17*z^2 + 105536256*a^4*b^5*c^10*d^10*e^9*z^2 - 98323200*a^6*b^5*c^8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^9*d^9*e^10*z^2 + 89464512*a^5*b^10*c^4*d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*d^2*e^17*z^2 + 68315904*a^5*b^8*c^6*d^5*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8*e^11*z^2 - 62951040*a^9*b^2*c^8*d^3*e^16*z^2 + 49056768*a^4*b^10*c^5*d^5*e^14*z^2 + 47614464*a^3*b^8*c^8*d^9*e^10*z^2 + 35604480*a^4*b^2*c^13*d^13*e^6*z^2 + 33983040*a^3*b^11*c^5*d^6*e^13*z^2 - 33515520*a^4*b^3*c^12*d^12*e^7*z^2 - 33463808*a^3*b^7*c^9*d^10*e^9*z^2 - 25128864*a^4*b^4*c^11*d^11*e^8*z^2 - 23193728*a^3*b^10*c^6*d^7*e^12*z^2 + 21015456*a^6*b^9*c^4*d^2*e^17*z^2 + 19924176*a^4*b^11*c^4*d^4*e^15*z^2 - 19251216*a^3*b^9*c^7*d^8*e^11*z^2 - 16434048*a^5*b^4*c^10*d^9*e^10*z^2 - 16289664*a^3*b^12*c^4*d^5*e^14*z^2 - 15059328*a^4*b^12*c^3*d^3*e^16*z^2 - 10766016*a^2*b^10*c^7*d^9*e^10*z^2 - 10453632*a^5*b^11*c^3*d^2*e^17*z^2 - 9940992*a^3*b^3*c^13*d^14*e^5*z^2 + 8373696*a^2*b^11*c^6*d^8*e^11*z^2 + 7776768*a^3*b^2*c^14*d^15*e^4*z^2 + 7077888*a^3*b^5*c^11*d^12*e^7*z^2 + 6798240*a^2*b^9*c^8*d^10*e^9*z^2 - 3589440*a^2*b^6*c^11*d^13*e^6*z^2 + 3544320*a^3*b^6*c^10*d^11*e^8*z^2 + 3128064*a^2*b^5*c^12*d^14*e^5*z^2 + 2346336*a^4*b^13*c^2*d^2*e^17*z^2 - 2261568*a^2*b^8*c^9*d^11*e^8*z^2 - 2125824*a^2*b^13*c^4*d^6*e^13*z^2 + 2002560*a^3*b^4*c^12*d^13*e^6*z^2 + 1927680*a^2*b^7*c^10*d^12*e^7*z^2 + 1814784*a^2*b^14*c^3*d^5*e^14*z^2 - 1807104*a^2*b^12*c^5*d^7*e^12*z^2 + 1637808*a^3*b^13*c^3*d^4*e^15*z^2 + 1083456*a^3*b^14*c^2*d^3*e^16*z^2 - 792384*a^2*b^4*c^13*d^15*e^4*z^2 - 657408*a^2*b^3*c^14*d^16*e^3*z^2 + 608256*a^7*b^7*c^5*d^2*e^17*z^2 + 595968*a^2*b^2*c^15*d^17*e^2*z^2 - 498624*a^2*b^15*c^2*d^4*e^15*z^2 - 3840*b^18*c*d^5*e^14*z^2 - 3840*b^5*c^14*d^18*e*z^2 + 2064384*a^11*c^8*d*e^18*z^2 - 4160*a^3*

b^16*d*e^18*z^2 - 4160*a*b^18*d^3*e^16*z^2 - 1290240*a^11*b*c^7*e^19*z^2 -
9840*a^5*b^13*c*e^19*z^2 - 5760*a*b^2*c^16*d^19*z^2 - 280581120*a^8*c^11*d^
7*e^12*z^2 + 110278656*a^9*c^10*d^5*e^14*z^2 - 89479168*a^7*c^12*d^9*e^10*z
^2 + 34464000*a^10*c^9*d^3*e^16*z^2 + 54240*b^15*c^4*d^8*e^11*z^2 + 54240*b
^8*c^11*d^15*e^4*z^2 - 49920*b^14*c^5*d^9*e^10*z^2 - 49920*b^9*c^10*d^14*e^
5*z^2 - 37376*b^16*c^3*d^7*e^12*z^2 - 37376*b^7*c^12*d^16*e^3*z^2 + 28480*b
^13*c^6*d^10*e^9*z^2 + 28480*b^10*c^9*d^13*e^6*z^2 + 15936*b^17*c^2*d^6*e^1
3*z^2 + 15936*b^6*c^13*d^17*e^2*z^2 - 7920*b^12*c^7*d^11*e^8*z^2 - 7920*b^1
1*c^8*d^12*e^7*z^2 + 7489536*a^5*c^14*d^13*e^6*z^2 + 6084096*a^6*c^13*d^11*
e^8*z^2 + 2280448*a^4*c^15*d^15*e^4*z^2 + 350208*a^3*c^16*d^17*e^2*z^2 + 11
616*a^2*b^17*d^2*e^17*z^2 - 3515904*a^9*b^5*c^5*e^19*z^2 + 3440640*a^10*b^3
*c^6*e^19*z^2 + 1870848*a^8*b^7*c^4*e^19*z^2 - 572272*a^7*b^9*c^3*e^19*z^2
+ 101856*a^6*b^11*c^2*e^19*z^2 + 400*b^19*d^4*e^15*z^2 + 400*b^4*c^15*d^19*
z^2 + 20736*a^2*c^17*d^19*z^2 + 400*a^4*b^15*e^19*z^2 - 3969216*a^4*b*c^10*
d^3*e^12 - 3001536*a^3*b*c^11*d^5*e^10 - 419904*a^2*b*c^12*d^7*e^8 + 184608
*a^4*b^3*c^8*d*e^14 - 153036*a*b^4*c^10*d^6*e^9 + 127008*a*b^3*c^11*d^7*e^8
 + 63108*a*b^6*c^8*d^4*e^11 - 29160*a*b^2*c^12*d^8*e^7 - 21060*a^3*b^5*c^7*
d*e^14 - 21060*a*b^7*c^7*d^3*e^12 + 5460*a*b^5*c^9*d^5*e^10 - 404544*a^5*b*
c^9*d*e^14 + 1251872*a^3*b^3*c^9*d^3*e^12 + 844224*a^4*b^2*c^9*d^2*e^13 + 8
20512*a^2*b^3*c^10*d^5*e^10 + 750672*a^3*b^2*c^10*d^4*e^11 - 657498*a^2*b^4
*c^9*d^4*e^11 - 487116*a^3*b^4*c^8*d^2*e^13 + 160704*a^2*b^2*c^11*d^6*e^9 +
 58806*a^2*b^6*c^7*d^2*e^13 + 13140*a^2*b^5*c^8*d^3*e^12 + 15286*b^6*c^9*d^
6*e^9 - 9540*b^7*c^8*d^5*e^10 - 9540*b^5*c^10*d^7*e^8 + 2025*b^8*c^7*d^4*e^
11 + 2025*b^4*c^11*d^8*e^7 + 3367008*a^4*c^11*d^4*e^11 + 1166400*a^3*c^12*d
^6*e^9 + 705600*a^5*c^10*d^2*e^13 + 104976*a^2*c^13*d^8*e^7 - 17640*a^5*b^2
*c^8*e^15 + 2025*a^4*b^4*c^7*e^15 + 38416*a^6*c^9*e^15, z, k)*((57344*a^12*
c^9*e^21 - 80*a^5*b^14*c^2*e^21 + 1824*a^6*b^12*c^3*e^21 - 17296*a^7*b^10*c
^4*e^21 + 87520*a^8*b^8*c^5*e^21 - 250880*a^9*b^6*c^6*e^21 + 394240*a^10*b^
4*c^7*e^21 - 290816*a^11*b^2*c^8*e^21 + 18432*a^3*c^18*d^18*e^3 + 210944*a^
4*c^17*d^16*e^5 + 878592*a^5*c^16*d^14*e^7 + 4749312*a^6*c^15*d^12*e^9 + 20
518912*a^7*c^14*d^10*e^11 + 12306432*a^8*c^13*d^8*e^13 - 22743040*a^9*c^12*
d^6*e^15 - 20076544*a^10*c^11*d^4*e^17 - 1425408*a^11*c^10*d^2*e^19 - 80*b^
5*c^16*d^19*e^2 + 704*b^6*c^15*d^18*e^3 - 2688*b^7*c^14*d^17*e^4 + 5824*b^8
*c^13*d^16*e^5 - 7840*b^9*c^12*d^15*e^6 + 6720*b^10*c^11*d^14*e^7 - 3728*b^
11*c^10*d^13*e^8 + 2176*b^12*c^9*d^12*e^9 - 3728*b^13*c^8*d^11*e^10 + 6720*
b^14*c^7*d^10*e^11 - 7840*b^15*c^6*d^9*e^12 + 5824*b^16*c^5*d^8*e^13 - 2688
*b^17*c^4*d^7*e^14 + 704*b^18*c^3*d^6*e^15 - 80*b^19*c^2*d^5*e^16 + 12288*a
^2*b^2*c^17*d^18*e^3 - 1536*a^2*b^3*c^16*d^17*e^4 - 131712*a^2*b^4*c^15*d^1
6*e^5 + 410112*a^2*b^5*c^14*d^15*e^6 - 576576*a^2*b^6*c^13*d^14*e^7 + 34272
0*a^2*b^7*c^12*d^13*e^8 + 298464*a^2*b^8*c^11*d^12*e^9 - 1248672*a^2*b^9*c^
10*d^11*e^10 + 2177920*a^2*b^10*c^9*d^10*e^11 - 2309120*a^2*b^11*c^8*d^9*e^
12 + 1389888*a^2*b^12*c^7*d^8*e^13 - 314048*a^2*b^13*c^6*d^7*e^14 - 120896*
a^2*b^14*c^5*d^6*e^15 + 88128*a^2*b^15*c^4*d^5*e^16 - 14240*a^2*b^16*c^3*d^
4*e^17 - 416*a^2*b^17*c^2*d^3*e^18 + 621568*a^3*b^2*c^16*d^16*e^5 - 953344*
a^3*b^3*c^15*d^15*e^6 + 196224*a^3*b^4*c^14*d^14*e^7 + 1667904*a^3*b^5*c^13
*d^13*e^8 - 3981824*a^3*b^6*c^12*d^12*e^9 + 7617920*a^3*b^7*c^11*d^11*e^10
- 11899456*a^3*b^8*c^10*d^10*e^11 + 11500496*a^3*b^9*c^9*d^9*e^12 - 4599536
*a^3*b^10*c^8*d^8*e^13 - 1951936*a^3*b^11*c^7*d^7*e^14 + 2953152*a^3*b^12*c
^6*d^6*e^15 - 1134960*a^3*b^13*c^5*d^5*e^16 + 98960*a^3*b^14*c^4*d^4*e^17 +
 21920*a^3*b^15*c^3*d^3*e^18 - 416*a^3*b^16*c^2*d^2*e^19 + 4509696*a^4*b^2*
c^15*d^14*e^7 - 6720000*a^4*b^3*c^14*d^13*e^8 + 8231808*a^4*b^4*c^13*d^12*e
^9 - 17138976*a^4*b^5*c^12*d^11*e^10 + 30880320*a^4*b^6*c^11*d^10*e^11 - 24
883456*a^4*b^7*c^10*d^9*e^12 - 6291360*a^4*b^8*c^9*d^8*e^13 + 28429152*a^4*
b^9*c^8*d^7*e^14 - 21523072*a^4*b^10*c^7*d^6*e^15 + 5834928*a^4*b^11*c^6*d^
5*e^16 + 339872*a^4*b^12*c^5*d^4*e^17 - 325216*a^4*b^13*c^4*d^3*e^18 + 1344
*a^4*b^14*c^3*d^2*e^19 + 5483520*a^5*b^2*c^14*d^12*e^9 + 14537472*a^5*b^3*c
^13*d^11*e^10 - 39383680*a^5*b^4*c^12*d^10*e^11 + 5513408*a^5*b^5*c^11*d^9*
e^12 + 84582144*a^5*b^6*c^10*d^8*e^13 - 124246848*a^5*b^7*c^9*d^7*e^14 + 70
979712*a^5*b^8*c^8*d^6*e^15 - 8326320*a^5*b^9*c^7*d^5*e^16 - 7484656*a^5*b^

10*c^6*d^4*e^17 + 2142272*a^5*b^11*c^5*d^3*e^18 + 83520*a^5*b^12*c^4*d^2*e^
19 + 25849856*a^6*b^2*c^13*d^10*e^11 + 67294720*a^6*b^3*c^12*d^9*e^12 - 216
767360*a^6*b^4*c^11*d^8*e^13 + 237211008*a^6*b^5*c^10*d^7*e^14 - 88839360*a
^6*b^6*c^9*d^6*e^15 - 35929920*a^6*b^7*c^8*d^5*e^16 + 37859616*a^6*b^8*c^7*
d^4*e^17 - 6475552*a^6*b^9*c^6*d^3*e^18 - 1055296*a^6*b^10*c^5*d^2*e^19 + 1
90669824*a^7*b^2*c^12*d^8*e^13 - 143425536*a^7*b^3*c^11*d^7*e^14 - 47908992
*a^7*b^4*c^10*d^6*e^15 + 154814400*a^7*b^5*c^9*d^5*e^16 - 83642880*a^7*b^6*
c^8*d^4*e^17 + 4534272*a^7*b^7*c^7*d^3*e^18 + 5525568*a^7*b^8*c^6*d^2*e^19
+ 165122048*a^8*b^2*c^11*d^6*e^15 - 187467264*a^8*b^3*c^10*d^5*e^16 + 66920
064*a^8*b^4*c^9*d^4*e^17 + 21356016*a^8*b^5*c^8*d^3*e^18 - 14644224*a^8*b^6
*c^7*d^2*e^19 + 16114688*a^9*b^2*c^10*d^4*e^17 - 48695936*a^9*b^3*c^9*d^3*e
^18 + 18757632*a^9*b^4*c^8*d^2*e^19 - 8060928*a^10*b^2*c^9*d^2*e^19 + 12574
72*a^11*b*c^9*d*e^20 + 896*a*b^3*c^17*d^19*e^2 - 7040*a*b^4*c^16*d^18*e^3 +
22080*a*b^5*c^15*d^17*e^4 - 32512*a*b^6*c^14*d^16*e^5 + 12736*a*b^7*c^13*d
^15*e^6 + 31104*a*b^8*c^12*d^14*e^7 - 51472*a*b^9*c^11*d^13*e^8 + 10864*a*b
^10*c^10*d^12*e^9 + 85440*a*b^11*c^9*d^11*e^10 - 186560*a*b^12*c^8*d^10*e^1
1 + 215904*a*b^13*c^7*d^9*e^12 - 151008*a*b^14*c^6*d^8*e^13 + 59776*a*b^15*
c^5*d^7*e^14 - 9408*a*b^16*c^4*d^6*e^15 - 1296*a*b^17*c^3*d^5*e^16 + 496*a*
b^18*c^2*d^4*e^17 - 2304*a^2*b*c^18*d^19*e^2 - 175104*a^3*b*c^17*d^17*e^4 -
1556480*a^4*b*c^16*d^15*e^6 + 496*a^4*b^15*c^2*d*e^20 - 4746240*a^5*b*c^15
*d^13*e^8 - 10256*a^5*b^13*c^3*d*e^20 - 24033792*a^6*b*c^14*d^11*e^10 + 845
12*a^6*b^11*c^4*d*e^20 - 100332544*a^7*b*c^13*d^9*e^12 - 341264*a^7*b^9*c^5
*d*e^20 - 65824768*a^8*b*c^12*d^7*e^14 + 621568*a^8*b^7*c^6*d*e^20 + 397383
68*a^9*b*c^11*d^5*e^16 - 68096*a^9*b^5*c^7*d*e^20 + 27159296*a^10*b*c^10*d^
3*e^18 - 1310720*a^10*b^3*c^8*d*e^20)/(32*(16*a^3*b^6*c^9*d^18 - a^2*b^8*c^
8*d^18 - 256*a^6*c^12*d^18 - 96*a^4*b^4*c^10*d^18 + 256*a^5*b^2*c^11*d^18 -
a^2*b^16*d^10*e^8 + 8*a^3*b^15*d^9*e^9 - 28*a^4*b^14*d^8*e^10 + 56*a^5*b^1
3*d^7*e^11 - 70*a^6*b^12*d^6*e^12 + 56*a^7*b^11*d^5*e^13 - 28*a^8*b^10*d^4*
e^14 + 8*a^9*b^9*d^3*e^15 - a^10*b^8*d^2*e^16 - 2048*a^7*c^11*d^16*e^2 - 71
68*a^8*c^10*d^14*e^4 - 14336*a^9*c^9*d^12*e^6 - 17920*a^10*c^8*d^10*e^8 - 1
4336*a^11*c^7*d^8*e^10 - 7168*a^12*c^6*d^6*e^12 - 2048*a^13*c^5*d^4*e^14 -
256*a^14*c^4*d^2*e^16 - 28*a^2*b^10*c^6*d^16*e^2 + 56*a^2*b^11*c^5*d^15*e^3
- 70*a^2*b^12*c^4*d^14*e^4 + 56*a^2*b^13*c^3*d^13*e^5 - 28*a^2*b^14*c^2*d^
12*e^6 + 440*a^3*b^8*c^7*d^16*e^2 - 840*a^3*b^9*c^6*d^15*e^3 + 952*a^3*b^10
*c^5*d^14*e^4 - 616*a^3*b^11*c^4*d^13*e^5 + 168*a^3*b^12*c^3*d^12*e^6 + 40*
a^3*b^13*c^2*d^11*e^7 - 2560*a^4*b^6*c^8*d^16*e^2 + 4480*a^4*b^7*c^7*d^15*e
^3 - 4060*a^4*b^8*c^6*d^14*e^4 + 1064*a^4*b^9*c^5*d^13*e^5 + 1372*a^4*b^10*
c^4*d^12*e^6 - 1360*a^4*b^11*c^3*d^11*e^7 + 380*a^4*b^12*c^2*d^10*e^8 + 640
0*a^5*b^4*c^9*d^16*e^2 - 8960*a^5*b^5*c^8*d^15*e^3 + 2240*a^5*b^6*c^7*d^14*
e^4 + 9856*a^5*b^7*c^6*d^13*e^5 - 13048*a^5*b^8*c^5*d^12*e^6 + 5400*a^5*b^9
*c^4*d^11*e^7 + 1040*a^5*b^10*c^3*d^10*e^8 - 1360*a^5*b^11*c^2*d^9*e^9 - 51
20*a^6*b^2*c^10*d^16*e^2 + 22400*a^6*b^4*c^8*d^14*e^4 - 41216*a^6*b^5*c^7*d
^13*e^5 + 25088*a^6*b^6*c^6*d^12*e^6 + 8320*a^6*b^7*c^5*d^11*e^7 - 17350*a^
6*b^8*c^4*d^10*e^8 + 5400*a^6*b^9*c^3*d^9*e^9 + 1372*a^6*b^10*c^2*d^8*e^10
- 35840*a^7*b^2*c^9*d^14*e^4 + 28672*a^7*b^3*c^8*d^13*e^5 + 30464*a^7*b^4*c
^7*d^12*e^6 - 73472*a^7*b^5*c^6*d^11*e^7 + 40544*a^7*b^6*c^5*d^10*e^8 + 832
0*a^7*b^7*c^4*d^9*e^9 - 13048*a^7*b^8*c^3*d^8*e^10 + 1064*a^7*b^9*c^2*d^7*e
^11 - 93184*a^8*b^2*c^8*d^12*e^6 + 71680*a^8*b^3*c^7*d^11*e^7 + 29120*a^8*b
^4*c^6*d^10*e^8 - 73472*a^8*b^5*c^5*d^9*e^9 + 25088*a^8*b^6*c^4*d^8*e^10 +
9856*a^8*b^7*c^3*d^7*e^11 - 4060*a^8*b^8*c^2*d^6*e^12 - 125440*a^9*b^2*c^7*
d^10*e^8 + 71680*a^9*b^3*c^6*d^9*e^9 + 30464*a^9*b^4*c^5*d^8*e^10 - 41216*a
^9*b^5*c^4*d^7*e^11 + 2240*a^9*b^6*c^3*d^6*e^12 + 4480*a^9*b^7*c^2*d^5*e^13
- 93184*a^10*b^2*c^6*d^8*e^10 + 28672*a^10*b^3*c^5*d^7*e^11 + 22400*a^10*b
^4*c^4*d^6*e^12 - 8960*a^10*b^5*c^3*d^5*e^13 - 2560*a^10*b^6*c^2*d^4*e^14 -
35840*a^11*b^2*c^5*d^6*e^12 + 6400*a^11*b^4*c^3*d^4*e^14 + 768*a^11*b^5*c^
2*d^3*e^15 - 5120*a^12*b^2*c^4*d^4*e^14 - 2048*a^12*b^3*c^3*d^3*e^15 - 96*a
^12*b^4*c^2*d^2*e^16 + 256*a^13*b^2*c^3*d^2*e^16 + 2048*a^6*b*c^11*d^17*e +
8*a^2*b^9*c^7*d^17*e + 8*a^2*b^15*c*d^11*e^7 - 128*a^3*b^7*c^8*d^17*e - 40
*a^3*b^14*c*d^10*e^8 + 768*a^4*b^5*c^9*d^17*e + 40*a^4*b^13*c*d^9*e^9 - 204

8*a^5*b^3*c^10*d^17*e + 168*a^5*b^12*c*d^8*e^10 - 616*a^6*b^11*c*d^7*e^11 +
14336*a^7*b*c^10*d^15*e^3 + 952*a^7*b^10*c*d^6*e^12 + 43008*a^8*b*c^9*d^13
*e^5 - 840*a^8*b^9*c*d^5*e^13 + 71680*a^9*b*c^8*d^11*e^7 + 440*a^9*b^8*c*d^
4*e^14 + 71680*a^10*b*c^7*d^9*e^9 - 128*a^10*b^7*c*d^3*e^15 + 43008*a^11*b*
c^6*d^7*e^11 + 16*a^11*b^6*c*d^2*e^16 + 14336*a^12*b*c^5*d^5*e^13 + 2048*a^
13*b*c^4*d^3*e^15)) - root(128723189760*a^14*b^4*c^9*d^13*e^14*z^6 + 128723
189760*a^12*b^4*c^11*d^17*e^10*z^6 - 8432455680*a^11*b^12*c^4*d^11*e^16*z^6
- 8432455680*a^7*b^12*c^8*d^19*e^8*z^6 + 12673351680*a^11*b^11*c^5*d^12*e^
15*z^6 + 12673351680*a^8*b^11*c^8*d^18*e^9*z^6 - 72637480960*a^12*b^9*c^6*d
^12*e^15*z^6 - 72637480960*a^9*b^9*c^9*d^18*e^9*z^6 - 21048344576*a^9*b^12*
c^6*d^15*e^12*z^6 - 16609443840*a^17*b^3*c^7*d^8*e^19*z^6 - 16609443840*a^1
0*b^3*c^14*d^22*e^5*z^6 + 145332633600*a^13*b^5*c^9*d^14*e^13*z^6 + 1453326
33600*a^12*b^5*c^10*d^16*e^11*z^6 + 123740356608*a^14*b^5*c^8*d^12*e^15*z^6
+ 123740356608*a^11*b^5*c^11*d^18*e^9*z^6 + 3460300800*a^17*b^5*c^5*d^6*e^
21*z^6 + 3460300800*a^8*b^5*c^14*d^24*e^3*z^6 - 7751073792*a^15*b^7*c^5*d^8
*e^19*z^6 - 7751073792*a^8*b^7*c^12*d^22*e^5*z^6 + 12041846784*a^14*b^7*c^6
*d^10*e^17*z^6 + 12041846784*a^9*b^7*c^11*d^20*e^7*z^6 - 325545099264*a^14*
b^3*c^10*d^14*e^13*z^6 - 325545099264*a^13*b^3*c^11*d^16*e^11*z^6 - 3330539
520*a^13*b^10*c^4*d^9*e^18*z^6 - 3330539520*a^7*b^10*c^10*d^21*e^6*z^6 + 15
7789716480*a^12*b^7*c^8*d^14*e^13*z^6 + 157789716480*a^11*b^7*c^9*d^16*e^11
*z^6 + 37492359168*a^11*b^10*c^6*d^13*e^14*z^6 + 37492359168*a^9*b^10*c^8*d
^17*e^10*z^6 + 301989888*a^8*b^3*c^16*d^26*e*z^6 - 7266631680*a^17*b^4*c^6*
d^7*e^20*z^6 - 7266631680*a^9*b^4*c^14*d^23*e^4*z^6 - 201326592*a^20*b*c^6*
d^4*e^23*z^6 - 188743680*a^7*b^5*c^15*d^26*e*z^6 + 45747339264*a^13*b^8*c^6
*d^11*e^16*z^6 + 45747339264*a^9*b^8*c^10*d^19*e^8*z^6 - 74612736*a^10*b^16
*c*d^9*e^18*z^6 - 2768240640*a^16*b^7*c^4*d^6*e^21*z^6 - 2768240640*a^7*b^7
*c^13*d^24*e^3*z^6 + 69746688*a^11*b^15*c*d^8*e^19*z^6 + 62914560*a^6*b^7*c
^14*d^26*e*z^6 + 2752020480*a^10*b^13*c^4*d^12*e^15*z^6 + 2752020480*a^7*b^
13*c^7*d^18*e^9*z^6 + 55148544*a^9*b^17*c*d^10*e^17*z^6 - 45957120*a^12*b^1
4*c*d^7*e^20*z^6 - 2724986880*a^14*b^9*c^4*d^8*e^19*z^6 - 2724986880*a^7*b^
9*c^11*d^22*e^5*z^6 - 25952256*a^8*b^18*c*d^11*e^16*z^6 + 21086208*a^13*b^1
3*c*d^6*e^21*z^6 - 11796480*a^5*b^9*c^13*d^26*e*z^6 - 6438912*a^14*b^12*c*d
^5*e^22*z^6 + 5406720*a^7*b^19*c*d^12*e^15*z^6 + 1622016*a^6*b^20*c*d^13*e^
14*z^6 - 1523712*a^5*b^21*c*d^14*e^13*z^6 + 1179648*a^15*b^11*c*d^4*e^23*z^
6 + 1179648*a^4*b^11*c^12*d^26*e*z^6 + 442368*a^4*b^22*c*d^15*e^12*z^6 - 98
304*a^16*b^10*c*d^3*e^24*z^6 - 49152*a^3*b^23*c*d^16*e^11*z^6 - 49152*a^3*b
^13*c^11*d^26*e*z^6 + 6897106944*a^9*b^13*c^5*d^14*e^13*z^6 + 6897106944*a^
8*b^13*c^6*d^16*e^11*z^6 - 2422210560*a^16*b^6*c^5*d^7*e^20*z^6 - 242221056
0*a^8*b^6*c^13*d^23*e^4*z^6 + 255785435136*a^14*b^2*c^11*d^15*e^12*z^6 + 41
004564480*a^15*b^4*c^8*d^11*e^16*z^6 + 41004564480*a^11*b^4*c^12*d^19*e^8*z
^6 + 2270822400*a^13*b^11*c^3*d^8*e^19*z^6 + 2270822400*a^6*b^11*c^10*d^22*
e^5*z^6 + 23677108224*a^14*b^8*c^5*d^9*e^18*z^6 + 23677108224*a^8*b^8*c^11*
d^21*e^6*z^6 + 212600881152*a^15*b^2*c^10*d^13*e^14*z^6 + 212600881152*a^13
*b^2*c^12*d^17*e^10*z^6 + 75157733376*a^15*b^5*c^7*d^10*e^17*z^6 + 75157733
376*a^10*b^5*c^12*d^20*e^7*z^6 - 251217838080*a^13*b^6*c^8*d^13*e^14*z^6 -
251217838080*a^11*b^6*c^10*d^17*e^10*z^6 - 1952907264*a^14*b^10*c^3*d^7*e^2
0*z^6 - 1952907264*a^6*b^10*c^11*d^23*e^4*z^6 - 27691057152*a^13*b^9*c^5*d^
10*e^17*z^6 - 27691057152*a^8*b^9*c^10*d^20*e^7*z^6 - 1902673920*a^8*b^15*c
^4*d^14*e^13*z^6 - 1902673920*a^7*b^15*c^5*d^16*e^11*z^6 + 10465050624*a^10
*b^11*c^6*d^14*e^13*z^6 + 10465050624*a^9*b^11*c^7*d^16*e^11*z^6 + 16139059
20*a^9*b^14*c^4*d^13*e^14*z^6 + 1613905920*a^7*b^14*c^6*d^17*e^10*z^6 - 332
18887680*a^17*b*c^9*d^10*e^17*z^6 - 33218887680*a^12*b*c^14*d^20*e^7*z^6 +
1524695040*a^10*b^14*c^3*d^11*e^16*z^6 + 1524695040*a^6*b^14*c^7*d^19*e^8*z
^6 - 1472200704*a^18*b^4*c^5*d^5*e^22*z^6 - 1472200704*a^8*b^4*c^15*d^25*e^
2*z^6 - 83047219200*a^16*b^3*c^8*d^10*e^17*z^6 - 83047219200*a^11*b^3*c^13*
d^20*e^7*z^6 + 44291850240*a^17*b^2*c^8*d^9*e^18*z^6 + 44291850240*a^11*b^2
*c^14*d^21*e^6*z^6 + 1308131328*a^8*b^14*c^5*d^15*e^12*z^6 - 201326592*a^9*
b*c^17*d^26*e*z^6 + 48530718720*a^12*b^8*c^7*d^13*e^14*z^6 + 48530718720*a^
10*b^8*c^9*d^17*e^10*z^6 - 1242644480*a^12*b^12*c^3*d^9*e^18*z^6 - 12426444

80*a^6*b^12*c^9*d^21*e^6*z^6 + 9813196800*a^12*b^10*c^5*d^11*e^16*z^6 + 981
3196800*a^8*b^10*c^9*d^19*e^8*z^6 - 93012885504*a^15*b*c^11*d^14*e^13*z^6 -
93012885504*a^14*b*c^12*d^16*e^11*z^6 + 177305812992*a^13*b^4*c^10*d^15*e^
12*z^6 + 52730658816*a^10*b^10*c^7*d^15*e^12*z^6 - 1180106752*a^9*b^15*c^3*
d^12*e^15*z^6 - 1180106752*a^6*b^15*c^6*d^18*e^9*z^6 + 1023672320*a^15*b^9*
c^3*d^6*e^21*z^6 + 1023672320*a^6*b^9*c^12*d^24*e^3*z^6 + 975175680*a^17*b^
6*c^4*d^5*e^22*z^6 + 975175680*a^7*b^6*c^14*d^25*e^2*z^6 - 11072962560*a^18
*b*c^8*d^8*e^19*z^6 - 11072962560*a^11*b*c^15*d^22*e^5*z^6 + 9412018176*a^1
8*b^2*c^7*d^7*e^20*z^6 + 9412018176*a^10*b^2*c^15*d^23*e^4*z^6 + 805306368*
a^19*b^2*c^6*d^5*e^22*z^6 + 805306368*a^9*b^2*c^16*d^25*e^2*z^6 - 133809831
936*a^14*b^6*c^7*d^11*e^16*z^6 - 133809831936*a^10*b^6*c^11*d^19*e^8*z^6 -
2214592512*a^19*b*c^7*d^6*e^21*z^6 - 2214592512*a^10*b*c^16*d^24*e^3*z^6 +
82216747008*a^13*b^7*c^7*d^12*e^15*z^6 + 82216747008*a^10*b^7*c^10*d^18*e^9
*z^6 - 586629120*a^12*b^13*c^2*d^8*e^19*z^6 - 586629120*a^5*b^13*c^9*d^22*e
^5*z^6 + 568565760*a^7*b^16*c^4*d^15*e^12*z^6 - 4844421120*a^16*b^4*c^7*d^9
*e^18*z^6 - 4844421120*a^10*b^4*c^13*d^21*e^6*z^6 + 531210240*a^11*b^14*c^2
*d^9*e^18*z^6 + 531210240*a^5*b^14*c^8*d^21*e^6*z^6 - 527155200*a^11*b^13*c
^3*d^10*e^17*z^6 - 527155200*a^6*b^13*c^8*d^20*e^7*z^6 + 43470028800*a^11*b
^8*c^8*d^15*e^12*z^6 - 107874877440*a^11*b^9*c^7*d^14*e^13*z^6 - 1078748774
40*a^10*b^9*c^8*d^16*e^11*z^6 + 9018408960*a^12*b^11*c^4*d^10*e^17*z^6 + 90
18408960*a^7*b^11*c^9*d^20*e^7*z^6 + 421994496*a^13*b^12*c^2*d^7*e^20*z^6 +
421994496*a^5*b^12*c^10*d^23*e^4*z^6 - 66437775360*a^16*b*c^10*d^12*e^15*z
^6 - 66437775360*a^13*b*c^13*d^18*e^9*z^6 + 26159874048*a^16*b^5*c^6*d^8*e^
19*z^6 + 26159874048*a^9*b^5*c^13*d^22*e^5*z^6 - 369098752*a^18*b^3*c^6*d^6
*e^21*z^6 - 369098752*a^9*b^3*c^15*d^24*e^3*z^6 + 351436800*a^8*b^16*c^3*d^
13*e^14*z^6 + 351436800*a^6*b^16*c^5*d^17*e^10*z^6 - 334233600*a^16*b^8*c^3
*d^5*e^22*z^6 - 334233600*a^6*b^8*c^13*d^25*e^2*z^6 + 301989888*a^19*b^3*c^
5*d^4*e^23*z^6 - 266010624*a^10*b^15*c^2*d^10*e^17*z^6 - 266010624*a^5*b^15
*c^7*d^20*e^7*z^6 - 305198530560*a^12*b^6*c^9*d^15*e^12*z^6 - 203292672*a^1
4*b^11*c^2*d^6*e^21*z^6 - 203292672*a^5*b^11*c^11*d^24*e^3*z^6 - 188743680*
a^18*b^5*c^4*d^4*e^23*z^6 + 120418467840*a^16*b^2*c^9*d^11*e^16*z^6 + 12041
8467840*a^12*b^2*c^13*d^19*e^8*z^6 - 17293934592*a^10*b^12*c^5*d^13*e^14*z^
6 - 17293934592*a^8*b^12*c^7*d^17*e^10*z^6 + 104890368*a^8*b^17*c^2*d^12*e^
15*z^6 + 104890368*a^5*b^17*c^5*d^18*e^9*z^6 + 4390256640*a^15*b^8*c^4*d^7*
e^20*z^6 + 4390256640*a^7*b^8*c^12*d^23*e^4*z^6 - 91750400*a^6*b^18*c^3*d^1
5*e^12*z^6 + 79134720*a^7*b^17*c^3*d^14*e^13*z^6 + 79134720*a^6*b^17*c^4*d^
16*e^11*z^6 - 74612736*a^4*b^16*c^7*d^21*e^6*z^6 - 72990720*a^7*b^18*c^2*d^
13*e^14*z^6 - 72990720*a^5*b^18*c^4*d^17*e^10*z^6 + 69746688*a^4*b^15*c^8*d
^22*e^5*z^6 + 63700992*a^15*b^10*c^2*d^5*e^22*z^6 + 63700992*a^5*b^10*c^12*
d^25*e^2*z^6 + 62914560*a^17*b^7*c^3*d^4*e^23*z^6 + 55148544*a^4*b^17*c^6*d
^20*e^7*z^6 - 45957120*a^4*b^14*c^9*d^23*e^4*z^6 - 25952256*a^4*b^18*c^5*d^
19*e^8*z^6 - 25165824*a^20*b^2*c^5*d^3*e^24*z^6 + 21086208*a^4*b^13*c^10*d^
24*e^3*z^6 + 20643840*a^6*b^19*c^2*d^14*e^13*z^6 + 20643840*a^5*b^19*c^3*d^
16*e^11*z^6 + 15728640*a^19*b^4*c^4*d^3*e^24*z^6 - 11796480*a^16*b^9*c^2*d^
4*e^23*z^6 - 6438912*a^4*b^12*c^11*d^25*e^2*z^6 + 5406720*a^4*b^19*c^4*d^18
*e^9*z^6 - 5242880*a^18*b^6*c^3*d^3*e^24*z^6 + 3784704*a^3*b^18*c^6*d^21*e^
6*z^6 - 3244032*a^3*b^19*c^5*d^20*e^7*z^6 - 3244032*a^3*b^17*c^7*d^22*e^5*z
^6 + 2027520*a^3*b^20*c^4*d^19*e^8*z^6 + 2027520*a^3*b^16*c^8*d^23*e^4*z^6
- 1622016*a^9*b^16*c^2*d^11*e^16*z^6 - 1622016*a^5*b^16*c^6*d^19*e^8*z^6 +
1622016*a^4*b^20*c^3*d^17*e^10*z^6 - 1523712*a^4*b^21*c^2*d^16*e^11*z^6 + 9
83040*a^17*b^8*c^2*d^3*e^24*z^6 - 901120*a^3*b^21*c^3*d^18*e^9*z^6 - 901120
*a^3*b^15*c^9*d^24*e^3*z^6 + 270336*a^3*b^22*c^2*d^17*e^10*z^6 + 270336*a^3
*b^14*c^10*d^25*e^2*z^6 + 172032*a^5*b^20*c^2*d^15*e^12*z^6 - 38593888256*a
^15*b^6*c^6*d^9*e^18*z^6 - 38593888256*a^9*b^6*c^12*d^21*e^6*z^6 - 21038628
8640*a^15*b^3*c^9*d^12*e^15*z^6 - 210386288640*a^12*b^3*c^12*d^18*e^9*z^6 +
15502147584*a^15*c^12*d^15*e^12*z^6 + 1107296256*a^19*c^8*d^7*e^20*z^6 + 1
107296256*a^11*c^16*d^23*e^4*z^6 + 13287555072*a^16*c^11*d^13*e^14*z^6 + 13
287555072*a^14*c^13*d^17*e^10*z^6 + 201326592*a^20*c^7*d^5*e^22*z^6 + 20132
6592*a^10*c^17*d^25*e^2*z^6 + 16777216*a^21*c^6*d^3*e^24*z^6 + 3784704*a^9*

b^18*d^9*e^18*z^6 - 3244032*a^10*b^17*d^8*e^19*z^6 - 3244032*a^8*b^19*d^10*
e^17*z^6 + 2027520*a^11*b^16*d^7*e^20*z^6 + 2027520*a^7*b^20*d^11*e^16*z^6
- 901120*a^12*b^15*d^6*e^21*z^6 - 901120*a^6*b^21*d^12*e^15*z^6 + 270336*a^
13*b^14*d^5*e^22*z^6 + 270336*a^5*b^22*d^13*e^14*z^6 - 49152*a^14*b^13*d^4*
e^23*z^6 - 49152*a^4*b^23*d^14*e^13*z^6 + 4096*a^15*b^12*d^3*e^24*z^6 + 409
6*a^3*b^24*d^15*e^12*z^6 - 25165824*a^8*b^2*c^17*d^27*z^6 + 15728640*a^7*b^
4*c^16*d^27*z^6 - 5242880*a^6*b^6*c^15*d^27*z^6 + 983040*a^5*b^8*c^14*d^27*
z^6 - 98304*a^4*b^10*c^13*d^27*z^6 + 4096*a^3*b^12*c^12*d^27*z^6 + 83047219
20*a^17*c^10*d^11*e^16*z^6 + 8304721920*a^13*c^14*d^19*e^8*z^6 + 3690987520
*a^18*c^9*d^9*e^18*z^6 + 3690987520*a^12*c^15*d^21*e^6*z^6 + 16777216*a^9*c
^18*d^27*z^6 - 8493371392*a^6*b^8*c^9*d^14*e^9*z^4 + 1458044928*a^8*b*c^14*
d^17*e^6*z^4 - 12604538880*a^11*b^4*c^8*d^8*e^15*z^4 - 8303067136*a^9*b^5*c
^9*d^11*e^12*z^4 - 5588058112*a^13*b*c^9*d^7*e^16*z^4 - 3892838400*a^8*b^2*
c^13*d^16*e^7*z^4 - 3611713536*a^8*b^8*c^7*d^10*e^13*z^4 + 7819006464*a^7*b
^9*c^7*d^11*e^12*z^4 - 7782137856*a^8*b^7*c^8*d^11*e^12*z^4 + 7780433920*a^
12*b^2*c^9*d^8*e^15*z^4 - 12020465664*a^7*b^5*c^11*d^15*e^8*z^4 + 317679206
4*a^8*b^3*c^12*d^15*e^8*z^4 - 322633728*a^15*b*c^7*d^3*e^20*z^4 + 210829312
*a^7*b*c^15*d^19*e^4*z^4 + 15623258112*a^9*b^6*c^8*d^10*e^13*z^4 + 25165824
*a^15*b^3*c^5*d*e^22*z^4 - 15728640*a^14*b^5*c^4*d*e^22*z^4 + 12582912*a^5*
b^2*c^16*d^22*e*z^4 - 11730944*a^4*b^4*c^15*d^22*e*z^4 + 5242880*a^13*b^7*c
^3*d*e^22*z^4 - 4561920*a*b^15*c^7*d^17*e^6*z^4 + 4521984*a^3*b^6*c^14*d^22
*e*z^4 + 4460544*a*b^14*c^8*d^18*e^5*z^4 + 3538944*a^6*b*c^16*d^21*e^2*z^4
+ 3108864*a*b^16*c^6*d^16*e^7*z^4 - 3027200*a*b^13*c^9*d^19*e^4*z^4 - 23454
72*a^5*b^17*c*d^7*e^16*z^4 - 2307072*a^8*b^14*c*d^4*e^19*z^4 + 1824768*a^6*
b^16*c*d^6*e^17*z^4 + 1734912*a^9*b^13*c*d^3*e^20*z^4 + 1419264*a*b^12*c^10
*d^20*e^3*z^4 - 1191168*a*b^17*c^5*d^15*e^8*z^4 - 983040*a^12*b^9*c^2*d*e^2
2*z^4 + 964608*a^4*b^18*c*d^8*e^15*z^4 - 866304*a^2*b^8*c^13*d^22*e*z^4 + 7
03488*a^7*b^15*c*d^5*e^18*z^4 - 608256*a^10*b^12*c*d^2*e^21*z^4 - 440832*a*
b^11*c^11*d^21*e^2*z^4 + 275968*a*b^19*c^3*d^13*e^10*z^4 - 159744*a^2*b^20*
c*d^10*e^13*z^4 - 153600*a*b^20*c^2*d^12*e^11*z^4 + 64512*a^3*b^19*c*d^9*e^
14*z^4 + 19746062336*a^8*b^6*c^9*d^12*e^11*z^4 - 15333588992*a^10*b^4*c^9*d
^10*e^13*z^4 + 6702170112*a^7*b^4*c^12*d^16*e^7*z^4 + 15167913984*a^10*b^3*
c^10*d^11*e^12*z^4 - 2256638976*a^5*b^11*c^7*d^13*e^10*z^4 + 2254307328*a^5
*b^7*c^11*d^17*e^6*z^4 - 2200633344*a^6*b^5*c^12*d^17*e^6*z^4 + 6457131008*
a^11*b^3*c^9*d^9*e^14*z^4 - 2128785408*a^5*b^8*c^10*d^16*e^7*z^4 - 21260574
72*a^6*b^11*c^6*d^11*e^12*z^4 + 2038349824*a^12*b^5*c^6*d^5*e^18*z^4 + 2037
841920*a^5*b^10*c^8*d^14*e^9*z^4 + 3615621120*a^9*b*c^13*d^15*e^8*z^4 + 190
0019712*a^11*b^2*c^10*d^10*e^13*z^4 + 1867698432*a^9*b^9*c^5*d^7*e^16*z^4 -
 6157369344*a^9*b^4*c^10*d^12*e^11*z^4 - 1856913408*a^7*b^10*c^6*d^10*e^13*
z^4 + 1789132800*a^6*b^4*c^13*d^18*e^5*z^4 + 6082658304*a^8*b^4*c^11*d^14*e
^9*z^4 + 6029549568*a^11*b^5*c^7*d^7*e^16*z^4 + 6010159104*a^6*b^7*c^10*d^1
5*e^8*z^4 + 1703182336*a^7*b^7*c^9*d^13*e^10*z^4 + 1658388480*a^11*b^6*c^6*
d^6*e^17*z^4 + 5917114368*a^10*b^6*c^7*d^8*e^15*z^4 - 1591197696*a^11*b^7*c
^5*d^5*e^18*z^4 - 1526464512*a^8*b^10*c^5*d^8*e^15*z^4 - 5772607488*a^12*b^
4*c^7*d^6*e^17*z^4 - 1423507456*a^13*b^4*c^6*d^4*e^19*z^4 - 1387266048*a^7*
b^3*c^13*d^17*e^6*z^4 + 2976120832*a^10*b*c^12*d^13*e^10*z^4 - 9906946048*a
^9*b^2*c^12*d^14*e^9*z^4 - 18421874688*a^8*b^5*c^10*d^13*e^10*z^4 + 1141217
280*a^6*b^12*c^5*d^10*e^13*z^4 - 9714364416*a^7*b^8*c^8*d^12*e^11*z^4 - 167
77216*a^16*b*c^6*d*e^22*z^4 + 98304*a^11*b^11*c*d*e^22*z^4 + 81920*a*b^10*c
^12*d^22*e*z^4 + 39168*a*b^21*c*d^11*e^12*z^4 - 1091829760*a^5*b^6*c^12*d^1
8*e^5*z^4 + 1046740992*a^14*b^2*c^7*d^4*e^19*z^4 - 6884425728*a^12*b*c^10*d
^9*e^14*z^4 + 987445248*a^4*b^10*c^9*d^16*e^7*z^4 + 984087552*a^5*b^12*c^6*
d^12*e^11*z^4 - 9564585984*a^9*b^7*c^7*d^9*e^14*z^4 - 5266857984*a^10*b^7*c
^6*d^7*e^16*z^4 - 892145664*a^7*b^11*c^5*d^9*e^14*z^4 - 2444623872*a^11*b*c
^11*d^11*e^12*z^4 + 768540672*a^12*b^3*c^8*d^7*e^16*z^4 + 5048322048*a^8*b^
9*c^6*d^9*e^14*z^4 + 5047612416*a^6*b^9*c^8*d^13*e^10*z^4 - 732492288*a^4*b
^11*c^8*d^15*e^8*z^4 + 9266921472*a^7*b^6*c^10*d^14*e^9*z^4 - 645857280*a^6
*b^6*c^11*d^16*e^7*z^4 - 623867904*a^4*b^9*c^10*d^17*e^6*z^4 - 622067712*a^
6*b^3*c^14*d^19*e^4*z^4 + 582617088*a^10*b^8*c^5*d^6*e^17*z^4 + 577119744*a

^7*b^12*c^4*d^8*e^15*z^4 + 552566784*a^12*b^6*c^5*d^4*e^19*z^4 + 549224448*
a^9*b^8*c^6*d^8*e^15*z^4 - 526565376*a^9*b^10*c^4*d^6*e^17*z^4 + 511520256*
a^10*b^9*c^4*d^5*e^18*z^4 + 13393723392*a^9*b^3*c^11*d^13*e^10*z^4 - 206635
0080*a^14*b*c^8*d^5*e^18*z^4 + 4718592000*a^13*b^2*c^8*d^6*e^17*z^4 - 31457
2800*a^7*b^2*c^14*d^18*e^5*z^4 + 287250432*a^4*b^13*c^6*d^13*e^10*z^4 + 456
5827584*a^10*b^5*c^8*d^9*e^14*z^4 - 250785792*a^4*b^14*c^5*d^12*e^11*z^4 +
235536384*a^13*b^3*c^7*d^5*e^18*z^4 - 232683264*a^8*b^11*c^4*d^7*e^16*z^4 -
199627776*a^5*b^14*c^4*d^10*e^13*z^4 - 190267392*a^12*b^7*c^4*d^3*e^20*z^4
+ 184891392*a^6*b^10*c^7*d^12*e^11*z^4 + 180502528*a^4*b^7*c^12*d^19*e^4*z
^4 + 178877952*a^3*b^13*c^7*d^15*e^8*z^4 + 172490752*a^14*b^3*c^6*d^3*e^20*
z^4 + 163946496*a^13*b^5*c^5*d^3*e^20*z^4 + 155839488*a^8*b^12*c^3*d^6*e^17
*z^4 + 155000832*a^5*b^5*c^13*d^19*e^4*z^4 - 152076288*a^4*b^6*c^13*d^20*e^
3*z^4 - 137592576*a^3*b^12*c^8*d^16*e^7*z^4 - 133693440*a^14*b^4*c^5*d^2*e^
21*z^4 - 116767488*a^3*b^9*c^11*d^19*e^4*z^4 - 108985344*a^3*b^14*c^6*d^14*
e^9*z^4 - 106223616*a^6*b^13*c^4*d^9*e^14*z^4 + 106119168*a^3*b^10*c^10*d^1
8*e^5*z^4 + 102432768*a^5*b^4*c^14*d^20*e^3*z^4 + 102113280*a^4*b^12*c^7*d^
14*e^9*z^4 + 100674048*a^5*b^9*c^9*d^15*e^8*z^4 + 90439680*a^13*b^6*c^4*d^2
*e^21*z^4 - 86808576*a^6*b^14*c^3*d^8*e^15*z^4 + 86245376*a^6*b^2*c^15*d^20
*e^3*z^4 + 79011840*a^4*b^8*c^11*d^18*e^5*z^4 + 78345216*a^4*b^15*c^4*d^11*
e^12*z^4 + 78006528*a^11*b^9*c^3*d^3*e^20*z^4 - 73253376*a^9*b^11*c^3*d^5*e
^18*z^4 + 67524608*a^3*b^8*c^12*d^20*e^3*z^4 + 67108864*a^15*b^2*c^6*d^2*e^
21*z^4 - 61590528*a^10*b^10*c^3*d^4*e^19*z^4 + 61559808*a^5*b^15*c^3*d^9*e^
14*z^4 - 59637760*a^5*b^3*c^15*d^21*e^2*z^4 + 58638336*a^4*b^5*c^14*d^21*e^
2*z^4 - 40828416*a^7*b^13*c^3*d^7*e^16*z^4 - 35639296*a^2*b^12*c^9*d^18*e^5
*z^4 - 31293440*a^12*b^8*c^3*d^2*e^21*z^4 + 29933568*a^5*b^13*c^5*d^11*e^12
*z^4 + 27793920*a^2*b^11*c^10*d^19*e^4*z^4 + 27168768*a^2*b^13*c^8*d^17*e^6
*z^4 - 23602176*a^7*b^14*c^2*d^6*e^17*z^4 - 23248896*a^3*b^7*c^13*d^21*e^2*
z^4 + 20929536*a^3*b^15*c^5*d^13*e^10*z^4 + 18428928*a^9*b^12*c^2*d^4*e^19*
z^4 + 18026496*a^6*b^15*c^2*d^7*e^16*z^4 - 16261632*a^10*b^11*c^2*d^3*e^20*
z^4 + 15128064*a^3*b^16*c^4*d^12*e^11*z^4 - 14060544*a^2*b^10*c^11*d^20*e^3
*z^4 + 13178880*a^2*b^16*c^5*d^14*e^9*z^4 - 11244288*a^3*b^17*c^3*d^11*e^12
*z^4 - 10509312*a^2*b^15*c^6*d^15*e^8*z^4 - 7262208*a^4*b^17*c^2*d^9*e^14*z
^4 - 7045632*a^2*b^17*c^4*d^13*e^10*z^4 - 6285312*a^2*b^14*c^7*d^16*e^7*z^4
+ 5996544*a^11*b^10*c^2*d^2*e^21*z^4 + 4558336*a^2*b^9*c^12*d^21*e^2*z^4 +
4478976*a^11*b^8*c^4*d^4*e^19*z^4 + 2850816*a^4*b^16*c^3*d^10*e^13*z^4 + 2
629632*a^3*b^11*c^9*d^17*e^6*z^4 + 2503680*a^3*b^18*c^2*d^10*e^13*z^4 + 162
7136*a^2*b^18*c^3*d^12*e^11*z^4 + 1605120*a^8*b^13*c^2*d^5*e^18*z^4 + 14837
76*a^5*b^16*c^2*d^8*e^15*z^4 + 139776*a^2*b^19*c^2*d^11*e^12*z^4 - 85422243
84*a^10*b^2*c^11*d^12*e^11*z^4 - 3072*b^22*c*d^12*e^11*z^4 - 3072*b^12*c^11
*d^22*e*z^4 - 1572864*a^6*c^17*d^22*e*z^4 - 4096*a^10*b^13*d*e^22*z^4 - 409
6*a*b^22*d^10*e^13*z^4 - 6144*a^12*b^10*c*e^23*z^4 - 983040*a^5*b*c^17*d^23
*z^4 - 6912*a*b^9*c^13*d^23*z^4 + 1824522240*a^13*c^10*d^8*e^15*z^4 + 17301
50400*a^12*c^11*d^10*e^13*z^4 + 958922752*a^14*c^9*d^6*e^17*z^4 - 537919488
*a^9*c^14*d^16*e^7*z^4 + 508559360*a^11*c^12*d^12*e^11*z^4 - 500170752*a^10
*c^13*d^14*e^9*z^4 + 246939648*a^15*c^8*d^4*e^19*z^4 - 199229440*a^8*c^15*d
^18*e^5*z^4 - 29884416*a^7*c^16*d^20*e^3*z^4 + 25165824*a^16*c^7*d^2*e^21*z
^4 + 236544*b^17*c^6*d^17*e^6*z^4 - 202752*b^18*c^5*d^16*e^7*z^4 - 202752*b
^16*c^7*d^18*e^5*z^4 + 126720*b^19*c^4*d^15*e^8*z^4 + 126720*b^15*c^8*d^19*
e^4*z^4 - 56320*b^20*c^3*d^14*e^9*z^4 - 56320*b^14*c^9*d^20*e^3*z^4 + 16896
*b^21*c^2*d^13*e^10*z^4 + 16896*b^13*c^10*d^21*e^2*z^4 + 110080*a^7*b^16*d^
4*e^19*z^4 + 110080*a^4*b^19*d^7*e^16*z^4 - 75520*a^8*b^15*d^3*e^20*z^4 - 7
5520*a^3*b^20*d^8*e^15*z^4 - 56320*a^6*b^17*d^5*e^18*z^4 - 56320*a^5*b^18*d
^6*e^17*z^4 + 25600*a^9*b^14*d^2*e^21*z^4 + 25600*a^2*b^21*d^9*e^14*z^4 - 1
572864*a^16*b^2*c^5*e^23*z^4 + 983040*a^15*b^4*c^4*e^23*z^4 - 327680*a^14*b
^6*c^3*e^23*z^4 + 61440*a^13*b^8*c^2*e^23*z^4 + 983040*a^4*b^3*c^16*d^23*z^
4 - 385024*a^3*b^5*c^15*d^23*z^4 + 73728*a^2*b^7*c^14*d^23*z^4 + 256*b^23*d
^11*e^12*z^4 + 1048576*a^17*c^6*e^23*z^4 + 256*b^11*c^12*d^23*z^4 + 256*a^1
1*b^12*e^23*z^4 + 948695040*a^8*b*c^10*d^6*e^13*z^2 + 348917760*a^7*b*c^11*
d^8*e^11*z^2 - 125030400*a^9*b*c^9*d^4*e^15*z^2 - 50728960*a^6*b*c^12*d^10*

e^9*z^2 - 44298240*a^5*b*c^13*d^12*e^7*z^2 - 36495360*a^10*b*c^8*d^2*e^17*z
^2 + 29675520*a^8*b^6*c^5*d*e^18*z^2 - 24170496*a^9*b^4*c^6*d*e^18*z^2 - 17
202816*a^7*b^8*c^4*d*e^18*z^2 - 14561280*a^4*b*c^14*d^14*e^5*z^2 + 5532416*
a^6*b^10*c^3*d*e^18*z^2 + 4128768*a^10*b^2*c^7*d*e^18*z^2 - 2662400*a^3*b*c
^15*d^16*e^3*z^2 + 1184512*a*b^12*c^6*d^9*e^10*z^2 - 1136160*a*b^13*c^5*d^8
*e^11*z^2 - 1017600*a^5*b^12*c^2*d*e^18*z^2 - 744768*a*b^11*c^7*d^10*e^9*z^
2 + 607872*a*b^14*c^4*d^7*e^12*z^2 - 424064*a*b^6*c^12*d^15*e^4*z^2 + 40857
6*a*b^5*c^13*d^16*e^3*z^2 + 361152*a*b^10*c^8*d^11*e^8*z^2 - 287408*a*b^9*c
^9*d^12*e^7*z^2 - 260448*a^3*b^15*c*d^2*e^17*z^2 - 203904*a*b^4*c^14*d^17*e
^2*z^2 + 200832*a*b^8*c^10*d^13*e^6*z^2 + 126720*a*b^7*c^11*d^14*e^5*z^2 -
123968*a*b^15*c^3*d^6*e^13*z^2 - 39168*a*b^16*c^2*d^5*e^14*z^2 + 11904*a^2*
b^16*c*d^3*e^16*z^2 + 1824135552*a^7*b^4*c^8*d^5*e^14*z^2 - 1457252352*a^8*
b^2*c^9*d^5*e^14*z^2 - 1405209600*a^7*b^5*c^7*d^4*e^15*z^2 - 184320*a^2*b*c
^16*d^18*e*z^2 + 100608*a^4*b^14*c*d*e^18*z^2 + 53248*a*b^3*c^15*d^18*e*z^2
 + 26448*a*b^17*c*d^4*e^15*z^2 + 1067599872*a^8*b^3*c^8*d^4*e^15*z^2 - 9308
28288*a^7*b^3*c^9*d^6*e^13*z^2 + 920760000*a^6*b^4*c^9*d^7*e^12*z^2 - 80663
9616*a^6*b^3*c^10*d^8*e^11*z^2 - 791052480*a^6*b^6*c^7*d^5*e^14*z^2 + 77223
7824*a^6*b^7*c^6*d^4*e^15*z^2 - 701025408*a^5*b^6*c^8*d^7*e^12*z^2 + 443340
288*a^5*b^5*c^9*d^8*e^11*z^2 + 433047552*a^7*b^6*c^6*d^3*e^16*z^2 + 4057413
12*a^5*b^7*c^7*d^6*e^13*z^2 + 293652480*a^6*b^2*c^11*d^9*e^10*z^2 - 2769626
88*a^6*b^8*c^5*d^3*e^16*z^2 - 247804272*a^8*b^4*c^7*d^3*e^16*z^2 + 21356438
4*a^4*b^8*c^7*d^7*e^12*z^2 - 202596816*a^5*b^9*c^5*d^4*e^15*z^2 - 182520896
*a^4*b^9*c^6*d^6*e^13*z^2 - 153489408*a^5*b^3*c^11*d^10*e^9*z^2 - 152151552
*a^7*b^2*c^10*d^7*e^12*z^2 + 115859712*a^5*b^2*c^12*d^11*e^8*z^2 + 10808524
8*a^9*b^3*c^7*d^2*e^17*z^2 + 105536256*a^4*b^5*c^10*d^10*e^9*z^2 - 98323200
*a^6*b^5*c^8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^9*d^9*e^10*z^2 + 89464512*a^
5*b^10*c^4*d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*d^2*e^17*z^2 + 68315904*a^5*
b^8*c^6*d^5*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8*e^11*z^2 - 62951040*a^9*b^2
*c^8*d^3*e^16*z^2 + 49056768*a^4*b^10*c^5*d^5*e^14*z^2 + 47614464*a^3*b^8*c
^8*d^9*e^10*z^2 + 35604480*a^4*b^2*c^13*d^13*e^6*z^2 + 33983040*a^3*b^11*c^
5*d^6*e^13*z^2 - 33515520*a^4*b^3*c^12*d^12*e^7*z^2 - 33463808*a^3*b^7*c^9*
d^10*e^9*z^2 - 25128864*a^4*b^4*c^11*d^11*e^8*z^2 - 23193728*a^3*b^10*c^6*d
^7*e^12*z^2 + 21015456*a^6*b^9*c^4*d^2*e^17*z^2 + 19924176*a^4*b^11*c^4*d^4
*e^15*z^2 - 19251216*a^3*b^9*c^7*d^8*e^11*z^2 - 16434048*a^5*b^4*c^10*d^9*e
^10*z^2 - 16289664*a^3*b^12*c^4*d^5*e^14*z^2 - 15059328*a^4*b^12*c^3*d^3*e^
16*z^2 - 10766016*a^2*b^10*c^7*d^9*e^10*z^2 - 10453632*a^5*b^11*c^3*d^2*e^1
7*z^2 - 9940992*a^3*b^3*c^13*d^14*e^5*z^2 + 8373696*a^2*b^11*c^6*d^8*e^11*z
^2 + 7776768*a^3*b^2*c^14*d^15*e^4*z^2 + 7077888*a^3*b^5*c^11*d^12*e^7*z^2
 + 6798240*a^2*b^9*c^8*d^10*e^9*z^2 - 3589440*a^2*b^6*c^11*d^13*e^6*z^2 + 35
44320*a^3*b^6*c^10*d^11*e^8*z^2 + 3128064*a^2*b^5*c^12*d^14*e^5*z^2 + 23463
36*a^4*b^13*c^2*d^2*e^17*z^2 - 2261568*a^2*b^8*c^9*d^11*e^8*z^2 - 2125824*a
^2*b^13*c^4*d^6*e^13*z^2 + 2002560*a^3*b^4*c^12*d^13*e^6*z^2 + 1927680*a^2*
b^7*c^10*d^12*e^7*z^2 + 1814784*a^2*b^14*c^3*d^5*e^14*z^2 - 1807104*a^2*b^1
2*c^5*d^7*e^12*z^2 + 1637808*a^3*b^13*c^3*d^4*e^15*z^2 + 1083456*a^3*b^14*c
^2*d^3*e^16*z^2 - 792384*a^2*b^4*c^13*d^15*e^4*z^2 - 657408*a^2*b^3*c^14*d^
16*e^3*z^2 + 608256*a^7*b^7*c^5*d^2*e^17*z^2 + 595968*a^2*b^2*c^15*d^17*e^2
*z^2 - 498624*a^2*b^15*c^2*d^4*e^15*z^2 - 3840*b^18*c*d^5*e^14*z^2 - 3840*b
^5*c^14*d^18*e*z^2 + 2064384*a^11*c^8*d*e^18*z^2 - 4160*a^3*b^16*d*e^18*z^2
 - 4160*a*b^18*d^3*e^16*z^2 - 1290240*a^11*b*c^7*e^19*z^2 - 9840*a^5*b^13*c
*e^19*z^2 - 5760*a*b^2*c^16*d^19*z^2 - 280581120*a^8*c^11*d^7*e^12*z^2 + 11
0278656*a^9*c^10*d^5*e^14*z^2 - 89479168*a^7*c^12*d^9*e^10*z^2 + 34464000*a
^10*c^9*d^3*e^16*z^2 + 54240*b^15*c^4*d^8*e^11*z^2 + 54240*b^8*c^11*d^15*e^
4*z^2 - 49920*b^14*c^5*d^9*e^10*z^2 - 49920*b^9*c^10*d^14*e^5*z^2 - 37376*b
^16*c^3*d^7*e^12*z^2 - 37376*b^7*c^12*d^16*e^3*z^2 + 28480*b^13*c^6*d^10*e^
9*z^2 + 28480*b^10*c^9*d^13*e^6*z^2 + 15936*b^17*c^2*d^6*e^13*z^2 + 15936*b
^6*c^13*d^17*e^2*z^2 - 7920*b^12*c^7*d^11*e^8*z^2 - 7920*b^11*c^8*d^12*e^7*
z^2 + 7489536*a^5*c^14*d^13*e^6*z^2 + 6084096*a^6*c^13*d^11*e^8*z^2 + 22804
48*a^4*c^15*d^15*e^4*z^2 + 350208*a^3*c^16*d^17*e^2*z^2 + 11616*a^2*b^17*d^
2*e^17*z^2 - 3515904*a^9*b^5*c^5*e^19*z^2 + 3440640*a^10*b^3*c^6*e^19*z^2 +

1870848*a^8*b^7*c^4*e^19*z^2 - 572272*a^7*b^9*c^3*e^19*z^2 + 101856*a^6*b^
11*c^2*e^19*z^2 + 400*b^19*d^4*e^15*z^2 + 400*b^4*c^15*d^19*z^2 + 20736*a^2
*c^17*d^19*z^2 + 400*a^4*b^15*e^19*z^2 - 3969216*a^4*b*c^10*d^3*e^12 - 3001
536*a^3*b*c^11*d^5*e^10 - 419904*a^2*b*c^12*d^7*e^8 + 184608*a^4*b^3*c^8*d*
e^14 - 153036*a*b^4*c^10*d^6*e^9 + 127008*a*b^3*c^11*d^7*e^8 + 63108*a*b^6*
c^8*d^4*e^11 - 29160*a*b^2*c^12*d^8*e^7 - 21060*a^3*b^5*c^7*d*e^14 - 21060*
a*b^7*c^7*d^3*e^12 + 5460*a*b^5*c^9*d^5*e^10 - 404544*a^5*b*c^9*d*e^14 + 12
51872*a^3*b^3*c^9*d^3*e^12 + 844224*a^4*b^2*c^9*d^2*e^13 + 820512*a^2*b^3*c
^10*d^5*e^10 + 750672*a^3*b^2*c^10*d^4*e^11 - 657498*a^2*b^4*c^9*d^4*e^11 -
487116*a^3*b^4*c^8*d^2*e^13 + 160704*a^2*b^2*c^11*d^6*e^9 + 58806*a^2*b^6*
c^7*d^2*e^13 + 13140*a^2*b^5*c^8*d^3*e^12 + 15286*b^6*c^9*d^6*e^9 - 9540*b^
7*c^8*d^5*e^10 - 9540*b^5*c^10*d^7*e^8 + 2025*b^8*c^7*d^4*e^11 + 2025*b^4*c
^11*d^8*e^7 + 3367008*a^4*c^11*d^4*e^11 + 1166400*a^3*c^12*d^6*e^9 + 705600
*a^5*c^10*d^2*e^13 + 104976*a^2*c^13*d^8*e^7 - 17640*a^5*b^2*c^8*e^15 + 202
5*a^4*b^4*c^7*e^15 + 38416*a^6*c^9*e^15, z, k)*(root(128723189760*a^14*b^4*
c^9*d^13*e^14*z^6 + 128723189760*a^12*b^4*c^11*d^17*e^10*z^6 - 8432455680*a
^11*b^12*c^4*d^11*e^16*z^6 - 8432455680*a^7*b^12*c^8*d^19*e^8*z^6 + 1267335
1680*a^11*b^11*c^5*d^12*e^15*z^6 + 12673351680*a^8*b^11*c^8*d^18*e^9*z^6 -
72637480960*a^12*b^9*c^6*d^12*e^15*z^6 - 72637480960*a^9*b^9*c^9*d^18*e^9*z
^6 - 21048344576*a^9*b^12*c^6*d^15*e^12*z^6 - 16609443840*a^17*b^3*c^7*d^8*
e^19*z^6 - 16609443840*a^10*b^3*c^14*d^22*e^5*z^6 + 145332633600*a^13*b^5*c
^9*d^14*e^13*z^6 + 145332633600*a^12*b^5*c^10*d^16*e^11*z^6 + 123740356608*
a^14*b^5*c^8*d^12*e^15*z^6 + 123740356608*a^11*b^5*c^11*d^18*e^9*z^6 + 3460
300800*a^17*b^5*c^5*d^6*e^21*z^6 + 3460300800*a^8*b^5*c^14*d^24*e^3*z^6 - 7
751073792*a^15*b^7*c^5*d^8*e^19*z^6 - 7751073792*a^8*b^7*c^12*d^22*e^5*z^6
+ 12041846784*a^14*b^7*c^6*d^10*e^17*z^6 + 12041846784*a^9*b^7*c^11*d^20*e^
7*z^6 - 325545099264*a^14*b^3*c^10*d^14*e^13*z^6 - 325545099264*a^13*b^3*c^
11*d^16*e^11*z^6 - 3330539520*a^13*b^10*c^4*d^9*e^18*z^6 - 3330539520*a^7*b
^10*c^10*d^21*e^6*z^6 + 157789716480*a^12*b^7*c^8*d^14*e^13*z^6 + 157789716
480*a^11*b^7*c^9*d^16*e^11*z^6 + 37492359168*a^11*b^10*c^6*d^13*e^14*z^6 +
37492359168*a^9*b^10*c^8*d^17*e^10*z^6 + 301989888*a^8*b^3*c^16*d^26*e*z^6
- 7266631680*a^17*b^4*c^6*d^7*e^20*z^6 - 7266631680*a^9*b^4*c^14*d^23*e^4*z
^6 - 201326592*a^20*b*c^6*d^4*e^23*z^6 - 188743680*a^7*b^5*c^15*d^26*e*z^6
+ 45747339264*a^13*b^8*c^6*d^11*e^16*z^6 + 45747339264*a^9*b^8*c^10*d^19*e^
8*z^6 - 74612736*a^10*b^16*c*d^9*e^18*z^6 - 2768240640*a^16*b^7*c^4*d^6*e^2
1*z^6 - 2768240640*a^7*b^7*c^13*d^24*e^3*z^6 + 69746688*a^11*b^15*c*d^8*e^1
9*z^6 + 62914560*a^6*b^7*c^14*d^26*e*z^6 + 2752020480*a^10*b^13*c^4*d^12*e^
15*z^6 + 2752020480*a^7*b^13*c^7*d^18*e^9*z^6 + 55148544*a^9*b^17*c*d^10*e^
17*z^6 - 45957120*a^12*b^14*c*d^7*e^20*z^6 - 2724986880*a^14*b^9*c^4*d^8*e^
19*z^6 - 2724986880*a^7*b^9*c^11*d^22*e^5*z^6 - 25952256*a^8*b^18*c*d^11*e^
16*z^6 + 21086208*a^13*b^13*c*d^6*e^21*z^6 - 11796480*a^5*b^9*c^13*d^26*e*z
^6 - 6438912*a^14*b^12*c*d^5*e^22*z^6 + 5406720*a^7*b^19*c*d^12*e^15*z^6 +
1622016*a^6*b^20*c*d^13*e^14*z^6 - 1523712*a^5*b^21*c*d^14*e^13*z^6 + 11796
48*a^15*b^11*c*d^4*e^23*z^6 + 1179648*a^4*b^11*c^12*d^26*e*z^6 + 442368*a^4
*b^22*c*d^15*e^12*z^6 - 98304*a^16*b^10*c*d^3*e^24*z^6 - 49152*a^3*b^23*c*d
^16*e^11*z^6 - 49152*a^3*b^13*c^11*d^26*e*z^6 + 6897106944*a^9*b^13*c^5*d^1
4*e^13*z^6 + 6897106944*a^8*b^13*c^6*d^16*e^11*z^6 - 2422210560*a^16*b^6*c^
5*d^7*e^20*z^6 - 2422210560*a^8*b^6*c^13*d^23*e^4*z^6 + 255785435136*a^14*b
^2*c^11*d^15*e^12*z^6 + 41004564480*a^15*b^4*c^8*d^11*e^16*z^6 + 4100456448
0*a^11*b^4*c^12*d^19*e^8*z^6 + 2270822400*a^13*b^11*c^3*d^8*e^19*z^6 + 2270
822400*a^6*b^11*c^10*d^22*e^5*z^6 + 23677108224*a^14*b^8*c^5*d^9*e^18*z^6 +
23677108224*a^8*b^8*c^11*d^21*e^6*z^6 + 212600881152*a^15*b^2*c^10*d^13*e^
14*z^6 + 212600881152*a^13*b^2*c^12*d^17*e^10*z^6 + 75157733376*a^15*b^5*c^
7*d^10*e^17*z^6 + 75157733376*a^10*b^5*c^12*d^20*e^7*z^6 - 251217838080*a^1
3*b^6*c^8*d^13*e^14*z^6 - 251217838080*a^11*b^6*c^10*d^17*e^10*z^6 - 195290
7264*a^14*b^10*c^3*d^7*e^20*z^6 - 1952907264*a^6*b^10*c^11*d^23*e^4*z^6 - 2
7691057152*a^13*b^9*c^5*d^10*e^17*z^6 - 27691057152*a^8*b^9*c^10*d^20*e^7*z
^6 - 1902673920*a^8*b^15*c^4*d^14*e^13*z^6 - 1902673920*a^7*b^15*c^5*d^16*e
^11*z^6 + 10465050624*a^10*b^11*c^6*d^14*e^13*z^6 + 10465050624*a^9*b^11*c^

7*d^16*e^11*z^6 + 1613905920*a^9*b^14*c^4*d^13*e^14*z^6 + 1613905920*a^7*b^
14*c^6*d^17*e^10*z^6 - 33218887680*a^17*b*c^9*d^10*e^17*z^6 - 33218887680*a
^12*b*c^14*d^20*e^7*z^6 + 1524695040*a^10*b^14*c^3*d^11*e^16*z^6 + 15246950
40*a^6*b^14*c^7*d^19*e^8*z^6 - 1472200704*a^18*b^4*c^5*d^5*e^22*z^6 - 14722
00704*a^8*b^4*c^15*d^25*e^2*z^6 - 83047219200*a^16*b^3*c^8*d^10*e^17*z^6 -
83047219200*a^11*b^3*c^13*d^20*e^7*z^6 + 44291850240*a^17*b^2*c^8*d^9*e^18*
z^6 + 44291850240*a^11*b^2*c^14*d^21*e^6*z^6 + 1308131328*a^8*b^14*c^5*d^15
*e^12*z^6 - 201326592*a^9*b*c^17*d^26*e*z^6 + 48530718720*a^12*b^8*c^7*d^13
*e^14*z^6 + 48530718720*a^10*b^8*c^9*d^17*e^10*z^6 - 1242644480*a^12*b^12*c
^3*d^9*e^18*z^6 - 1242644480*a^6*b^12*c^9*d^21*e^6*z^6 + 9813196800*a^12*b^
10*c^5*d^11*e^16*z^6 + 9813196800*a^8*b^10*c^9*d^19*e^8*z^6 - 93012885504*a
^15*b*c^11*d^14*e^13*z^6 - 93012885504*a^14*b*c^12*d^16*e^11*z^6 + 17730581
2992*a^13*b^4*c^10*d^15*e^12*z^6 + 52730658816*a^10*b^10*c^7*d^15*e^12*z^6
- 1180106752*a^9*b^15*c^3*d^12*e^15*z^6 - 1180106752*a^6*b^15*c^6*d^18*e^9*
z^6 + 1023672320*a^15*b^9*c^3*d^6*e^21*z^6 + 1023672320*a^6*b^9*c^12*d^24*e
^3*z^6 + 975175680*a^17*b^6*c^4*d^5*e^22*z^6 + 975175680*a^7*b^6*c^14*d^25*
e^2*z^6 - 11072962560*a^18*b*c^8*d^8*e^19*z^6 - 11072962560*a^11*b*c^15*d^2
2*e^5*z^6 + 9412018176*a^18*b^2*c^7*d^7*e^20*z^6 + 9412018176*a^10*b^2*c^15
*d^23*e^4*z^6 + 805306368*a^19*b^2*c^6*d^5*e^22*z^6 + 805306368*a^9*b^2*c^1
6*d^25*e^2*z^6 - 133809831936*a^14*b^6*c^7*d^11*e^16*z^6 - 133809831936*a^1
0*b^6*c^11*d^19*e^8*z^6 - 2214592512*a^19*b*c^7*d^6*e^21*z^6 - 2214592512*a
^10*b*c^16*d^24*e^3*z^6 + 82216747008*a^13*b^7*c^7*d^12*e^15*z^6 + 82216747
008*a^10*b^7*c^10*d^18*e^9*z^6 - 586629120*a^12*b^13*c^2*d^8*e^19*z^6 - 586
629120*a^5*b^13*c^9*d^22*e^5*z^6 + 568565760*a^7*b^16*c^4*d^15*e^12*z^6 - 4
844421120*a^16*b^4*c^7*d^9*e^18*z^6 - 4844421120*a^10*b^4*c^13*d^21*e^6*z^6
+ 531210240*a^11*b^14*c^2*d^9*e^18*z^6 + 531210240*a^5*b^14*c^8*d^21*e^6*z
^6 - 527155200*a^11*b^13*c^3*d^10*e^17*z^6 - 527155200*a^6*b^13*c^8*d^20*e^
7*z^6 + 43470028800*a^11*b^8*c^8*d^15*e^12*z^6 - 107874877440*a^11*b^9*c^7*
d^14*e^13*z^6 - 107874877440*a^10*b^9*c^8*d^16*e^11*z^6 + 9018408960*a^12*b
^11*c^4*d^10*e^17*z^6 + 9018408960*a^7*b^11*c^9*d^20*e^7*z^6 + 421994496*a^
13*b^12*c^2*d^7*e^20*z^6 + 421994496*a^5*b^12*c^10*d^23*e^4*z^6 - 664377753
60*a^16*b*c^10*d^12*e^15*z^6 - 66437775360*a^13*b*c^13*d^18*e^9*z^6 + 26159
874048*a^16*b^5*c^6*d^8*e^19*z^6 + 26159874048*a^9*b^5*c^13*d^22*e^5*z^6 -
369098752*a^18*b^3*c^6*d^6*e^21*z^6 - 369098752*a^9*b^3*c^15*d^24*e^3*z^6 +
351436800*a^8*b^16*c^3*d^13*e^14*z^6 + 351436800*a^6*b^16*c^5*d^17*e^10*z^
6 - 334233600*a^16*b^8*c^3*d^5*e^22*z^6 - 334233600*a^6*b^8*c^13*d^25*e^2*z
^6 + 301989888*a^19*b^3*c^5*d^4*e^23*z^6 - 266010624*a^10*b^15*c^2*d^10*e^1
7*z^6 - 266010624*a^5*b^15*c^7*d^20*e^7*z^6 - 305198530560*a^12*b^6*c^9*d^1
5*e^12*z^6 - 203292672*a^14*b^11*c^2*d^6*e^21*z^6 - 203292672*a^5*b^11*c^11
*d^24*e^3*z^6 - 188743680*a^18*b^5*c^4*d^4*e^23*z^6 + 120418467840*a^16*b^2
*c^9*d^11*e^16*z^6 + 120418467840*a^12*b^2*c^13*d^19*e^8*z^6 - 17293934592*
a^10*b^12*c^5*d^13*e^14*z^6 - 17293934592*a^8*b^12*c^7*d^17*e^10*z^6 + 1048
90368*a^8*b^17*c^2*d^12*e^15*z^6 + 104890368*a^5*b^17*c^5*d^18*e^9*z^6 + 43
90256640*a^15*b^8*c^4*d^7*e^20*z^6 + 4390256640*a^7*b^8*c^12*d^23*e^4*z^6 -
91750400*a^6*b^18*c^3*d^15*e^12*z^6 + 79134720*a^7*b^17*c^3*d^14*e^13*z^6
+ 79134720*a^6*b^17*c^4*d^16*e^11*z^6 - 74612736*a^4*b^16*c^7*d^21*e^6*z^6
- 72990720*a^7*b^18*c^2*d^13*e^14*z^6 - 72990720*a^5*b^18*c^4*d^17*e^10*z^6
+ 69746688*a^4*b^15*c^8*d^22*e^5*z^6 + 63700992*a^15*b^10*c^2*d^5*e^22*z^6
+ 63700992*a^5*b^10*c^12*d^25*e^2*z^6 + 62914560*a^17*b^7*c^3*d^4*e^23*z^6
+ 55148544*a^4*b^17*c^6*d^20*e^7*z^6 - 45957120*a^4*b^14*c^9*d^23*e^4*z^6
- 25952256*a^4*b^18*c^5*d^19*e^8*z^6 - 25165824*a^20*b^2*c^5*d^3*e^24*z^6 +
21086208*a^4*b^13*c^10*d^24*e^3*z^6 + 20643840*a^6*b^19*c^2*d^14*e^13*z^6
+ 20643840*a^5*b^19*c^3*d^16*e^11*z^6 + 15728640*a^19*b^4*c^4*d^3*e^24*z^6
- 11796480*a^16*b^9*c^2*d^4*e^23*z^6 - 6438912*a^4*b^12*c^11*d^25*e^2*z^6 +
5406720*a^4*b^19*c^4*d^18*e^9*z^6 - 5242880*a^18*b^6*c^3*d^3*e^24*z^6 + 37
84704*a^3*b^18*c^6*d^21*e^6*z^6 - 3244032*a^3*b^19*c^5*d^20*e^7*z^6 - 32440
32*a^3*b^17*c^7*d^22*e^5*z^6 + 2027520*a^3*b^20*c^4*d^19*e^8*z^6 + 2027520*
a^3*b^16*c^8*d^23*e^4*z^6 - 1622016*a^9*b^16*c^2*d^11*e^16*z^6 - 1622016*a^
5*b^16*c^6*d^19*e^8*z^6 + 1622016*a^4*b^20*c^3*d^17*e^10*z^6 - 1523712*a^4*

b^21*c^2*d^16*e^11*z^6 + 983040*a^17*b^8*c^2*d^3*e^24*z^6 - 901120*a^3*b^21
*c^3*d^18*e^9*z^6 - 901120*a^3*b^15*c^9*d^24*e^3*z^6 + 270336*a^3*b^22*c^2*
d^17*e^10*z^6 + 270336*a^3*b^14*c^10*d^25*e^2*z^6 + 172032*a^5*b^20*c^2*d^1
5*e^12*z^6 - 38593888256*a^15*b^6*c^6*d^9*e^18*z^6 - 38593888256*a^9*b^6*c^
12*d^21*e^6*z^6 - 210386288640*a^15*b^3*c^9*d^12*e^15*z^6 - 210386288640*a^
12*b^3*c^12*d^18*e^9*z^6 + 15502147584*a^15*c^12*d^15*e^12*z^6 + 1107296256
*a^19*c^8*d^7*e^20*z^6 + 1107296256*a^11*c^16*d^23*e^4*z^6 + 13287555072*a^
16*c^11*d^13*e^14*z^6 + 13287555072*a^14*c^13*d^17*e^10*z^6 + 201326592*a^2
0*c^7*d^5*e^22*z^6 + 201326592*a^10*c^17*d^25*e^2*z^6 + 16777216*a^21*c^6*d
^3*e^24*z^6 + 3784704*a^9*b^18*d^9*e^18*z^6 - 3244032*a^10*b^17*d^8*e^19*z^
6 - 3244032*a^8*b^19*d^10*e^17*z^6 + 2027520*a^11*b^16*d^7*e^20*z^6 + 20275
20*a^7*b^20*d^11*e^16*z^6 - 901120*a^12*b^15*d^6*e^21*z^6 - 901120*a^6*b^21
*d^12*e^15*z^6 + 270336*a^13*b^14*d^5*e^22*z^6 + 270336*a^5*b^22*d^13*e^14*
z^6 - 49152*a^14*b^13*d^4*e^23*z^6 - 49152*a^4*b^23*d^14*e^13*z^6 + 4096*a^
15*b^12*d^3*e^24*z^6 + 4096*a^3*b^24*d^15*e^12*z^6 - 25165824*a^8*b^2*c^17*
d^27*z^6 + 15728640*a^7*b^4*c^16*d^27*z^6 - 5242880*a^6*b^6*c^15*d^27*z^6 +
983040*a^5*b^8*c^14*d^27*z^6 - 98304*a^4*b^10*c^13*d^27*z^6 + 4096*a^3*b^1
2*c^12*d^27*z^6 + 8304721920*a^17*c^10*d^11*e^16*z^6 + 8304721920*a^13*c^14
*d^19*e^8*z^6 + 3690987520*a^18*c^9*d^9*e^18*z^6 + 3690987520*a^12*c^15*d^2
1*e^6*z^6 + 16777216*a^9*c^18*d^27*z^6 - 8493371392*a^6*b^8*c^9*d^14*e^9*z^
4 + 1458044928*a^8*b*c^14*d^17*e^6*z^4 - 12604538880*a^11*b^4*c^8*d^8*e^15*
z^4 - 8303067136*a^9*b^5*c^9*d^11*e^12*z^4 - 5588058112*a^13*b*c^9*d^7*e^16
*z^4 - 3892838400*a^8*b^2*c^13*d^16*e^7*z^4 - 3611713536*a^8*b^8*c^7*d^10*e
^13*z^4 + 7819006464*a^7*b^9*c^7*d^11*e^12*z^4 - 7782137856*a^8*b^7*c^8*d^1
1*e^12*z^4 + 7780433920*a^12*b^2*c^9*d^8*e^15*z^4 - 12020465664*a^7*b^5*c^1
1*d^15*e^8*z^4 + 3176792064*a^8*b^3*c^12*d^15*e^8*z^4 - 322633728*a^15*b*c^
7*d^3*e^20*z^4 + 210829312*a^7*b*c^15*d^19*e^4*z^4 + 15623258112*a^9*b^6*c^
8*d^10*e^13*z^4 + 25165824*a^15*b^3*c^5*d*e^22*z^4 - 15728640*a^14*b^5*c^4*
d*e^22*z^4 + 12582912*a^5*b^2*c^16*d^22*e*z^4 - 11730944*a^4*b^4*c^15*d^22*
e*z^4 + 5242880*a^13*b^7*c^3*d*e^22*z^4 - 4561920*a*b^15*c^7*d^17*e^6*z^4 +
4521984*a^3*b^6*c^14*d^22*e*z^4 + 4460544*a*b^14*c^8*d^18*e^5*z^4 + 353894
4*a^6*b*c^16*d^21*e^2*z^4 + 3108864*a*b^16*c^6*d^16*e^7*z^4 - 3027200*a*b^1
3*c^9*d^19*e^4*z^4 - 2345472*a^5*b^17*c*d^7*e^16*z^4 - 2307072*a^8*b^14*c*d
^4*e^19*z^4 + 1824768*a^6*b^16*c*d^6*e^17*z^4 + 1734912*a^9*b^13*c*d^3*e^20
*z^4 + 1419264*a*b^12*c^10*d^20*e^3*z^4 - 1191168*a*b^17*c^5*d^15*e^8*z^4 -
983040*a^12*b^9*c^2*d*e^22*z^4 + 964608*a^4*b^18*c*d^8*e^15*z^4 - 866304*a
^2*b^8*c^13*d^22*e*z^4 + 703488*a^7*b^15*c*d^5*e^18*z^4 - 608256*a^10*b^12*
c*d^2*e^21*z^4 - 440832*a*b^11*c^11*d^21*e^2*z^4 + 275968*a*b^19*c^3*d^13*e
^10*z^4 - 159744*a^2*b^20*c*d^10*e^13*z^4 - 153600*a*b^20*c^2*d^12*e^11*z^4
+ 64512*a^3*b^19*c*d^9*e^14*z^4 + 19746062336*a^8*b^6*c^9*d^12*e^11*z^4 -
15333588992*a^10*b^4*c^9*d^10*e^13*z^4 + 6702170112*a^7*b^4*c^12*d^16*e^7*z
^4 + 15167913984*a^10*b^3*c^10*d^11*e^12*z^4 - 2256638976*a^5*b^11*c^7*d^13
*e^10*z^4 + 2254307328*a^5*b^7*c^11*d^17*e^6*z^4 - 2200633344*a^6*b^5*c^12*
d^17*e^6*z^4 + 6457131008*a^11*b^3*c^9*d^9*e^14*z^4 - 2128785408*a^5*b^8*c^
10*d^16*e^7*z^4 - 2126057472*a^6*b^11*c^6*d^11*e^12*z^4 + 2038349824*a^12*b
^5*c^6*d^5*e^18*z^4 + 2037841920*a^5*b^10*c^8*d^14*e^9*z^4 + 3615621120*a^9
*b*c^13*d^15*e^8*z^4 + 1900019712*a^11*b^2*c^10*d^10*e^13*z^4 + 1867698432*
a^9*b^9*c^5*d^7*e^16*z^4 - 6157369344*a^9*b^4*c^10*d^12*e^11*z^4 - 18569134
08*a^7*b^10*c^6*d^10*e^13*z^4 + 1789132800*a^6*b^4*c^13*d^18*e^5*z^4 + 6082
658304*a^8*b^4*c^11*d^14*e^9*z^4 + 6029549568*a^11*b^5*c^7*d^7*e^16*z^4 + 6
010159104*a^6*b^7*c^10*d^15*e^8*z^4 + 1703182336*a^7*b^7*c^9*d^13*e^10*z^4
+ 1658388480*a^11*b^6*c^6*d^6*e^17*z^4 + 5917114368*a^10*b^6*c^7*d^8*e^15*z
^4 - 1591197696*a^11*b^7*c^5*d^5*e^18*z^4 - 1526464512*a^8*b^10*c^5*d^8*e^1
5*z^4 - 5772607488*a^12*b^4*c^7*d^6*e^17*z^4 - 1423507456*a^13*b^4*c^6*d^4*
e^19*z^4 - 1387266048*a^7*b^3*c^13*d^17*e^6*z^4 + 2976120832*a^10*b*c^12*d^
13*e^10*z^4 - 9906946048*a^9*b^2*c^12*d^14*e^9*z^4 - 18421874688*a^8*b^5*c^
10*d^13*e^10*z^4 + 1141217280*a^6*b^12*c^5*d^10*e^13*z^4 - 9714364416*a^7*b
^8*c^8*d^12*e^11*z^4 - 16777216*a^16*b*c^6*d*e^22*z^4 + 98304*a^11*b^11*c*d
*e^22*z^4 + 81920*a*b^10*c^12*d^22*e*z^4 + 39168*a*b^21*c*d^11*e^12*z^4 - 1

091829760*a^5*b^6*c^12*d^18*e^5*z^4 + 1046740992*a^14*b^2*c^7*d^4*e^19*z^4
- 6884425728*a^12*b*c^10*d^9*e^14*z^4 + 987445248*a^4*b^10*c^9*d^16*e^7*z^4
 + 984087552*a^5*b^12*c^6*d^12*e^11*z^4 - 9564585984*a^9*b^7*c^7*d^9*e^14*z
^4 - 5266857984*a^10*b^7*c^6*d^7*e^16*z^4 - 892145664*a^7*b^11*c^5*d^9*e^14
*z^4 - 2444623872*a^11*b*c^11*d^11*e^12*z^4 + 768540672*a^12*b^3*c^8*d^7*e^
16*z^4 + 5048322048*a^8*b^9*c^6*d^9*e^14*z^4 + 5047612416*a^6*b^9*c^8*d^13*
e^10*z^4 - 732492288*a^4*b^11*c^8*d^15*e^8*z^4 + 9266921472*a^7*b^6*c^10*d^
14*e^9*z^4 - 645857280*a^6*b^6*c^11*d^16*e^7*z^4 - 623867904*a^4*b^9*c^10*d
^17*e^6*z^4 - 622067712*a^6*b^3*c^14*d^19*e^4*z^4 + 582617088*a^10*b^8*c^5*
d^6*e^17*z^4 + 577119744*a^7*b^12*c^4*d^8*e^15*z^4 + 552566784*a^12*b^6*c^5
*d^4*e^19*z^4 + 549224448*a^9*b^8*c^6*d^8*e^15*z^4 - 526565376*a^9*b^10*c^4
*d^6*e^17*z^4 + 511520256*a^10*b^9*c^4*d^5*e^18*z^4 + 13393723392*a^9*b^3*c
^11*d^13*e^10*z^4 - 2066350080*a^14*b*c^8*d^5*e^18*z^4 + 4718592000*a^13*b^
2*c^8*d^6*e^17*z^4 - 314572800*a^7*b^2*c^14*d^18*e^5*z^4 + 287250432*a^4*b^
13*c^6*d^13*e^10*z^4 + 4565827584*a^10*b^5*c^8*d^9*e^14*z^4 - 250785792*a^4
*b^14*c^5*d^12*e^11*z^4 + 235536384*a^13*b^3*c^7*d^5*e^18*z^4 - 232683264*a
^8*b^11*c^4*d^7*e^16*z^4 - 199627776*a^5*b^14*c^4*d^10*e^13*z^4 - 190267392
*a^12*b^7*c^4*d^3*e^20*z^4 + 184891392*a^6*b^10*c^7*d^12*e^11*z^4 + 1805025
28*a^4*b^7*c^12*d^19*e^4*z^4 + 178877952*a^3*b^13*c^7*d^15*e^8*z^4 + 172490
752*a^14*b^3*c^6*d^3*e^20*z^4 + 163946496*a^13*b^5*c^5*d^3*e^20*z^4 + 15583
9488*a^8*b^12*c^3*d^6*e^17*z^4 + 155000832*a^5*b^5*c^13*d^19*e^4*z^4 - 1520
76288*a^4*b^6*c^13*d^20*e^3*z^4 - 137592576*a^3*b^12*c^8*d^16*e^7*z^4 - 133
693440*a^14*b^4*c^5*d^2*e^21*z^4 - 116767488*a^3*b^9*c^11*d^19*e^4*z^4 - 10
8985344*a^3*b^14*c^6*d^14*e^9*z^4 - 106223616*a^6*b^13*c^4*d^9*e^14*z^4 + 1
06119168*a^3*b^10*c^10*d^18*e^5*z^4 + 102432768*a^5*b^4*c^14*d^20*e^3*z^4 +
 102113280*a^4*b^12*c^7*d^14*e^9*z^4 + 100674048*a^5*b^9*c^9*d^15*e^8*z^4 +
 90439680*a^13*b^6*c^4*d^2*e^21*z^4 - 86808576*a^6*b^14*c^3*d^8*e^15*z^4 +
86245376*a^6*b^2*c^15*d^20*e^3*z^4 + 79011840*a^4*b^8*c^11*d^18*e^5*z^4 + 7
8345216*a^4*b^15*c^4*d^11*e^12*z^4 + 78006528*a^11*b^9*c^3*d^3*e^20*z^4 - 7
3253376*a^9*b^11*c^3*d^5*e^18*z^4 + 67524608*a^3*b^8*c^12*d^20*e^3*z^4 + 67
108864*a^15*b^2*c^6*d^2*e^21*z^4 - 61590528*a^10*b^10*c^3*d^4*e^19*z^4 + 61
559808*a^5*b^15*c^3*d^9*e^14*z^4 - 59637760*a^5*b^3*c^15*d^21*e^2*z^4 + 586
38336*a^4*b^5*c^14*d^21*e^2*z^4 - 40828416*a^7*b^13*c^3*d^7*e^16*z^4 - 3563
9296*a^2*b^12*c^9*d^18*e^5*z^4 - 31293440*a^12*b^8*c^3*d^2*e^21*z^4 + 29933
568*a^5*b^13*c^5*d^11*e^12*z^4 + 27793920*a^2*b^11*c^10*d^19*e^4*z^4 + 2716
8768*a^2*b^13*c^8*d^17*e^6*z^4 - 23602176*a^7*b^14*c^2*d^6*e^17*z^4 - 23248
896*a^3*b^7*c^13*d^21*e^2*z^4 + 20929536*a^3*b^15*c^5*d^13*e^10*z^4 + 18428
928*a^9*b^12*c^2*d^4*e^19*z^4 + 18026496*a^6*b^15*c^2*d^7*e^16*z^4 - 162616
32*a^10*b^11*c^2*d^3*e^20*z^4 + 15128064*a^3*b^16*c^4*d^12*e^11*z^4 - 14060
544*a^2*b^10*c^11*d^20*e^3*z^4 + 13178880*a^2*b^16*c^5*d^14*e^9*z^4 - 11244
288*a^3*b^17*c^3*d^11*e^12*z^4 - 10509312*a^2*b^15*c^6*d^15*e^8*z^4 - 72622
08*a^4*b^17*c^2*d^9*e^14*z^4 - 7045632*a^2*b^17*c^4*d^13*e^10*z^4 - 6285312
*a^2*b^14*c^7*d^16*e^7*z^4 + 5996544*a^11*b^10*c^2*d^2*e^21*z^4 + 4558336*a
^2*b^9*c^12*d^21*e^2*z^4 + 4478976*a^11*b^8*c^4*d^4*e^19*z^4 + 2850816*a^4*
b^16*c^3*d^10*e^13*z^4 + 2629632*a^3*b^11*c^9*d^17*e^6*z^4 + 2503680*a^3*b^
18*c^2*d^10*e^13*z^4 + 1627136*a^2*b^18*c^3*d^12*e^11*z^4 + 1605120*a^8*b^1
3*c^2*d^5*e^18*z^4 + 1483776*a^5*b^16*c^2*d^8*e^15*z^4 + 139776*a^2*b^19*c^
2*d^11*e^12*z^4 - 8542224384*a^10*b^2*c^11*d^12*e^11*z^4 - 3072*b^22*c*d^12
*e^11*z^4 - 3072*b^12*c^11*d^22*e*z^4 - 1572864*a^6*c^17*d^22*e*z^4 - 4096*
a^10*b^13*d*e^22*z^4 - 4096*a*b^22*d^10*e^13*z^4 - 6144*a^12*b^10*c*e^23*z^
4 - 983040*a^5*b*c^17*d^23*z^4 - 6912*a*b^9*c^13*d^23*z^4 + 1824522240*a^13
*c^10*d^8*e^15*z^4 + 1730150400*a^12*c^11*d^10*e^13*z^4 + 958922752*a^14*c^
9*d^6*e^17*z^4 - 537919488*a^9*c^14*d^16*e^7*z^4 + 508559360*a^11*c^12*d^12
*e^11*z^4 - 500170752*a^10*c^13*d^14*e^9*z^4 + 246939648*a^15*c^8*d^4*e^19*
z^4 - 199229440*a^8*c^15*d^18*e^5*z^4 - 29884416*a^7*c^16*d^20*e^3*z^4 + 25
165824*a^16*c^7*d^2*e^21*z^4 + 236544*b^17*c^6*d^17*e^6*z^4 - 202752*b^18*c
^5*d^16*e^7*z^4 - 202752*b^16*c^7*d^18*e^5*z^4 + 126720*b^19*c^4*d^15*e^8*z
^4 + 126720*b^15*c^8*d^19*e^4*z^4 - 56320*b^20*c^3*d^14*e^9*z^4 - 56320*b^1
4*c^9*d^20*e^3*z^4 + 16896*b^21*c^2*d^13*e^10*z^4 + 16896*b^13*c^10*d^21*e^

2*z^4 + 110080*a^7*b^16*d^4*e^19*z^4 + 110080*a^4*b^19*d^7*e^16*z^4 - 75520
*a^8*b^15*d^3*e^20*z^4 - 75520*a^3*b^20*d^8*e^15*z^4 - 56320*a^6*b^17*d^5*e
^18*z^4 - 56320*a^5*b^18*d^6*e^17*z^4 + 25600*a^9*b^14*d^2*e^21*z^4 + 25600
*a^2*b^21*d^9*e^14*z^4 - 1572864*a^16*b^2*c^5*e^23*z^4 + 983040*a^15*b^4*c^
4*e^23*z^4 - 327680*a^14*b^6*c^3*e^23*z^4 + 61440*a^13*b^8*c^2*e^23*z^4 + 9
83040*a^4*b^3*c^16*d^23*z^4 - 385024*a^3*b^5*c^15*d^23*z^4 + 73728*a^2*b^7*
c^14*d^23*z^4 + 256*b^23*d^11*e^12*z^4 + 1048576*a^17*c^6*e^23*z^4 + 256*b^
11*c^12*d^23*z^4 + 256*a^11*b^12*e^23*z^4 + 948695040*a^8*b*c^10*d^6*e^13*z
^2 + 348917760*a^7*b*c^11*d^8*e^11*z^2 - 125030400*a^9*b*c^9*d^4*e^15*z^2 -
50728960*a^6*b*c^12*d^10*e^9*z^2 - 44298240*a^5*b*c^13*d^12*e^7*z^2 - 3649
5360*a^10*b*c^8*d^2*e^17*z^2 + 29675520*a^8*b^6*c^5*d*e^18*z^2 - 24170496*a
^9*b^4*c^6*d*e^18*z^2 - 17202816*a^7*b^8*c^4*d*e^18*z^2 - 14561280*a^4*b*c^
14*d^14*e^5*z^2 + 5532416*a^6*b^10*c^3*d*e^18*z^2 + 4128768*a^10*b^2*c^7*d*
e^18*z^2 - 2662400*a^3*b*c^15*d^16*e^3*z^2 + 1184512*a*b^12*c^6*d^9*e^10*z^
2 - 1136160*a*b^13*c^5*d^8*e^11*z^2 - 1017600*a^5*b^12*c^2*d*e^18*z^2 - 744
768*a*b^11*c^7*d^10*e^9*z^2 + 607872*a*b^14*c^4*d^7*e^12*z^2 - 424064*a*b^6
*c^12*d^15*e^4*z^2 + 408576*a*b^5*c^13*d^16*e^3*z^2 + 361152*a*b^10*c^8*d^1
1*e^8*z^2 - 287408*a*b^9*c^9*d^12*e^7*z^2 - 260448*a^3*b^15*c*d^2*e^17*z^2
- 203904*a*b^4*c^14*d^17*e^2*z^2 + 200832*a*b^8*c^10*d^13*e^6*z^2 + 126720*
a*b^7*c^11*d^14*e^5*z^2 - 123968*a*b^15*c^3*d^6*e^13*z^2 - 39168*a*b^16*c^2
*d^5*e^14*z^2 + 11904*a^2*b^16*c*d^3*e^16*z^2 + 1824135552*a^7*b^4*c^8*d^5*
e^14*z^2 - 1457252352*a^8*b^2*c^9*d^5*e^14*z^2 - 1405209600*a^7*b^5*c^7*d^4
*e^15*z^2 - 184320*a^2*b*c^16*d^18*e*z^2 + 100608*a^4*b^14*c*d*e^18*z^2 + 5
3248*a*b^3*c^15*d^18*e*z^2 + 26448*a*b^17*c*d^4*e^15*z^2 + 1067599872*a^8*b
^3*c^8*d^4*e^15*z^2 - 930828288*a^7*b^3*c^9*d^6*e^13*z^2 + 920760000*a^6*b^
4*c^9*d^7*e^12*z^2 - 806639616*a^6*b^3*c^10*d^8*e^11*z^2 - 791052480*a^6*b^
6*c^7*d^5*e^14*z^2 + 772237824*a^6*b^7*c^6*d^4*e^15*z^2 - 701025408*a^5*b^6
*c^8*d^7*e^12*z^2 + 443340288*a^5*b^5*c^9*d^8*e^11*z^2 + 433047552*a^7*b^6*
c^6*d^3*e^16*z^2 + 405741312*a^5*b^7*c^7*d^6*e^13*z^2 + 293652480*a^6*b^2*c
^11*d^9*e^10*z^2 - 276962688*a^6*b^8*c^5*d^3*e^16*z^2 - 247804272*a^8*b^4*c
^7*d^3*e^16*z^2 + 213564384*a^4*b^8*c^7*d^7*e^12*z^2 - 202596816*a^5*b^9*c^
5*d^4*e^15*z^2 - 182520896*a^4*b^9*c^6*d^6*e^13*z^2 - 153489408*a^5*b^3*c^1
1*d^10*e^9*z^2 - 152151552*a^7*b^2*c^10*d^7*e^12*z^2 + 115859712*a^5*b^2*c^
12*d^11*e^8*z^2 + 108085248*a^9*b^3*c^7*d^2*e^17*z^2 + 105536256*a^4*b^5*c^
10*d^10*e^9*z^2 - 98323200*a^6*b^5*c^8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^9*
d^9*e^10*z^2 + 89464512*a^5*b^10*c^4*d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*d^
2*e^17*z^2 + 68315904*a^5*b^8*c^6*d^5*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8*e
^11*z^2 - 62951040*a^9*b^2*c^8*d^3*e^16*z^2 + 49056768*a^4*b^10*c^5*d^5*e^1
4*z^2 + 47614464*a^3*b^8*c^8*d^9*e^10*z^2 + 35604480*a^4*b^2*c^13*d^13*e^6*
z^2 + 33983040*a^3*b^11*c^5*d^6*e^13*z^2 - 33515520*a^4*b^3*c^12*d^12*e^7*z
^2 - 33463808*a^3*b^7*c^9*d^10*e^9*z^2 - 25128864*a^4*b^4*c^11*d^11*e^8*z^2
- 23193728*a^3*b^10*c^6*d^7*e^12*z^2 + 21015456*a^6*b^9*c^4*d^2*e^17*z^2 +
19924176*a^4*b^11*c^4*d^4*e^15*z^2 - 19251216*a^3*b^9*c^7*d^8*e^11*z^2 - 1
6434048*a^5*b^4*c^10*d^9*e^10*z^2 - 16289664*a^3*b^12*c^4*d^5*e^14*z^2 - 15
059328*a^4*b^12*c^3*d^3*e^16*z^2 - 10766016*a^2*b^10*c^7*d^9*e^10*z^2 - 104
53632*a^5*b^11*c^3*d^2*e^17*z^2 - 9940992*a^3*b^3*c^13*d^14*e^5*z^2 + 83736
96*a^2*b^11*c^6*d^8*e^11*z^2 + 7776768*a^3*b^2*c^14*d^15*e^4*z^2 + 7077888*
a^3*b^5*c^11*d^12*e^7*z^2 + 6798240*a^2*b^9*c^8*d^10*e^9*z^2 - 3589440*a^2*
b^6*c^11*d^13*e^6*z^2 + 3544320*a^3*b^6*c^10*d^11*e^8*z^2 + 3128064*a^2*b^5
*c^12*d^14*e^5*z^2 + 2346336*a^4*b^13*c^2*d^2*e^17*z^2 - 2261568*a^2*b^8*c^
9*d^11*e^8*z^2 - 2125824*a^2*b^13*c^4*d^6*e^13*z^2 + 2002560*a^3*b^4*c^12*d
^13*e^6*z^2 + 1927680*a^2*b^7*c^10*d^12*e^7*z^2 + 1814784*a^2*b^14*c^3*d^5*
e^14*z^2 - 1807104*a^2*b^12*c^5*d^7*e^12*z^2 + 1637808*a^3*b^13*c^3*d^4*e^1
5*z^2 + 1083456*a^3*b^14*c^2*d^3*e^16*z^2 - 792384*a^2*b^4*c^13*d^15*e^4*z^
2 - 657408*a^2*b^3*c^14*d^16*e^3*z^2 + 608256*a^7*b^7*c^5*d^2*e^17*z^2 + 59
5968*a^2*b^2*c^15*d^17*e^2*z^2 - 498624*a^2*b^15*c^2*d^4*e^15*z^2 - 3840*b^
18*c*d^5*e^14*z^2 - 3840*b^5*c^14*d^18*e*z^2 + 2064384*a^11*c^8*d*e^18*z^2
- 4160*a^3*b^16*d*e^18*z^2 - 4160*a*b^18*d^3*e^16*z^2 - 1290240*a^11*b*c^7*
e^19*z^2 - 9840*a^5*b^13*c*e^19*z^2 - 5760*a*b^2*c^16*d^19*z^2 - 280581120*

a^8*c^11*d^7*e^12*z^2 + 110278656*a^9*c^10*d^5*e^14*z^2 - 89479168*a^7*c^12
*d^9*e^10*z^2 + 34464000*a^10*c^9*d^3*e^16*z^2 + 54240*b^15*c^4*d^8*e^11*z^
2 + 54240*b^8*c^11*d^15*e^4*z^2 - 49920*b^14*c^5*d^9*e^10*z^2 - 49920*b^9*c
^10*d^14*e^5*z^2 - 37376*b^16*c^3*d^7*e^12*z^2 - 37376*b^7*c^12*d^16*e^3*z^
2 + 28480*b^13*c^6*d^10*e^9*z^2 + 28480*b^10*c^9*d^13*e^6*z^2 + 15936*b^17*
c^2*d^6*e^13*z^2 + 15936*b^6*c^13*d^17*e^2*z^2 - 7920*b^12*c^7*d^11*e^8*z^2
 - 7920*b^11*c^8*d^12*e^7*z^2 + 7489536*a^5*c^14*d^13*e^6*z^2 + 6084096*a^6
*c^13*d^11*e^8*z^2 + 2280448*a^4*c^15*d^15*e^4*z^2 + 350208*a^3*c^16*d^17*e
^2*z^2 + 11616*a^2*b^17*d^2*e^17*z^2 - 3515904*a^9*b^5*c^5*e^19*z^2 + 34406
40*a^10*b^3*c^6*e^19*z^2 + 1870848*a^8*b^7*c^4*e^19*z^2 - 572272*a^7*b^9*c^
3*e^19*z^2 + 101856*a^6*b^11*c^2*e^19*z^2 + 400*b^19*d^4*e^15*z^2 + 400*b^4
*c^15*d^19*z^2 + 20736*a^2*c^17*d^19*z^2 + 400*a^4*b^15*e^19*z^2 - 3969216*
a^4*b*c^10*d^3*e^12 - 3001536*a^3*b*c^11*d^5*e^10 - 419904*a^2*b*c^12*d^7*e
^8 + 184608*a^4*b^3*c^8*d*e^14 - 153036*a*b^4*c^10*d^6*e^9 + 127008*a*b^3*c
^11*d^7*e^8 + 63108*a*b^6*c^8*d^4*e^11 - 29160*a*b^2*c^12*d^8*e^7 - 21060*a
^3*b^5*c^7*d*e^14 - 21060*a*b^7*c^7*d^3*e^12 + 5460*a*b^5*c^9*d^5*e^10 - 40
4544*a^5*b*c^9*d*e^14 + 1251872*a^3*b^3*c^9*d^3*e^12 + 844224*a^4*b^2*c^9*d
^2*e^13 + 820512*a^2*b^3*c^10*d^5*e^10 + 750672*a^3*b^2*c^10*d^4*e^11 - 657
498*a^2*b^4*c^9*d^4*e^11 - 487116*a^3*b^4*c^8*d^2*e^13 + 160704*a^2*b^2*c^1
1*d^6*e^9 + 58806*a^2*b^6*c^7*d^2*e^13 + 13140*a^2*b^5*c^8*d^3*e^12 + 15286
*b^6*c^9*d^6*e^9 - 9540*b^7*c^8*d^5*e^10 - 9540*b^5*c^10*d^7*e^8 + 2025*b^8
*c^7*d^4*e^11 + 2025*b^4*c^11*d^8*e^7 + 3367008*a^4*c^11*d^4*e^11 + 1166400
*a^3*c^12*d^6*e^9 + 705600*a^5*c^10*d^2*e^13 + 104976*a^2*c^13*d^8*e^7 - 17
640*a^5*b^2*c^8*e^15 + 2025*a^4*b^4*c^7*e^15 + 38416*a^6*c^9*e^15, z, k)*((
1048576*a^17*c^8*d*e^24 - 393216*a^6*c^19*d^23*e^2 - 3407872*a^7*c^18*d^21*
e^4 - 5636096*a^8*c^17*d^19*e^6 + 31457280*a^9*c^16*d^17*e^8 + 175374336*a^
10*c^15*d^15*e^10 + 407371776*a^11*c^14*d^13*e^12 + 556007424*a^12*c^13*d^1
1*e^14 + 481296384*a^13*c^12*d^9*e^16 + 265420800*a^14*c^11*d^7*e^18 + 8886
6816*a^15*c^10*d^5*e^20 + 15859712*a^16*c^9*d^3*e^22 - 5632*a^2*b^8*c^15*d^
23*e^2 + 67584*a^2*b^9*c^14*d^22*e^3 - 368640*a^2*b^10*c^13*d^21*e^4 + 1205
248*a^2*b^11*c^12*d^20*e^5 - 2618880*a^2*b^12*c^11*d^19*e^6 + 3953664*a^2*b
^13*c^10*d^18*e^7 - 4190208*a^2*b^14*c^9*d^17*e^8 + 3041280*a^2*b^15*c^8*d^
16*e^9 - 1368576*a^2*b^16*c^7*d^15*e^10 + 225280*a^2*b^17*c^6*d^14*e^11 + 1
35168*a^2*b^18*c^5*d^13*e^12 - 101376*a^2*b^19*c^4*d^12*e^13 + 28160*a^2*b^
20*c^3*d^11*e^14 - 3072*a^2*b^21*c^2*d^10*e^15 + 49152*a^3*b^6*c^16*d^23*e^
2 - 589824*a^3*b^7*c^15*d^22*e^3 + 3181568*a^3*b^8*c^14*d^21*e^4 - 10121216
*a^3*b^9*c^13*d^20*e^5 + 20854016*a^3*b^10*c^12*d^19*e^6 - 28504064*a^3*b^1
1*c^11*d^18*e^7 + 24727808*a^3*b^12*c^10*d^17*e^8 - 10510336*a^3*b^13*c^9*d
^16*e^9 - 3040768*a^3*b^14*c^8*d^15*e^10 + 7405568*a^3*b^15*c^7*d^14*e^11 -
 4684288*a^3*b^16*c^6*d^13*e^12 + 1314816*a^3*b^17*c^5*d^12*e^13 - 12032*a^
3*b^18*c^4*d^11*e^14 - 86016*a^3*b^19*c^3*d^10*e^15 + 15616*a^3*b^20*c^2*d^
9*e^16 - 212992*a^4*b^4*c^17*d^23*e^2 + 2555904*a^4*b^5*c^16*d^22*e^3 - 135
49568*a^4*b^6*c^15*d^21*e^4 + 41189376*a^4*b^7*c^14*d^20*e^5 - 76867072*a^4
*b^8*c^13*d^19*e^6 + 83304448*a^4*b^9*c^12*d^18*e^7 - 29710336*a^4*b^10*c^1
1*d^17*e^8 - 53473280*a^4*b^11*c^10*d^16*e^9 + 94751744*a^4*b^12*c^9*d^15*e
^10 - 68968448*a^4*b^13*c^8*d^14*e^11 + 20899840*a^4*b^14*c^7*d^13*e^12 + 4
022272*a^4*b^15*c^6*d^12*e^13 - 5248512*a^4*b^16*c^5*d^11*e^14 + 1310720*a^
4*b^17*c^4*d^10*e^15 + 40960*a^4*b^18*c^3*d^9*e^16 - 45056*a^4*b^19*c^2*d^8
*e^17 + 458752*a^5*b^2*c^18*d^23*e^2 - 5505024*a^5*b^3*c^17*d^22*e^3 + 2821
3248*a^5*b^4*c^16*d^21*e^4 - 77725696*a^5*b^5*c^15*d^20*e^5 + 109985792*a^5
*b^6*c^14*d^19*e^6 - 16252928*a^5*b^7*c^13*d^18*e^7 - 236929024*a^5*b^8*c^1
2*d^17*e^8 + 460423168*a^5*b^9*c^11*d^16*e^9 - 412556800*a^5*b^10*c^10*d^15
*e^10 + 137754624*a^5*b^11*c^9*d^14*e^11 + 80635904*a^5*b^12*c^8*d^13*e^12
 - 102774784*a^5*b^13*c^7*d^12*e^13 + 36015104*a^5*b^14*c^6*d^11*e^14 + 1345
536*a^5*b^15*c^5*d^10*e^15 - 3577856*a^5*b^16*c^4*d^9*e^16 + 407552*a^5*b^1
7*c^3*d^8*e^17 + 82432*a^5*b^18*c^2*d^7*e^18 - 21757952*a^6*b^2*c^17*d^21*e
^4 + 39059456*a^6*b^3*c^16*d^20*e^5 + 44351488*a^6*b^4*c^15*d^19*e^6 - 3816
81664*a^6*b^5*c^14*d^18*e^7 + 872808448*a^6*b^6*c^13*d^17*e^8 - 981073920*a
^6*b^7*c^12*d^16*e^9 + 329307136*a^6*b^8*c^11*d^15*e^10 + 558870528*a^6*b^9

*c^10*d^14*e^11 - 809418752*a^6*b^10*c^9*d^13*e^12 + 394459136*a^6*b^11*c^8
*d^12*e^13 + 10594304*a^6*b^12*c^7*d^11*e^14 - 84887552*a^6*b^13*c^6*d^10*e
^15 + 23650304*a^6*b^14*c^5*d^9*e^16 + 2762752*a^6*b^15*c^4*d^8*e^17 - 1268
736*a^6*b^16*c^3*d^7*e^18 - 100352*a^6*b^17*c^2*d^6*e^19 - 192217088*a^7*b^
2*c^16*d^19*e^6 + 514850816*a^7*b^3*c^15*d^18*e^7 - 691208192*a^7*b^4*c^14*
d^17*e^8 + 8388608*a^7*b^5*c^13*d^16*e^9 + 1583054848*a^7*b^6*c^12*d^15*e^1
0 - 2597715968*a^7*b^7*c^11*d^14*e^11 + 1705592832*a^7*b^8*c^10*d^13*e^12 +
65314816*a^7*b^9*c^9*d^12*e^13 - 792112640*a^7*b^10*c^8*d^11*e^14 + 396832
768*a^7*b^11*c^7*d^10*e^15 + 5305856*a^7*b^12*c^6*d^9*e^16 - 47955968*a^7*b
^13*c^5*d^8*e^17 + 4476416*a^7*b^14*c^4*d^7*e^18 + 1921024*a^7*b^15*c^3*d^6
*e^19 + 82432*a^7*b^16*c^2*d^5*e^20 - 472383488*a^8*b^2*c^15*d^17*e^8 + 155
2941056*a^8*b^3*c^14*d^16*e^9 - 2815066112*a^8*b^4*c^13*d^15*e^10 + 2329542
656*a^8*b^5*c^12*d^14*e^11 + 631472128*a^8*b^6*c^11*d^13*e^12 - 3123511296*
a^8*b^7*c^10*d^12*e^13 + 2406024192*a^8*b^8*c^9*d^11*e^14 - 253763584*a^8*b
^9*c^8*d^10*e^15 - 535957504*a^8*b^10*c^7*d^9*e^16 + 196169728*a^8*b^11*c^6
*d^8*e^17 + 27567104*a^8*b^12*c^5*d^7*e^18 - 13180928*a^8*b^13*c^4*d^6*e^19
 - 1767424*a^8*b^14*c^3*d^5*e^20 - 45056*a^8*b^15*c^2*d^4*e^21 - 26345472*a
^9*b^2*c^14*d^15*e^10 + 1757937664*a^9*b^3*c^13*d^14*e^11 - 4680646656*a^9*
b^4*c^12*d^13*e^12 + 4978376704*a^9*b^5*c^11*d^12*e^13 - 1037008896*a^9*b^6
*c^10*d^11*e^14 - 2360082432*a^9*b^7*c^9*d^10*e^15 + 1791750144*a^9*b^8*c^8
*d^9*e^16 - 76677120*a^9*b^9*c^7*d^8*e^17 - 263758592*a^9*b^10*c^6*d^7*e^18
 + 28357632*a^9*b^11*c^5*d^6*e^19 + 14978560*a^9*b^12*c^4*d^5*e^20 + 102912
0*a^9*b^13*c^3*d^4*e^21 + 15616*a^9*b^14*c^2*d^3*e^22 + 1853358080*a^10*b^2
*c^13*d^13*e^12 + 106430464*a^10*b^3*c^12*d^12*e^13 - 4433149952*a^10*b^4*c
^11*d^11*e^14 + 5213257728*a^10*b^5*c^10*d^10*e^15 - 1239613440*a^10*b^6*c^
9*d^9*e^16 - 1399455744*a^10*b^7*c^8*d^8*e^17 + 721519104*a^10*b^8*c^7*d^7*
e^18 + 92768256*a^10*b^9*c^6*d^6*e^19 - 60235776*a^10*b^10*c^5*d^5*e^20 - 9
616384*a^10*b^11*c^4*d^4*e^21 - 369152*a^10*b^12*c^3*d^3*e^22 - 3072*a^10*b
^13*c^2*d^2*e^23 + 3744333824*a^11*b^2*c^12*d^11*e^14 - 1445986304*a^11*b^3
*c^11*d^10*e^15 - 2945974272*a^11*b^4*c^10*d^9*e^16 + 3180331008*a^11*b^5*c
^9*d^8*e^17 - 344997888*a^11*b^6*c^8*d^7*e^18 - 607715328*a^11*b^7*c^7*d^6*
e^19 + 91261952*a^11*b^8*c^6*d^5*e^20 + 46288896*a^11*b^9*c^5*d^4*e^21 + 36
19072*a^11*b^10*c^4*d^3*e^22 + 73728*a^11*b^11*c^3*d^2*e^23 + 3567255552*a^
12*b^2*c^11*d^9*e^16 - 1152385024*a^12*b^3*c^10*d^8*e^17 - 1550467072*a^12*
b^4*c^9*d^7*e^18 + 1052180480*a^12*b^5*c^8*d^6*e^19 + 114114560*a^12*b^6*c^
7*d^5*e^20 - 115572736*a^12*b^7*c^6*d^4*e^21 - 18767360*a^12*b^8*c^5*d^3*e^
22 - 737280*a^12*b^9*c^4*d^2*e^23 + 1821048832*a^13*b^2*c^10*d^7*e^18 - 236
191744*a^13*b^3*c^9*d^6*e^19 - 544571392*a^13*b^4*c^8*d^5*e^20 + 114688000*
a^13*b^5*c^7*d^4*e^21 + 53821440*a^13*b^6*c^6*d^3*e^22 + 3932160*a^13*b^7*c
^5*d^2*e^23 + 460587008*a^14*b^2*c^9*d^5*e^20 + 57933824*a^14*b^3*c^8*d^4*e
^21 - 78659584*a^14*b^4*c^7*d^3*e^22 - 11796480*a^14*b^5*c^6*d^2*e^23 + 382
07488*a^15*b^2*c^8*d^3*e^22 + 18874368*a^15*b^3*c^7*d^2*e^23 + 256*a*b^10*c
^14*d^23*e^2 - 3072*a*b^11*c^13*d^22*e^3 + 16896*a*b^12*c^12*d^21*e^4 - 563
20*a*b^13*c^11*d^20*e^5 + 126720*a*b^14*c^10*d^19*e^6 - 202752*a*b^15*c^9*d
^18*e^7 + 236544*a*b^16*c^8*d^17*e^8 - 202752*a*b^17*c^7*d^16*e^9 + 126720*
a*b^18*c^6*d^15*e^10 - 56320*a*b^19*c^5*d^14*e^11 + 16896*a*b^20*c^4*d^13*e
^12 - 3072*a*b^21*c^3*d^12*e^13 + 256*a*b^22*c^2*d^11*e^14 + 4718592*a^6*b*
c^18*d^22*e^3 + 38797312*a^7*b*c^17*d^20*e^5 + 77594624*a^8*b*c^16*d^18*e^7
 - 159383552*a^9*b*c^15*d^16*e^9 - 1020264448*a^10*b*c^14*d^14*e^11 - 21286
09280*a^11*b*c^13*d^12*e^13 + 256*a^11*b^12*c^2*d*e^24 - 2451570688*a^12*b*
c^12*d^10*e^15 - 6144*a^12*b^10*c^3*d*e^24 - 1694498816*a^13*b*c^11*d^8*e^1
7 + 61440*a^13*b^8*c^4*d*e^24 - 691535872*a^14*b*c^10*d^6*e^19 - 327680*a^1
4*b^6*c^5*d*e^24 - 149946368*a^15*b*c^9*d^4*e^21 + 983040*a^15*b^4*c^6*d*e^
24 - 12582912*a^16*b*c^8*d^2*e^23 - 1572864*a^16*b^2*c^7*d*e^24)/(32*(16*a^
3*b^6*c^9*d^18 - a^2*b^8*c^8*d^18 - 256*a^6*c^12*d^18 - 96*a^4*b^4*c^10*d^1
8 + 256*a^5*b^2*c^11*d^18 - a^2*b^16*d^10*e^8 + 8*a^3*b^15*d^9*e^9 - 28*a^4
*b^14*d^8*e^10 + 56*a^5*b^13*d^7*e^11 - 70*a^6*b^12*d^6*e^12 + 56*a^7*b^11*
d^5*e^13 - 28*a^8*b^10*d^4*e^14 + 8*a^9*b^9*d^3*e^15 - a^10*b^8*d^2*e^16 -
2048*a^7*c^11*d^16*e^2 - 7168*a^8*c^10*d^14*e^4 - 14336*a^9*c^9*d^12*e^6 -

17920*a^10*c^8*d^10*e^8 - 14336*a^11*c^7*d^8*e^10 - 7168*a^12*c^6*d^6*e^12 - 2048*a^13*c^5*d^4*e^14 - 256*a^14*c^4*d^2*e^16 - 28*a^2*b^10*c^6*d^16*e^2 + 56*a^2*b^11*c^5*d^15*e^3 - 70*a^2*b^12*c^4*d^14*e^4 + 56*a^2*b^13*c^3*d^13*e^5 - 28*a^2*b^14*c^2*d^12*e^6 + 440*a^3*b^8*c^7*d^16*e^2 - 840*a^3*b^9*c^6*d^15*e^3 + 952*a^3*b^10*c^5*d^14*e^4 - 616*a^3*b^11*c^4*d^13*e^5 + 168*a^3*b^12*c^3*d^12*e^6 + 40*a^3*b^13*c^2*d^11*e^7 - 2560*a^4*b^6*c^8*d^16*e^2 + 4480*a^4*b^7*c^7*d^15*e^3 - 4060*a^4*b^8*c^6*d^14*e^4 + 1064*a^4*b^9*c^5*d^13*e^5 + 1372*a^4*b^10*c^4*d^12*e^6 - 1360*a^4*b^11*c^3*d^11*e^7 + 380*a^4*b^12*c^2*d^10*e^8 + 6400*a^5*b^4*c^9*d^16*e^2 - 8960*a^5*b^5*c^8*d^15*e^3 + 2240*a^5*b^6*c^7*d^14*e^4 + 9856*a^5*b^7*c^6*d^13*e^5 - 13048*a^5*b^8*c^5*d^12*e^6 + 5400*a^5*b^9*c^4*d^11*e^7 + 1040*a^5*b^10*c^3*d^10*e^8 - 1360*a^5*b^11*c^2*d^9*e^9 - 5120*a^6*b^2*c^10*d^16*e^2 + 22400*a^6*b^4*c^8*d^14*e^4 - 41216*a^6*b^5*c^7*d^13*e^5 + 25088*a^6*b^6*c^6*d^12*e^6 + 8320*a^6*b^7*c^5*d^11*e^7 - 17350*a^6*b^8*c^4*d^10*e^8 + 5400*a^6*b^9*c^3*d^9*e^9 + 1372*a^6*b^10*c^2*d^8*e^10 - 35840*a^7*b^2*c^9*d^14*e^4 + 28672*a^7*b^3*c^8*d^13*e^5 + 30464*a^7*b^4*c^7*d^12*e^6 - 73472*a^7*b^5*c^6*d^11*e^7 + 40544*a^7*b^6*c^5*d^10*e^8 + 8320*a^7*b^7*c^4*d^9*e^9 - 13048*a^7*b^8*c^3*d^8*e^10 + 1064*a^7*b^9*c^2*d^7*e^11 - 93184*a^8*b^2*c^8*d^12*e^6 + 71680*a^8*b^3*c^7*d^11*e^7 + 29120*a^8*b^4*c^6*d^10*e^8 - 73472*a^8*b^5*c^5*d^9*e^9 + 25088*a^8*b^6*c^4*d^8*e^10 + 9856*a^8*b^7*c^3*d^7*e^11 - 4060*a^8*b^8*c^2*d^6*e^12 - 125440*a^9*b^2*c^7*d^10*e^8 + 71680*a^9*b^3*c^6*d^9*e^9 + 30464*a^9*b^4*c^5*d^8*e^10 - 41216*a^9*b^5*c^4*d^7*e^11 + 2240*a^9*b^6*c^3*d^6*e^12 + 4480*a^9*b^7*c^2*d^5*e^13 - 93184*a^10*b^2*c^6*d^8*e^10 + 28672*a^10*b^3*c^5*d^7*e^11 + 22400*a^10*b^4*c^4*d^6*e^12 - 8960*a^10*b^5*c^3*d^5*e^13 - 2560*a^10*b^6*c^2*d^4*e^14 - 35840*a^11*b^2*c^5*d^6*e^12 + 6400*a^11*b^4*c^3*d^4*e^14 + 768*a^11*b^5*c^2*d^3*e^15 - 5120*a^12*b^2*c^4*d^4*e^14 - 2048*a^12*b^3*c^3*d^3*e^15 - 96*a^12*b^4*c^2*d^2*e^16 + 256*a^13*b^2*c^3*d^2*e^16 + 2048*a^6*b*c^11*d^17*e + 8*a^2*b^9*c^7*d^17*e + 8*a^2*b^15*c*d^11*e^7 - 128*a^3*b^7*c^8*d^17*e - 40*a^3*b^14*c*d^10*e^8 + 768*a^4*b^5*c^9*d^17*e + 40*a^4*b^13*c*d^9*e^9 - 2048*a^5*b^3*c^10*d^17*e + 168*a^5*b^12*c*d^8*e^10 - 616*a^6*b^11*c*d^7*e^11 + 14336*a^7*b*c^10*d^15*e^3 + 952*a^7*b^10*c*d^6*e^12 + 43008*a^8*b*c^9*d^13*e^5 - 840*a^8*b^9*c*d^5*e^13 + 71680*a^9*b*c^8*d^11*e^7 + 440*a^9*b^8*c*d^4*e^14 + 71680*a^10*b*c^7*d^9*e^9 - 128*a^10*b^7*c*d^3*e^15 + 43008*a^11*b*c^6*d^7*e^11 + 16*a^11*b^6*c*d^2*e^16 + 14336*a^12*b*c^5*d^5*e^13 + 2048*a^13*b*c^4*d^3*e^15)) + (root(128723189760*a^14*b^4*c^9*d^13*e^14*z^6 + 128723189760*a^12*b^4*c^11*d^17*e^10*z^6 - 8432455680*a^11*b^12*c^4*d^11*e^16*z^6 - 8432455680*a^7*b^12*c^8*d^19*e^8*z^6 + 12673351680*a^11*b^11*c^5*d^12*e^15*z^6 + 12673351680*a^8*b^11*c^8*d^18*e^9*z^6 - 72637480960*a^12*b^9*c^6*d^12*e^15*z^6 - 72637480960*a^9*b^9*c^9*d^18*e^9*z^6 - 21048344576*a^9*b^12*c^6*d^15*e^12*z^6 - 16609443840*a^17*b^3*c^7*d^8*e^19*z^6 - 16609443840*a^10*b^3*c^14*d^22*e^5*z^6 + 145332633600*a^13*b^5*c^9*d^14*e^13*z^6 + 145332633600*a^12*b^5*c^10*d^16*e^11*z^6 + 123740356608*a^14*b^5*c^8*d^12*e^15*z^6 + 123740356608*a^11*b^5*c^11*d^18*e^9*z^6 + 3460300800*a^17*b^5*c^5*d^6*e^21*z^6 + 3460300800*a^8*b^5*c^14*d^24*e^3*z^6 - 7751073792*a^15*b^7*c^5*d^8*e^19*z^6 - 7751073792*a^8*b^7*c^12*d^22*e^5*z^6 + 12041846784*a^14*b^7*c^6*d^10*e^17*z^6 + 12041846784*a^9*b^7*c^11*d^20*e^7*z^6 - 325545099264*a^14*b^3*c^10*d^14*e^13*z^6 - 325545099264*a^13*b^3*c^11*d^16*e^11*z^6 - 3330539520*a^13*b^10*c^4*d^9*e^18*z^6 - 3330539520*a^7*b^10*c^10*d^21*e^6*z^6 + 157789716480*a^12*b^7*c^8*d^14*e^13*z^6 + 157789716480*a^11*b^7*c^9*d^16*e^11*z^6 + 37492359168*a^11*b^10*c^6*d^13*e^14*z^6 + 37492359168*a^9*b^10*c^8*d^17*e^10*z^6 + 301989888*a^8*b^3*c^16*d^26*e*z^6 - 7266631680*a^17*b^4*c^6*d^7*e^20*z^6 - 7266631680*a^9*b^4*c^14*d^23*e^4*z^6 - 201326592*a^20*b*c^6*d^4*e^23*z^6 - 188743680*a^7*b^5*c^15*d^26*e*z^6 + 45747339264*a^13*b^8*c^6*d^11*e^16*z^6 + 45747339264*a^9*b^8*c^10*d^19*e^8*z^6 - 74612736*a^10*b^16*c*d^9*e^18*z^6 - 2768240640*a^16*b^7*c^4*d^6*e^21*z^6 - 2768240640*a^7*b^7*c^13*d^24*e^3*z^6 + 69746688*a^11*b^15*c*d^8*e^19*z^6 + 62914560*a^6*b^7*c^14*d^26*e*z^6 + 2752020480*a^10*b^13*c^4*d^12*e^15*z^6 + 2752020480*a^7*b^13*c^7*d^18*e^9*z^6 + 55148544*a^9*b^17*c*d^10*e^17*z^6 - 45957120*a^12*b^14*c*d^7*e^20*z^6 - 2724986880*a^14*b^9*c^4*d^8* e^19*z^6 - 2724986880*a^7*b^9*c^11*d^22*e^5*z^6 - 25952256*a^8*b^18*c*d^11*e^16*z^6 + 21086208*a^13*b^13*c*d^6*e^21*z^6 - 11796480*a^5*b^9*c^13*d^26*e*z^6 - 6438912*a^14*b^12*c*d^5*e^22*z^6 + 5406720*a^7*b^19*c*d^12*e^15*z^6 + 1622016*a^6*b^20*c*d^13*e^14*z^6 - 1523712*a^5*b^21*c*d^14*e^13*z^6 + 1179648*a^15*b^11*c*d^4*e^23*z^6 + 1179648*a^4*b^11*c^12*d^26*e*z^6 + 442368*a^4*b^22*c*d^15*e^12*z^6 - 98304*a^16*b^10*c*d^3*e^24*z^6 - 49152*a^3*b^23*c*d^16*e^11*z^6 - 49152*a^3*b^13*c^11*d^26*e*z^6 + 6897106944*a^9*b^13*c^5*d^14*e^13*z^6 + 6897106944*a^8*b^13*c^6*d^16*e^11*z^6 - 2422210560*a^16*b^6*c^5*d^7*e^20*z^6 - 2422210560*a^8*b^6*c^13*d^23*e^4*z^6 + 255785435136*a^14*b^2*c^11*d^15*e^12*z^6 + 41004564480*a^15*b^4*c^8*d^11*e^16*z^6 + 41004564480*a^11*b^4*c^12*d^19*e^8*z^6 + 2270822400*a^13*b^11*c^3*d^8*e^19*z^6 + 2270822400*a^6*b^11*c^10*d^22*e^5*z^6 + 23677108224*a^14*b^8*c^5*d^9*e^18*z^6 + 23677108224*a^8*b^8*c^11*d^21*e^6*z^6 + 212600881152*a^15*b^2*c^10*d^13*e^14*z^6 + 212600881152*a^13*b^2*c^12*d^17*e^10*z^6 + 75157733376*a^15*b^5*c^7*d^10*e^17*z^6 + 75157733376*a^10*b^5*c^12*d^20*e^7*z^6 - 251217838080*a^13*b^6*c^8*d^13*e^14*z^6 - 251217838080*a^11*b^6*c^10*d^17*e^10*z^6 - 1952907264*a^14*b^10*c^3*d^7*e^20*z^6 - 1952907264*a^6*b^10*c^11*d^23*e^4*z^6 - 27691057152*a^13*b^9*c^5*d^10*e^17*z^6 - 27691057152*a^8*b^9*c^10*d^20*e^7*z^6 - 1902673920*a^8*b^15*c^4*d^14*e^13*z^6 - 1902673920*a^7*b^15*c^5*d^16*e^11*z^6 + 10465050624*a^10*b^11*c^6*d^14*e^13*z^6 + 10465050624*a^9*b^11*c^7*d^16*e^11*z^6 + 1613905920*a^9*b^14*c^4*d^13*e^14*z^6 + 1613905920*a^7*b^14*c^6*d^17*e^10*z^6 - 33218887680*a^17*b*c^9*d^10*e^17*z^6 - 33218887680*a^12*b*c^14*d^20*e^7*z^6 + 1524695040*a^10*b^14*c^3*d^11*e^16*z^6 + 1524695040*a^6*b^14*c^7*d^19*e^8*z^6 - 1472200704*a^18*b^4*c^5*d^5*e^22*z^6 - 1472200704*a^8*b^4*c^15*d^25*e^2*z^6 - 83047219200*a^16*b^3*c^8*d^10*e^17*z^6 - 83047219200*a^11*b^3*c^13*d^20*e^7*z^6 + 44291850240*a^17*b^2*c^8*d^9*e^18*z^6 + 44291850240*a^11*b^2*c^14*d^21*e^6*z^6 + 1308131328*a^8*b^14*c^5*d^15*e^12*z^6 - 201326592*a^9*b*c^17*d^26*e*z^6 + 48530718720*a^12*b^8*c^7*d^13*e^14*z^6 + 48530718720*a^10*b^8*c^9*d^17*e^10*z^6 - 1242644480*a^12*b^12*c^3*d^9*e^18*z^6 - 1242644480*a^6*b^12*c^9*d^21*e^6*z^6 + 9813196800*a^12*b^10*c^5*d^11*e^16*z^6 + 9813196800*a^8*b^10*c^9*d^19*e^8*z^6 - 93012885504*a^15*b*c^11*d^14*e^13*z^6 - 93012885504*a^14*b*c^12*d^16*e^11*z^6 + 177305812992*a^13*b^4*c^10*d^15*e^12*z^6 + 52730658816*a^10*b^10*c^7*d^15*e^12*z^6 - 1180106752*a^9*b^15*c^3*d^12*e^15*z^6 - 1180106752*a^6*b^15*c^6*d^18*e^9*z^6 + 1023672320*a^15*b^9*c^3*d^6*e^21*z^6 + 1023672320*a^6*b^9*c^12*d^24*e^3*z^6 + 975175680*a^17*b^6*c^4*d^5*e^22*z^6 + 975175680*a^7*b^6*c^14*d^25*e^2*z^6 - 11072962560*a^18*b*c^8*d^8*e^19*z^6 - 11072962560*a^11*b*c^15*d^22*e^5*z^6 + 9412018176*a^18*b^2*c^7*d^7*e^20*z^6 + 9412018176*a^10*b^2*c^15*d^23*e^4*z^6 + 805306368*a^19*b^2*c^6*d^5*e^22*z^6 + 805306368*a^9*b^2*c^16*d^25*e^2*z^6 - 133809831936*a^14*b^6*c^7*d^11*e^16*z^6 - 133809831936*a^10*b^6*c^11*d^19*e^8*z^6 - 2214592512*a^19*b*c^7*d^6*e^21*z^6 - 2214592512*a^10*b*c^16*d^24*e^3*z^6 + 82216747008*a^13*b^7*c^7*d^12*e^15*z^6 + 82216747008*a^10*b^7*c^10*d^18*e^9*z^6 - 586629120*a^12*b^13*c^2*d^8*e^19*z^6 - 586629120*a^5*b^13*c^9*d^22*e^5*z^6 + 568565760*a^7*b^16*c^4*d^15*e^12*z^6 - 4844421120*a^16*b^4*c^7*d^9*e^18*z^6 - 4844421120*a^10*b^4*c^13*d^21*e^6*z^6 + 531210240*a^11*b^14*c^2*d^9*e^18*z^6 + 531210240*a^5*b^14*c^8*d^21*e^6*z^6 - 527155200*a^11*b^13*c^3*d^10*e^17*z^6 - 527155200*a^6*b^13*c^8*d^20*e^7*z^6 + 43470028800*a^11*b^8*c^8*d^15*e^12*z^6 - 107874877440*a^11*b^9*c^7*d^14*e^13*z^6 - 107874877440*a^10*b^9*c^8*d^16*e^11*z^6 + 9018408960*a^12*b^11*c^4*d^10*e^17*z^6 + 9018408960*a^7*b^11*c^9*d^20*e^7*z^6 + 421994496*a^13*b^12*c^2*d^7*e^20*z^6 + 421994496*a^5*b^12*c^10*d^23*e^4*z^6 - 66437775360*a^16*b*c^10*d^12*e^15*z^6 - 66437775360*a^13*b*c^13*d^18*e^9*z^6 + 26159874048*a^16*b^5*c^6*d^8*e^19*z^6 + 26159874048*a^9*b^5*c^13*d^22*e^5*z^6 - 369098752*a^18*b^3*c^6*d^6*e^21*z^6 - 369098752*a^9*b^3*c^15*d^24*e^3*z^6 + 351436800*a^8*b^16*c^3*d^13*e^14*z^6 + 351436800*a^6*b^16*c^5*d^17*e^10*z^6 - 334233600*a^16*b^8*c^3*d^5*e^22*z^6 - 334233600*a^6*b^8*c^13*d^25*e^2*z^6 + 301989888*a^19*b^3*c^5*d^4*e^23*z^6 - 266010624*a^10*b^15*c^2*d^10*e^17*z^6 - 266010624*a^5*b^15*c^7*d^20*e^7*z^6 - 305198530560*a^12*b^6*c^9*d^15*e^12*z^6 - 203292672*a^14*b^11*c^2*d^6*e^21*z^6 - 203292672*a^5*b^11*c^

11*d^24*e^3*z^6 - 188743680*a^18*b^5*c^4*d^4*e^23*z^6 + 120418467840*a^16*b
^2*c^9*d^11*e^16*z^6 + 120418467840*a^12*b^2*c^13*d^19*e^8*z^6 - 1729393459
2*a^10*b^12*c^5*d^13*e^14*z^6 - 17293934592*a^8*b^12*c^7*d^17*e^10*z^6 + 10
4890368*a^8*b^17*c^2*d^12*e^15*z^6 + 104890368*a^5*b^17*c^5*d^18*e^9*z^6 +
4390256640*a^15*b^8*c^4*d^7*e^20*z^6 + 4390256640*a^7*b^8*c^12*d^23*e^4*z^6
 - 91750400*a^6*b^18*c^3*d^15*e^12*z^6 + 79134720*a^7*b^17*c^3*d^14*e^13*z^
6 + 79134720*a^6*b^17*c^4*d^16*e^11*z^6 - 74612736*a^4*b^16*c^7*d^21*e^6*z^
6 - 72990720*a^7*b^18*c^2*d^13*e^14*z^6 - 72990720*a^5*b^18*c^4*d^17*e^10*z
^6 + 69746688*a^4*b^15*c^8*d^22*e^5*z^6 + 63700992*a^15*b^10*c^2*d^5*e^22*z
^6 + 63700992*a^5*b^10*c^12*d^25*e^2*z^6 + 62914560*a^17*b^7*c^3*d^4*e^23*z
^6 + 55148544*a^4*b^17*c^6*d^20*e^7*z^6 - 45957120*a^4*b^14*c^9*d^23*e^4*z^
6 - 25952256*a^4*b^18*c^5*d^19*e^8*z^6 - 25165824*a^20*b^2*c^5*d^3*e^24*z^6
 + 21086208*a^4*b^13*c^10*d^24*e^3*z^6 + 20643840*a^6*b^19*c^2*d^14*e^13*z^
6 + 20643840*a^5*b^19*c^3*d^16*e^11*z^6 + 15728640*a^19*b^4*c^4*d^3*e^24*z^
6 - 11796480*a^16*b^9*c^2*d^4*e^23*z^6 - 6438912*a^4*b^12*c^11*d^25*e^2*z^6
 + 5406720*a^4*b^19*c^4*d^18*e^9*z^6 - 5242880*a^18*b^6*c^3*d^3*e^24*z^6 +
3784704*a^3*b^18*c^6*d^21*e^6*z^6 - 3244032*a^3*b^19*c^5*d^20*e^7*z^6 - 324
4032*a^3*b^17*c^7*d^22*e^5*z^6 + 2027520*a^3*b^20*c^4*d^19*e^8*z^6 + 202752
0*a^3*b^16*c^8*d^23*e^4*z^6 - 1622016*a^9*b^16*c^2*d^11*e^16*z^6 - 1622016*
a^5*b^16*c^6*d^19*e^8*z^6 + 1622016*a^4*b^20*c^3*d^17*e^10*z^6 - 1523712*a^
4*b^21*c^2*d^16*e^11*z^6 + 983040*a^17*b^8*c^2*d^3*e^24*z^6 - 901120*a^3*b^
21*c^3*d^18*e^9*z^6 - 901120*a^3*b^15*c^9*d^24*e^3*z^6 + 270336*a^3*b^22*c^
2*d^17*e^10*z^6 + 270336*a^3*b^14*c^10*d^25*e^2*z^6 + 172032*a^5*b^20*c^2*d
^15*e^12*z^6 - 38593888256*a^15*b^6*c^6*d^9*e^18*z^6 - 38593888256*a^9*b^6*
c^12*d^21*e^6*z^6 - 210386288640*a^15*b^3*c^9*d^12*e^15*z^6 - 210386288640*
a^12*b^3*c^12*d^18*e^9*z^6 + 15502147584*a^15*c^12*d^15*e^12*z^6 + 11072962
56*a^19*c^8*d^7*e^20*z^6 + 1107296256*a^11*c^16*d^23*e^4*z^6 + 13287555072*
a^16*c^11*d^13*e^14*z^6 + 13287555072*a^14*c^13*d^17*e^10*z^6 + 201326592*a
^20*c^7*d^5*e^22*z^6 + 201326592*a^10*c^17*d^25*e^2*z^6 + 16777216*a^21*c^6
*d^3*e^24*z^6 + 3784704*a^9*b^18*d^9*e^18*z^6 - 3244032*a^10*b^17*d^8*e^19*
z^6 - 3244032*a^8*b^19*d^10*e^17*z^6 + 2027520*a^11*b^16*d^7*e^20*z^6 + 202
7520*a^7*b^20*d^11*e^16*z^6 - 901120*a^12*b^15*d^6*e^21*z^6 - 901120*a^6*b^
21*d^12*e^15*z^6 + 270336*a^13*b^14*d^5*e^22*z^6 + 270336*a^5*b^22*d^13*e^1
4*z^6 - 49152*a^14*b^13*d^4*e^23*z^6 - 49152*a^4*b^23*d^14*e^13*z^6 + 4096*
a^15*b^12*d^3*e^24*z^6 + 4096*a^3*b^24*d^15*e^12*z^6 - 25165824*a^8*b^2*c^1
7*d^27*z^6 + 15728640*a^7*b^4*c^16*d^27*z^6 - 5242880*a^6*b^6*c^15*d^27*z^6
 + 983040*a^5*b^8*c^14*d^27*z^6 - 98304*a^4*b^10*c^13*d^27*z^6 + 4096*a^3*b
^12*c^12*d^27*z^6 + 8304721920*a^17*c^10*d^11*e^16*z^6 + 8304721920*a^13*c^
14*d^19*e^8*z^6 + 3690987520*a^18*c^9*d^9*e^18*z^6 + 3690987520*a^12*c^15*d
^21*e^6*z^6 + 16777216*a^9*c^18*d^27*z^6 - 8493371392*a^6*b^8*c^9*d^14*e^9*
z^4 + 1458044928*a^8*b*c^14*d^17*e^6*z^4 - 12604538880*a^11*b^4*c^8*d^8*e^1
5*z^4 - 8303067136*a^9*b^5*c^9*d^11*e^12*z^4 - 5588058112*a^13*b*c^9*d^7*e^
16*z^4 - 3892838400*a^8*b^2*c^13*d^16*e^7*z^4 - 3611713536*a^8*b^8*c^7*d^10
*e^13*z^4 + 7819006464*a^7*b^9*c^7*d^11*e^12*z^4 - 7782137856*a^8*b^7*c^8*d
^11*e^12*z^4 + 7780433920*a^12*b^2*c^9*d^8*e^15*z^4 - 12020465664*a^7*b^5*c
^11*d^15*e^8*z^4 + 3176792064*a^8*b^3*c^12*d^15*e^8*z^4 - 322633728*a^15*b*
c^7*d^3*e^20*z^4 + 210829312*a^7*b*c^15*d^19*e^4*z^4 + 15623258112*a^9*b^6*
c^8*d^10*e^13*z^4 + 25165824*a^15*b^3*c^5*d*e^22*z^4 - 15728640*a^14*b^5*c^
4*d*e^22*z^4 + 12582912*a^5*b^2*c^16*d^22*e*z^4 - 11730944*a^4*b^4*c^15*d^2
2*e*z^4 + 5242880*a^13*b^7*c^3*d*e^22*z^4 - 4561920*a*b^15*c^7*d^17*e^6*z^4
 + 4521984*a^3*b^6*c^14*d^22*e*z^4 + 4460544*a*b^14*c^8*d^18*e^5*z^4 + 3538
944*a^6*b*c^16*d^21*e^2*z^4 + 3108864*a*b^16*c^6*d^16*e^7*z^4 - 3027200*a*b
^13*c^9*d^19*e^4*z^4 - 2345472*a^5*b^17*c*d^7*e^16*z^4 - 2307072*a^8*b^14*c
*d^4*e^19*z^4 + 1824768*a^6*b^16*c*d^6*e^17*z^4 + 1734912*a^9*b^13*c*d^3*e^
20*z^4 + 1419264*a*b^12*c^10*d^20*e^3*z^4 - 1191168*a*b^17*c^5*d^15*e^8*z^4
 - 983040*a^12*b^9*c^2*d*e^22*z^4 + 964608*a^4*b^18*c*d^8*e^15*z^4 - 866304
*a^2*b^8*c^13*d^22*e*z^4 + 703488*a^7*b^15*c*d^5*e^18*z^4 - 608256*a^10*b^1
2*c*d^2*e^21*z^4 - 440832*a*b^11*c^11*d^21*e^2*z^4 + 275968*a*b^19*c^3*d^13
*e^10*z^4 - 159744*a^2*b^20*c*d^10*e^13*z^4 - 153600*a*b^20*c^2*d^12*e^11*z

^4 + 64512*a^3*b^19*c*d^9*e^14*z^4 + 19746062336*a^8*b^6*c^9*d^12*e^11*z^4
- 15333588992*a^10*b^4*c^9*d^10*e^13*z^4 + 6702170112*a^7*b^4*c^12*d^16*e^7
*z^4 + 15167913984*a^10*b^3*c^10*d^11*e^12*z^4 - 2256638976*a^5*b^11*c^7*d^
13*e^10*z^4 + 2254307328*a^5*b^7*c^11*d^17*e^6*z^4 - 2200633344*a^6*b^5*c^1
2*d^17*e^6*z^4 + 6457131008*a^11*b^3*c^9*d^9*e^14*z^4 - 2128785408*a^5*b^8*
c^10*d^16*e^7*z^4 - 2126057472*a^6*b^11*c^6*d^11*e^12*z^4 + 2038349824*a^12
*b^5*c^6*d^5*e^18*z^4 + 2037841920*a^5*b^10*c^8*d^14*e^9*z^4 + 3615621120*a
^9*b*c^13*d^15*e^8*z^4 + 1900019712*a^11*b^2*c^10*d^10*e^13*z^4 + 186769843
2*a^9*b^9*c^5*d^7*e^16*z^4 - 6157369344*a^9*b^4*c^10*d^12*e^11*z^4 - 185691
3408*a^7*b^10*c^6*d^10*e^13*z^4 + 1789132800*a^6*b^4*c^13*d^18*e^5*z^4 + 60
82658304*a^8*b^4*c^11*d^14*e^9*z^4 + 6029549568*a^11*b^5*c^7*d^7*e^16*z^4 +
6010159104*a^6*b^7*c^10*d^15*e^8*z^4 + 1703182336*a^7*b^7*c^9*d^13*e^10*z^
4 + 1658388480*a^11*b^6*c^6*d^6*e^17*z^4 + 5917114368*a^10*b^6*c^7*d^8*e^15
*z^4 - 1591197696*a^11*b^7*c^5*d^5*e^18*z^4 - 1526464512*a^8*b^10*c^5*d^8*e
^15*z^4 - 5772607488*a^12*b^4*c^7*d^6*e^17*z^4 - 1423507456*a^13*b^4*c^6*d^
4*e^19*z^4 - 1387266048*a^7*b^3*c^13*d^17*e^6*z^4 + 2976120832*a^10*b*c^12*
d^13*e^10*z^4 - 9906946048*a^9*b^2*c^12*d^14*e^9*z^4 - 18421874688*a^8*b^5*
c^10*d^13*e^10*z^4 + 1141217280*a^6*b^12*c^5*d^10*e^13*z^4 - 9714364416*a^7
*b^8*c^8*d^12*e^11*z^4 - 16777216*a^16*b*c^6*d*e^22*z^4 + 98304*a^11*b^11*c
*d*e^22*z^4 + 81920*a*b^10*c^12*d^22*e*z^4 + 39168*a*b^21*c*d^11*e^12*z^4 -
1091829760*a^5*b^6*c^12*d^18*e^5*z^4 + 1046740992*a^14*b^2*c^7*d^4*e^19*z^
4 - 6884425728*a^12*b*c^10*d^9*e^14*z^4 + 987445248*a^4*b^10*c^9*d^16*e^7*z
^4 + 984087552*a^5*b^12*c^6*d^12*e^11*z^4 - 9564585984*a^9*b^7*c^7*d^9*e^14
*z^4 - 5266857984*a^10*b^7*c^6*d^7*e^16*z^4 - 892145664*a^7*b^11*c^5*d^9*e^
14*z^4 - 2444623872*a^11*b*c^11*d^11*e^12*z^4 + 768540672*a^12*b^3*c^8*d^7*
e^16*z^4 + 5048322048*a^8*b^9*c^6*d^9*e^14*z^4 + 5047612416*a^6*b^9*c^8*d^1
3*e^10*z^4 - 732492288*a^4*b^11*c^8*d^15*e^8*z^4 + 9266921472*a^7*b^6*c^10*
d^14*e^9*z^4 - 645857280*a^6*b^6*c^11*d^16*e^7*z^4 - 623867904*a^4*b^9*c^10
*d^17*e^6*z^4 - 622067712*a^6*b^3*c^14*d^19*e^4*z^4 + 582617088*a^10*b^8*c^
5*d^6*e^17*z^4 + 577119744*a^7*b^12*c^4*d^8*e^15*z^4 + 552566784*a^12*b^6*c
^5*d^4*e^19*z^4 + 549224448*a^9*b^8*c^6*d^8*e^15*z^4 - 526565376*a^9*b^10*c
^4*d^6*e^17*z^4 + 511520256*a^10*b^9*c^4*d^5*e^18*z^4 + 13393723392*a^9*b^3
*c^11*d^13*e^10*z^4 - 2066350080*a^14*b*c^8*d^5*e^18*z^4 + 4718592000*a^13*
b^2*c^8*d^6*e^17*z^4 - 314572800*a^7*b^2*c^14*d^18*e^5*z^4 + 287250432*a^4*
b^13*c^6*d^13*e^10*z^4 + 4565827584*a^10*b^5*c^8*d^9*e^14*z^4 - 250785792*a
^4*b^14*c^5*d^12*e^11*z^4 + 235536384*a^13*b^3*c^7*d^5*e^18*z^4 - 232683264
*a^8*b^11*c^4*d^7*e^16*z^4 - 199627776*a^5*b^14*c^4*d^10*e^13*z^4 - 1902673
92*a^12*b^7*c^4*d^3*e^20*z^4 + 184891392*a^6*b^10*c^7*d^12*e^11*z^4 + 18050
2528*a^4*b^7*c^12*d^19*e^4*z^4 + 178877952*a^3*b^13*c^7*d^15*e^8*z^4 + 1724
90752*a^14*b^3*c^6*d^3*e^20*z^4 + 163946496*a^13*b^5*c^5*d^3*e^20*z^4 + 155
839488*a^8*b^12*c^3*d^6*e^17*z^4 + 155000832*a^5*b^5*c^13*d^19*e^4*z^4 - 15
2076288*a^4*b^6*c^13*d^20*e^3*z^4 - 137592576*a^3*b^12*c^8*d^16*e^7*z^4 - 1
33693440*a^14*b^4*c^5*d^2*e^21*z^4 - 116767488*a^3*b^9*c^11*d^19*e^4*z^4 -
108985344*a^3*b^14*c^6*d^14*e^9*z^4 - 106223616*a^6*b^13*c^4*d^9*e^14*z^4 +
106119168*a^3*b^10*c^10*d^18*e^5*z^4 + 102432768*a^5*b^4*c^14*d^20*e^3*z^4
+ 102113280*a^4*b^12*c^7*d^14*e^9*z^4 + 100674048*a^5*b^9*c^9*d^15*e^8*z^4
+ 90439680*a^13*b^6*c^4*d^2*e^21*z^4 - 86808576*a^6*b^14*c^3*d^8*e^15*z^4
+ 86245376*a^6*b^2*c^15*d^20*e^3*z^4 + 79011840*a^4*b^8*c^11*d^18*e^5*z^4 +
78345216*a^4*b^15*c^4*d^11*e^12*z^4 + 78006528*a^11*b^9*c^3*d^3*e^20*z^4 -
73253376*a^9*b^11*c^3*d^5*e^18*z^4 + 67524608*a^3*b^8*c^12*d^20*e^3*z^4 +
67108864*a^15*b^2*c^6*d^2*e^21*z^4 - 61590528*a^10*b^10*c^3*d^4*e^19*z^4 +
61559808*a^5*b^15*c^3*d^9*e^14*z^4 - 59637760*a^5*b^3*c^15*d^21*e^2*z^4 + 5
8638336*a^4*b^5*c^14*d^21*e^2*z^4 - 40828416*a^7*b^13*c^3*d^7*e^16*z^4 - 35
639296*a^2*b^12*c^9*d^18*e^5*z^4 - 31293440*a^12*b^8*c^3*d^2*e^21*z^4 + 299
33568*a^5*b^13*c^5*d^11*e^12*z^4 + 27793920*a^2*b^11*c^10*d^19*e^4*z^4 + 27
168768*a^2*b^13*c^8*d^17*e^6*z^4 - 23602176*a^7*b^14*c^2*d^6*e^17*z^4 - 232
48896*a^3*b^7*c^13*d^21*e^2*z^4 + 20929536*a^3*b^15*c^5*d^13*e^10*z^4 + 184
28928*a^9*b^12*c^2*d^4*e^19*z^4 + 18026496*a^6*b^15*c^2*d^7*e^16*z^4 - 1626
1632*a^10*b^11*c^2*d^3*e^20*z^4 + 15128064*a^3*b^16*c^4*d^12*e^11*z^4 - 140

60544*a^2*b^10*c^11*d^20*e^3*z^4 + 13178880*a^2*b^16*c^5*d^14*e^9*z^4 - 112
44288*a^3*b^17*c^3*d^11*e^12*z^4 - 10509312*a^2*b^15*c^6*d^15*e^8*z^4 - 726
2208*a^4*b^17*c^2*d^9*e^14*z^4 - 7045632*a^2*b^17*c^4*d^13*e^10*z^4 - 62853
12*a^2*b^14*c^7*d^16*e^7*z^4 + 5996544*a^11*b^10*c^2*d^2*e^21*z^4 + 4558336
*a^2*b^9*c^12*d^21*e^2*z^4 + 4478976*a^11*b^8*c^4*d^4*e^19*z^4 + 2850816*a^
4*b^16*c^3*d^10*e^13*z^4 + 2629632*a^3*b^11*c^9*d^17*e^6*z^4 + 2503680*a^3*
b^18*c^2*d^10*e^13*z^4 + 1627136*a^2*b^18*c^3*d^12*e^11*z^4 + 1605120*a^8*b
^13*c^2*d^5*e^18*z^4 + 1483776*a^5*b^16*c^2*d^8*e^15*z^4 + 139776*a^2*b^19*
c^2*d^11*e^12*z^4 - 8542224384*a^10*b^2*c^11*d^12*e^11*z^4 - 3072*b^22*c*d^
12*e^11*z^4 - 3072*b^12*c^11*d^22*e*z^4 - 1572864*a^6*c^17*d^22*e*z^4 - 409
6*a^10*b^13*d*e^22*z^4 - 4096*a*b^22*d^10*e^13*z^4 - 6144*a^12*b^10*c*e^23*
z^4 - 983040*a^5*b*c^17*d^23*z^4 - 6912*a*b^9*c^13*d^23*z^4 + 1824522240*a^
13*c^10*d^8*e^15*z^4 + 1730150400*a^12*c^11*d^10*e^13*z^4 + 958922752*a^14*
c^9*d^6*e^17*z^4 - 537919488*a^9*c^14*d^16*e^7*z^4 + 508559360*a^11*c^12*d^
12*e^11*z^4 - 500170752*a^10*c^13*d^14*e^9*z^4 + 246939648*a^15*c^8*d^4*e^1
9*z^4 - 199229440*a^8*c^15*d^18*e^5*z^4 - 29884416*a^7*c^16*d^20*e^3*z^4 +
25165824*a^16*c^7*d^2*e^21*z^4 + 236544*b^17*c^6*d^17*e^6*z^4 - 202752*b^18
*c^5*d^16*e^7*z^4 - 202752*b^16*c^7*d^18*e^5*z^4 + 126720*b^19*c^4*d^15*e^8
*z^4 + 126720*b^15*c^8*d^19*e^4*z^4 - 56320*b^20*c^3*d^14*e^9*z^4 - 56320*b
^14*c^9*d^20*e^3*z^4 + 16896*b^21*c^2*d^13*e^10*z^4 + 16896*b^13*c^10*d^21*
e^2*z^4 + 110080*a^7*b^16*d^4*e^19*z^4 + 110080*a^4*b^19*d^7*e^16*z^4 - 755
20*a^8*b^15*d^3*e^20*z^4 - 75520*a^3*b^20*d^8*e^15*z^4 - 56320*a^6*b^17*d^5
*e^18*z^4 - 56320*a^5*b^18*d^6*e^17*z^4 + 25600*a^9*b^14*d^2*e^21*z^4 + 256
00*a^2*b^21*d^9*e^14*z^4 - 1572864*a^16*b^2*c^5*e^23*z^4 + 983040*a^15*b^4*
c^4*e^23*z^4 - 327680*a^14*b^6*c^3*e^23*z^4 + 61440*a^13*b^8*c^2*e^23*z^4 +
983040*a^4*b^3*c^16*d^23*z^4 - 385024*a^3*b^5*c^15*d^23*z^4 + 73728*a^2*b^
7*c^14*d^23*z^4 + 256*b^23*d^11*e^12*z^4 + 1048576*a^17*c^6*e^23*z^4 + 256*
b^11*c^12*d^23*z^4 + 256*a^11*b^12*e^23*z^4 + 948695040*a^8*b*c^10*d^6*e^13
*z^2 + 348917760*a^7*b*c^11*d^8*e^11*z^2 - 125030400*a^9*b*c^9*d^4*e^15*z^2
- 50728960*a^6*b*c^12*d^10*e^9*z^2 - 44298240*a^5*b*c^13*d^12*e^7*z^2 - 36
495360*a^10*b*c^8*d^2*e^17*z^2 + 29675520*a^8*b^6*c^5*d*e^18*z^2 - 24170496
*a^9*b^4*c^6*d*e^18*z^2 - 17202816*a^7*b^8*c^4*d*e^18*z^2 - 14561280*a^4*b*
c^14*d^14*e^5*z^2 + 5532416*a^6*b^10*c^3*d*e^18*z^2 + 4128768*a^10*b^2*c^7*
d*e^18*z^2 - 2662400*a^3*b*c^15*d^16*e^3*z^2 + 1184512*a*b^12*c^6*d^9*e^10*
z^2 - 1136160*a*b^13*c^5*d^8*e^11*z^2 - 1017600*a^5*b^12*c^2*d*e^18*z^2 - 7
44768*a*b^11*c^7*d^10*e^9*z^2 + 607872*a*b^14*c^4*d^7*e^12*z^2 - 424064*a*b
^6*c^12*d^15*e^4*z^2 + 408576*a*b^5*c^13*d^16*e^3*z^2 + 361152*a*b^10*c^8*d
^11*e^8*z^2 - 287408*a*b^9*c^9*d^12*e^7*z^2 - 260448*a^3*b^15*c*d^2*e^17*z^
2 - 203904*a*b^4*c^14*d^17*e^2*z^2 + 200832*a*b^8*c^10*d^13*e^6*z^2 + 12672
0*a*b^7*c^11*d^14*e^5*z^2 - 123968*a*b^15*c^3*d^6*e^13*z^2 - 39168*a*b^16*c
^2*d^5*e^14*z^2 + 11904*a^2*b^16*c*d^3*e^16*z^2 + 1824135552*a^7*b^4*c^8*d^
5*e^14*z^2 - 1457252352*a^8*b^2*c^9*d^5*e^14*z^2 - 1405209600*a^7*b^5*c^7*d
^4*e^15*z^2 - 184320*a^2*b*c^16*d^18*e*z^2 + 100608*a^4*b^14*c*d*e^18*z^2 +
53248*a*b^3*c^15*d^18*e*z^2 + 26448*a*b^17*c*d^4*e^15*z^2 + 1067599872*a^8
*b^3*c^8*d^4*e^15*z^2 - 930828288*a^7*b^3*c^9*d^6*e^13*z^2 + 920760000*a^6*
b^4*c^9*d^7*e^12*z^2 - 806639616*a^6*b^3*c^10*d^8*e^11*z^2 - 791052480*a^6*
b^6*c^7*d^5*e^14*z^2 + 772237824*a^6*b^7*c^6*d^4*e^15*z^2 - 701025408*a^5*b
^6*c^8*d^7*e^12*z^2 + 443340288*a^5*b^5*c^9*d^8*e^11*z^2 + 433047552*a^7*b^
6*c^6*d^3*e^16*z^2 + 405741312*a^5*b^7*c^7*d^6*e^13*z^2 + 293652480*a^6*b^2
*c^11*d^9*e^10*z^2 - 276962688*a^6*b^8*c^5*d^3*e^16*z^2 - 247804272*a^8*b^4
*c^7*d^3*e^16*z^2 + 213564384*a^4*b^8*c^7*d^7*e^12*z^2 - 202596816*a^5*b^9*
c^5*d^4*e^15*z^2 - 182520896*a^4*b^9*c^6*d^6*e^13*z^2 - 153489408*a^5*b^3*c
^11*d^10*e^9*z^2 - 152151552*a^7*b^2*c^10*d^7*e^12*z^2 + 115859712*a^5*b^2*
c^12*d^11*e^8*z^2 + 108085248*a^9*b^3*c^7*d^2*e^17*z^2 + 105536256*a^4*b^5*
c^10*d^10*e^9*z^2 - 98323200*a^6*b^5*c^8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^
9*d^9*e^10*z^2 + 89464512*a^5*b^10*c^4*d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*
d^2*e^17*z^2 + 68315904*a^5*b^8*c^6*d^5*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8
*e^11*z^2 - 62951040*a^9*b^2*c^8*d^3*e^16*z^2 + 49056768*a^4*b^10*c^5*d^5*e
^14*z^2 + 47614464*a^3*b^8*c^8*d^9*e^10*z^2 + 35604480*a^4*b^2*c^13*d^13*e^

6*z^2 + 33983040*a^3*b^11*c^5*d^6*e^13*z^2 - 33515520*a^4*b^3*c^12*d^12*e^7
*z^2 - 33463808*a^3*b^7*c^9*d^10*e^9*z^2 - 25128864*a^4*b^4*c^11*d^11*e^8*z
^2 - 23193728*a^3*b^10*c^6*d^7*e^12*z^2 + 21015456*a^6*b^9*c^4*d^2*e^17*z^2
 + 19924176*a^4*b^11*c^4*d^4*e^15*z^2 - 19251216*a^3*b^9*c^7*d^8*e^11*z^2 -
 16434048*a^5*b^4*c^10*d^9*e^10*z^2 - 16289664*a^3*b^12*c^4*d^5*e^14*z^2 -
15059328*a^4*b^12*c^3*d^3*e^16*z^2 - 10766016*a^2*b^10*c^7*d^9*e^10*z^2 - 1
0453632*a^5*b^11*c^3*d^2*e^17*z^2 - 9940992*a^3*b^3*c^13*d^14*e^5*z^2 + 837
3696*a^2*b^11*c^6*d^8*e^11*z^2 + 7776768*a^3*b^2*c^14*d^15*e^4*z^2 + 707788
8*a^3*b^5*c^11*d^12*e^7*z^2 + 6798240*a^2*b^9*c^8*d^10*e^9*z^2 - 3589440*a^
2*b^6*c^11*d^13*e^6*z^2 + 3544320*a^3*b^6*c^10*d^11*e^8*z^2 + 3128064*a^2*b
^5*c^12*d^14*e^5*z^2 + 2346336*a^4*b^13*c^2*d^2*e^17*z^2 - 2261568*a^2*b^8*
c^9*d^11*e^8*z^2 - 2125824*a^2*b^13*c^4*d^6*e^13*z^2 + 2002560*a^3*b^4*c^12
*d^13*e^6*z^2 + 1927680*a^2*b^7*c^10*d^12*e^7*z^2 + 1814784*a^2*b^14*c^3*d^
5*e^14*z^2 - 1807104*a^2*b^12*c^5*d^7*e^12*z^2 + 1637808*a^3*b^13*c^3*d^4*e
^15*z^2 + 1083456*a^3*b^14*c^2*d^3*e^16*z^2 - 792384*a^2*b^4*c^13*d^15*e^4*
z^2 - 657408*a^2*b^3*c^14*d^16*e^3*z^2 + 608256*a^7*b^7*c^5*d^2*e^17*z^2 +
595968*a^2*b^2*c^15*d^17*e^2*z^2 - 498624*a^2*b^15*c^2*d^4*e^15*z^2 - 3840*
b^18*c*d^5*e^14*z^2 - 3840*b^5*c^14*d^18*e*z^2 + 2064384*a^11*c^8*d*e^18*z^
2 - 4160*a^3*b^16*d*e^18*z^2 - 4160*a*b^18*d^3*e^16*z^2 - 1290240*a^11*b*c^
7*e^19*z^2 - 9840*a^5*b^13*c*e^19*z^2 - 5760*a*b^2*c^16*d^19*z^2 - 28058112
0*a^8*c^11*d^7*e^12*z^2 + 110278656*a^9*c^10*d^5*e^14*z^2 - 89479168*a^7*c^
12*d^9*e^10*z^2 + 34464000*a^10*c^9*d^3*e^16*z^2 + 54240*b^15*c^4*d^8*e^11*
z^2 + 54240*b^8*c^11*d^15*e^4*z^2 - 49920*b^14*c^5*d^9*e^10*z^2 - 49920*b^9
*c^10*d^14*e^5*z^2 - 37376*b^16*c^3*d^7*e^12*z^2 - 37376*b^7*c^12*d^16*e^3*
z^2 + 28480*b^13*c^6*d^10*e^9*z^2 + 28480*b^10*c^9*d^13*e^6*z^2 + 15936*b^1
7*c^2*d^6*e^13*z^2 + 15936*b^6*c^13*d^17*e^2*z^2 - 7920*b^12*c^7*d^11*e^8*z
^2 - 7920*b^11*c^8*d^12*e^7*z^2 + 7489536*a^5*c^14*d^13*e^6*z^2 + 6084096*a
^6*c^13*d^11*e^8*z^2 + 2280448*a^4*c^15*d^15*e^4*z^2 + 350208*a^3*c^16*d^17
*e^2*z^2 + 11616*a^2*b^17*d^2*e^17*z^2 - 3515904*a^9*b^5*c^5*e^19*z^2 + 344
0640*a^10*b^3*c^6*e^19*z^2 + 1870848*a^8*b^7*c^4*e^19*z^2 - 572272*a^7*b^9*
c^3*e^19*z^2 + 101856*a^6*b^11*c^2*e^19*z^2 + 400*b^19*d^4*e^15*z^2 + 400*b
^4*c^15*d^19*z^2 + 20736*a^2*c^17*d^19*z^2 + 400*a^4*b^15*e^19*z^2 - 396921
6*a^4*b*c^10*d^3*e^12 - 3001536*a^3*b*c^11*d^5*e^10 - 419904*a^2*b*c^12*d^7
*e^8 + 184608*a^4*b^3*c^8*d*e^14 - 153036*a*b^4*c^10*d^6*e^9 + 127008*a*b^3
*c^11*d^7*e^8 + 63108*a*b^6*c^8*d^4*e^11 - 29160*a*b^2*c^12*d^8*e^7 - 21060
*a^3*b^5*c^7*d*e^14 - 21060*a*b^7*c^7*d^3*e^12 + 5460*a*b^5*c^9*d^5*e^10 -
404544*a^5*b*c^9*d*e^14 + 1251872*a^3*b^3*c^9*d^3*e^12 + 844224*a^4*b^2*c^9
*d^2*e^13 + 820512*a^2*b^3*c^10*d^5*e^10 + 750672*a^3*b^2*c^10*d^4*e^11 - 6
57498*a^2*b^4*c^9*d^4*e^11 - 487116*a^3*b^4*c^8*d^2*e^13 + 160704*a^2*b^2*c
^11*d^6*e^9 + 58806*a^2*b^6*c^7*d^2*e^13 + 13140*a^2*b^5*c^8*d^3*e^12 + 152
86*b^6*c^9*d^6*e^9 - 9540*b^7*c^8*d^5*e^10 - 9540*b^5*c^10*d^7*e^8 + 2025*b
^8*c^7*d^4*e^11 + 2025*b^4*c^11*d^8*e^7 + 3367008*a^4*c^11*d^4*e^11 + 11664
00*a^3*c^12*d^6*e^9 + 705600*a^5*c^10*d^2*e^13 + 104976*a^2*c^13*d^8*e^7 -
17640*a^5*b^2*c^8*e^15 + 2025*a^4*b^4*c^7*e^15 + 38416*a^6*c^9*e^15, z, k)*
x*(1048576*a^8*c^19*d^24*e^3 + 9437184*a^9*c^18*d^22*e^5 + 36700160*a^10*c^
17*d^20*e^7 + 78643200*a^11*c^16*d^18*e^9 + 94371840*a^12*c^15*d^16*e^11 +
44040192*a^13*c^14*d^14*e^13 - 44040192*a^14*c^13*d^12*e^15 - 94371840*a^15
*c^12*d^10*e^17 - 78643200*a^16*c^11*d^8*e^19 - 36700160*a^17*c^10*d^6*e^21
 - 9437184*a^18*c^9*d^4*e^23 - 1048576*a^19*c^8*d^2*e^25 - 256*a^2*b^11*c^1
4*d^25*e^2 + 3072*a^2*b^12*c^13*d^24*e^3 - 16896*a^2*b^13*c^12*d^23*e^4 + 5
6320*a^2*b^14*c^11*d^22*e^5 - 126720*a^2*b^15*c^10*d^21*e^6 + 202752*a^2*b^
16*c^9*d^20*e^7 - 236544*a^2*b^17*c^8*d^19*e^8 + 202752*a^2*b^18*c^7*d^18*e
^9 - 126720*a^2*b^19*c^6*d^17*e^10 + 56320*a^2*b^20*c^5*d^16*e^11 - 16896*a
^2*b^21*c^4*d^15*e^12 + 3072*a^2*b^22*c^3*d^14*e^13 - 256*a^2*b^23*c^2*d^13
*e^14 + 5120*a^3*b^9*c^15*d^25*e^2 - 62464*a^3*b^10*c^14*d^24*e^3 + 346368*
a^3*b^11*c^13*d^23*e^4 - 1152256*a^3*b^12*c^12*d^22*e^5 + 2553600*a^3*b^13*
c^11*d^21*e^6 - 3951360*a^3*b^14*c^10*d^20*e^7 + 4336128*a^3*b^15*c^9*d^19*
e^8 - 3334656*a^3*b^16*c^8*d^18*e^9 + 1700352*a^3*b^17*c^7*d^17*e^10 - 4736
00*a^3*b^18*c^6*d^16*e^11 - 8960*a^3*b^19*c^5*d^15*e^12 + 59136*a^3*b^20*c^

4*d^14*e^13 - 19712*a^3*b^21*c^3*d^13*e^14 + 2304*a^3*b^22*c^2*d^12*e^15 -
40960*a^4*b^7*c^16*d^25*e^2 + 512000*a^4*b^8*c^15*d^24*e^3 - 2872320*a^4*b^
9*c^14*d^23*e^4 + 9519104*a^4*b^10*c^13*d^22*e^5 - 20581120*a^4*b^11*c^12*d
^21*e^6 + 30087680*a^4*b^12*c^11*d^20*e^7 - 29433600*a^4*b^13*c^10*d^19*e^8
 + 17602560*a^4*b^14*c^9*d^18*e^9 - 3798528*a^4*b^15*c^8*d^17*e^10 - 307712
0*a^4*b^16*c^7*d^16*e^11 + 3028480*a^4*b^17*c^6*d^15*e^12 - 1075200*a^4*b^1
8*c^5*d^14*e^13 + 98560*a^4*b^19*c^4*d^13*e^14 + 39424*a^4*b^20*c^3*d^12*e^
15 - 8960*a^4*b^21*c^2*d^11*e^16 + 163840*a^5*b^5*c^17*d^25*e^2 - 2129920*a
^5*b^6*c^16*d^24*e^3 + 12165120*a^5*b^7*c^15*d^23*e^4 - 39997440*a^5*b^8*c^
14*d^22*e^5 + 82611200*a^5*b^9*c^13*d^21*e^6 - 107627520*a^5*b^10*c^12*d^20
*e^7 + 78140160*a^5*b^11*c^11*d^19*e^8 - 6831360*a^5*b^12*c^10*d^18*e^9 - 4
6586880*a^5*b^13*c^9*d^17*e^10 + 47436800*a^5*b^14*c^8*d^16*e^11 - 20088320
*a^5*b^15*c^7*d^15*e^12 + 1128960*a^5*b^16*c^6*d^14*e^13 + 2365440*a^5*b^17
*c^5*d^13*e^14 - 788480*a^5*b^18*c^4*d^12*e^15 + 19200*a^5*b^19*c^3*d^11*e^
16 + 19200*a^5*b^20*c^2*d^10*e^17 - 327680*a^6*b^3*c^18*d^25*e^2 + 4587520*
a^6*b^4*c^17*d^24*e^3 - 27033600*a^6*b^5*c^16*d^23*e^4 + 87162880*a^6*b^6*c
^15*d^22*e^5 - 161996800*a^6*b^7*c^14*d^21*e^6 + 149237760*a^6*b^8*c^13*d^2
0*e^7 + 27202560*a^6*b^9*c^12*d^19*e^8 - 251750400*a^6*b^10*c^11*d^18*e^9 +
 305948160*a^6*b^11*c^10*d^17*e^10 - 160153600*a^6*b^12*c^9*d^16*e^11 + 143
360*a^6*b^13*c^8*d^15*e^12 + 46018560*a^6*b^14*c^7*d^14*e^13 - 21683200*a^6
*b^15*c^6*d^13*e^14 + 1576960*a^6*b^16*c^5*d^12*e^15 + 1305600*a^6*b^17*c^4
*d^11*e^16 - 215040*a^6*b^18*c^3*d^10*e^17 - 23040*a^6*b^19*c^2*d^9*e^18 -
4456448*a^7*b^2*c^18*d^24*e^3 + 28114944*a^7*b^3*c^17*d^23*e^4 - 84869120*a
^7*b^4*c^16*d^22*e^5 + 104366080*a^7*b^5*c^15*d^21*e^6 + 97943552*a^7*b^6*c
^14*d^20*e^7 - 549986304*a^7*b^7*c^13*d^19*e^8 + 841961472*a^7*b^8*c^12*d^1
8*e^9 - 549795840*a^7*b^9*c^11*d^17*e^10 - 68823040*a^7*b^10*c^10*d^16*e^11
 + 375613952*a^7*b^11*c^9*d^15*e^12 - 240167424*a^7*b^12*c^8*d^14*e^13 + 32
840192*a^7*b^13*c^7*d^13*e^14 + 27399680*a^7*b^14*c^6*d^12*e^15 - 10703360*
a^7*b^15*c^5*d^11*e^16 - 81408*a^7*b^16*c^4*d^10*e^17 + 370176*a^7*b^17*c^3
*d^9*e^18 + 10752*a^7*b^18*c^2*d^8*e^19 + 14680064*a^8*b^2*c^17*d^22*e^5 +
80281600*a^8*b^3*c^16*d^21*e^6 - 440401920*a^8*b^4*c^15*d^20*e^7 + 88837324
8*a^8*b^5*c^14*d^19*e^8 - 703266816*a^8*b^6*c^13*d^18*e^9 - 394149888*a^8*b
^7*c^12*d^17*e^10 + 1358438400*a^8*b^8*c^11*d^16*e^11 - 1129891840*a^8*b^9*
c^10*d^15*e^12 + 225189888*a^8*b^10*c^9*d^14*e^13 + 246045184*a^8*b^11*c^8*
d^13*e^14 - 164082688*a^8*b^12*c^7*d^12*e^15 + 18009600*a^8*b^13*c^6*d^11*e
^16 + 10659840*a^8*b^14*c^5*d^10*e^17 - 2099712*a^8*b^15*c^4*d^9*e^18 - 193
536*a^8*b^16*c^3*d^8*e^19 + 10752*a^8*b^17*c^2*d^7*e^20 + 239861760*a^9*b^2
*c^16*d^20*e^7 - 172032000*a^9*b^3*c^15*d^19*e^8 - 704839680*a^9*b^4*c^14*d
^18*e^9 + 2013069312*a^9*b^5*c^13*d^17*e^10 - 2086993920*a^9*b^6*c^12*d^16*
e^11 + 424427520*a^9*b^7*c^11*d^15*e^12 + 1074585600*a^9*b^8*c^10*d^14*e^13
 - 997877760*a^9*b^9*c^9*d^13*e^14 + 234493952*a^9*b^10*c^8*d^12*e^15 + 957
61920*a^9*b^11*c^7*d^11*e^16 - 55288320*a^9*b^12*c^6*d^10*e^17 + 3916800*a^
9*b^13*c^5*d^9*e^18 + 1704960*a^9*b^14*c^4*d^8*e^19 - 250368*a^9*b^15*c^3*d
^7*e^20 - 23040*a^9*b^16*c^2*d^6*e^21 + 857210880*a^10*b^2*c^15*d^18*e^9 -
1036124160*a^10*b^3*c^14*d^17*e^10 - 255590400*a^10*b^4*c^13*d^16*e^11 + 21
95128320*a^10*b^5*c^12*d^15*e^12 - 2422210560*a^10*b^6*c^11*d^14*e^13 + 813
711360*a^10*b^7*c^10*d^13*e^14 + 420372480*a^10*b^8*c^9*d^12*e^15 - 4285952
00*a^10*b^9*c^8*d^11*e^16 + 106106880*a^10*b^10*c^7*d^10*e^17 + 8866560*a^1
0*b^11*c^6*d^9*e^18 - 11074560*a^10*b^12*c^5*d^8*e^19 + 1989120*a^10*b^13*c
^4*d^7*e^20 + 537600*a^10*b^14*c^3*d^6*e^21 + 19200*a^10*b^15*c^2*d^5*e^22
+ 1454899200*a^11*b^2*c^14*d^16*e^11 - 1747845120*a^11*b^3*c^13*d^15*e^12 +
 454164480*a^11*b^4*c^12*d^14*e^13 + 1135411200*a^11*b^5*c^11*d^13*e^14 - 1
286799360*a^11*b^6*c^10*d^12*e^15 + 527155200*a^11*b^7*c^9*d^11*e^16 - 4190
2080*a^11*b^8*c^8*d^10*e^17 - 74849280*a^11*b^9*c^7*d^9*e^18 + 53222400*a^1
1*b^10*c^6*d^8*e^19 - 4023040*a^11*b^11*c^5*d^7*e^20 - 4972800*a^11*b^12*c^
4*d^6*e^21 - 456960*a^11*b^13*c^3*d^5*e^22 - 8960*a^11*b^14*c^2*d^4*e^23 +
1189085184*a^12*b^2*c^13*d^14*e^13 - 1241382912*a^12*b^3*c^12*d^13*e^14 + 6
05552640*a^12*b^4*c^11*d^12*e^15 - 97320960*a^12*b^5*c^10*d^11*e^16 - 14273
7408*a^12*b^6*c^9*d^10*e^17 + 278716416*a^12*b^7*c^8*d^9*e^18 - 144764928*a

^12*b^8*c^7*d^8*e^19 - 28779520*a^12*b^9*c^6*d^7*e^20 + 22077440*a^12*b^10*
c^5*d^6*e^21 + 4456704*a^12*b^11*c^4*d^5*e^22 + 215552*a^12*b^12*c^3*d^4*e^
23 + 2304*a^12*b^13*c^2*d^3*e^24 + 121110528*a^13*b^2*c^12*d^12*e^15 - 1081
34400*a^13*b^3*c^11*d^11*e^16 + 454164480*a^13*b^4*c^10*d^10*e^17 - 5871697
92*a^13*b^5*c^9*d^9*e^18 + 98402304*a^13*b^6*c^8*d^8*e^19 + 184819712*a^13*
b^7*c^7*d^7*e^20 - 39424000*a^13*b^8*c^6*d^6*e^21 - 22471680*a^13*b^9*c^5*d
^5*e^22 - 2151424*a^13*b^10*c^4*d^4*e^23 - 55552*a^13*b^11*c^3*d^3*e^24 - 2
56*a^13*b^12*c^2*d^2*e^25 - 644874240*a^14*b^2*c^11*d^10*e^17 + 339148800*a
^14*b^3*c^10*d^9*e^18 + 371589120*a^14*b^4*c^9*d^8*e^19 - 367689728*a^14*b^
5*c^8*d^7*e^20 - 32112640*a^14*b^6*c^7*d^6*e^21 + 59351040*a^14*b^7*c^6*d^5
*e^22 + 11366400*a^14*b^8*c^5*d^4*e^23 + 558080*a^14*b^9*c^4*d^3*e^24 + 614
4*a^14*b^10*c^3*d^2*e^25 - 578027520*a^15*b^2*c^10*d^8*e^19 + 135331840*a^1
5*b^3*c^9*d^7*e^20 + 217907200*a^15*b^4*c^8*d^6*e^21 - 65372160*a^15*b^5*c^
7*d^5*e^22 - 33259520*a^15*b^6*c^6*d^4*e^23 - 2990080*a^15*b^7*c^5*d^3*e^24
 - 61440*a^15*b^8*c^4*d^2*e^25 - 209715200*a^16*b^2*c^9*d^6*e^21 - 20643840
*a^16*b^3*c^8*d^5*e^22 + 49807360*a^16*b^4*c^7*d^4*e^23 + 9011200*a^16*b^5*
c^6*d^3*e^24 + 327680*a^16*b^6*c^5*d^2*e^25 - 25427968*a^17*b^2*c^8*d^4*e^2
3 - 14483456*a^17*b^3*c^7*d^3*e^24 - 983040*a^17*b^4*c^6*d^2*e^25 + 1572864
*a^18*b^2*c^7*d^2*e^25 + 262144*a^7*b*c^19*d^25*e^2 - 8650752*a^8*b*c^18*d^
23*e^4 - 79953920*a^9*b*c^17*d^21*e^6 - 287047680*a^10*b*c^16*d^19*e^8 - 54
2638080*a^11*b*c^15*d^17*e^10 - 539492352*a^12*b*c^14*d^15*e^12 - 143130624
*a^13*b*c^13*d^13*e^14 + 306708480*a^14*b*c^12*d^11*e^16 + 420741120*a^15*b
*c^11*d^9*e^18 + 250347520*a^16*b*c^10*d^7*e^20 + 76283904*a^17*b*c^9*d^5*e
^22 + 9699328*a^18*b*c^8*d^3*e^24))/(8*(16*a^3*b^6*c^9*d^18 - a^2*b^8*c^8*d
^18 - 256*a^6*c^12*d^18 - 96*a^4*b^4*c^10*d^18 + 256*a^5*b^2*c^11*d^18 - a^
2*b^16*d^10*e^8 + 8*a^3*b^15*d^9*e^9 - 28*a^4*b^14*d^8*e^10 + 56*a^5*b^13*d
^7*e^11 - 70*a^6*b^12*d^6*e^12 + 56*a^7*b^11*d^5*e^13 - 28*a^8*b^10*d^4*e^1
4 + 8*a^9*b^9*d^3*e^15 - a^10*b^8*d^2*e^16 - 2048*a^7*c^11*d^16*e^2 - 7168*
a^8*c^10*d^14*e^4 - 14336*a^9*c^9*d^12*e^6 - 17920*a^10*c^8*d^10*e^8 - 1433
6*a^11*c^7*d^8*e^10 - 7168*a^12*c^6*d^6*e^12 - 2048*a^13*c^5*d^4*e^14 - 256
*a^14*c^4*d^2*e^16 - 28*a^2*b^10*c^6*d^16*e^2 + 56*a^2*b^11*c^5*d^15*e^3 -
70*a^2*b^12*c^4*d^14*e^4 + 56*a^2*b^13*c^3*d^13*e^5 - 28*a^2*b^14*c^2*d^12*
e^6 + 440*a^3*b^8*c^7*d^16*e^2 - 840*a^3*b^9*c^6*d^15*e^3 + 952*a^3*b^10*c^
5*d^14*e^4 - 616*a^3*b^11*c^4*d^13*e^5 + 168*a^3*b^12*c^3*d^12*e^6 + 40*a^3
*b^13*c^2*d^11*e^7 - 2560*a^4*b^6*c^8*d^16*e^2 + 4480*a^4*b^7*c^7*d^15*e^3
 - 4060*a^4*b^8*c^6*d^14*e^4 + 1064*a^4*b^9*c^5*d^13*e^5 + 1372*a^4*b^10*c^4
*d^12*e^6 - 1360*a^4*b^11*c^3*d^11*e^7 + 380*a^4*b^12*c^2*d^10*e^8 + 6400*a
^5*b^4*c^9*d^16*e^2 - 8960*a^5*b^5*c^8*d^15*e^3 + 2240*a^5*b^6*c^7*d^14*e^4
 + 9856*a^5*b^7*c^6*d^13*e^5 - 13048*a^5*b^8*c^5*d^12*e^6 + 5400*a^5*b^9*c^
4*d^11*e^7 + 1040*a^5*b^10*c^3*d^10*e^8 - 1360*a^5*b^11*c^2*d^9*e^9 - 5120*
a^6*b^2*c^10*d^16*e^2 + 22400*a^6*b^4*c^8*d^14*e^4 - 41216*a^6*b^5*c^7*d^13
*e^5 + 25088*a^6*b^6*c^6*d^12*e^6 + 8320*a^6*b^7*c^5*d^11*e^7 - 17350*a^6*b
^8*c^4*d^10*e^8 + 5400*a^6*b^9*c^3*d^9*e^9 + 1372*a^6*b^10*c^2*d^8*e^10 - 3
5840*a^7*b^2*c^9*d^14*e^4 + 28672*a^7*b^3*c^8*d^13*e^5 + 30464*a^7*b^4*c^7*
d^12*e^6 - 73472*a^7*b^5*c^6*d^11*e^7 + 40544*a^7*b^6*c^5*d^10*e^8 + 8320*a
^7*b^7*c^4*d^9*e^9 - 13048*a^7*b^8*c^3*d^8*e^10 + 1064*a^7*b^9*c^2*d^7*e^11
 - 93184*a^8*b^2*c^8*d^12*e^6 + 71680*a^8*b^3*c^7*d^11*e^7 + 29120*a^8*b^4*
c^6*d^10*e^8 - 73472*a^8*b^5*c^5*d^9*e^9 + 25088*a^8*b^6*c^4*d^8*e^10 + 985
6*a^8*b^7*c^3*d^7*e^11 - 4060*a^8*b^8*c^2*d^6*e^12 - 125440*a^9*b^2*c^7*d^1
0*e^8 + 71680*a^9*b^3*c^6*d^9*e^9 + 30464*a^9*b^4*c^5*d^8*e^10 - 41216*a^9*
b^5*c^4*d^7*e^11 + 2240*a^9*b^6*c^3*d^6*e^12 + 4480*a^9*b^7*c^2*d^5*e^13 -
93184*a^10*b^2*c^6*d^8*e^10 + 28672*a^10*b^3*c^5*d^7*e^11 + 22400*a^10*b^4*
c^4*d^6*e^12 - 8960*a^10*b^5*c^3*d^5*e^13 - 2560*a^10*b^6*c^2*d^4*e^14 - 35
840*a^11*b^2*c^5*d^6*e^12 + 6400*a^11*b^4*c^3*d^4*e^14 + 768*a^11*b^5*c^2*d
^3*e^15 - 5120*a^12*b^2*c^4*d^4*e^14 - 2048*a^12*b^3*c^3*d^3*e^15 - 96*a^12
*b^4*c^2*d^2*e^16 + 256*a^13*b^2*c^3*d^2*e^16 + 2048*a^6*b*c^11*d^17*e + 8*
a^2*b^9*c^7*d^17*e + 8*a^2*b^15*c*d^11*e^7 - 128*a^3*b^7*c^8*d^17*e - 40*a^
3*b^14*c*d^10*e^8 + 768*a^4*b^5*c^9*d^17*e + 40*a^4*b^13*c*d^9*e^9 - 2048*a
^5*b^3*c^10*d^17*e + 168*a^5*b^12*c*d^8*e^10 - 616*a^6*b^11*c*d^7*e^11 + 14

336*a^7*b*c^10*d^15*e^3 + 952*a^7*b^10*c*d^6*e^12 + 43008*a^8*b*c^9*d^13*e^
5 - 840*a^8*b^9*c*d^5*e^13 + 71680*a^9*b*c^8*d^11*e^7 + 440*a^9*b^8*c*d^4*e
^14 + 71680*a^10*b*c^7*d^9*e^9 - 128*a^10*b^7*c*d^3*e^15 + 43008*a^11*b*c^6
*d^7*e^11 + 16*a^11*b^6*c*d^2*e^16 + 14336*a^12*b*c^5*d^5*e^13 + 2048*a^13*
b*c^4*d^3*e^15))) - (x*(49152*a^14*b*c^8*e^23 - 65536*a^14*c^9*d*e^22 + 16*
a^8*b^13*c^2*e^23 - 368*a^9*b^11*c^3*e^23 + 3520*a^10*b^9*c^4*e^23 - 17920*
a^11*b^7*c^5*e^23 + 51200*a^12*b^5*c^6*e^23 - 77824*a^13*b^3*c^7*e^23 + 184
32*a^4*c^19*d^21*e^2 + 243712*a^5*c^18*d^19*e^4 + 1253376*a^6*c^17*d^17*e^6
 + 2252800*a^7*c^16*d^15*e^8 - 7835648*a^8*c^15*d^13*e^10 - 35516416*a^9*c^
14*d^11*e^12 - 50487296*a^10*c^13*d^9*e^14 - 30416896*a^11*c^12*d^7*e^16 -
5797888*a^12*c^11*d^5*e^18 + 522240*a^13*c^10*d^3*e^20 + 16*b^8*c^15*d^21*e
^2 - 160*b^9*c^14*d^20*e^3 + 720*b^10*c^13*d^19*e^4 - 1904*b^11*c^12*d^18*e
^5 + 3200*b^12*c^11*d^17*e^6 - 3312*b^13*c^10*d^16*e^7 + 1440*b^14*c^9*d^15
*e^8 + 1440*b^15*c^8*d^14*e^9 - 3312*b^16*c^7*d^13*e^10 + 3200*b^17*c^6*d^1
2*e^11 - 1904*b^18*c^5*d^11*e^12 + 720*b^19*c^4*d^10*e^13 - 160*b^20*c^3*d^
9*e^14 + 16*b^21*c^2*d^8*e^15 + 3200*a^2*b^4*c^17*d^21*e^2 - 30336*a^2*b^5*
c^16*d^20*e^3 + 123296*a^2*b^6*c^15*d^19*e^4 - 269568*a^2*b^7*c^14*d^18*e^5
 + 295872*a^2*b^8*c^13*d^17*e^6 + 16576*a^2*b^9*c^12*d^16*e^7 - 582688*a^2*
b^10*c^11*d^15*e^8 + 944640*a^2*b^11*c^10*d^14*e^9 - 761856*a^2*b^12*c^9*d^
13*e^10 + 243456*a^2*b^13*c^8*d^12*e^11 + 126048*a^2*b^14*c^7*d^11*e^12 - 1
64096*a^2*b^15*c^6*d^10*e^13 + 58304*a^2*b^16*c^5*d^9*e^14 + 3264*a^2*b^17*
c^4*d^8*e^15 - 7648*a^2*b^18*c^3*d^7*e^16 + 1536*a^2*b^19*c^2*d^6*e^17 - 12
800*a^3*b^2*c^18*d^21*e^2 + 119296*a^3*b^3*c^17*d^20*e^3 - 448896*a^3*b^4*c
^16*d^19*e^4 + 783872*a^3*b^5*c^15*d^18*e^5 - 197504*a^3*b^6*c^14*d^17*e^6
- 1977216*a^3*b^7*c^13*d^16*e^7 + 4413568*a^3*b^8*c^12*d^15*e^8 - 4435520*a
^3*b^9*c^11*d^14*e^9 + 1422432*a^3*b^10*c^10*d^13*e^10 + 1795872*a^3*b^11*c
^9*d^12*e^11 - 2349888*a^3*b^12*c^8*d^11*e^12 + 800352*a^3*b^13*c^7*d^10*e^
13 + 426688*a^3*b^14*c^6*d^9*e^14 - 478112*a^3*b^15*c^5*d^8*e^15 + 145344*a
^3*b^16*c^4*d^7*e^16 - 3104*a^3*b^17*c^3*d^6*e^17 - 4384*a^3*b^18*c^2*d^5*e
^18 + 519680*a^4*b^2*c^17*d^19*e^4 - 122880*a^4*b^3*c^16*d^18*e^5 - 3229184
*a^4*b^4*c^15*d^17*e^6 + 9323008*a^4*b^5*c^14*d^16*e^7 - 11702656*a^4*b^6*c
^13*d^15*e^8 + 3460864*a^4*b^7*c^12*d^14*e^9 + 10917472*a^4*b^8*c^11*d^13*e
^10 - 16615488*a^4*b^9*c^10*d^12*e^11 + 7102272*a^4*b^10*c^9*d^11*e^12 + 58
42272*a^4*b^11*c^8*d^10*e^13 - 8942080*a^4*b^12*c^7*d^9*e^14 + 4203232*a^4*
b^13*c^6*d^8*e^15 - 364736*a^4*b^14*c^5*d^7*e^16 - 309472*a^4*b^15*c^4*d^6*
e^17 + 63136*a^4*b^16*c^3*d^5*e^18 + 6112*a^4*b^17*c^2*d^4*e^19 + 6961152*a
^5*b^2*c^16*d^17*e^6 - 10246144*a^5*b^3*c^15*d^16*e^7 - 747008*a^5*b^4*c^14
*d^15*e^8 + 29979648*a^5*b^5*c^13*d^14*e^9 - 52869952*a^5*b^6*c^12*d^13*e^1
0 + 32791616*a^5*b^7*c^11*d^12*e^11 + 25176960*a^5*b^8*c^10*d^11*e^12 - 629
55552*a^5*b^9*c^9*d^10*e^13 + 45989472*a^5*b^10*c^8*d^9*e^14 - 9362688*a^5*
b^11*c^7*d^8*e^15 - 5824480*a^5*b^12*c^6*d^7*e^16 + 3196768*a^5*b^13*c^5*d^
6*e^17 - 132768*a^5*b^14*c^4*d^5*e^18 - 119680*a^5*b^15*c^3*d^4*e^19 - 4384
*a^5*b^16*c^2*d^3*e^20 + 32086016*a^6*b^2*c^15*d^15*e^8 - 57880576*a^6*b^3*
c^14*d^14*e^9 + 44683008*a^6*b^4*c^13*d^13*e^10 + 49481984*a^6*b^5*c^12*d^1
2*e^11 - 175788864*a^6*b^6*c^11*d^11*e^12 + 194611968*a^6*b^7*c^10*d^10*e^1
3 - 73867584*a^6*b^8*c^9*d^9*e^14 - 38225280*a^6*b^9*c^8*d^8*e^15 + 4545014
4*a^6*b^10*c^7*d^7*e^16 - 10588672*a^6*b^11*c^6*d^6*e^17 - 2519296*a^6*b^12
*c^5*d^5*e^18 + 864384*a^6*b^13*c^4*d^4*e^19 + 96224*a^6*b^14*c^3*d^3*e^20
+ 1536*a^6*b^15*c^2*d^2*e^21 + 67527680*a^7*b^2*c^14*d^13*e^10 - 181466112*
a^7*b^3*c^13*d^12*e^11 + 278696704*a^7*b^4*c^12*d^11*e^12 - 171431936*a^7*b
^5*c^11*d^10*e^13 - 104909184*a^7*b^6*c^10*d^9*e^14 + 231100032*a^7*b^7*c^9
*d^8*e^15 - 116105856*a^7*b^8*c^8*d^7*e^16 - 5653568*a^7*b^9*c^7*d^6*e^17 +
 19556768*a^7*b^10*c^6*d^5*e^18 - 2291488*a^7*b^11*c^5*d^4*e^19 - 855936*a^
7*b^12*c^4*d^3*e^20 - 35168*a^7*b^13*c^3*d^2*e^21 - 40418304*a^8*b^2*c^13*d
^11*e^12 - 155127808*a^8*b^3*c^12*d^10*e^13 + 421659136*a^8*b^4*c^11*d^9*e^
14 - 366294528*a^8*b^5*c^10*d^8*e^15 + 42953856*a^8*b^6*c^9*d^7*e^16 + 1158
41280*a^8*b^7*c^8*d^6*e^17 - 54301680*a^8*b^8*c^7*d^5*e^18 - 3139616*a^8*b^
9*c^6*d^4*e^19 + 3850352*a^8*b^10*c^5*d^3*e^20 + 333840*a^8*b^11*c^4*d^2*e^
21 - 262465536*a^9*b^2*c^12*d^9*e^14 + 49444864*a^9*b^3*c^11*d^8*e^15 + 255

840768*a^9*b^4*c^10*d^7*e^16 - 241492992*a^9*b^5*c^9*d^6*e^17 + 41574816*a^9*b^6*c^8*d^5*e^18 + 32344416*a^9*b^7*c^7*d^4*e^19 - 8542208*a^9*b^8*c^6*d^3*e^20 - 1677872*a^9*b^9*c^5*d^2*e^21 - 270632960*a^10*b^2*c^11*d^7*e^16 + 105492480*a^10*b^3*c^10*d^6*e^17 + 71796864*a^10*b^4*c^9*d^5*e^18 - 66791040*a^10*b^5*c^8*d^4*e^19 + 5437088*a^10*b^6*c^7*d^3*e^20 + 4684288*a^10*b^7*c^6*d^2*e^21 - 105693696*a^11*b^2*c^10*d^5*e^18 + 38220288*a^11*b^3*c^9*d^4*e^19 + 10967680*a^11*b^4*c^8*d^3*e^20 - 6778368*a^11*b^5*c^7*d^2*e^21 - 15811072*a^12*b^2*c^9*d^3*e^20 + 3633152*a^12*b^3*c^8*d^2*e^21 - 352*a*b^6*c^16*d^21*e^2 + 3424*a*b^7*c^15*d^20*e^3 - 14720*a*b^8*c^14*d^19*e^4 + 36048*a*b^9*c^13*d^18*e^5 - 52384*a*b^10*c^12*d^17*e^6 + 36464*a*b^11*c^11*d^16*e^7 + 17952*a*b^12*c^10*d^15*e^8 - 75360*a*b^13*c^9*d^14*e^9 + 91104*a*b^14*c^8*d^13*e^10 - 60992*a*b^15*c^7*d^12*e^11 + 20288*a*b^16*c^6*d^11*e^12 + 1424*a*b^17*c^5*d^10*e^13 - 4320*a*b^18*c^4*d^9*e^14 + 1648*a*b^19*c^3*d^8*e^15 - 224*a*b^20*c^2*d^7*e^16 - 169984*a^4*b*c^18*d^20*e^3 - 2076672*a^5*b*c^17*d^18*e^5 - 9658368*a^6*b*c^16*d^16*e^7 - 16384000*a^7*b*c^15*d^14*e^9 - 224*a^7*b^14*c^2*d*e^22 + 42463232*a^8*b*c^14*d^12*e^11 + 5120*a^8*b^12*c^3*d*e^22 + 170631168*a^9*b*c^13*d^10*e^13 - 48576*a^9*b^10*c^4*d*e^22 + 199843840*a^10*b*c^12*d^8*e^15 + 244480*a^10*b^8*c^5*d*e^22 + 95387648*a^11*b*c^11*d^6*e^17 - 686080*a^11*b^6*c^6*d*e^22 + 15722496*a^12*b*c^10*d^4*e^19 + 1007616*a^12*b^4*c^7*d*e^22 + 692224*a^13*b*c^9*d^2*e^21 - 573440*a^13*b^2*c^8*d*e^22))/(8*(16*a^3*b^6*c^9*d^18 - a^2*b^8*c^8*d^18 - 256*a^6*c^12*d^18 - 96*a^4*b^4*c^10*d^18 + 256*a^5*b^2*c^11*d^18 - a^2*b^16*d^10*e^8 + 8*a^3*b^15*d^9*e^9 - 28*a^4*b^14*d^8*e^10 + 56*a^5*b^13*d^7*e^11 - 70*a^6*b^12*d^6*e^12 + 56*a^7*b^11*d^5*e^13 - 28*a^8*b^10*d^4*e^14 + 8*a^9*b^9*d^3*e^15 - a^10*b^8*d^2*e^16 - 2048*a^7*c^11*d^16*e^2 - 7168*a^8*c^10*d^14*e^4 - 14336*a^9*c^9*d^12*e^6 - 17920*a^10*c^8*d^10*e^8 - 14336*a^11*c^7*d^8*e^10 - 7168*a^12*c^6*d^6*e^12 - 2048*a^13*c^5*d^4*e^14 - 256*a^14*c^4*d^2*e^16 - 28*a^2*b^10*c^6*d^16*e^2 + 56*a^2*b^11*c^5*d^15*e^3 - 70*a^2*b^12*c^4*d^14*e^4 + 56*a^2*b^13*c^3*d^13*e^5 - 28*a^2*b^14*c^2*d^12*e^6 + 440*a^3*b^8*c^7*d^16*e^2 - 840*a^3*b^9*c^6*d^15*e^3 + 952*a^3*b^10*c^5*d^14*e^4 - 616*a^3*b^11*c^4*d^13*e^5 + 168*a^3*b^12*c^3*d^12*e^6 + 40*a^3*b^13*c^2*d^11*e^7 - 2560*a^4*b^6*c^8*d^16*e^2 + 4480*a^4*b^7*c^7*d^15*e^3 - 4060*a^4*b^8*c^6*d^14*e^4 + 1064*a^4*b^9*c^5*d^13*e^5 + 1372*a^4*b^10*c^4*d^12*e^6 - 1360*a^4*b^11*c^3*d^11*e^7 + 380*a^4*b^12*c^2*d^10*e^8 + 6400*a^5*b^4*c^9*d^16*e^2 - 8960*a^5*b^5*c^8*d^15*e^3 + 2240*a^5*b^6*c^7*d^14*e^4 + 9856*a^5*b^7*c^6*d^13*e^5 - 13048*a^5*b^8*c^5*d^12*e^6 + 5400*a^5*b^9*c^4*d^11*e^7 + 1040*a^5*b^10*c^3*d^10*e^8 - 1360*a^5*b^11*c^2*d^9*e^9 - 5120*a^6*b^2*c^10*d^16*e^2 + 22400*a^6*b^4*c^8*d^14*e^4 - 41216*a^6*b^5*c^7*d^13*e^5 + 25088*a^6*b^6*c^6*d^12*e^6 + 8320*a^6*b^7*c^5*d^11*e^7 - 17350*a^6*b^8*c^4*d^10*e^8 + 5400*a^6*b^9*c^3*d^9*e^9 + 1372*a^6*b^10*c^2*d^8*e^10 - 35840*a^7*b^2*c^9*d^14*e^4 + 28672*a^7*b^3*c^8*d^13*e^5 + 30464*a^7*b^4*c^7*d^12*e^6 - 73472*a^7*b^5*c^6*d^11*e^7 + 40544*a^7*b^6*c^5*d^10*e^8 + 8320*a^7*b^7*c^4*d^9*e^9 - 13048*a^7*b^8*c^3*d^8*e^10 + 1064*a^7*b^9*c^2*d^7*e^11 - 93184*a^8*b^2*c^8*d^12*e^6 + 71680*a^8*b^3*c^7*d^11*e^7 + 29120*a^8*b^4*c^6*d^10*e^8 - 73472*a^8*b^5*c^5*d^9*e^9 + 25088*a^8*b^6*c^4*d^8*e^10 + 9856*a^8*b^7*c^3*d^7*e^11 - 4060*a^8*b^8*c^2*d^6*e^12 - 125440*a^9*b^2*c^7*d^10*e^8 + 71680*a^9*b^3*c^6*d^9*e^9 + 30464*a^9*b^4*c^5*d^8*e^10 - 41216*a^9*b^5*c^4*d^7*e^11 + 2240*a^9*b^6*c^3*d^6*e^12 + 4480*a^9*b^7*c^2*d^5*e^13 - 93184*a^10*b^2*c^6*d^8*e^10 + 28672*a^10*b^3*c^5*d^7*e^11 + 22400*a^10*b^4*c^4*d^6*e^12 - 8960*a^10*b^5*c^3*d^5*e^13 - 2560*a^10*b^6*c^2*d^4*e^14 - 35840*a^11*b^2*c^5*d^6*e^12 + 6400*a^11*b^4*c^3*d^4*e^14 + 768*a^11*b^5*c^2*d^3*e^15 - 5120*a^12*b^2*c^4*d^4*e^14 - 2048*a^12*b^3*c^3*d^3*e^15 - 96*a^12*b^4*c^2*d^2*e^16 + 256*a^13*b^2*c^3*d^2*e^16 + 2048*a^6*b*c^11*d^17*e + 8*a^2*b^9*c^7*d^17*e + 8*a^2*b^15*c*d^11*e^7 - 128*a^3*b^7*c^8*d^17*e - 40*a^3*b^14*c*d^10*e^8 + 768*a^4*b^5*c^9*d^17*e + 40*a^4*b^13*c*d^9*e^9 - 2048*a^5*b^3*c^10*d^17*e + 168*a^5*b^12*c*d^8*e^10 - 616*a^6*b^11*c*d^7*e^11 + 14336*a^7*b*c^10*d^15*e^3 + 952*a^7*b^10*c*d^6*e^12 + 43008*a^8*b*c^9*d^13*e^5 - 840*a^8*b^9*c*d^5*e^13 + 71680*a^9*b*c^8*d^11*e^7 + 440*a^9*b^8*c*d^4*e^14 + 71680*a^10*b*c^7*d^9*e^9 - 128*a^10*b^7*c*d^3*e^15 + 43008*a^11*b*c^6*d^7*e^11 + 16*a^11*

b^6*c*d^2*e^16 + 14336*a^12*b*c^5*d^5*e^13 + 2048*a^13*b*c^4*d^3*e^15)))) +
(x*(25*a^4*b^10*c^5*e^19 - 6272*a^9*c^10*e^19 - 440*a^5*b^8*c^6*e^19 + 298
6*a^6*b^6*c^7*e^19 - 9560*a^7*b^4*c^8*e^19 + 13792*a^8*b^2*c^9*e^19 + 1296*
a^2*c^17*d^14*e^5 + 19296*a^3*c^16*d^12*e^7 + 195952*a^4*c^15*d^10*e^9 + 93
8176*a^5*c^14*d^8*e^11 + 1838832*a^6*c^13*d^6*e^13 - 20896*a^7*c^12*d^4*e^1
5 - 57200*a^8*c^11*d^2*e^17 + 25*b^4*c^15*d^14*e^5 - 190*b^5*c^14*d^13*e^6
+ 591*b^6*c^13*d^12*e^7 - 964*b^7*c^12*d^11*e^8 + 952*b^8*c^11*d^10*e^9 - 8
28*b^9*c^10*d^9*e^10 + 952*b^10*c^9*d^8*e^11 - 964*b^11*c^8*d^7*e^12 + 591*
b^12*c^7*d^6*e^13 - 190*b^13*c^6*d^5*e^14 + 25*b^14*c^5*d^4*e^15 + 18816*a^
2*b^2*c^15*d^12*e^7 - 464*a^2*b^3*c^14*d^11*e^8 - 33441*a^2*b^4*c^13*d^10*e
^9 - 9780*a^2*b^5*c^12*d^9*e^10 + 98620*a^2*b^6*c^11*d^8*e^11 - 74420*a^2*b
^7*c^10*d^7*e^12 - 25327*a^2*b^8*c^9*d^6*e^13 + 51944*a^2*b^9*c^8*d^5*e^14
- 19162*a^2*b^10*c^7*d^4*e^15 + 376*a^2*b^11*c^6*d^3*e^16 + 726*a^2*b^12*c^
5*d^2*e^17 + 132104*a^3*b^2*c^14*d^10*e^9 + 202944*a^3*b^3*c^13*d^9*e^10 -
496916*a^3*b^4*c^12*d^8*e^11 + 62420*a^3*b^5*c^11*d^7*e^12 + 477560*a^3*b^6
*c^10*d^6*e^13 - 367184*a^3*b^7*c^9*d^5*e^14 + 42920*a^3*b^8*c^8*d^4*e^15 +
41584*a^3*b^9*c^7*d^3*e^16 - 11716*a^3*b^10*c^6*d^2*e^17 + 774624*a^4*b^2*
c^13*d^8*e^11 + 1091488*a^4*b^3*c^12*d^7*e^12 - 2078409*a^4*b^4*c^11*d^6*e^
13 + 759546*a^4*b^5*c^10*d^5*e^14 + 436579*a^4*b^6*c^9*d^4*e^15 - 373848*a^
4*b^7*c^8*d^3*e^16 + 68053*a^4*b^8*c^7*d^2*e^17 + 2519400*a^5*b^2*c^12*d^6*
e^13 + 1051760*a^5*b^3*c^11*d^5*e^14 - 2494242*a^5*b^4*c^10*d^4*e^15 + 1223
634*a^5*b^5*c^9*d^3*e^16 - 153022*a^5*b^6*c^8*d^2*e^17 + 3717952*a^6*b^2*c^
11*d^4*e^15 - 1366224*a^6*b^3*c^10*d^3*e^16 + 23697*a^6*b^4*c^9*d^2*e^17 +
268408*a^7*b^2*c^10*d^2*e^17 + 43136*a^8*b*c^10*d*e^18 - 360*a*b^2*c^16*d^1
4*e^5 + 2608*a*b^3*c^15*d^13*e^6 - 7218*a*b^4*c^14*d^12*e^7 + 8922*a*b^5*c^
13*d^11*e^8 - 4786*a*b^6*c^12*d^10*e^9 + 4722*a*b^7*c^11*d^9*e^10 - 12250*a
*b^8*c^10*d^8*e^11 + 13434*a*b^9*c^9*d^7*e^12 - 4918*a*b^10*c^8*d^6*e^13 -
1202*a*b^11*c^7*d^5*e^14 + 1308*a*b^12*c^6*d^4*e^15 - 260*a*b^13*c^5*d^3*e^
16 - 8928*a^2*b*c^16*d^13*e^6 - 107360*a^3*b*c^15*d^11*e^8 - 260*a^3*b^11*c
^5*d*e^18 - 846912*a^4*b*c^14*d^9*e^10 + 4518*a^4*b^9*c^6*d*e^18 - 3155136*
a^5*b*c^13*d^7*e^12 - 30034*a^5*b^7*c^7*d*e^18 - 4176736*a^6*b*c^12*d^5*e^1
4 + 92664*a^6*b^5*c^8*d*e^18 - 154080*a^7*b*c^11*d^3*e^16 - 123488*a^7*b^3*
c^9*d*e^18))/(8*(16*a^3*b^6*c^9*d^18 - a^2*b^8*c^8*d^18 - 256*a^6*c^12*d^18
- 96*a^4*b^4*c^10*d^18 + 256*a^5*b^2*c^11*d^18 - a^2*b^16*d^10*e^8 + 8*a^3
*b^15*d^9*e^9 - 28*a^4*b^14*d^8*e^10 + 56*a^5*b^13*d^7*e^11 - 70*a^6*b^12*d
^6*e^12 + 56*a^7*b^11*d^5*e^13 - 28*a^8*b^10*d^4*e^14 + 8*a^9*b^9*d^3*e^15
- a^10*b^8*d^2*e^16 - 2048*a^7*c^11*d^16*e^2 - 7168*a^8*c^10*d^14*e^4 - 143
36*a^9*c^9*d^12*e^6 - 17920*a^10*c^8*d^10*e^8 - 14336*a^11*c^7*d^8*e^10 - 7
168*a^12*c^6*d^6*e^12 - 2048*a^13*c^5*d^4*e^14 - 256*a^14*c^4*d^2*e^16 - 28
*a^2*b^10*c^6*d^16*e^2 + 56*a^2*b^11*c^5*d^15*e^3 - 70*a^2*b^12*c^4*d^14*e^
4 + 56*a^2*b^13*c^3*d^13*e^5 - 28*a^2*b^14*c^2*d^12*e^6 + 440*a^3*b^8*c^7*d
^16*e^2 - 840*a^3*b^9*c^6*d^15*e^3 + 952*a^3*b^10*c^5*d^14*e^4 - 616*a^3*b^
11*c^4*d^13*e^5 + 168*a^3*b^12*c^3*d^12*e^6 + 40*a^3*b^13*c^2*d^11*e^7 - 25
60*a^4*b^6*c^8*d^16*e^2 + 4480*a^4*b^7*c^7*d^15*e^3 - 4060*a^4*b^8*c^6*d^14
*e^4 + 1064*a^4*b^9*c^5*d^13*e^5 + 1372*a^4*b^10*c^4*d^12*e^6 - 1360*a^4*b^
11*c^3*d^11*e^7 + 380*a^4*b^12*c^2*d^10*e^8 + 6400*a^5*b^4*c^9*d^16*e^2 - 8
960*a^5*b^5*c^8*d^15*e^3 + 2240*a^5*b^6*c^7*d^14*e^4 + 9856*a^5*b^7*c^6*d^1
3*e^5 - 13048*a^5*b^8*c^5*d^12*e^6 + 5400*a^5*b^9*c^4*d^11*e^7 + 1040*a^5*b
^10*c^3*d^10*e^8 - 1360*a^5*b^11*c^2*d^9*e^9 - 5120*a^6*b^2*c^10*d^16*e^2 +
22400*a^6*b^4*c^8*d^14*e^4 - 41216*a^6*b^5*c^7*d^13*e^5 + 25088*a^6*b^6*c^
6*d^12*e^6 + 8320*a^6*b^7*c^5*d^11*e^7 - 17350*a^6*b^8*c^4*d^10*e^8 + 5400*
a^6*b^9*c^3*d^9*e^9 + 1372*a^6*b^10*c^2*d^8*e^10 - 35840*a^7*b^2*c^9*d^14*e
^4 + 28672*a^7*b^3*c^8*d^13*e^5 + 30464*a^7*b^4*c^7*d^12*e^6 - 73472*a^7*b^
5*c^6*d^11*e^7 + 40544*a^7*b^6*c^5*d^10*e^8 + 8320*a^7*b^7*c^4*d^9*e^9 - 13
048*a^7*b^8*c^3*d^8*e^10 + 1064*a^7*b^9*c^2*d^7*e^11 - 93184*a^8*b^2*c^8*d^
12*e^6 + 71680*a^8*b^3*c^7*d^11*e^7 + 29120*a^8*b^4*c^6*d^10*e^8 - 73472*a^
8*b^5*c^5*d^9*e^9 + 25088*a^8*b^6*c^4*d^8*e^10 + 9856*a^8*b^7*c^3*d^7*e^11
- 4060*a^8*b^8*c^2*d^6*e^12 - 125440*a^9*b^2*c^7*d^10*e^8 + 71680*a^9*b^3*c
^6*d^9*e^9 + 30464*a^9*b^4*c^5*d^8*e^10 - 41216*a^9*b^5*c^4*d^7*e^11 + 2240

*a^9*b^6*c^3*d^6*e^12 + 4480*a^9*b^7*c^2*d^5*e^13 - 93184*a^10*b^2*c^6*d^8*
e^10 + 28672*a^10*b^3*c^5*d^7*e^11 + 22400*a^10*b^4*c^4*d^6*e^12 - 8960*a^1
0*b^5*c^3*d^5*e^13 - 2560*a^10*b^6*c^2*d^4*e^14 - 35840*a^11*b^2*c^5*d^6*e^
12 + 6400*a^11*b^4*c^3*d^4*e^14 + 768*a^11*b^5*c^2*d^3*e^15 - 5120*a^12*b^2
*c^4*d^4*e^14 - 2048*a^12*b^3*c^3*d^3*e^15 - 96*a^12*b^4*c^2*d^2*e^16 + 256
*a^13*b^2*c^3*d^2*e^16 + 2048*a^6*b*c^11*d^17*e + 8*a^2*b^9*c^7*d^17*e + 8*
a^2*b^15*c*d^11*e^7 - 128*a^3*b^7*c^8*d^17*e - 40*a^3*b^14*c*d^10*e^8 + 768
*a^4*b^5*c^9*d^17*e + 40*a^4*b^13*c*d^9*e^9 - 2048*a^5*b^3*c^10*d^17*e + 16
8*a^5*b^12*c*d^8*e^10 - 616*a^6*b^11*c*d^7*e^11 + 14336*a^7*b*c^10*d^15*e^3
 + 952*a^7*b^10*c*d^6*e^12 + 43008*a^8*b*c^9*d^13*e^5 - 840*a^8*b^9*c*d^5*e
^13 + 71680*a^9*b*c^8*d^11*e^7 + 440*a^9*b^8*c*d^4*e^14 + 71680*a^10*b*c^7*
d^9*e^9 - 128*a^10*b^7*c*d^3*e^15 + 43008*a^11*b*c^6*d^7*e^11 + 16*a^11*b^6
*c*d^2*e^16 + 14336*a^12*b*c^5*d^5*e^13 + 2048*a^13*b*c^4*d^3*e^15))) - (39
20*a^6*b*c^10*e^17 + 32144*a^6*c^11*d*e^16 + 225*a^4*b^5*c^8*e^17 - 1880*a^
5*b^3*c^9*e^17 + 11664*a^2*c^15*d^9*e^8 + 46656*a^3*c^14*d^7*e^10 - 40608*a
^4*c^13*d^5*e^12 + 284224*a^5*c^12*d^3*e^14 + 225*b^4*c^13*d^9*e^8 - 755*b^
5*c^12*d^8*e^9 + 530*b^6*c^11*d^7*e^10 + 530*b^7*c^10*d^6*e^11 - 755*b^8*c^
9*d^5*e^12 + 225*b^9*c^8*d^4*e^13 + 27648*a^2*b^2*c^13*d^7*e^10 + 4576*a^2*
b^3*c^12*d^6*e^11 + 24438*a^2*b^4*c^11*d^5*e^12 - 44262*a^2*b^5*c^10*d^4*e^
13 + 4042*a^2*b^6*c^9*d^3*e^14 + 6534*a^2*b^7*c^8*d^2*e^15 - 23408*a^3*b^2*
c^12*d^5*e^12 + 41872*a^3*b^3*c^11*d^4*e^13 + 100948*a^3*b^4*c^10*d^3*e^14
 - 60416*a^3*b^5*c^9*d^2*e^15 - 384384*a^4*b^2*c^11*d^3*e^14 + 165216*a^4*b^
3*c^10*d^2*e^15 - 3240*a*b^2*c^14*d^9*e^8 + 11016*a*b^3*c^13*d^8*e^9 - 8812
*a*b^4*c^12*d^7*e^10 - 1992*a*b^5*c^11*d^6*e^11 + 408*a*b^6*c^10*d^5*e^12 +
 5216*a*b^7*c^9*d^4*e^13 - 2340*a*b^8*c^8*d^3*e^14 - 40176*a^2*b*c^14*d^8*e
^9 - 63360*a^3*b*c^13*d^6*e^11 - 2340*a^3*b^6*c^8*d*e^16 + 120608*a^4*b*c^1
2*d^4*e^13 + 21281*a^4*b^4*c^9*d*e^16 - 114432*a^5*b*c^11*d^2*e^15 - 55656*
a^5*b^2*c^10*d*e^16)/(32*(16*a^3*b^6*c^9*d^18 - a^2*b^8*c^8*d^18 - 256*a^6*
c^12*d^18 - 96*a^4*b^4*c^10*d^18 + 256*a^5*b^2*c^11*d^18 - a^2*b^16*d^10*e^
8 + 8*a^3*b^15*d^9*e^9 - 28*a^4*b^14*d^8*e^10 + 56*a^5*b^13*d^7*e^11 - 70*a
^6*b^12*d^6*e^12 + 56*a^7*b^11*d^5*e^13 - 28*a^8*b^10*d^4*e^14 + 8*a^9*b^9*
d^3*e^15 - a^10*b^8*d^2*e^16 - 2048*a^7*c^11*d^16*e^2 - 7168*a^8*c^10*d^14*
e^4 - 14336*a^9*c^9*d^12*e^6 - 17920*a^10*c^8*d^10*e^8 - 14336*a^11*c^7*d^8
*e^10 - 7168*a^12*c^6*d^6*e^12 - 2048*a^13*c^5*d^4*e^14 - 256*a^14*c^4*d^2*
e^16 - 28*a^2*b^10*c^6*d^16*e^2 + 56*a^2*b^11*c^5*d^15*e^3 - 70*a^2*b^12*c^
4*d^14*e^4 + 56*a^2*b^13*c^3*d^13*e^5 - 28*a^2*b^14*c^2*d^12*e^6 + 440*a^3*
b^8*c^7*d^16*e^2 - 840*a^3*b^9*c^6*d^15*e^3 + 952*a^3*b^10*c^5*d^14*e^4 - 6
16*a^3*b^11*c^4*d^13*e^5 + 168*a^3*b^12*c^3*d^12*e^6 + 40*a^3*b^13*c^2*d^11
*e^7 - 2560*a^4*b^6*c^8*d^16*e^2 + 4480*a^4*b^7*c^7*d^15*e^3 - 4060*a^4*b^8
*c^6*d^14*e^4 + 1064*a^4*b^9*c^5*d^13*e^5 + 1372*a^4*b^10*c^4*d^12*e^6 - 13
60*a^4*b^11*c^3*d^11*e^7 + 380*a^4*b^12*c^2*d^10*e^8 + 6400*a^5*b^4*c^9*d^1
6*e^2 - 8960*a^5*b^5*c^8*d^15*e^3 + 2240*a^5*b^6*c^7*d^14*e^4 + 9856*a^5*b^
7*c^6*d^13*e^5 - 13048*a^5*b^8*c^5*d^12*e^6 + 5400*a^5*b^9*c^4*d^11*e^7 + 1
040*a^5*b^10*c^3*d^10*e^8 - 1360*a^5*b^11*c^2*d^9*e^9 - 5120*a^6*b^2*c^10*d
^16*e^2 + 22400*a^6*b^4*c^8*d^14*e^4 - 41216*a^6*b^5*c^7*d^13*e^5 + 25088*a
^6*b^6*c^6*d^12*e^6 + 8320*a^6*b^7*c^5*d^11*e^7 - 17350*a^6*b^8*c^4*d^10*e^
8 + 5400*a^6*b^9*c^3*d^9*e^9 + 1372*a^6*b^10*c^2*d^8*e^10 - 35840*a^7*b^2*c
^9*d^14*e^4 + 28672*a^7*b^3*c^8*d^13*e^5 + 30464*a^7*b^4*c^7*d^12*e^6 - 734
72*a^7*b^5*c^6*d^11*e^7 + 40544*a^7*b^6*c^5*d^10*e^8 + 8320*a^7*b^7*c^4*d^9
*e^9 - 13048*a^7*b^8*c^3*d^8*e^10 + 1064*a^7*b^9*c^2*d^7*e^11 - 93184*a^8*b
^2*c^8*d^12*e^6 + 71680*a^8*b^3*c^7*d^11*e^7 + 29120*a^8*b^4*c^6*d^10*e^8 -
 73472*a^8*b^5*c^5*d^9*e^9 + 25088*a^8*b^6*c^4*d^8*e^10 + 9856*a^8*b^7*c^3*
d^7*e^11 - 4060*a^8*b^8*c^2*d^6*e^12 - 125440*a^9*b^2*c^7*d^10*e^8 + 71680*
a^9*b^3*c^6*d^9*e^9 + 30464*a^9*b^4*c^5*d^8*e^10 - 41216*a^9*b^5*c^4*d^7*e^
11 + 2240*a^9*b^6*c^3*d^6*e^12 + 4480*a^9*b^7*c^2*d^5*e^13 - 93184*a^10*b^2
*c^6*d^8*e^10 + 28672*a^10*b^3*c^5*d^7*e^11 + 22400*a^10*b^4*c^4*d^6*e^12 -
 8960*a^10*b^5*c^3*d^5*e^13 - 2560*a^10*b^6*c^2*d^4*e^14 - 35840*a^11*b^2*c
^5*d^6*e^12 + 6400*a^11*b^4*c^3*d^4*e^14 + 768*a^11*b^5*c^2*d^3*e^15 - 5120
*a^12*b^2*c^4*d^4*e^14 - 2048*a^12*b^3*c^3*d^3*e^15 - 96*a^12*b^4*c^2*d^2*e

^16 + 256*a^13*b^2*c^3*d^2*e^16 + 2048*a^6*b*c^11*d^17*e + 8*a^2*b^9*c^7*d^
17*e + 8*a^2*b^15*c*d^11*e^7 - 128*a^3*b^7*c^8*d^17*e - 40*a^3*b^14*c*d^10*
e^8 + 768*a^4*b^5*c^9*d^17*e + 40*a^4*b^13*c*d^9*e^9 - 2048*a^5*b^3*c^10*d^
17*e + 168*a^5*b^12*c*d^8*e^10 - 616*a^6*b^11*c*d^7*e^11 + 14336*a^7*b*c^10
*d^15*e^3 + 952*a^7*b^10*c*d^6*e^12 + 43008*a^8*b*c^9*d^13*e^5 - 840*a^8*b^
9*c*d^5*e^13 + 71680*a^9*b*c^8*d^11*e^7 + 440*a^9*b^8*c*d^4*e^14 + 71680*a^
10*b*c^7*d^9*e^9 - 128*a^10*b^7*c*d^3*e^15 + 43008*a^11*b*c^6*d^7*e^11 + 16
*a^11*b^6*c*d^2*e^16 + 14336*a^12*b*c^5*d^5*e^13 + 2048*a^13*b*c^4*d^3*e^15
)))*root(128723189760*a^14*b^4*c^9*d^13*e^14*z^6 + 128723189760*a^12*b^4*c^
11*d^17*e^10*z^6 - 8432455680*a^11*b^12*c^4*d^11*e^16*z^6 - 8432455680*a^7*
b^12*c^8*d^19*e^8*z^6 + 12673351680*a^11*b^11*c^5*d^12*e^15*z^6 + 126733516
80*a^8*b^11*c^8*d^18*e^9*z^6 - 72637480960*a^12*b^9*c^6*d^12*e^15*z^6 - 726
37480960*a^9*b^9*c^9*d^18*e^9*z^6 - 21048344576*a^9*b^12*c^6*d^15*e^12*z^6
- 16609443840*a^17*b^3*c^7*d^8*e^19*z^6 - 16609443840*a^10*b^3*c^14*d^22*e^
5*z^6 + 145332633600*a^13*b^5*c^9*d^14*e^13*z^6 + 145332633600*a^12*b^5*c^1
0*d^16*e^11*z^6 + 123740356608*a^14*b^5*c^8*d^12*e^15*z^6 + 123740356608*a^
11*b^5*c^11*d^18*e^9*z^6 + 3460300800*a^17*b^5*c^5*d^6*e^21*z^6 + 346030080
0*a^8*b^5*c^14*d^24*e^3*z^6 - 7751073792*a^15*b^7*c^5*d^8*e^19*z^6 - 775107
3792*a^8*b^7*c^12*d^22*e^5*z^6 + 12041846784*a^14*b^7*c^6*d^10*e^17*z^6 + 1
2041846784*a^9*b^7*c^11*d^20*e^7*z^6 - 325545099264*a^14*b^3*c^10*d^14*e^13
*z^6 - 325545099264*a^13*b^3*c^11*d^16*e^11*z^6 - 3330539520*a^13*b^10*c^4*
d^9*e^18*z^6 - 3330539520*a^7*b^10*c^10*d^21*e^6*z^6 + 157789716480*a^12*b^
7*c^8*d^14*e^13*z^6 + 157789716480*a^11*b^7*c^9*d^16*e^11*z^6 + 37492359168
*a^11*b^10*c^6*d^13*e^14*z^6 + 37492359168*a^9*b^10*c^8*d^17*e^10*z^6 + 301
989888*a^8*b^3*c^16*d^26*e*z^6 - 7266631680*a^17*b^4*c^6*d^7*e^20*z^6 - 726
6631680*a^9*b^4*c^14*d^23*e^4*z^6 - 201326592*a^20*b*c^6*d^4*e^23*z^6 - 188
743680*a^7*b^5*c^15*d^26*e*z^6 + 45747339264*a^13*b^8*c^6*d^11*e^16*z^6 + 4
5747339264*a^9*b^8*c^10*d^19*e^8*z^6 - 74612736*a^10*b^16*c*d^9*e^18*z^6 -
2768240640*a^16*b^7*c^4*d^6*e^21*z^6 - 2768240640*a^7*b^7*c^13*d^24*e^3*z^6
+ 69746688*a^11*b^15*c*d^8*e^19*z^6 + 62914560*a^6*b^7*c^14*d^26*e*z^6 + 2
752020480*a^10*b^13*c^4*d^12*e^15*z^6 + 2752020480*a^7*b^13*c^7*d^18*e^9*z^
6 + 55148544*a^9*b^17*c*d^10*e^17*z^6 - 45957120*a^12*b^14*c*d^7*e^20*z^6 -
2724986880*a^14*b^9*c^4*d^8*e^19*z^6 - 2724986880*a^7*b^9*c^11*d^22*e^5*z^
6 - 25952256*a^8*b^18*c*d^11*e^16*z^6 + 21086208*a^13*b^13*c*d^6*e^21*z^6 -
11796480*a^5*b^9*c^13*d^26*e*z^6 - 6438912*a^14*b^12*c*d^5*e^22*z^6 + 5406
720*a^7*b^19*c*d^12*e^15*z^6 + 1622016*a^6*b^20*c*d^13*e^14*z^6 - 1523712*a
^5*b^21*c*d^14*e^13*z^6 + 1179648*a^15*b^11*c*d^4*e^23*z^6 + 1179648*a^4*b^
11*c^12*d^26*e*z^6 + 442368*a^4*b^22*c*d^15*e^12*z^6 - 98304*a^16*b^10*c*d^
3*e^24*z^6 - 49152*a^3*b^23*c*d^16*e^11*z^6 - 49152*a^3*b^13*c^11*d^26*e*z^
6 + 6897106944*a^9*b^13*c^5*d^14*e^13*z^6 + 6897106944*a^8*b^13*c^6*d^16*e^
11*z^6 - 2422210560*a^16*b^6*c^5*d^7*e^20*z^6 - 2422210560*a^8*b^6*c^13*d^2
3*e^4*z^6 + 255785435136*a^14*b^2*c^11*d^15*e^12*z^6 + 41004564480*a^15*b^4
*c^8*d^11*e^16*z^6 + 41004564480*a^11*b^4*c^12*d^19*e^8*z^6 + 2270822400*a^
13*b^11*c^3*d^8*e^19*z^6 + 2270822400*a^6*b^11*c^10*d^22*e^5*z^6 + 23677108
224*a^14*b^8*c^5*d^9*e^18*z^6 + 23677108224*a^8*b^8*c^11*d^21*e^6*z^6 + 212
600881152*a^15*b^2*c^10*d^13*e^14*z^6 + 212600881152*a^13*b^2*c^12*d^17*e^1
0*z^6 + 75157733376*a^15*b^5*c^7*d^10*e^17*z^6 + 75157733376*a^10*b^5*c^12*
d^20*e^7*z^6 - 251217838080*a^13*b^6*c^8*d^13*e^14*z^6 - 251217838080*a^11*
b^6*c^10*d^17*e^10*z^6 - 1952907264*a^14*b^10*c^3*d^7*e^20*z^6 - 1952907264
*a^6*b^10*c^11*d^23*e^4*z^6 - 27691057152*a^13*b^9*c^5*d^10*e^17*z^6 - 2769
1057152*a^8*b^9*c^10*d^20*e^7*z^6 - 1902673920*a^8*b^15*c^4*d^14*e^13*z^6 -
1902673920*a^7*b^15*c^5*d^16*e^11*z^6 + 10465050624*a^10*b^11*c^6*d^14*e^1
3*z^6 + 10465050624*a^9*b^11*c^7*d^16*e^11*z^6 + 1613905920*a^9*b^14*c^4*d^
13*e^14*z^6 + 1613905920*a^7*b^14*c^6*d^17*e^10*z^6 - 33218887680*a^17*b*c^
9*d^10*e^17*z^6 - 33218887680*a^12*b*c^14*d^20*e^7*z^6 + 1524695040*a^10*b^
14*c^3*d^11*e^16*z^6 + 1524695040*a^6*b^14*c^7*d^19*e^8*z^6 - 1472200704*a^
18*b^4*c^5*d^5*e^22*z^6 - 1472200704*a^8*b^4*c^15*d^25*e^2*z^6 - 8304721920
0*a^16*b^3*c^8*d^10*e^17*z^6 - 83047219200*a^11*b^3*c^13*d^20*e^7*z^6 + 442
91850240*a^17*b^2*c^8*d^9*e^18*z^6 + 44291850240*a^11*b^2*c^14*d^21*e^6*z^6

+ 1308131328*a^8*b^14*c^5*d^15*e^12*z^6 - 201326592*a^9*b*c^17*d^26*e*z^6
+ 48530718720*a^12*b^8*c^7*d^13*e^14*z^6 + 48530718720*a^10*b^8*c^9*d^17*e^
10*z^6 - 1242644480*a^12*b^12*c^3*d^9*e^18*z^6 - 1242644480*a^6*b^12*c^9*d^
21*e^6*z^6 + 9813196800*a^12*b^10*c^5*d^11*e^16*z^6 + 9813196800*a^8*b^10*c
^9*d^19*e^8*z^6 - 93012885504*a^15*b*c^11*d^14*e^13*z^6 - 93012885504*a^14*
b*c^12*d^16*e^11*z^6 + 177305812992*a^13*b^4*c^10*d^15*e^12*z^6 + 527306588
16*a^10*b^10*c^7*d^15*e^12*z^6 - 1180106752*a^9*b^15*c^3*d^12*e^15*z^6 - 11
80106752*a^6*b^15*c^6*d^18*e^9*z^6 + 1023672320*a^15*b^9*c^3*d^6*e^21*z^6 +
1023672320*a^6*b^9*c^12*d^24*e^3*z^6 + 975175680*a^17*b^6*c^4*d^5*e^22*z^6
+ 975175680*a^7*b^6*c^14*d^25*e^2*z^6 - 11072962560*a^18*b*c^8*d^8*e^19*z^
6 - 11072962560*a^11*b*c^15*d^22*e^5*z^6 + 9412018176*a^18*b^2*c^7*d^7*e^20
*z^6 + 9412018176*a^10*b^2*c^15*d^23*e^4*z^6 + 805306368*a^19*b^2*c^6*d^5*e
^22*z^6 + 805306368*a^9*b^2*c^16*d^25*e^2*z^6 - 133809831936*a^14*b^6*c^7*d
^11*e^16*z^6 - 133809831936*a^10*b^6*c^11*d^19*e^8*z^6 - 2214592512*a^19*b*
c^7*d^6*e^21*z^6 - 2214592512*a^10*b*c^16*d^24*e^3*z^6 + 82216747008*a^13*b
^7*c^7*d^12*e^15*z^6 + 82216747008*a^10*b^7*c^10*d^18*e^9*z^6 - 586629120*a
^12*b^13*c^2*d^8*e^19*z^6 - 586629120*a^5*b^13*c^9*d^22*e^5*z^6 + 568565760
*a^7*b^16*c^4*d^15*e^12*z^6 - 4844421120*a^16*b^4*c^7*d^9*e^18*z^6 - 484442
1120*a^10*b^4*c^13*d^21*e^6*z^6 + 531210240*a^11*b^14*c^2*d^9*e^18*z^6 + 53
1210240*a^5*b^14*c^8*d^21*e^6*z^6 - 527155200*a^11*b^13*c^3*d^10*e^17*z^6 -
527155200*a^6*b^13*c^8*d^20*e^7*z^6 + 43470028800*a^11*b^8*c^8*d^15*e^12*z
^6 - 107874877440*a^11*b^9*c^7*d^14*e^13*z^6 - 107874877440*a^10*b^9*c^8*d^
16*e^11*z^6 + 9018408960*a^12*b^11*c^4*d^10*e^17*z^6 + 9018408960*a^7*b^11*
c^9*d^20*e^7*z^6 + 421994496*a^13*b^12*c^2*d^7*e^20*z^6 + 421994496*a^5*b^1
2*c^10*d^23*e^4*z^6 - 66437775360*a^16*b*c^10*d^12*e^15*z^6 - 66437775360*a
^13*b*c^13*d^18*e^9*z^6 + 26159874048*a^16*b^5*c^6*d^8*e^19*z^6 + 261598740
48*a^9*b^5*c^13*d^22*e^5*z^6 - 369098752*a^18*b^3*c^6*d^6*e^21*z^6 - 369098
752*a^9*b^3*c^15*d^24*e^3*z^6 + 351436800*a^8*b^16*c^3*d^13*e^14*z^6 + 3514
36800*a^6*b^16*c^5*d^17*e^10*z^6 - 334233600*a^16*b^8*c^3*d^5*e^22*z^6 - 33
4233600*a^6*b^8*c^13*d^25*e^2*z^6 + 301989888*a^19*b^3*c^5*d^4*e^23*z^6 - 2
66010624*a^10*b^15*c^2*d^10*e^17*z^6 - 266010624*a^5*b^15*c^7*d^20*e^7*z^6
- 305198530560*a^12*b^6*c^9*d^15*e^12*z^6 - 203292672*a^14*b^11*c^2*d^6*e^2
1*z^6 - 203292672*a^5*b^11*c^11*d^24*e^3*z^6 - 188743680*a^18*b^5*c^4*d^4*e
^23*z^6 + 120418467840*a^16*b^2*c^9*d^11*e^16*z^6 + 120418467840*a^12*b^2*c
^13*d^19*e^8*z^6 - 17293934592*a^10*b^12*c^5*d^13*e^14*z^6 - 17293934592*a^
8*b^12*c^7*d^17*e^10*z^6 + 104890368*a^8*b^17*c^2*d^12*e^15*z^6 + 104890368
*a^5*b^17*c^5*d^18*e^9*z^6 + 4390256640*a^15*b^8*c^4*d^7*e^20*z^6 + 4390256
640*a^7*b^8*c^12*d^23*e^4*z^6 - 91750400*a^6*b^18*c^3*d^15*e^12*z^6 + 79134
720*a^7*b^17*c^3*d^14*e^13*z^6 + 79134720*a^6*b^17*c^4*d^16*e^11*z^6 - 7461
2736*a^4*b^16*c^7*d^21*e^6*z^6 - 72990720*a^7*b^18*c^2*d^13*e^14*z^6 - 7299
0720*a^5*b^18*c^4*d^17*e^10*z^6 + 69746688*a^4*b^15*c^8*d^22*e^5*z^6 + 6370
0992*a^15*b^10*c^2*d^5*e^22*z^6 + 63700992*a^5*b^10*c^12*d^25*e^2*z^6 + 629
14560*a^17*b^7*c^3*d^4*e^23*z^6 + 55148544*a^4*b^17*c^6*d^20*e^7*z^6 - 4595
7120*a^4*b^14*c^9*d^23*e^4*z^6 - 25952256*a^4*b^18*c^5*d^19*e^8*z^6 - 25165
824*a^20*b^2*c^5*d^3*e^24*z^6 + 21086208*a^4*b^13*c^10*d^24*e^3*z^6 + 20643
840*a^6*b^19*c^2*d^14*e^13*z^6 + 20643840*a^5*b^19*c^3*d^16*e^11*z^6 + 1572
8640*a^19*b^4*c^4*d^3*e^24*z^6 - 11796480*a^16*b^9*c^2*d^4*e^23*z^6 - 64389
12*a^4*b^12*c^11*d^25*e^2*z^6 + 5406720*a^4*b^19*c^4*d^18*e^9*z^6 - 5242880
*a^18*b^6*c^3*d^3*e^24*z^6 + 3784704*a^3*b^18*c^6*d^21*e^6*z^6 - 3244032*a^
3*b^19*c^5*d^20*e^7*z^6 - 3244032*a^3*b^17*c^7*d^22*e^5*z^6 + 2027520*a^3*b
^20*c^4*d^19*e^8*z^6 + 2027520*a^3*b^16*c^8*d^23*e^4*z^6 - 1622016*a^9*b^16
*c^2*d^11*e^16*z^6 - 1622016*a^5*b^16*c^6*d^19*e^8*z^6 + 1622016*a^4*b^20*c
^3*d^17*e^10*z^6 - 1523712*a^4*b^21*c^2*d^16*e^11*z^6 + 983040*a^17*b^8*c^2
*d^3*e^24*z^6 - 901120*a^3*b^21*c^3*d^18*e^9*z^6 - 901120*a^3*b^15*c^9*d^24
*e^3*z^6 + 270336*a^3*b^22*c^2*d^17*e^10*z^6 + 270336*a^3*b^14*c^10*d^25*e^
2*z^6 + 172032*a^5*b^20*c^2*d^15*e^12*z^6 - 38593888256*a^15*b^6*c^6*d^9*e^
18*z^6 - 38593888256*a^9*b^6*c^12*d^21*e^6*z^6 - 210386288640*a^15*b^3*c^9*
d^12*e^15*z^6 - 210386288640*a^12*b^3*c^12*d^18*e^9*z^6 + 15502147584*a^15*
c^12*d^15*e^12*z^6 + 1107296256*a^19*c^8*d^7*e^20*z^6 + 1107296256*a^11*c^1

6*d^23*e^4*z^6 + 13287555072*a^16*c^11*d^13*e^14*z^6 + 13287555072*a^14*c^1
3*d^17*e^10*z^6 + 201326592*a^20*c^7*d^5*e^22*z^6 + 201326592*a^10*c^17*d^2
5*e^2*z^6 + 16777216*a^21*c^6*d^3*e^24*z^6 + 3784704*a^9*b^18*d^9*e^18*z^6
- 3244032*a^10*b^17*d^8*e^19*z^6 - 3244032*a^8*b^19*d^10*e^17*z^6 + 2027520
*a^11*b^16*d^7*e^20*z^6 + 2027520*a^7*b^20*d^11*e^16*z^6 - 901120*a^12*b^15
*d^6*e^21*z^6 - 901120*a^6*b^21*d^12*e^15*z^6 + 270336*a^13*b^14*d^5*e^22*z
^6 + 270336*a^5*b^22*d^13*e^14*z^6 - 49152*a^14*b^13*d^4*e^23*z^6 - 49152*a
^4*b^23*d^14*e^13*z^6 + 4096*a^15*b^12*d^3*e^24*z^6 + 4096*a^3*b^24*d^15*e^
12*z^6 - 25165824*a^8*b^2*c^17*d^27*z^6 + 15728640*a^7*b^4*c^16*d^27*z^6 -
5242880*a^6*b^6*c^15*d^27*z^6 + 983040*a^5*b^8*c^14*d^27*z^6 - 98304*a^4*b^
10*c^13*d^27*z^6 + 4096*a^3*b^12*c^12*d^27*z^6 + 8304721920*a^17*c^10*d^11*
e^16*z^6 + 8304721920*a^13*c^14*d^19*e^8*z^6 + 3690987520*a^18*c^9*d^9*e^18
*z^6 + 3690987520*a^12*c^15*d^21*e^6*z^6 + 16777216*a^9*c^18*d^27*z^6 - 849
3371392*a^6*b^8*c^9*d^14*e^9*z^4 + 1458044928*a^8*b*c^14*d^17*e^6*z^4 - 126
04538880*a^11*b^4*c^8*d^8*e^15*z^4 - 8303067136*a^9*b^5*c^9*d^11*e^12*z^4 -
5588058112*a^13*b*c^9*d^7*e^16*z^4 - 3892838400*a^8*b^2*c^13*d^16*e^7*z^4
- 3611713536*a^8*b^8*c^7*d^10*e^13*z^4 + 7819006464*a^7*b^9*c^7*d^11*e^12*z
^4 - 7782137856*a^8*b^7*c^8*d^11*e^12*z^4 + 7780433920*a^12*b^2*c^9*d^8*e^1
5*z^4 - 12020465664*a^7*b^5*c^11*d^15*e^8*z^4 + 3176792064*a^8*b^3*c^12*d^1
5*e^8*z^4 - 322633728*a^15*b*c^7*d^3*e^20*z^4 + 210829312*a^7*b*c^15*d^19*e
^4*z^4 + 15623258112*a^9*b^6*c^8*d^10*e^13*z^4 + 25165824*a^15*b^3*c^5*d*e^
22*z^4 - 15728640*a^14*b^5*c^4*d*e^22*z^4 + 12582912*a^5*b^2*c^16*d^22*e*z^
4 - 11730944*a^4*b^4*c^15*d^22*e*z^4 + 5242880*a^13*b^7*c^3*d*e^22*z^4 - 45
61920*a*b^15*c^7*d^17*e^6*z^4 + 4521984*a^3*b^6*c^14*d^22*e*z^4 + 4460544*a
*b^14*c^8*d^18*e^5*z^4 + 3538944*a^6*b*c^16*d^21*e^2*z^4 + 3108864*a*b^16*c
^6*d^16*e^7*z^4 - 3027200*a*b^13*c^9*d^19*e^4*z^4 - 2345472*a^5*b^17*c*d^7*
e^16*z^4 - 2307072*a^8*b^14*c*d^4*e^19*z^4 + 1824768*a^6*b^16*c*d^6*e^17*z^
4 + 1734912*a^9*b^13*c*d^3*e^20*z^4 + 1419264*a*b^12*c^10*d^20*e^3*z^4 - 11
91168*a*b^17*c^5*d^15*e^8*z^4 - 983040*a^12*b^9*c^2*d*e^22*z^4 + 964608*a^4
*b^18*c*d^8*e^15*z^4 - 866304*a^2*b^8*c^13*d^22*e*z^4 + 703488*a^7*b^15*c*d
^5*e^18*z^4 - 608256*a^10*b^12*c*d^2*e^21*z^4 - 440832*a*b^11*c^11*d^21*e^2
*z^4 + 275968*a*b^19*c^3*d^13*e^10*z^4 - 159744*a^2*b^20*c*d^10*e^13*z^4 -
153600*a*b^20*c^2*d^12*e^11*z^4 + 64512*a^3*b^19*c*d^9*e^14*z^4 + 197460623
36*a^8*b^6*c^9*d^12*e^11*z^4 - 15333588992*a^10*b^4*c^9*d^10*e^13*z^4 + 670
2170112*a^7*b^4*c^12*d^16*e^7*z^4 + 15167913984*a^10*b^3*c^10*d^11*e^12*z^4
- 2256638976*a^5*b^11*c^7*d^13*e^10*z^4 + 2254307328*a^5*b^7*c^11*d^17*e^6
*z^4 - 2200633344*a^6*b^5*c^12*d^17*e^6*z^4 + 6457131008*a^11*b^3*c^9*d^9*e
^14*z^4 - 2128785408*a^5*b^8*c^10*d^16*e^7*z^4 - 2126057472*a^6*b^11*c^6*d^
11*e^12*z^4 + 2038349824*a^12*b^5*c^6*d^5*e^18*z^4 + 2037841920*a^5*b^10*c^
8*d^14*e^9*z^4 + 3615621120*a^9*b*c^13*d^15*e^8*z^4 + 1900019712*a^11*b^2*c
^10*d^10*e^13*z^4 + 1867698432*a^9*b^9*c^5*d^7*e^16*z^4 - 6157369344*a^9*b^
4*c^10*d^12*e^11*z^4 - 1856913408*a^7*b^10*c^6*d^10*e^13*z^4 + 1789132800*a
^6*b^4*c^13*d^18*e^5*z^4 + 6082658304*a^8*b^4*c^11*d^14*e^9*z^4 + 602954956
8*a^11*b^5*c^7*d^7*e^16*z^4 + 6010159104*a^6*b^7*c^10*d^15*e^8*z^4 + 170318
2336*a^7*b^7*c^9*d^13*e^10*z^4 + 1658388480*a^11*b^6*c^6*d^6*e^17*z^4 + 591
7114368*a^10*b^6*c^7*d^8*e^15*z^4 - 1591197696*a^11*b^7*c^5*d^5*e^18*z^4 -
1526464512*a^8*b^10*c^5*d^8*e^15*z^4 - 5772607488*a^12*b^4*c^7*d^6*e^17*z^4
- 1423507456*a^13*b^4*c^6*d^4*e^19*z^4 - 1387266048*a^7*b^3*c^13*d^17*e^6*
z^4 + 2976120832*a^10*b*c^12*d^13*e^10*z^4 - 9906946048*a^9*b^2*c^12*d^14*e
^9*z^4 - 18421874688*a^8*b^5*c^10*d^13*e^10*z^4 + 1141217280*a^6*b^12*c^5*d
^10*e^13*z^4 - 9714364416*a^7*b^8*c^8*d^12*e^11*z^4 - 16777216*a^16*b*c^6*d
*e^22*z^4 + 98304*a^11*b^11*c*d*e^22*z^4 + 81920*a*b^10*c^12*d^22*e*z^4 + 3
9168*a*b^21*c*d^11*e^12*z^4 - 1091829760*a^5*b^6*c^12*d^18*e^5*z^4 + 104674
0992*a^14*b^2*c^7*d^4*e^19*z^4 - 6884425728*a^12*b*c^10*d^9*e^14*z^4 + 9874
45248*a^4*b^10*c^9*d^16*e^7*z^4 + 984087552*a^5*b^12*c^6*d^12*e^11*z^4 - 95
64585984*a^9*b^7*c^7*d^9*e^14*z^4 - 5266857984*a^10*b^7*c^6*d^7*e^16*z^4 -
892145664*a^7*b^11*c^5*d^9*e^14*z^4 - 2444623872*a^11*b*c^11*d^11*e^12*z^4
+ 768540672*a^12*b^3*c^8*d^7*e^16*z^4 + 5048322048*a^8*b^9*c^6*d^9*e^14*z^4
+ 5047612416*a^6*b^9*c^8*d^13*e^10*z^4 - 732492288*a^4*b^11*c^8*d^15*e^8*z

^4 + 9266921472*a^7*b^6*c^10*d^14*e^9*z^4 - 645857280*a^6*b^6*c^11*d^16*e^7
*z^4 - 623867904*a^4*b^9*c^10*d^17*e^6*z^4 - 622067712*a^6*b^3*c^14*d^19*e^
4*z^4 + 582617088*a^10*b^8*c^5*d^6*e^17*z^4 + 577119744*a^7*b^12*c^4*d^8*e^
15*z^4 + 552566784*a^12*b^6*c^5*d^4*e^19*z^4 + 549224448*a^9*b^8*c^6*d^8*e^
15*z^4 - 526565376*a^9*b^10*c^4*d^6*e^17*z^4 + 511520256*a^10*b^9*c^4*d^5*e
^18*z^4 + 13393723392*a^9*b^3*c^11*d^13*e^10*z^4 - 2066350080*a^14*b*c^8*d^
5*e^18*z^4 + 4718592000*a^13*b^2*c^8*d^6*e^17*z^4 - 314572800*a^7*b^2*c^14*
d^18*e^5*z^4 + 287250432*a^4*b^13*c^6*d^13*e^10*z^4 + 4565827584*a^10*b^5*c
^8*d^9*e^14*z^4 - 250785792*a^4*b^14*c^5*d^12*e^11*z^4 + 235536384*a^13*b^3
*c^7*d^5*e^18*z^4 - 232683264*a^8*b^11*c^4*d^7*e^16*z^4 - 199627776*a^5*b^1
4*c^4*d^10*e^13*z^4 - 190267392*a^12*b^7*c^4*d^3*e^20*z^4 + 184891392*a^6*b
^10*c^7*d^12*e^11*z^4 + 180502528*a^4*b^7*c^12*d^19*e^4*z^4 + 178877952*a^3
*b^13*c^7*d^15*e^8*z^4 + 172490752*a^14*b^3*c^6*d^3*e^20*z^4 + 163946496*a^
13*b^5*c^5*d^3*e^20*z^4 + 155839488*a^8*b^12*c^3*d^6*e^17*z^4 + 155000832*a
^5*b^5*c^13*d^19*e^4*z^4 - 152076288*a^4*b^6*c^13*d^20*e^3*z^4 - 137592576*
a^3*b^12*c^8*d^16*e^7*z^4 - 133693440*a^14*b^4*c^5*d^2*e^21*z^4 - 116767488
*a^3*b^9*c^11*d^19*e^4*z^4 - 108985344*a^3*b^14*c^6*d^14*e^9*z^4 - 10622361
6*a^6*b^13*c^4*d^9*e^14*z^4 + 106119168*a^3*b^10*c^10*d^18*e^5*z^4 + 102432
768*a^5*b^4*c^14*d^20*e^3*z^4 + 102113280*a^4*b^12*c^7*d^14*e^9*z^4 + 10067
4048*a^5*b^9*c^9*d^15*e^8*z^4 + 90439680*a^13*b^6*c^4*d^2*e^21*z^4 - 868085
76*a^6*b^14*c^3*d^8*e^15*z^4 + 86245376*a^6*b^2*c^15*d^20*e^3*z^4 + 7901184
0*a^4*b^8*c^11*d^18*e^5*z^4 + 78345216*a^4*b^15*c^4*d^11*e^12*z^4 + 7800652
8*a^11*b^9*c^3*d^3*e^20*z^4 - 73253376*a^9*b^11*c^3*d^5*e^18*z^4 + 67524608
*a^3*b^8*c^12*d^20*e^3*z^4 + 67108864*a^15*b^2*c^6*d^2*e^21*z^4 - 61590528*
a^10*b^10*c^3*d^4*e^19*z^4 + 61559808*a^5*b^15*c^3*d^9*e^14*z^4 - 59637760*
a^5*b^3*c^15*d^21*e^2*z^4 + 58638336*a^4*b^5*c^14*d^21*e^2*z^4 - 40828416*a
^7*b^13*c^3*d^7*e^16*z^4 - 35639296*a^2*b^12*c^9*d^18*e^5*z^4 - 31293440*a^
12*b^8*c^3*d^2*e^21*z^4 + 29933568*a^5*b^13*c^5*d^11*e^12*z^4 + 27793920*a^
2*b^11*c^10*d^19*e^4*z^4 + 27168768*a^2*b^13*c^8*d^17*e^6*z^4 - 23602176*a^
7*b^14*c^2*d^6*e^17*z^4 - 23248896*a^3*b^7*c^13*d^21*e^2*z^4 + 20929536*a^3
*b^15*c^5*d^13*e^10*z^4 + 18428928*a^9*b^12*c^2*d^4*e^19*z^4 + 18026496*a^6
*b^15*c^2*d^7*e^16*z^4 - 16261632*a^10*b^11*c^2*d^3*e^20*z^4 + 15128064*a^3
*b^16*c^4*d^12*e^11*z^4 - 14060544*a^2*b^10*c^11*d^20*e^3*z^4 + 13178880*a^
2*b^16*c^5*d^14*e^9*z^4 - 11244288*a^3*b^17*c^3*d^11*e^12*z^4 - 10509312*a^
2*b^15*c^6*d^15*e^8*z^4 - 7262208*a^4*b^17*c^2*d^9*e^14*z^4 - 7045632*a^2*b
^17*c^4*d^13*e^10*z^4 - 6285312*a^2*b^14*c^7*d^16*e^7*z^4 + 5996544*a^11*b^
10*c^2*d^2*e^21*z^4 + 4558336*a^2*b^9*c^12*d^21*e^2*z^4 + 4478976*a^11*b^8*
c^4*d^4*e^19*z^4 + 2850816*a^4*b^16*c^3*d^10*e^13*z^4 + 2629632*a^3*b^11*c^
9*d^17*e^6*z^4 + 2503680*a^3*b^18*c^2*d^10*e^13*z^4 + 1627136*a^2*b^18*c^3*
d^12*e^11*z^4 + 1605120*a^8*b^13*c^2*d^5*e^18*z^4 + 1483776*a^5*b^16*c^2*d^
8*e^15*z^4 + 139776*a^2*b^19*c^2*d^11*e^12*z^4 - 8542224384*a^10*b^2*c^11*d
^12*e^11*z^4 - 3072*b^22*c*d^12*e^11*z^4 - 3072*b^12*c^11*d^22*e*z^4 - 1572
864*a^6*c^17*d^22*e*z^4 - 4096*a^10*b^13*d*e^22*z^4 - 4096*a*b^22*d^10*e^13
*z^4 - 6144*a^12*b^10*c*e^23*z^4 - 983040*a^5*b*c^17*d^23*z^4 - 6912*a*b^9*
c^13*d^23*z^4 + 1824522240*a^13*c^10*d^8*e^15*z^4 + 1730150400*a^12*c^11*d^
10*e^13*z^4 + 958922752*a^14*c^9*d^6*e^17*z^4 - 537919488*a^9*c^14*d^16*e^7
*z^4 + 508559360*a^11*c^12*d^12*e^11*z^4 - 500170752*a^10*c^13*d^14*e^9*z^4
 + 246939648*a^15*c^8*d^4*e^19*z^4 - 199229440*a^8*c^15*d^18*e^5*z^4 - 2988
4416*a^7*c^16*d^20*e^3*z^4 + 25165824*a^16*c^7*d^2*e^21*z^4 + 236544*b^17*c
^6*d^17*e^6*z^4 - 202752*b^18*c^5*d^16*e^7*z^4 - 202752*b^16*c^7*d^18*e^5*z
^4 + 126720*b^19*c^4*d^15*e^8*z^4 + 126720*b^15*c^8*d^19*e^4*z^4 - 56320*b^
20*c^3*d^14*e^9*z^4 - 56320*b^14*c^9*d^20*e^3*z^4 + 16896*b^21*c^2*d^13*e^1
0*z^4 + 16896*b^13*c^10*d^21*e^2*z^4 + 110080*a^7*b^16*d^4*e^19*z^4 + 11008
0*a^4*b^19*d^7*e^16*z^4 - 75520*a^8*b^15*d^3*e^20*z^4 - 75520*a^3*b^20*d^8*
e^15*z^4 - 56320*a^6*b^17*d^5*e^18*z^4 - 56320*a^5*b^18*d^6*e^17*z^4 + 2560
0*a^9*b^14*d^2*e^21*z^4 + 25600*a^2*b^21*d^9*e^14*z^4 - 1572864*a^16*b^2*c^
5*e^23*z^4 + 983040*a^15*b^4*c^4*e^23*z^4 - 327680*a^14*b^6*c^3*e^23*z^4 +
61440*a^13*b^8*c^2*e^23*z^4 + 983040*a^4*b^3*c^16*d^23*z^4 - 385024*a^3*b^5
*c^15*d^23*z^4 + 73728*a^2*b^7*c^14*d^23*z^4 + 256*b^23*d^11*e^12*z^4 + 104

8576*a^17*c^6*e^23*z^4 + 256*b^11*c^12*d^23*z^4 + 256*a^11*b^12*e^23*z^4 +
948695040*a^8*b*c^10*d^6*e^13*z^2 + 348917760*a^7*b*c^11*d^8*e^11*z^2 - 125
030400*a^9*b*c^9*d^4*e^15*z^2 - 50728960*a^6*b*c^12*d^10*e^9*z^2 - 44298240
*a^5*b*c^13*d^12*e^7*z^2 - 36495360*a^10*b*c^8*d^2*e^17*z^2 + 29675520*a^8*
b^6*c^5*d*e^18*z^2 - 24170496*a^9*b^4*c^6*d*e^18*z^2 - 17202816*a^7*b^8*c^4
*d*e^18*z^2 - 14561280*a^4*b*c^14*d^14*e^5*z^2 + 5532416*a^6*b^10*c^3*d*e^1
8*z^2 + 4128768*a^10*b^2*c^7*d*e^18*z^2 - 2662400*a^3*b*c^15*d^16*e^3*z^2 +
1184512*a*b^12*c^6*d^9*e^10*z^2 - 1136160*a*b^13*c^5*d^8*e^11*z^2 - 101760
0*a^5*b^12*c^2*d*e^18*z^2 - 744768*a*b^11*c^7*d^10*e^9*z^2 + 607872*a*b^14*
c^4*d^7*e^12*z^2 - 424064*a*b^6*c^12*d^15*e^4*z^2 + 408576*a*b^5*c^13*d^16*
e^3*z^2 + 361152*a*b^10*c^8*d^11*e^8*z^2 - 287408*a*b^9*c^9*d^12*e^7*z^2 -
260448*a^3*b^15*c*d^2*e^17*z^2 - 203904*a*b^4*c^14*d^17*e^2*z^2 + 200832*a*
b^8*c^10*d^13*e^6*z^2 + 126720*a*b^7*c^11*d^14*e^5*z^2 - 123968*a*b^15*c^3*
d^6*e^13*z^2 - 39168*a*b^16*c^2*d^5*e^14*z^2 + 11904*a^2*b^16*c*d^3*e^16*z^
2 + 1824135552*a^7*b^4*c^8*d^5*e^14*z^2 - 1457252352*a^8*b^2*c^9*d^5*e^14*z
^2 - 1405209600*a^7*b^5*c^7*d^4*e^15*z^2 - 184320*a^2*b*c^16*d^18*e*z^2 + 1
00608*a^4*b^14*c*d*e^18*z^2 + 53248*a*b^3*c^15*d^18*e*z^2 + 26448*a*b^17*c*
d^4*e^15*z^2 + 1067599872*a^8*b^3*c^8*d^4*e^15*z^2 - 930828288*a^7*b^3*c^9*
d^6*e^13*z^2 + 920760000*a^6*b^4*c^9*d^7*e^12*z^2 - 806639616*a^6*b^3*c^10*
d^8*e^11*z^2 - 791052480*a^6*b^6*c^7*d^5*e^14*z^2 + 772237824*a^6*b^7*c^6*d
^4*e^15*z^2 - 701025408*a^5*b^6*c^8*d^7*e^12*z^2 + 443340288*a^5*b^5*c^9*d^
8*e^11*z^2 + 433047552*a^7*b^6*c^6*d^3*e^16*z^2 + 405741312*a^5*b^7*c^7*d^6
*e^13*z^2 + 293652480*a^6*b^2*c^11*d^9*e^10*z^2 - 276962688*a^6*b^8*c^5*d^3
*e^16*z^2 - 247804272*a^8*b^4*c^7*d^3*e^16*z^2 + 213564384*a^4*b^8*c^7*d^7*
e^12*z^2 - 202596816*a^5*b^9*c^5*d^4*e^15*z^2 - 182520896*a^4*b^9*c^6*d^6*e
^13*z^2 - 153489408*a^5*b^3*c^11*d^10*e^9*z^2 - 152151552*a^7*b^2*c^10*d^7*
e^12*z^2 + 115859712*a^5*b^2*c^12*d^11*e^8*z^2 + 108085248*a^9*b^3*c^7*d^2*
e^17*z^2 + 105536256*a^4*b^5*c^10*d^10*e^9*z^2 - 98323200*a^6*b^5*c^8*d^6*e
^13*z^2 - 93564992*a^4*b^6*c^9*d^9*e^10*z^2 + 89464512*a^5*b^10*c^4*d^3*e^1
6*z^2 - 75930624*a^8*b^5*c^6*d^2*e^17*z^2 + 68315904*a^5*b^8*c^6*d^5*e^14*z
^2 - 64157184*a^4*b^7*c^8*d^8*e^11*z^2 - 62951040*a^9*b^2*c^8*d^3*e^16*z^2
+ 49056768*a^4*b^10*c^5*d^5*e^14*z^2 + 47614464*a^3*b^8*c^8*d^9*e^10*z^2 +
35604480*a^4*b^2*c^13*d^13*e^6*z^2 + 33983040*a^3*b^11*c^5*d^6*e^13*z^2 - 3
3515520*a^4*b^3*c^12*d^12*e^7*z^2 - 33463808*a^3*b^7*c^9*d^10*e^9*z^2 - 251
28864*a^4*b^4*c^11*d^11*e^8*z^2 - 23193728*a^3*b^10*c^6*d^7*e^12*z^2 + 2101
5456*a^6*b^9*c^4*d^2*e^17*z^2 + 19924176*a^4*b^11*c^4*d^4*e^15*z^2 - 192512
16*a^3*b^9*c^7*d^8*e^11*z^2 - 16434048*a^5*b^4*c^10*d^9*e^10*z^2 - 16289664
*a^3*b^12*c^4*d^5*e^14*z^2 - 15059328*a^4*b^12*c^3*d^3*e^16*z^2 - 10766016*
a^2*b^10*c^7*d^9*e^10*z^2 - 10453632*a^5*b^11*c^3*d^2*e^17*z^2 - 9940992*a^
3*b^3*c^13*d^14*e^5*z^2 + 8373696*a^2*b^11*c^6*d^8*e^11*z^2 + 7776768*a^3*b
^2*c^14*d^15*e^4*z^2 + 7077888*a^3*b^5*c^11*d^12*e^7*z^2 + 6798240*a^2*b^9*
c^8*d^10*e^9*z^2 - 3589440*a^2*b^6*c^11*d^13*e^6*z^2 + 3544320*a^3*b^6*c^10
*d^11*e^8*z^2 + 3128064*a^2*b^5*c^12*d^14*e^5*z^2 + 2346336*a^4*b^13*c^2*d^
2*e^17*z^2 - 2261568*a^2*b^8*c^9*d^11*e^8*z^2 - 2125824*a^2*b^13*c^4*d^6*e^
13*z^2 + 2002560*a^3*b^4*c^12*d^13*e^6*z^2 + 1927680*a^2*b^7*c^10*d^12*e^7*
z^2 + 1814784*a^2*b^14*c^3*d^5*e^14*z^2 - 1807104*a^2*b^12*c^5*d^7*e^12*z^2
+ 1637808*a^3*b^13*c^3*d^4*e^15*z^2 + 1083456*a^3*b^14*c^2*d^3*e^16*z^2 -
792384*a^2*b^4*c^13*d^15*e^4*z^2 - 657408*a^2*b^3*c^14*d^16*e^3*z^2 + 60825
6*a^7*b^7*c^5*d^2*e^17*z^2 + 595968*a^2*b^2*c^15*d^17*e^2*z^2 - 498624*a^2*
b^15*c^2*d^4*e^15*z^2 - 3840*b^18*c*d^5*e^14*z^2 - 3840*b^5*c^14*d^18*e*z^2
+ 2064384*a^11*c^8*d*e^18*z^2 - 4160*a^3*b^16*d*e^18*z^2 - 4160*a*b^18*d^3
*e^16*z^2 - 1290240*a^11*b*c^7*e^19*z^2 - 9840*a^5*b^13*c*e^19*z^2 - 5760*a
*b^2*c^16*d^19*z^2 - 280581120*a^8*c^11*d^7*e^12*z^2 + 110278656*a^9*c^10*d
^5*e^14*z^2 - 89479168*a^7*c^12*d^9*e^10*z^2 + 34464000*a^10*c^9*d^3*e^16*z
^2 + 54240*b^15*c^4*d^8*e^11*z^2 + 54240*b^8*c^11*d^15*e^4*z^2 - 49920*b^14
*c^5*d^9*e^10*z^2 - 49920*b^9*c^10*d^14*e^5*z^2 - 37376*b^16*c^3*d^7*e^12*z
^2 - 37376*b^7*c^12*d^16*e^3*z^2 + 28480*b^13*c^6*d^10*e^9*z^2 + 28480*b^10
*c^9*d^13*e^6*z^2 + 15936*b^17*c^2*d^6*e^13*z^2 + 15936*b^6*c^13*d^17*e^2*z
^2 - 7920*b^12*c^7*d^11*e^8*z^2 - 7920*b^11*c^8*d^12*e^7*z^2 + 7489536*a^5*

```
c^14*d^13*e^6*z^2 + 6084096*a^6*c^13*d^11*e^8*z^2 + 2280448*a^4*c^15*d^15*e
^4*z^2 + 350208*a^3*c^16*d^17*e^2*z^2 + 11616*a^2*b^17*d^2*e^17*z^2 - 35159
04*a^9*b^5*c^5*e^19*z^2 + 3440640*a^10*b^3*c^6*e^19*z^2 + 1870848*a^8*b^7*c
^4*e^19*z^2 - 572272*a^7*b^9*c^3*e^19*z^2 + 101856*a^6*b^11*c^2*e^19*z^2 +
400*b^19*d^4*e^15*z^2 + 400*b^4*c^15*d^19*z^2 + 20736*a^2*c^17*d^19*z^2 + 4
00*a^4*b^15*e^19*z^2 - 3969216*a^4*b*c^10*d^3*e^12 - 3001536*a^3*b*c^11*d^5
*e^10 - 419904*a^2*b*c^12*d^7*e^8 + 184608*a^4*b^3*c^8*d*e^14 - 153036*a*b^
4*c^10*d^6*e^9 + 127008*a*b^3*c^11*d^7*e^8 + 63108*a*b^6*c^8*d^4*e^11 - 291
60*a*b^2*c^12*d^8*e^7 - 21060*a^3*b^5*c^7*d*e^14 - 21060*a*b^7*c^7*d^3*e^12
 + 5460*a*b^5*c^9*d^5*e^10 - 404544*a^5*b*c^9*d*e^14 + 1251872*a^3*b^3*c^9*
d^3*e^12 + 844224*a^4*b^2*c^9*d^2*e^13 + 820512*a^2*b^3*c^10*d^5*e^10 + 750
672*a^3*b^2*c^10*d^4*e^11 - 657498*a^2*b^4*c^9*d^4*e^11 - 487116*a^3*b^4*c^
8*d^2*e^13 + 160704*a^2*b^2*c^11*d^6*e^9 + 58806*a^2*b^6*c^7*d^2*e^13 + 131
40*a^2*b^5*c^8*d^3*e^12 + 15286*b^6*c^9*d^6*e^9 - 9540*b^7*c^8*d^5*e^10 - 9
540*b^5*c^10*d^7*e^8 + 2025*b^8*c^7*d^4*e^11 + 2025*b^4*c^11*d^8*e^7 + 3367
008*a^4*c^11*d^4*e^11 + 1166400*a^3*c^12*d^6*e^9 + 705600*a^5*c^10*d^2*e^13
 + 104976*a^2*c^13*d^8*e^7 - 17640*a^5*b^2*c^8*e^15 + 2025*a^4*b^4*c^7*e^15
 + 38416*a^6*c^9*e^15, z, k), k, 1, 6) - ((x*(a^2*b^2*e^4 - 4*a^3*c*e^4 - 2
*a*c^3*d^4 + b^2*c^2*d^4 + b^4*d^2*e^2 + 2*a^2*c^2*d^2*e^2 - 2*b^3*c*d^3*e
+ 6*a*b*c^2*d^3*e - 4*a*b^2*c*d^2*e^2))/(2*a*d*(4*a*c^3*d^4 + 4*a^3*c*e^4 -
 a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^
3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2))
 + (x^3*(a*b^3*e^4 + b*c^3*d^4 + b^4*d*e^3 + 2*a^2*c^2*d*e^3 - b^2*c^2*d^3*
e - b^3*c*d^2*e^2 - 4*a^2*b*c*e^4 + 2*a*c^3*d^3*e - 4*a*b^2*c*d*e^3 + 3*a*b
*c^2*d^2*e^2))/(2*a*d*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^
4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b
*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) + (c*e*x^5*(a*b^2*e^3 +
b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b
*c*d*e^2))/(2*a*d*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 -
b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2
*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)))/(a*d + x^2*(a*e + b*d) + x^
4*(b*e + c*d) + c*e*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

3.276 $\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=215

$$\frac{x\left(d+ex^2\right)^{5/2}\left(80ae^2-10bde+3cd^2\right)}{480e^2}+\frac{dx\left(d+ex^2\right)^{3/2}\left(80ae^2-10bde+3cd^2\right)}{384e^2}+\frac{d^2x\sqrt{d+ex^2}\left(80ae^2-10bde+\right.}{256e^2}$$

```
[Out] 1/384*d*(80*a*e^2-10*b*d*e+3*c*d^2)*x*(e*x^2+d)^(3/2)/e^2+1/480*(80*a*e^2-1
0*b*d*e+3*c*d^2)*x*(e*x^2+d)^(5/2)/e^2-1/80*(-10*b*e+3*c*d)*x*(e*x^2+d)^(7/
2)/e^2+1/10*c*x^3*(e*x^2+d)^(7/2)/e+1/256*d^3*(80*a*e^2-10*b*d*e+3*c*d^2)*a
rctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/256*d^2*(80*a*e^2-10*b*d*e+3*c*
d^2)*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] time = 0.16, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1159, 388, 195, 217, 206}

$$\frac{x\left(d+ex^2\right)^{5/2}\left(80ae^2-10bde+3cd^2\right)}{480e^2}+\frac{dx\left(d+ex^2\right)^{3/2}\left(80ae^2-10bde+3cd^2\right)}{384e^2}+\frac{d^2x\sqrt{d+ex^2}\left(80ae^2-10bde+\right.}{256e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d^2*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*Sqrt[d + e*x^2])/(256*e^2) + (d*(3*c
*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(3/2))/(384*e^2) + ((3*c*d^2 - 10
*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(5/2))/(480*e^2) - ((3*c*d - 10*b*e)*x*(d
+ e*x^2)^(7/2))/(80*e^2) + (c*x^3*(d + e*x^2)^(7/2))/(10*e) + (d^3*(3*c*d^2
 - 10*b*d*e + 80*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(256*e^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)dx = \frac{cx^3\left(d+ex^2\right)^{7/2}}{10e}+\frac{\int \left(d+ex^2\right)^{5/2}\left(10ae-(3cd-10be)x^2\right)dx}{10e}$$

$$= -\frac{(3cd-10be)x\left(d+ex^2\right)^{7/2}}{80e^2}+\frac{cx^3\left(d+ex^2\right)^{7/2}}{10e}-\frac{1}{80}\left(-80a-\frac{d(3cd-10be)}{e^2}\right)$$

$$= \frac{1}{480}\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{5/2}-\frac{(3cd-10be)x\left(d+ex^2\right)^{7/2}}{80e^2}+\frac{cx^3}{}$$

$$= \frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{3/2}+\frac{1}{480}\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\,(d$$

$$= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\,(d$$

$$= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\,(d$$

$$= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\,(d$$

Mathematica [A] time = 0.39, size = 190, normalized size = 0.88

$$\frac{\sqrt{d+ex^2}\left(\frac{15d^{5/2}\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(10e(8ae-bd)+3cd^2\right)}{\sqrt{\frac{ex^2}{d}+1}}+\sqrt{e}\,x\left(10e\left(8ae\left(33d^2+26dex^2+8e^2x^4\right)+b\left(15d^3+118d^2ex^2+136d\right.\right.\right.\right.}{3840e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2]*(Sqrt[e]*x*(c*(-45*d^4 + 30*d^3*e*x^2 + 744*d^2*e^2*x^4 +
1008*d*e^3*x^6 + 384*e^4*x^8) + 10*e*(8*a*e*(33*d^2 + 26*d*e*x^2 + 8*e^2*x^
4) + b*(15*d^3 + 118*d^2*e*x^2 + 136*d*e^2*x^4 + 48*e^3*x^6))) + (15*d^(5/2
)*(3*c*d^2 + 10*e*(-(b*d) + 8*a*e))*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[1 +
(e*x^2)/d]))/(3840*e^(5/2))
```

fricas [A] time = 1.13, size = 370, normalized size = 1.72

$$\left[\frac{15\left(3\,cd^5-10\,bd^4e+80\,ad^3e^2\right)\sqrt{e}\,\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\,\sqrt{e}\,x-d\right)+2\left(384\,ce^5x^9+48\left(21\,cde^4+10\,be^5\right)x^7+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*sqrt(e)*log(-2*e*x^2 - 2*
sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(384*c*e^5*x^9 + 48*(21*c*d*e^4 + 10*b*e
^5)*x^7 + 8*(93*c*d^2*e^3 + 170*b*d*e^4 + 80*a*e^5)*x^5 + 10*(3*c*d^3*e^2 +
 118*b*d^2*e^3 + 208*a*d*e^4)*x^3 - 15*(3*c*d^4*e - 10*b*d^3*e^2 - 176*a*d^
2*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/3840*(15*(3*c*d^5 - 10*b*d^4*e + 80*a*d^
3*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (384*c*e^5*x^9 + 48*(2
1*c*d*e^4 + 10*b*e^5)*x^7 + 8*(93*c*d^2*e^3 + 170*b*d*e^4 + 80*a*e^5)*x^5 +
 10*(3*c*d^3*e^2 + 118*b*d^2*e^3 + 208*a*d*e^4)*x^3 - 15*(3*c*d^4*e - 10*b*
d^3*e^2 - 176*a*d^2*e^3)*x)*sqrt(e*x^2 + d))/e^3]
```

giac [A] time = 0.23, size = 180, normalized size = 0.84

$$-\frac{1}{256}\left(3\,cd^5-10\,bd^4e+80\,ad^3e^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{3840}\left(2\left(4\left(6\left(8\,cx^2e^2+\left(21\,cde^9+10\,be^{10}\right)e\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/256*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) +
sqrt(x^2*e + d))) + 1/3840*(2*(4*(6*(8*c*x^2*e^2 + (21*c*d*e^9 + 10*b*e^10)
*e^(-8))*x^2 + (93*c*d^2*e^8 + 170*b*d*e^9 + 80*a*e^10)*e^(-8))*x^2 + 5*(3*
c*d^3*e^7 + 118*b*d^2*e^8 + 208*a*d*e^9)*e^(-8))*x^2 - 15*(3*c*d^4*e^6 - 10
*b*d^3*e^7 - 176*a*d^2*e^8)*e^(-8))*sqrt(x^2*e + d)*x
```

maple [A] time = 0.01, size = 283, normalized size = 1.32

$$\frac{5a\,d^3\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{16\sqrt{e}}-\frac{5b\,d^4\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{128e^{\frac{3}{2}}}+\frac{3c\,d^5\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{256e^{\frac{5}{2}}}+\frac{5\sqrt{e\,x^2+d}\;a\,d^2x}{16}-5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/10*c*x^3*(e*x^2+d)^(7/2)/e-3/80*c*d/e^2*x*(e*x^2+d)^(7/2)+1/160*c*d^2/e^2
*x*(e*x^2+d)^(5/2)+1/128*c*d^3/e^2*x*(e*x^2+d)^(3/2)+3/256*c*d^4/e^2*x*(e*x
^2+d)^(1/2)+3/256*c*d^5/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/8*b*x*(e*x^
2+d)^(7/2)/e-1/48*b*d/e*x*(e*x^2+d)^(5/2)-5/192*b*d^2/e*x*(e*x^2+d)^(3/2)-5
/128*b*d^3/e*x*(e*x^2+d)^(1/2)-5/128*b*d^4/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(
1/2))+1/6*a*x*(e*x^2+d)^(5/2)+5/24*a*d*x*(e*x^2+d)^(3/2)+5/16*a*d^2*x*(e*x^
2+d)^(1/2)+5/16*a*d^3/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

maxima [A] time = 1.12, size = 261, normalized size = 1.21

$$\frac{\left(ex^2+d\right)^{\frac{7}{2}}cx^3}{10\,e}+\frac{1}{6}\left(ex^2+d\right)^{\frac{5}{2}}ax+\frac{5}{24}\left(ex^2+d\right)^{\frac{3}{2}}adx+\frac{5}{16}\sqrt{ex^2+d}\,ad^2x-\frac{3\left(ex^2+d\right)^{\frac{7}{2}}cdx}{80\,e^2}+\frac{\left(ex^2+d\right)^{\frac{5}{2}}cd^2x}{160\,e^2}+\left(ex\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/10*(e*x^2 + d)^(7/2)*c*x^3/e + 1/6*(e*x^2 + d)^(5/2)*a*x + 5/24*(e*x^2 +
d)^(3/2)*a*d*x + 5/16*sqrt(e*x^2 + d)*a*d^2*x - 3/80*(e*x^2 + d)^(7/2)*c*d*
x/e^2 + 1/160*(e*x^2 + d)^(5/2)*c*d^2*x/e^2 + 1/128*(e*x^2 + d)^(3/2)*c*d^3
*x/e^2 + 3/256*sqrt(e*x^2 + d)*c*d^4*x/e^2 + 1/8*(e*x^2 + d)^(7/2)*b*x/e -
1/48*(e*x^2 + d)^(5/2)*b*d*x/e - 5/192*(e*x^2 + d)^(3/2)*b*d^2*x/e - 5/128*
sqrt(e*x^2 + d)*b*d^3*x/e + 3/256*c*d^5*arcsinh(e*x/sqrt(d*e))/e^(5/2) - 5/
128*b*d^4*arcsinh(e*x/sqrt(d*e))/e^(3/2) + 5/16*a*d^3*arcsinh(e*x/sqrt(d*e)
)/sqrt(e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x^2+d\right)^{5/2}\,\left(c\,x^4+b\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4), x)
```

sympy [B] time = 63.83, size = 505, normalized size = 2.35

$$\frac{ad^{\frac{5}{2}}x\sqrt{1+\frac{ex^2}{d}}}{2}+\frac{3ad^{\frac{5}{2}}x}{16\sqrt{1+\frac{ex^2}{d}}}+\frac{35ad^{\frac{3}{2}}ex^3}{48\sqrt{1+\frac{ex^2}{d}}}+\frac{17a\sqrt{d}\,e^2x^5}{24\sqrt{1+\frac{ex^2}{d}}}+\frac{5ad^3\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16\sqrt{e}}+\frac{ae^3x^7}{6\sqrt{d}\,\sqrt{1+\frac{ex^2}{d}}}+\frac{5bd^{\frac{7}{2}}x}{128e\sqrt{1+\frac{ex^2}{d}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**(5/2)*x*sqrt(1 + e*x**2/d)/2 + 3*a*d**(5/2)*x/(16*sqrt(1 + e*x**2/d))
+ 35*a*d**(3/2)*e*x**3/(48*sqrt(1 + e*x**2/d)) + 17*a*sqrt(d)*e**2*x**5/(24
*sqrt(1 + e*x**2/d)) + 5*a*d**3*asinh(sqrt(e)*x/sqrt(d))/(16*sqrt(e)) + a*e
**3*x**7/(6*sqrt(d)*sqrt(1 + e*x**2/d)) + 5*b*d**(7/2)*x/(128*e*sqrt(1 + e*
x**2/d)) + 133*b*d**(5/2)*x**3/(384*sqrt(1 + e*x**2/d)) + 127*b*d**(3/2)*e*
x**5/(192*sqrt(1 + e*x**2/d)) + 23*b*sqrt(d)*e**2*x**7/(48*sqrt(1 + e*x**2/
d)) - 5*b*d**4*asinh(sqrt(e)*x/sqrt(d))/(128*e**(3/2)) + b*e**3*x**9/(8*sqr
t(d)*sqrt(1 + e*x**2/d)) - 3*c*d**(9/2)*x/(256*e**2*sqrt(1 + e*x**2/d)) - c
*d**(7/2)*x**3/(256*e*sqrt(1 + e*x**2/d)) + 129*c*d**(5/2)*x**5/(640*sqrt(1
 + e*x**2/d)) + 73*c*d**(3/2)*e*x**7/(160*sqrt(1 + e*x**2/d)) + 29*c*sqrt(d
)*e**2*x**9/(80*sqrt(1 + e*x**2/d)) + 3*c*d**5*asinh(sqrt(e)*x/sqrt(d))/(25
6*e**(5/2)) + c*e**3*x**11/(10*sqrt(d)*sqrt(1 + e*x**2/d))
```

3.277 $\int \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=175

$$\frac{x\left(d+ex^2\right)^{3/2}\left(48ae^2-8bde+3cd^2\right)}{192e^2}+\frac{dx\sqrt{d+ex^2}\left(48ae^2-8bde+3cd^2\right)}{128e^2}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(48ae^2-8bde\right.}{128e^{5/2}}$$

```
[Out] 1/192*(48*a*e^2-8*b*d*e+3*c*d^2)*x*(e*x^2+d)^(3/2)/e^2-1/48*(-8*b*e+3*c*d)*
x*(e*x^2+d)^(5/2)/e^2+1/8*c*x^3*(e*x^2+d)^(5/2)/e+1/128*d^2*(48*a*e^2-8*b*d
*e+3*c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/128*d*(48*a*e^2-8*
b*d*e+3*c*d^2)*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1159, 388, 195, 217, 206}

$$\frac{x\left(d+ex^2\right)^{3/2}\left(48ae^2-8bde+3cd^2\right)}{192e^2}+\frac{dx\sqrt{d+ex^2}\left(48ae^2-8bde+3cd^2\right)}{128e^2}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(48ae^2-8bde\right.}{128e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*Sqrt[d + e*x^2])/(128*e^2) + ((3*c*d^2
- 8*b*d*e + 48*a*e^2)*x*(d + e*x^2)^(3/2))/(192*e^2) - ((3*c*d - 8*b*e)*x*(
d + e*x^2)^(5/2))/(48*e^2) + (c*x^3*(d + e*x^2)^(5/2))/(8*e) + (d^2*(3*c*d^
2 - 8*b*d*e + 48*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
```

```
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]
```

Rubi steps

$$\int \left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)dx = \frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}+\frac{\int \left(d+ex^2\right)^{3/2}\left(8ae-(3cd-8be)x^2\right)dx}{8e}$$

$$= -\frac{(3cd-8be)x\left(d+ex^2\right)^{5/2}}{48e^2}+\frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}-\frac{1}{48}\left(-48a-\frac{d(3cd-8be)}{e^2}\right)\int$$

$$= \frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\left(d+ex^2\right)^{3/2}-\frac{(3cd-8be)x\left(d+ex^2\right)^{5/2}}{48e^2}+\frac{cx^3(d}{}$$

$$= \frac{1}{128}d\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\left(d+ex\right.$$

$$= \frac{1}{128}d\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\left(d+ex\right.$$

$$= \frac{1}{128}d\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\left(d+ex\right.$$

Mathematica [A] time = 0.32, size = 157, normalized size = 0.90

$$\frac{\sqrt{d+ex^2}\left(\frac{3d^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(8e(6ae-bd)+3cd^2\right)}{\sqrt{\frac{ex^2}{d}+1}}+\sqrt{e}x\left(8e\left(6ae\left(5d+2ex^2\right)+b\left(3d^2+14dex^2+8e^2x^4\right)\right)+c\left(-9d^3+6d^2\right.\right.}{384e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2]*(Sqrt[e]*x*(c*(-9*d^3 + 6*d^2*e*x^2 + 72*d*e^2*x^4 + 48*e^
3*x^6) + 8*e*(6*a*e*(5*d + 2*e*x^2) + b*(3*d^2 + 14*d*e*x^2 + 8*e^2*x^4)))
+ (3*d^(3/2)*(3*c*d^2 + 8*e*(-(b*d) + 6*a*e))*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])
/Sqrt[1 + (e*x^2)/d]))/(384*e^(5/2))
```

fricas [A] time = 1.13, size = 304, normalized size = 1.74

$$\left[\frac{3\left(3cd^4-8bd^3e+48ad^2e^2\right)\sqrt{e}\log\left(-2ex^2-2\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(48ce^4x^7+8\left(9cde^3+8be^4\right)x^5+2\left(3cd\right.\right.}{768e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqr
t(e*x^2 + d)*sqrt(e)*x - d) + 2*(48*c*e^4*x^7 + 8*(9*c*d*e^3 + 8*b*e^4)*x^5
 + 2*(3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*x^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2
 - 80*a*d*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/384*(3*(3*c*d^4 - 8*b*d^3*e + 48
*a*d^2*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (48*c*e^4*x^7 + 8
*(9*c*d*e^3 + 8*b*e^4)*x^5 + 2*(3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*x^3 -
3*(3*c*d^3*e - 8*b*d^2*e^2 - 80*a*d*e^3)*x)*sqrt(e*x^2 + d))/e^3]
```

giac [A] time = 0.22, size = 145, normalized size = 0.83

$$-\frac{1}{128}\left(3\,cd^4-8\,bd^3e+48\,ad^2e^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{384}\left(2\left(4\left(6\,cx^2e+\left(9\,cde^6+8\,be^7\right)e^{(-6)}\right)x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/128*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + s
qrt(x^2*e + d))) + 1/384*(2*(4*(6*c*x^2*e + (9*c*d*e^6 + 8*b*e^7)*e^(-6))*x
^2 + (3*c*d^2*e^5 + 56*b*d*e^6 + 48*a*e^7)*e^(-6))*x^2 - 3*(3*c*d^3*e^4 - 8
*b*d^2*e^5 - 80*a*d*e^6)*e^(-6))*sqrt(x^2*e + d)*x
```

maple [A] time = 0.01, size = 229, normalized size = 1.31

$$\frac{3a\,d^2\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{8\sqrt{e}}-\frac{b\,d^3\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{16e^{\frac{3}{2}}}+\frac{3c\,d^4\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{128e^{\frac{5}{2}}}+\frac{3\sqrt{e\,x^2+d}\,adx}{8}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/8*c*x^3*(e*x^2+d)^(5/2)/e-1/16*c*d/e^2*x*(e*x^2+d)^(5/2)+1/64*c*d^2/e^2*x
*(e*x^2+d)^(3/2)+3/128*c*d^3/e^2*x*(e*x^2+d)^(1/2)+3/128*c*d^4/e^(5/2)*ln(e
^(1/2)*x+(e*x^2+d)^(1/2))+1/6*b*x*(e*x^2+d)^(5/2)/e-1/24*b*d/e*x*(e*x^2+d)^
(3/2)-1/16*b*d^2/e*x*(e*x^2+d)^(1/2)-1/16*b*d^3/e^(3/2)*ln(e^(1/2)*x+(e*x^2
+d)^(1/2))+1/4*a*x*(e*x^2+d)^(3/2)+3/8*a*d*x*(e*x^2+d)^(1/2)+3/8*a*d^2/e^(1
/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [A] time = 1.02, size = 207, normalized size = 1.18

$$\frac{\left(ex^2+d\right)^{\frac{5}{2}}cx^3}{8\,e}+\frac{1}{4}\left(ex^2+d\right)^{\frac{3}{2}}ax+\frac{3}{8}\sqrt{ex^2+d}\,adx-\frac{\left(ex^2+d\right)^{\frac{5}{2}}cdx}{16\,e^2}+\frac{\left(ex^2+d\right)^{\frac{3}{2}}cd^2x}{64\,e^2}+\frac{3\sqrt{ex^2+d}\,cd^3x}{128\,e^2}+\frac{\left(ex^2+d\right.}{6\,e}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*(e*x^2 + d)^(5/2)*c*x^3/e + 1/4*(e*x^2 + d)^(3/2)*a*x + 3/8*sqrt(e*x^2
+ d)*a*d*x - 1/16*(e*x^2 + d)^(5/2)*c*d*x/e^2 + 1/64*(e*x^2 + d)^(3/2)*c*d^
2*x/e^2 + 3/128*sqrt(e*x^2 + d)*c*d^3*x/e^2 + 1/6*(e*x^2 + d)^(5/2)*b*x/e -
 1/24*(e*x^2 + d)^(3/2)*b*d*x/e - 1/16*sqrt(e*x^2 + d)*b*d^2*x/e + 3/128*c*
d^4*arcsinh(e*x/sqrt(d*e))/e^(5/2) - 1/16*b*d^3*arcsinh(e*x/sqrt(d*e))/e^(3
/2) + 3/8*a*d^2*arcsinh(e*x/sqrt(d*e))/sqrt(e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(e\,x^2+d\right)^{3/2}\,\left(c\,x^4+b\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4), x)
```

sympy [B] time = 31.10, size = 413, normalized size = 2.36

$$\frac{ad^{\frac{3}{2}}x\sqrt{1+\frac{ex^2}{d}}}{2}+\frac{ad^{\frac{3}{2}}x}{8\sqrt{1+\frac{ex^2}{d}}}+\frac{3a\sqrt{d}\,ex^3}{8\sqrt{1+\frac{ex^2}{d}}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{e}}+\frac{ae^2x^5}{4\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}+\frac{bd^{\frac{5}{2}}x}{16e\sqrt{1+\frac{ex^2}{d}}}+\frac{17bd^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{ex^2}{d}}}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**(3/2)*x*sqrt(1 + e*x**2/d)/2 + a*d**(3/2)*x/(8*sqrt(1 + e*x**2/d)) + 3
*a*sqrt(d)*e*x**3/(8*sqrt(1 + e*x**2/d)) + 3*a*d**2*asinh(sqrt(e)*x/sqrt(d)
)/(8*sqrt(e)) + a*e**2*x**5/(4*sqrt(d)*sqrt(1 + e*x**2/d)) + b*d**(5/2)*x/(
16*e*sqrt(1 + e*x**2/d)) + 17*b*d**(3/2)*x**3/(48*sqrt(1 + e*x**2/d)) + 11*
b*sqrt(d)*e*x**5/(24*sqrt(1 + e*x**2/d)) - b*d**3*asinh(sqrt(e)*x/sqrt(d))/
(16*e**(3/2)) + b*e**2*x**7/(6*sqrt(d)*sqrt(1 + e*x**2/d)) - 3*c*d**(7/2)*x
/(128*e**2*sqrt(1 + e*x**2/d)) - c*d**(5/2)*x**3/(128*e*sqrt(1 + e*x**2/d))
 + 13*c*d**(3/2)*x**5/(64*sqrt(1 + e*x**2/d)) + 5*c*sqrt(d)*e*x**7/(16*sqrt
(1 + e*x**2/d)) + 3*c*d**4*asinh(sqrt(e)*x/sqrt(d))/(128*e**(5/2)) + c*e**2
*x**9/(8*sqrt(d)*sqrt(1 + e*x**2/d))
```

3.278 $\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)dx$

Optimal. Leaf size=132

$$\frac{x\sqrt{d+ex^2}\left(8ae^2-2bde+cd^2\right)}{16e^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(8ae^2-2bde+cd^2\right)}{16e^{5/2}}-\frac{x\left(d+ex^2\right)^{3/2}(cd-2be)}{8e^2}+\frac{cx^3\left(d+ex\right.}{6e}$$

```
[Out] -1/8*(-2*b*e+c*d)*x*(e*x^2+d)^(3/2)/e^2+1/6*c*x^3*(e*x^2+d)^(3/2)/e+1/16*d*
(8*a*e^2-2*b*d*e+c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/16*(8*
a*e^2-2*b*d*e+c*d^2)*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1159, 388, 195, 217, 206}

$$\frac{x\sqrt{d+ex^2}\left(8ae^2-2bde+cd^2\right)}{16e^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(8ae^2-2bde+cd^2\right)}{16e^{5/2}}-\frac{x\left(d+ex^2\right)^{3/2}(cd-2be)}{8e^2}+\frac{cx^3\left(d+ex\right.}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d^2 - 2*b*d*e + 8*a*e^2)*x*Sqrt[d + e*x^2])/(16*e^2) - ((c*d - 2*b*e)*x
*(d + e*x^2)^(3/2))/(8*e^2) + (c*x^3*(d + e*x^2)^(3/2))/(6*e) + (d*(c*d^2 -
 2*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(16*e^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)dx = \frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}+\frac{\int \sqrt{d+ex^2}\left(6ae-3(cd-2be)x^2\right)dx}{6e}$$

$$= -\frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}+\frac{1}{8}\left(8a+\frac{d(cd-2be)}{e^2}\right)\int \sqrt{d+e}$$

$$= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2}-\frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2}-\frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2}-\frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

Mathematica [A] time = 0.23, size = 121, normalized size = 0.92

$$\frac{\sqrt{d+ex^2}\left(\frac{3\sqrt{d}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(8ae^2-2bde+cd^2\right)}{\sqrt{\frac{ex^2}{d}+1}}+\sqrt{e}x\left(6e\left(4ae+b\left(d+2ex^2\right)\right)+c\left(-3d^2+2dex^2+8e^2x^4\right)\right)\right)}{48e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2]*(Sqrt[e]*x*(c*(-3*d^2 + 2*d*e*x^2 + 8*e^2*x^4) + 6*e*(4*a*
e + b*(d + 2*e*x^2))) + (3*Sqrt[d]*(c*d^2 - 2*b*d*e + 8*a*e^2)*ArcSinh[(Sqr
t[e]*x)/Sqrt[d]])/Sqrt[1 + (e*x^2)/d]))/(48*e^(5/2))
```

fricas [A] time = 0.99, size = 232, normalized size = 1.76

$$\left[\frac{3\left(cd^3-2\,bd^2e+8\,ade^2\right)\sqrt{e}\,\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\,\sqrt{e}\,x-d\right)+2\left(8\,ce^3x^5+2\left(cde^2+6\,be^3\right)x^3-3\left(cd^2e-2\,bd\right.\right.}{96\,e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^
2 + d)*sqrt(e)*x - d) + 2*(8*c*e^3*x^5 + 2*(c*d*e^2 + 6*b*e^3)*x^3 - 3*(c*d
^2*e - 2*b*d*e^2 - 8*a*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/48*(3*(c*d^3 - 2*b*
d^2*e + 8*a*d*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (8*c*e^3*x
^5 + 2*(c*d*e^2 + 6*b*e^3)*x^3 - 3*(c*d^2*e - 2*b*d*e^2 - 8*a*e^3)*x)*sqrt(
e*x^2 + d))/e^3]
```

giac [A] time = 0.22, size = 106, normalized size = 0.80

$$-\frac{1}{16}\left(cd^3-2\,bd^2e+8\,ade^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{48}\left(2\left(4\,cx^2+\left(cde^3+6\,be^4\right)e^{(-4)}\right)x^2-3\left(cd^2e^2-2\,bd\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/16*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^
2*e + d))) + 1/48*(2*(4*c*x^2 + (c*d*e^3 + 6*b*e^4)*e^(-4))*x^2 - 3*(c*d^2*
e^2 - 2*b*d*e^3 - 8*a*e^4)*e^(-4))*sqrt(x^2*e + d)*x
```

maple [A] time = 0.01, size = 175, normalized size = 1.33

$$\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} c\,x^{3}}{6e}+\frac{a d \ln\left(\sqrt{e}\;x+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}-\frac{b\,d^{2} \ln\left(\sqrt{e}\;x+\sqrt{e\,x^{2}+d}\right)}{8e^{\frac{3}{2}}}+\frac{c\,d^{3} \ln\left(\sqrt{e}\;x+\sqrt{e\,x^{2}+d}\right)}{16e^{\frac{5}{2}}}+\frac{\sqrt{e\,x^{2}+}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/6*c*x^3*(e*x^2+d)^(3/2)/e-1/8*c*d/e^2*x*(e*x^2+d)^(3/2)+1/16*c*d^2/e^2*x*
(e*x^2+d)^(1/2)+1/16*c*d^3/e^(5/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))+1/4*b*x*(e
*x^2+d)^(3/2)/e-1/8*b*d/e*x*(e*x^2+d)^(1/2)-1/8*b*d^2/e^(3/2)*ln(e^(1/2)*x+
(e*x^2+d)^(1/2))+1/2*a*x*(e*x^2+d)^(1/2)+1/2*a*d/e^(1/2)*ln(e^(1/2)*x+(e*x^
2+d)^(1/2))
```

maxima [A] time = 0.98, size = 153, normalized size = 1.16

$$\frac{\left(ex^{2}+d\right)^{\frac{3}{2}} cx^{3}}{6\,e}+\frac{1}{2}\,\sqrt{ex^{2}+d}\,ax-\frac{\left(ex^{2}+d\right)^{\frac{3}{2}} cdx}{8\,e^{2}}+\frac{\sqrt{ex^{2}+d}\,cd^{2}x}{16\,e^{2}}+\frac{\left(ex^{2}+d\right)^{\frac{3}{2}} bx}{4\,e}-\frac{\sqrt{ex^{2}+d}\,bdx}{8\,e}+\frac{cd^{3}\operatorname{arsinh}\left(\frac{ex}{\sqrt{d}}\right.}{16\,e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*(e*x^2 + d)^(3/2)*c*x^3/e + 1/2*sqrt(e*x^2 + d)*a*x - 1/8*(e*x^2 + d)^(
3/2)*c*d*x/e^2 + 1/16*sqrt(e*x^2 + d)*c*d^2*x/e^2 + 1/4*(e*x^2 + d)^(3/2)*b
*x/e - 1/8*sqrt(e*x^2 + d)*b*d*x/e + 1/16*c*d^3*arcsinh(e*x/sqrt(d*e))/e^(5
/2) - 1/8*b*d^2*arcsinh(e*x/sqrt(d*e))/e^(3/2) + 1/2*a*d*arcsinh(e*x/sqrt(d
*e))/sqrt(e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4), x)
```

sympy [B] time = 12.27, size = 272, normalized size = 2.06

$$\frac{a\sqrt{d}\,x\sqrt{1+\frac{ex^{2}}{d}}}{2}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{e}}+\frac{bd^{\frac{3}{2}}x}{8e\sqrt{1+\frac{ex^{2}}{d}}}+\frac{3b\sqrt{d}\,x^{3}}{8\sqrt{1+\frac{ex^{2}}{d}}}-\frac{bd^{2}\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8e^{\frac{3}{2}}}+\frac{bex^{5}}{4\sqrt{d}\sqrt{1+\frac{ex^{2}}{d}}}-\frac{cd^{\frac{5}{2}}x}{16e^{2}\sqrt{1+\frac{ex^{2}}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*sqrt(d)*x*sqrt(1 + e*x**2/d)/2 + a*d*asinh(sqrt(e)*x/sqrt(d))/(2*sqrt(e))
 + b*d**(3/2)*x/(8*e*sqrt(1 + e*x**2/d)) + 3*b*sqrt(d)*x**3/(8*sqrt(1 + e*x
**2/d)) - b*d**2*asinh(sqrt(e)*x/sqrt(d))/(8*e**(3/2)) + b*e*x**5/(4*sqrt(d
)*sqrt(1 + e*x**2/d)) - c*d**(5/2)*x/(16*e**2*sqrt(1 + e*x**2/d)) - c*d**(3
/2)*x**3/(48*e*sqrt(1 + e*x**2/d)) + 5*c*sqrt(d)*x**5/(24*sqrt(1 + e*x**2/d
)) + c*d**3*asinh(sqrt(e)*x/sqrt(d))/(16*e**(5/2)) + c*e*x**7/(6*sqrt(d)*sq
rt(1 + e*x**2/d))
```

3.279 $\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\left(8ae^2-4bde+3cd^2\right)}{8e^{5/2}}-\frac{x\sqrt{d+ex^2}\,(3cd-4be)}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

[Out] 1/8*(8*a*e^2-4*b*d*e+3*c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)-1/8*(-4*b*e+3*c*d)*x*(e*x^2+d)^(1/2)/e^2+1/4*c*x^3*(e*x^2+d)^(1/2)/e

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1159, 388, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\left(8ae^2-4bde+3cd^2\right)}{8e^{5/2}}-\frac{x\sqrt{d+ex^2}\,(3cd-4be)}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]

[Out] -((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*e^2) + (c*x^3*Sqrt[d + e*x^2])/(4*e) + ((3*c*d^2 - 4*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*e^(5/2))

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]
```

Rule steps

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx = \frac{cx^3\sqrt{d+ex^2}}{4e} + \frac{\int \frac{4ae-(3cd-4be)x^2}{\sqrt{d+ex^2}}dx}{4e}$$
$$= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2} + \frac{cx^3\sqrt{d+ex^2}}{4e} - \frac{1}{8}\left(-8a - \frac{d(3cd-4be)}{e^2}\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx$$
$$= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2} + \frac{cx^3\sqrt{d+ex^2}}{4e} - \frac{1}{8}\left(-8a - \frac{d(3cd-4be)}{e^2}\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx\right.$$
$$= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2} + \frac{cx^3\sqrt{d+ex^2}}{4e} + \frac{(3cd^2-4bde+8ae^2)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{5/2}}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.85

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(8ae^2-4bde+3cd^2)+\sqrt{e}x\sqrt{d+ex^2}\,(4be-3cd+2cex^2)}{8e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*c*d + 4*b*e + 2*c*e*x^2) + (3*c*d^2 - 4*b*d*
e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*e^(5/2))
```

fricas [A] time = 1.07, size = 174, normalized size = 1.79

$$\left[\frac{(3\,cd^2-4\,bde+8\,ae^2)\sqrt{e}\,\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\,\sqrt{e}\,x-d\right)+2\left(2\,ce^2x^3-(3\,cde-4\,be^2)x\right)\sqrt{ex^2+d}}{16\,e^3}, -\frac{(3\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*d^2 - 4*b*d*e + 8*a*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 +
d)*sqrt(e)*x - d) + 2*(2*c*e^2*x^3 - (3*c*d*e - 4*b*e^2)*x)*sqrt(e*x^2 + d)
)/e^3, -1/8*((3*c*d^2 - 4*b*d*e + 8*a*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(
e*x^2 + d)) - (2*c*e^2*x^3 - (3*c*d*e - 4*b*e^2)*x)*sqrt(e*x^2 + d))/e^3]
```

giac [A] time = 0.19, size = 79, normalized size = 0.81

$$-\frac{1}{8}\left(3\,cd^2-4\,bde+8\,ae^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{8}\left(2\,cx^2e^{(-1)}-\left(3\,cde-4\,be^2\right)e^{(-3)}\right)\sqrt{x^2e+d}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*c*d^2 - 4*b*d*e + 8*a*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e
 + d))) + 1/8*(2*c*x^2*e^(-1) - (3*c*d*e - 4*b*e^2)*e^(-3))*sqrt(x^2*e + d)
*x
```

maple [A] time = 0.01, size = 122, normalized size = 1.26

$$\frac{\sqrt{e\,x^2+d}\;c\,x^3}{4e}+\frac{a\ln\left(\sqrt{e}\;x+\sqrt{e\,x^2+d}\right)}{\sqrt{e}}-\frac{bd\ln\left(\sqrt{e}\;x+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}+\frac{3c\,d^2\ln\left(\sqrt{e}\;x+\sqrt{e\,x^2+d}\right)}{8e^{\frac{5}{2}}}+\frac{\sqrt{e\,x^2+d}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/4*c*x^3*(e*x^2+d)^(1/2)/e-3/8*c*d/e^2*x*(e*x^2+d)^(1/2)+3/8*c*d^2/e^(5/2)
*ln(e^(1/2)*x+(e*x^2+d)^(1/2))+1/2*b*x/e*(e*x^2+d)^(1/2)-1/2*b*d/e^(3/2)*ln
(e^(1/2)*x+(e*x^2+d)^(1/2))+a*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)
```

maxima [A] time = 1.07, size = 100, normalized size = 1.03

$$\frac{\sqrt{ex^2+d}\,cx^3}{4\,e}-\frac{3\,\sqrt{ex^2+d}\,cdx}{8\,e^2}+\frac{\sqrt{ex^2+d}\,bx}{2\,e}+\frac{3\,cd^2\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{8\,e^{\frac{5}{2}}}-\frac{bd\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{2\,e^{\frac{3}{2}}}+\frac{a\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(e*x^2 + d)*c*x^3/e - 3/8*sqrt(e*x^2 + d)*c*d*x/e^2 + 1/2*sqrt(e*x^
2 + d)*b*x/e + 3/8*c*d^2*arcsinh(e*x/sqrt(d*e))/e^(5/2) - 1/2*b*d*arcsinh(e
*x/sqrt(d*e))/e^(3/2) + a*arcsinh(e*x/sqrt(d*e))/sqrt(e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^4+b\,x^2+a}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(1/2), x)
```

sympy [A] time = 7.05, size = 230, normalized size = 2.37

$$a\left(\begin{cases}\frac{\sqrt{-\frac{d}{e}}\,\mathrm{asin}\left(x\sqrt{-\frac{e}{d}}\right)}{\sqrt{d}} & \text{for}\ d>0\wedge e<0\\ \frac{\sqrt{\frac{d}{e}}\,\mathrm{asinh}\left(x\sqrt{\frac{e}{d}}\right)}{\sqrt{d}} & \text{for}\ d>0\wedge e>0\\ \frac{\sqrt{-\frac{d}{e}}\,\mathrm{acosh}\left(x\sqrt{-\frac{e}{d}}\right)}{\sqrt{-d}} & \text{for}\ e>0\wedge d<0\end{cases}\right)+\frac{b\sqrt{d}\,x\sqrt{1+\frac{ex^2}{d}}}{2e}-\frac{bd\,\mathrm{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2e^{\frac{3}{2}}}-\frac{3cd^{\frac{3}{2}}x}{8e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{c\sqrt{d}\,x^3}{8e\sqrt{1+\frac{ex^2}{d}}}+\frac{3cd^2\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] a*Piecewise((sqrt(-d/e)*asin(x*sqrt(-e/d))/sqrt(d), (d > 0) & (e < 0)), (sq
rt(d/e)*asinh(x*sqrt(e/d))/sqrt(d), (d > 0) & (e > 0)), (sqrt(-d/e)*acosh(x
*sqrt(-e/d))/sqrt(-d), (e > 0) & (d < 0))) + b*sqrt(d)*x*sqrt(1 + e*x**2/d)
/(2*e) - b*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(3/2)) - 3*c*d**(3/2)*x/(8*e**2
*sqrt(1 + e*x**2/d)) - c*sqrt(d)*x**3/(8*e*sqrt(1 + e*x**2/d)) + 3*c*d**2*a
sinh(sqrt(e)*x/sqrt(d))/(8*e**(5/2)) + c*x**5/(4*sqrt(d)*sqrt(1 + e*x**2/d)
)
```

3.280 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}$$

```
[Out] -1/2*(-2*b*e+3*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+(a+d*(-b*e+c
*d)/e^2)*x/d/(e*x^2+d)^(1/2)+1/2*c*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1157, 388, 217, 206}

$$\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(d*Sqrt[d + e*x^2]) + (c*x*Sqrt[d + e*x^2])/(
2*e^2) - ((3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}} - \frac{\int \frac{\frac{d(cd-be)}{e^2}-\frac{cdx^2}{e}}{\sqrt{d+ex^2}}\,dx}{d}$$

$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}} + \frac{cx\sqrt{d+ex^2}}{2e^2} - \frac{(3cd-2be)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2e^2}$$

$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}} + \frac{cx\sqrt{d+ex^2}}{2e^2} - \frac{(3cd-2be)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2e^2}$$

$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}} + \frac{cx\sqrt{d+ex^2}}{2e^2} - \frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}$$

Mathematica [A] time = 0.11, size = 98, normalized size = 1.10

$$\frac{\sqrt{e}x\left(2e(ae-bd)+cd\left(3d+ex^2\right)\right)-d^{3/2}\sqrt{\frac{ex^2}{d}+1}\,(3cd-2be)\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2de^{5/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

```
[Out] (Sqrt[e]*x*(2*e*(-(b*d) + a*e) + c*d*(3*d + e*x^2)) - d^(3/2)*(3*c*d - 2*b*
e)*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/(2*d*e^(5/2)*Sqrt[d +
e*x^2])
```

fricas [A] time = 0.88, size = 249, normalized size = 2.80

$$\left[-\frac{\left(3\,cd^3-2\,bd^2e+\left(3\,cd^2e-2\,bde^2\right)x^2\right)\sqrt{e}\,\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\,\sqrt{e}\,x-d\right)-2\left(cde^2x^3+\left(3\,cd^2e-2\,bde^2+2\right.\right.}{4\left(de^4x^2+d^2e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*c*d^3 - 2*b*d^2*e + (3*c*d^2*e - 2*b*d*e^2)*x^2)*sqrt(e)*log(-2*e
*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(c*d*e^2*x^3 + (3*c*d^2*e - 2*b
*d*e^2 + 2*a*e^3)*x)*sqrt(e*x^2 + d))/(d*e^4*x^2 + d^2*e^3), 1/2*((3*c*d^3
- 2*b*d^2*e + (3*c*d^2*e - 2*b*d*e^2)*x^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(
e*x^2 + d)) + (c*d*e^2*x^3 + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*x)*sqrt(e*x^
2 + d))/(d*e^4*x^2 + d^2*e^3)]
```

giac [A] time = 0.20, size = 80, normalized size = 0.90

$$\frac{1}{2}\,(3\,cd-2\,be)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{\left(cx^2e^{(-1)}+\frac{\left(3\,cd^2e-2\,bde^2+2\,ae^3\right)e^{(-3)}}{d}\right)x}{2\,\sqrt{x^2e+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(3*c*d - 2*b*e)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) + 1/2*(
c*x^2*e^(-1) + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*e^(-3)/d)*x/sqrt(x^2*e + d
)
```

maple [A] time = 0.01, size = 112, normalized size = 1.26

$$\frac{c\,x^3}{2\sqrt{e\,x^2+d}\;e}+\frac{ax}{\sqrt{e\,x^2+d}\;d}-\frac{bx}{\sqrt{e\,x^2+d}\;e}+\frac{3cdx}{2\sqrt{e\,x^2+d}\;e^2}+\frac{b\ln\left(\sqrt{e}\;x+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}-\frac{3cd\ln\left(\sqrt{e}\;x+\sqrt{e\,x^2+d}\right)}{2e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x)
```

```
[Out] 1/2*c*x^3/e/(e*x^2+d)^(1/2)+3/2*c*d/e^2*x/(e*x^2+d)^(1/2)-3/2*c*d/e^(5/2)*l
n(e^(1/2)*x+(e*x^2+d)^(1/2))-b*x/e/(e*x^2+d)^(1/2)+b/e^(3/2)*ln(e^(1/2)*x+(
e*x^2+d)^(1/2))+a*x/d/(e*x^2+d)^(1/2)
```

maxima [A] time = 1.13, size = 97, normalized size = 1.09

$$\frac{cx^3}{2\sqrt{ex^2+d}\,e}+\frac{ax}{\sqrt{ex^2+d}\,d}+\frac{3\,cdx}{2\sqrt{ex^2+d}\,e^2}-\frac{bx}{\sqrt{ex^2+d}\,e}-\frac{3\,cd\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{2\,e^{\frac{5}{2}}}+\frac{b\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*x^3/(sqrt(e*x^2 + d)*e) + a*x/(sqrt(e*x^2 + d)*d) + 3/2*c*d*x/(sqrt(e
*x^2 + d)*e^2) - b*x/(sqrt(e*x^2 + d)*e) - 3/2*c*d*arcsinh(e*x/sqrt(d*e))/e
^(5/2) + b*arcsinh(e*x/sqrt(d*e))/e^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^4+b\,x^2+a}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2), x)
```

sympy [A] time = 9.98, size = 134, normalized size = 1.51

$$\frac{ax}{d^{\frac{3}{2}}\sqrt{1+\frac{ex^2}{d}}}+b\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{\frac{3}{2}}}-\frac{x}{\sqrt{d}\,e\sqrt{1+\frac{ex^2}{d}}}\right)+c\left(\frac{3\sqrt{d}\,x}{2e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{3d\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2e^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{d}\,e\sqrt{1+\frac{ex^2}{d}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(3/2),x)
```

```
[Out] a*x/(d**(3/2)*sqrt(1 + e*x**2/d)) + b*(asinh(sqrt(e)*x/sqrt(d))/e**(3/2) -
x/(sqrt(d)*e*sqrt(1 + e*x**2/d))) + c*(3*sqrt(d)*x/(2*e**2*sqrt(1 + e*x**2/
d)) - 3*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(5/2)) + x**3/(2*sqrt(d)*e*sqrt(1
+ e*x**2/d)))
```

3.281 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{x\left(4cd^2-e(2ae+bd)\right)}{3d^2e^2\sqrt{d+ex^2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

```
[Out] 1/3*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^(3/2)+c*arctanh(x*e^(1/2)/(e*x^2+d)^
(1/2))/e^(5/2)-1/3*(4*c*d^2-e*(2*a*e+b*d))*x/d^2/e^2/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1157, 385, 217, 206}

$$-\frac{x\left(4cd^2-e(2ae+bd)\right)}{3d^2e^2\sqrt{d+ex^2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) - ((4*c*d^2 - e*(b*d
+ 2*a*e))*x)/(3*d^2*e^2*Sqrt[d + e*x^2]) + (c*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
e*x^2]])/e^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}} - \frac{\int \frac{-2a+\frac{d(cd-be)}{e^2}-\frac{3cdx^2}{e}}{\left(d+ex^2\right)^{3/2}}\,dx}{3d}$$

$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}} - \frac{\left(4cd^2-e(bd+2ae)\right)x}{3d^2e^2\sqrt{d+ex^2}} + \frac{c\int \frac{1}{\sqrt{d+ex^2}}\,dx}{e^2}$$

$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}} - \frac{\left(4cd^2-e(bd+2ae)\right)x}{3d^2e^2\sqrt{d+ex^2}} + \frac{c\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{e^2}$$

$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}} - \frac{\left(4cd^2-e(bd+2ae)\right)x}{3d^2e^2\sqrt{d+ex^2}} + \frac{c\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

Mathematica [A] time = 0.19, size = 112, normalized size = 1.11

$$\frac{\sqrt{e}x\left(e^2\left(3ad+2aex^2+bdx^2\right)-cd^2\left(3d+4ex^2\right)\right)+3cd^{5/2}\left(d+ex^2\right)\sqrt{\frac{ex^2}{d}+1}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^2e^{5/2}\left(d+ex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

```
[Out] (Sqrt[e]*x*(-(c*d^2*(3*d + 4*e*x^2)) + e^2*(3*a*d + b*d*x^2 + 2*a*e*x^2)) +
 3*c*d^(5/2)*(d + e*x^2)*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/
(3*d^2*e^(5/2)*(d + e*x^2)^(3/2))
```

fricas [A] time = 0.75, size = 289, normalized size = 2.86

$$\left[\frac{3\left(cd^2e^2x^4+2\,cd^3ex^2+cd^4\right)\sqrt{e}\,\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\,\sqrt{e}x-d\right)-2\left(\left(4\,cd^2e^2-bde^3-2\,ae^4\right)x^3+3\left(cd^3e-\right.\right.}{6\left(d^2e^5x^4+2\,d^3e^4x^2+d^4e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(e)*log(-2*e*x^2 - 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) - 2*((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^3 + 3
*(c*d^3*e - a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4
*e^3), -1/3*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(-e)*arctan(sqrt
(-e)*x/sqrt(e*x^2 + d)) + ((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^3 + 3*(c*d^3
*e - a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4*e^3)]
```

giac [A] time = 0.23, size = 88, normalized size = 0.87

$$-ce^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)-\frac{\left(\frac{\left(4\,cd^2e^2-bde^3-2\,ae^4\right)x^2e^{(-3)}}{d^2}+\frac{3\left(cd^3e-ade^3\right)e^{(-3)}}{d^2}\right)x}{3\left(x^2e+d\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] -c*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) - 1/3*((4*c*d^2*e^2 - b*
d*e^3 - 2*a*e^4)*x^2*e^(-3)/d^2 + 3*(c*d^3*e - a*d*e^3)*e^(-3)/d^2)*x/(x^2*
e + d)^(3/2)
```

maple [A] time = 0.01, size = 124, normalized size = 1.23

$$-\frac{c x^3}{3\left(e x^2+d\right)^{\frac{3}{2}} e}+\frac{a x}{3\left(e x^2+d\right)^{\frac{3}{2}} d}-\frac{b x}{3\left(e x^2+d\right)^{\frac{3}{2}} e}+\frac{2 a x}{3\sqrt{e x^2+d}\, d^2}+\frac{b x}{3\sqrt{e x^2+d}\, d e}-\frac{c x}{\sqrt{e x^2+d}\, e^2}+\frac{c \ln\left(\sqrt{e}\, x+\sqrt{e}\right.}{e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x)
```

```
[Out] -1/3*c*x^3/e/(e*x^2+d)^(3/2)-c/e^2*x/(e*x^2+d)^(1/2)+c/e^(5/2)*ln(e^(1/2)*x
+(e*x^2+d)^(1/2))-1/3*b/e*x/(e*x^2+d)^(3/2)+1/3*b/d/e*x/(e*x^2+d)^(1/2)+1/3
*a*x/d/(e*x^2+d)^(3/2)+2/3*a/d^2*x/(e*x^2+d)^(1/2)
```

maxima [A] time = 1.01, size = 135, normalized size = 1.34

$$-\frac{1}{3}\, c x\left(\frac{3\, x^2}{\left(e x^2+d\right)^{\frac{3}{2}} e}+\frac{2\, d}{\left(e x^2+d\right)^{\frac{3}{2}} e^2}\right)+\frac{2\, a x}{3\, \sqrt{e x^2+d}\, d^2}+\frac{a x}{3\, \left(e x^2+d\right)^{\frac{3}{2}} d}-\frac{c x}{3\, \sqrt{e x^2+d}\, e^2}-\frac{b x}{3\, \left(e x^2+d\right)^{\frac{3}{2}} e}+\frac{b x}{3\, \sqrt{e x^2+d}\, d e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*c*x*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + 2/3*
a*x/(sqrt(e*x^2 + d)*d^2) + 1/3*a*x/((e*x^2 + d)^(3/2)*d) - 1/3*c*x/(sqrt(e
*x^2 + d)*e^2) - 1/3*b*x/((e*x^2 + d)^(3/2)*e) + 1/3*b*x/(sqrt(e*x^2 + d)*d
*e) + c*arcsinh(e*x/sqrt(d*e))/e^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\, x^4+b\, x^2+a}{\left(e\, x^2+d\right)^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2), x)
```

sympy [B] time = 18.95, size = 450, normalized size = 4.46

$$a\left(\frac{3dx}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}+\frac{2ex^3}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}\right)+\frac{bx^3}{3d^{\frac{5}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{3}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}+c\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(5/2),x)
```

```
[Out] a*(3*d*x/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 + e*x**2
/d)) + 2*e*x**3/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 +
 e*x**2/d))) + b*x**3/(3*d**(5/2)*sqrt(1 + e*x**2/d) + 3*d**(3/2)*e*x**2*sq
rt(1 + e*x**2/d)) + c*(3*d**(39/2)*e**11*sqrt(1 + e*x**2/d)*asinh(sqrt(e)*x
/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)
*x**2*sqrt(1 + e*x**2/d)) + 3*d**(37/2)*e**12*x**2*sqrt(1 + e*x**2/d)*asinh
```

```
(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)
*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 3*d**19*e**(23/2)*x/(3*d**(39/2)*e**(
27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) -
 4*d**18*e**(25/2)*x**3/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(3
7/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)))
```

3.282 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx$

Optimal. Leaf size=86

$$\frac{x^5\left(2e(4ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{5/2}}+\frac{x^3(4ae+bd)}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

```
[Out] a*x/d/(e*x^2+d)^(5/2)+1/3*(4*a*e+b*d)*x^3/d^2/(e*x^2+d)^(5/2)+1/15*(3*c*d^2
+2*e*(4*a*e+b*d))*x^5/d^3/(e*x^2+d)^(5/2)
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1155, 1803, 12, 264}

$$\frac{x^5\left(2e(4ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{5/2}}+\frac{x^3(4ae+bd)}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(5/2)) + ((b*d + 4*a*e)*x^3)/(3*d^2*(d + e*x^2)^(5/2))
 + ((3*c*d^2 + 2*e*(b*d + 4*a*e))*x^5)/(15*d^3*(d + e*x^2)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1155

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(a^p*x*(d + e*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx = \frac{ax}{d\left(d+ex^2\right)^{5/2}} + \frac{\int \frac{x^2\left(4ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{7/2}}\,dx}{d}$$
$$= \frac{ax}{d\left(d+ex^2\right)^{5/2}} + \frac{(bd+4ae)x^3}{3d^2\left(d+ex^2\right)^{5/2}} + \frac{\int \frac{\left(3cd^2+2e(bd+4ae)\right)x^4}{\left(d+ex^2\right)^{7/2}}\,dx}{3d^2}$$
$$= \frac{ax}{d\left(d+ex^2\right)^{5/2}} + \frac{(bd+4ae)x^3}{3d^2\left(d+ex^2\right)^{5/2}} + \frac{1}{3}\left(3c+\frac{2e(bd+4ae)}{d^2}\right)\int \frac{x^4}{\left(d+ex^2\right)^{7/2}}\,dx$$
$$= \frac{ax}{d\left(d+ex^2\right)^{5/2}} + \frac{(bd+4ae)x^3}{3d^2\left(d+ex^2\right)^{5/2}} + \frac{\left(3cd^2+2e(bd+4ae)\right)x^5}{15d^3\left(d+ex^2\right)^{5/2}}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.78

$$\frac{a\left(15d^2x+20dex^3+8e^2x^5\right)+dx^3\left(5bd+2bex^2+3cdx^2\right)}{15d^3\left(d+ex^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

```
[Out] (d*x^3*(5*b*d + 3*c*d*x^2 + 2*b*e*x^2) + a*(15*d^2*x + 20*d*e*x^3 + 8*e^2*x
^5))/(15*d^3*(d + e*x^2)^(5/2))
```

fricas [A] time = 0.67, size = 93, normalized size = 1.08

$$\frac{\left(\left(3\,cd^2+2\,bde+8\,ae^2\right)x^5+15\,ad^2x+5\left(bd^2+4\,ade\right)x^3\right)\sqrt{ex^2+d}}{15\left(d^3e^3x^6+3\,d^4e^2x^4+3\,d^5ex^2+d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*((3*c*d^2 + 2*b*d*e + 8*a*e^2)*x^5 + 15*a*d^2*x + 5*(b*d^2 + 4*a*d*e)*
x^3)*sqrt(e*x^2 + d)/(d^3*e^3*x^6 + 3*d^4*e^2*x^4 + 3*d^5*e*x^2 + d^6)
```

giac [A] time = 0.21, size = 75, normalized size = 0.87

$$\frac{\left(x^2\left(\frac{\left(3\,cd^2e^2+2\,bde^3+8\,ae^4\right)x^2e^{(-2)}}{d^3}+\frac{5\left(bd^2e^2+4\,ade^3\right)e^{(-2)}}{d^3}\right)+\frac{15\,a}{d}\right)x}{15\left(x^2e+d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^2*((3*c*d^2*e^2 + 2*b*d*e^3 + 8*a*e^4)*x^2*e^(-2)/d^3 + 5*(b*d^2*e^
2 + 4*a*d*e^3)*e^(-2)/d^3) + 15*a/d)*x/(x^2*e + d)^(5/2)
```

maple [A] time = 0.00, size = 66, normalized size = 0.77

$$\frac{\left(8a\,e^2x^4+2bde\,x^4+3c\,d^2x^4+20ade\,x^2+5b\,d^2x^2+15a\,d^2\right)x}{15\left(e\,x^2+d\right)^{\frac{5}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x)
```

```
[Out] 1/15*x*(8*a*e^2*x^4+2*b*d*e*x^4+3*c*d^2*x^4+20*a*d*e*x^2+5*b*d^2*x^2+15*a*d
^2)/(e*x^2+d)^(5/2)/d^3
```

maxima [B] time = 1.16, size = 173, normalized size = 2.01

$$-\frac{cx^3}{2\left(ex^2+d\right)^{\frac{5}{2}}e}+\frac{8\,ax}{15\,\sqrt{ex^2+d}\,d^3}+\frac{4\,ax}{15\left(ex^2+d\right)^{\frac{3}{2}}d^2}+\frac{ax}{5\left(ex^2+d\right)^{\frac{5}{2}}d}+\frac{cx}{10\left(ex^2+d\right)^{\frac{3}{2}}e^2}+\frac{cx}{5\,\sqrt{ex^2+d}\,de^2}-\frac{3\,cdx}{10\left(ex^2+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*c*x^3/((e*x^2 + d)^(5/2)*e) + 8/15*a*x/(sqrt(e*x^2 + d)*d^3) + 4/15*a*
x/((e*x^2 + d)^(3/2)*d^2) + 1/5*a*x/((e*x^2 + d)^(5/2)*d) + 1/10*c*x/((e*x^
2 + d)^(3/2)*e^2) + 1/5*c*x/(sqrt(e*x^2 + d)*d*e^2) - 3/10*c*d*x/((e*x^2 +
d)^(5/2)*e^2) - 1/5*b*x/((e*x^2 + d)^(5/2)*e) + 2/15*b*x/(sqrt(e*x^2 + d)*d
^2*e) + 1/15*b*x/((e*x^2 + d)^(3/2)*d*e)
```

mupad [B] time = 4.70, size = 133, normalized size = 1.55

$$\frac{3\,c\,d^4\,x-6\,c\,d^3\,x\,\left(e\,x^2+d\right)-3\,b\,d^3\,e\,x+8\,a\,e^2\,x\,{\left(e\,x^2+d\right)}^2+3\,c\,d^2\,x\,{\left(e\,x^2+d\right)}^2+3\,a\,d^2\,e^2\,x+4\,a\,d\,e^2\,x\,\left(e\,x^2+\right.}{15\,d^3\,e^2\,{\left(e\,x^2+d\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x)
```

```
[Out] (3*c*d^4*x - 6*c*d^3*x*(d + e*x^2) - 3*b*d^3*e*x + 8*a*e^2*x*(d + e*x^2)^2
+ 3*c*d^2*x*(d + e*x^2)^2 + 3*a*d^2*e^2*x + 4*a*d*e^2*x*(d + e*x^2) + 2*b*d
*e*x*(d + e*x^2)^2 + b*d^2*e*x*(d + e*x^2))/(15*d^3*e^2*(d + e*x^2)^(5/2))
```

sympy [B] time = 45.98, size = 639, normalized size = 7.43

$$a\left(\frac{15d^5x}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}+\frac{}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(7/2),x)
```

```
[Out] a*(15*d**5*x/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1
+ e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3
*x**6*sqrt(1 + e*x**2/d)) + 35*d**4*e*x**3/(15*d**(17/2)*sqrt(1 + e*x**2/d)
 + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 +
 e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 28*d**3*e**2*x**5
/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d)
+ 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1
 + e*x**2/d)) + 8*d**2*e**3*x**7/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(
15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d)
 + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d))) + b*(5*d*x**3/(15*d**(9/2)*s
qrt(1 + e*x**2/d) + 30*d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**
2*x**4*sqrt(1 + e*x**2/d)) + 2*e*x**5/(15*d**(9/2)*sqrt(1 + e*x**2/d) + 30*
d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**2*x**4*sqrt(1 + e*x**2/
d))) + c*x**5/(5*d**(7/2)*sqrt(1 + e*x**2/d) + 10*d**(5/2)*e*x**2*sqrt(1 +
e*x**2/d) + 5*d**(3/2)*e**2*x**4*sqrt(1 + e*x**2/d))
```

3.283 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{9/2}} \, dx$

Optimal. Leaf size=126

$$\frac{2ex^7\left(4e(6ae+bd)+3cd^2\right)}{105d^4\left(d+ex^2\right)^{7/2}}+\frac{x^5\left(4e(6ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{x^3(6ae+bd)}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

```
[Out] a*x/d/(e*x^2+d)^(7/2)+1/3*(6*a*e+b*d)*x^3/d^2/(e*x^2+d)^(7/2)+1/15*(3*c*d^2
+4*e*(6*a*e+b*d))*x^5/d^3/(e*x^2+d)^(7/2)+2/105*e*(3*c*d^2+4*e*(6*a*e+b*d))
*x^7/d^4/(e*x^2+d)^(7/2)
```

Rubi [A] time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1155, 1803, 12, 271, 264}

$$\frac{2ex^7\left(4e(6ae+bd)+3cd^2\right)}{105d^4\left(d+ex^2\right)^{7/2}}+\frac{x^5\left(4e(6ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{x^3(6ae+bd)}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(7/2)) + ((b*d + 6*a*e)*x^3)/(3*d^2*(d + e*x^2)^(7/2))
 + ((3*c*d^2 + 4*e*(b*d + 6*a*e))*x^5)/(15*d^3*(d + e*x^2)^(7/2)) + (2*e*(3
*c*d^2 + 4*e*(b*d + 6*a*e))*x^7)/(105*d^4*(d + e*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1155

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(a^p*x*(d + e*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
```

```
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx &= \frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{\int \frac{x^2\left(6ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{9/2}}\,dx}{d}\\
&= \frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{\int \frac{\left(3cd^2+4e(bd+6ae)\right)x^4}{\left(d+ex^2\right)^{9/2}}\,dx}{3d^2}\\
&= \frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{1}{3}\left(3c+\frac{4e(bd+6ae)}{d^2}\right)\int \frac{x^4}{\left(d+ex^2\right)^{9/2}}\,dx\\
&= \frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{\left(3cd^2+4e(bd+6ae)\right)x^5}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{\left(2e\left(3cd^2+4e(bd+6ae)\right)\right)}{15d^3}\\
&= \frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{\left(3cd^2+4e(bd+6ae)\right)x^5}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{2e\left(3cd^2+4e(bd+6ae)\right)x^7}{105d^4\left(d+ex^2\right)^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 101, normalized size = 0.80

$$\frac{3a\left(35d^3x+70d^2ex^3+56de^2x^5+16e^3x^7\right)+dx^3\left(b\left(35d^2+28dex^2+8e^2x^4\right)+3cdx^2\left(7d+2ex^2\right)\right)}{105d^4\left(d+ex^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

```
[Out] (3*a*(35*d^3*x + 70*d^2*e*x^3 + 56*d*e^2*x^5 + 16*e^3*x^7) + d*x^3*(3*c*d*x
^2*(7*d + 2*e*x^2) + b*(35*d^2 + 28*d*e*x^2 + 8*e^2*x^4)))/(105*d^4*(d + e*
x^2)^(7/2))
```

fricas [A] time = 0.90, size = 136, normalized size = 1.08

$$\frac{\left(2\left(3\,cd^2e+4\,bde^2+24\,ae^3\right)x^7+7\left(3\,cd^3+4\,bd^2e+24\,ade^2\right)x^5+105\,ad^3x+35\left(bd^3+6\,ad^2e\right)x^3\right)\sqrt{ex^2+d}}{105\left(d^4e^4x^8+4\,d^5e^3x^6+6\,d^6e^2x^4+4\,d^7ex^2+d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*c*d^2*e + 4*b*d*e^2 + 24*a*e^3)*x^7 + 7*(3*c*d^3 + 4*b*d^2*e +
24*a*d*e^2)*x^5 + 105*a*d^3*x + 35*(b*d^3 + 6*a*d^2*e)*x^3)*sqrt(e*x^2 + d)
/(d^4*e^4*x^8 + 4*d^5*e^3*x^6 + 6*d^6*e^2*x^4 + 4*d^7*e*x^2 + d^8)
```

giac [A] time = 0.27, size = 113, normalized size = 0.90

$$\frac{\left(\left(x^2\left(\frac{2\left(3\,cd^2e^4+4\,bde^5+24\,ae^6\right)x^2e^{(-3)}}{d^4}+\frac{7\left(3\,cd^3e^3+4\,bd^2e^4+24\,ade^5\right)e^{(-3)}}{d^4}\right)+\frac{35\left(bd^3e^3+6\,ad^2e^4\right)e^{(-3)}}{d^4}\right)x^2+\frac{105\,a}{d}\right)x}{105\left(x^2e+d\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*(2*(3*c*d^2*e^4 + 4*b*d*e^5 + 24*a*e^6)*x^2*e^(-3)/d^4 + 7*(3*c
*d^3*e^3 + 4*b*d^2*e^4 + 24*a*d*e^5)*e^(-3)/d^4) + 35*(b*d^3*e^3 + 6*a*d^2*
e^4)*e^(-3)/d^4)*x^2 + 105*a/d)*x/(x^2*e + d)^(7/2)
```

maple [A] time = 0.00, size = 100, normalized size = 0.79

$$\frac{\left(48a\,e^{3}x^{6}+8bd\,e^{2}x^{6}+6c\,d^{2}e\,x^{6}+168ad\,e^{2}x^{4}+28b\,d^{2}e\,x^{4}+21c\,d^{3}x^{4}+210a\,d^{2}e\,x^{2}+35b\,d^{3}x^{2}+105a\,d^{3}\right)x}{105\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x)
```

```
[Out] 1/105*x*(48*a*e^3*x^6+8*b*d*e^2*x^6+6*c*d^2*e*x^6+168*a*d*e^2*x^4+28*b*d^2*
e*x^4+21*c*d^3*x^4+210*a*d^2*e*x^2+35*b*d^3*x^2+105*a*d^3)/(e*x^2+d)^(7/2)/
d^4
```

maxima [B] time = 1.20, size = 227, normalized size = 1.80

$$-\frac{cx^{3}}{4\left(ex^{2}+d\right)^{\frac{7}{2}}e}+\frac{16\,ax}{35\,\sqrt{ex^{2}+d}\,d^{4}}+\frac{8\,ax}{35\left(ex^{2}+d\right)^{\frac{3}{2}}d^{3}}+\frac{6\,ax}{35\left(ex^{2}+d\right)^{\frac{5}{2}}d^{2}}+\frac{ax}{7\left(ex^{2}+d\right)^{\frac{7}{2}}d}+\frac{3\,cx}{140\left(ex^{2}+d\right)^{\frac{5}{2}}e^{2}}+\frac{}{35\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*c*x^3/((e*x^2 + d)^(7/2)*e) + 16/35*a*x/(sqrt(e*x^2 + d)*d^4) + 8/35*a
*x/((e*x^2 + d)^(3/2)*d^3) + 6/35*a*x/((e*x^2 + d)^(5/2)*d^2) + 1/7*a*x/((e
*x^2 + d)^(7/2)*d) + 3/140*c*x/((e*x^2 + d)^(5/2)*e^2) + 2/35*c*x/(sqrt(e*x
^2 + d)*d^2*e^2) + 1/35*c*x/((e*x^2 + d)^(3/2)*d*e^2) - 3/28*c*d*x/((e*x^2
+ d)^(7/2)*e^2) - 1/7*b*x/((e*x^2 + d)^(7/2)*e) + 8/105*b*x/(sqrt(e*x^2 + d
)*d^3*e) + 4/105*b*x/((e*x^2 + d)^(3/2)*d^2*e) + 1/35*b*x/((e*x^2 + d)^(5/2
)*d*e)
```

mupad [B] time = 4.67, size = 154, normalized size = 1.22

$$\frac{x\left(\frac{a}{7\,d}-\frac{d\left(\frac{b}{7\,d}-\frac{c}{7\,e}\right)}{e}\right)}{\left(e\,x^{2}+d\right)^{7/2}}-\frac{x\left(\frac{c}{5\,e^{2}}-\frac{-c\,d^{2}+b\,d\,e+6\,a\,e^{2}}{35\,d^{2}\,e^{2}}\right)}{\left(e\,x^{2}+d\right)^{5/2}}+\frac{x\left(3\,c\,d^{2}+4\,b\,d\,e+24\,a\,e^{2}\right)}{105\,d^{3}\,e^{2}\left(e\,x^{2}+d\right)^{3/2}}+\frac{x\left(6\,c\,d^{2}+8\,b\,d\,e+48\,a\,e^{2}\right)}{105\,d^{4}\,e^{2}\,\sqrt{e\,x^{2}+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x)
```

```
[Out] (x*(a/(7*d) - (d*(b/(7*d) - c/(7*e)))/e))/(d + e*x^2)^(7/2) - (x*(c/(5*e^2)
 - (6*a*e^2 - c*d^2 + b*d*e)/(35*d^2*e^2)))/(d + e*x^2)^(5/2) + (x*(24*a*e^
2 + 3*c*d^2 + 4*b*d*e))/(105*d^3*e^2*(d + e*x^2)^(3/2)) + (x*(48*a*e^2 + 6*
c*d^2 + 8*b*d*e))/(105*d^4*e^2*(d + e*x^2)^(1/2))
```

sympy [B] time = 119.19, size = 1989, normalized size = 15.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(9/2),x)
```

```
[Out] a*(35*d**14*x/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(
1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*
e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) +
 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt
(1 + e*x**2/d)) + 175*d**13*e*x**3/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d
**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**
2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8
*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(
25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 371*d**12*e**2*x**5/(35*d**(37/2)*sq
rt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*
e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) +
 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt
(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 429*d**11*e*
*3*x**7/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*
x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x
**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d
**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e
*x**2/d)) + 286*d**10*e**4*x**9/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(
35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d
) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sq
rt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/
2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 104*d**9*e**5*x**11/(35*d**(37/2)*sqrt(
1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**
2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 52
5*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1
+ e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 16*d**8*e**6*x*
*13/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2
/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*
sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(2
7/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**
2/d))) + b*(35*d**5*x**3/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420*d**(17/2)*
e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 42
0*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x**8*sqrt(1 +
 e*x**2/d)) + 63*d**4*e*x**5/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420*d**(17
/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x**2/d)
+ 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x**8*sqrt
(1 + e*x**2/d)) + 36*d**3*e**2*x**7/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420
*d**(17/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x
**2/d) + 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x*
*8*sqrt(1 + e*x**2/d)) + 8*d**2*e**3*x**9/(105*d**(19/2)*sqrt(1 + e*x**2/d)
 + 420*d**(17/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1
 + e*x**2/d) + 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e
**4*x**8*sqrt(1 + e*x**2/d))) + c*(7*d*x**5/(35*d**(11/2)*sqrt(1 + e*x**2/d
) + 105*d**(9/2)*e*x**2*sqrt(1 + e*x**2/d) + 105*d**(7/2)*e**2*x**4*sqrt(1
+ e*x**2/d) + 35*d**(5/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 2*e*x**7/(35*d**(
11/2)*sqrt(1 + e*x**2/d) + 105*d**(9/2)*e*x**2*sqrt(1 + e*x**2/d) + 105*d**
(7/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 35*d**(5/2)*e**3*x**6*sqrt(1 + e*x**2/
d)))
```

3.284 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{11/2}} dx$

Optimal. Leaf size=165

$$\frac{8e^2x^9\left(2e(8ae+bd)+cd^2\right)}{315d^5\left(d+ex^2\right)^{9/2}}+\frac{4ex^7\left(2e(8ae+bd)+cd^2\right)}{35d^4\left(d+ex^2\right)^{9/2}}+\frac{x^5\left(2e(8ae+bd)+cd^2\right)}{5d^3\left(d+ex^2\right)^{9/2}}+\frac{x^3(8ae+bd)}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

```
[Out] a*x/d/(e*x^2+d)^(9/2)+1/3*(8*a*e+b*d)*x^3/d^2/(e*x^2+d)^(9/2)+1/5*(c*d^2+2*
e*(8*a*e+b*d))*x^5/d^3/(e*x^2+d)^(9/2)+4/35*e*(c*d^2+2*e*(8*a*e+b*d))*x^7/d
^4/(e*x^2+d)^(9/2)+8/315*e^2*(c*d^2+2*e*(8*a*e+b*d))*x^9/d^5/(e*x^2+d)^(9/2
)
```

Rubi [A] time = 0.21, antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1155, 1803, 12, 271, 264}

$$\frac{8e^2x^9\left(2e(8ae+bd)+cd^2\right)}{315d^5\left(d+ex^2\right)^{9/2}}+\frac{4ex^7\left(2e(8ae+bd)+cd^2\right)}{35d^4\left(d+ex^2\right)^{9/2}}+\frac{x^5\left(\frac{2e(8ae+bd)}{d^2}+c\right)}{5d\left(d+ex^2\right)^{9/2}}+\frac{x^3(8ae+bd)}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(9/2)) + ((b*d + 8*a*e)*x^3)/(3*d^2*(d + e*x^2)^(9/2))
 + ((c + (2*e*(b*d + 8*a*e))/d^2)*x^5)/(5*d*(d + e*x^2)^(9/2)) + (4*e*(c*d^
2 + 2*e*(b*d + 8*a*e))*x^7)/(35*d^4*(d + e*x^2)^(9/2)) + (8*e^2*(c*d^2 + 2*
e*(b*d + 8*a*e))*x^9)/(315*d^5*(d + e*x^2)^(9/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1155

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(a^p*x*(d + e*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx = \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{\int \frac{x^2\left(8ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{11/2}}\,dx}{d}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{\int \frac{\left(3cd^2+6e(bd+8ae)\right)x^4}{\left(d+ex^2\right)^{11/2}}\,dx}{3d^2}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}}+\left(c+\frac{2e(bd+8ae)}{d^2}\right)\int \frac{x^4}{\left(d+ex^2\right)^{11/2}}\,dx$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\left(d+ex^2\right)^{9/2}}+\frac{\left(4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)\right)\int \frac{x^6}{\left(d+ex^2\right)^{11/2}}\,dx}{5d}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\left(d+ex^2\right)^{9/2}}+\frac{4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^7}{35d^2\left(d+ex^2\right)^{9/2}}+\frac{\left(8e^2\left(c+\right.\right.}{}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\left(d+ex^2\right)^{9/2}}+\frac{4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^7}{35d^2\left(d+ex^2\right)^{9/2}}+\frac{8e^2\left(c+\frac{2e}{}\right.}{315d^3\left(d\right.}$$

Mathematica [A] time = 0.12, size = 132, normalized size = 0.80

$$\frac{a\left(315d^4x+840d^3ex^3+1008d^2e^2x^5+576de^3x^7+128e^4x^9\right)+dx^3\left(b\left(105d^3+126d^2ex^2+72de^2x^4+16e^3x^6\right)+c\right.}{315d^5\left(d+ex^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

```
[Out] (a*(315*d^4*x + 840*d^3*e*x^3 + 1008*d^2*e^2*x^5 + 576*d*e^3*x^7 + 128*e^4*
x^9) + d*x^3*(c*d*x^2*(63*d^2 + 36*d*e*x^2 + 8*e^2*x^4) + b*(105*d^3 + 126*
d^2*e*x^2 + 72*d*e^2*x^4 + 16*e^3*x^6)))/(315*d^5*(d + e*x^2)^(9/2))
```

fricas [A] time = 0.73, size = 177, normalized size = 1.07

$$\frac{\left(8\left(cd^2e^2+2\,bde^3+16\,ae^4\right)x^9+36\left(cd^3e+2\,bd^2e^2+16\,ade^3\right)x^7+315\,ad^4x+63\left(cd^4+2\,bd^3e+16\,ad^2e^2\right)x^5+10\right.}{315\left(d^5e^5x^{10}+5\,d^6e^4x^8+10\,d^7e^3x^6+10\,d^8e^2x^4+5\,d^9ex^2+d^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(8*(c*d^2*e^2 + 2*b*d*e^3 + 16*a*e^4)*x^9 + 36*(c*d^3*e + 2*b*d^2*e^2
 + 16*a*d*e^3)*x^7 + 315*a*d^4*x + 63*(c*d^4 + 2*b*d^3*e + 16*a*d^2*e^2)*x^
```

```
5 + 105*(b*d^4 + 8*a*d^3*e)*x^3)*sqrt(e*x^2 + d)/(d^5*e^5*x^10 + 5*d^6*e^4*
x^8 + 10*d^7*e^3*x^6 + 10*d^8*e^2*x^4 + 5*d^9*e*x^2 + d^10)
```

giac [A] time = 0.23, size = 148, normalized size = 0.90

$$\frac{\left(\left(\left(4\,x^2\left(\frac{2\left(cd^2e^6+2\,bde^7+16\,ae^8\right)x^2e^{(-4)}}{d^5}+\frac{9\left(cd^3e^5+2\,bd^2e^6+16\,ade^7\right)e^{(-4)}}{d^5}\right)+\frac{63\left(cd^4e^4+2\,bd^3e^5+16\,ad^2e^6\right)e^{(-4)}}{d^5}\right)x^2+\frac{105\left(bd^4e^4+8\,ad^3e^5\right)}{d^5}\cdots}{315\left(x^2e+d\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((4*x^2*(2*(c*d^2*e^6 + 2*b*d*e^7 + 16*a*e^8)*x^2*e^(-4)/d^5 + 9*(c*
d^3*e^5 + 2*b*d^2*e^6 + 16*a*d*e^7)*e^(-4)/d^5) + 63*(c*d^4*e^4 + 2*b*d^3*e
^5 + 16*a*d^2*e^6)*e^(-4)/d^5)*x^2 + 105*(b*d^4*e^4 + 8*a*d^3*e^5)*e^(-4)/d
^5)*x^2 + 315*a/d)*x/(x^2*e + d)^(9/2)
```

maple [A] time = 0.01, size = 136, normalized size = 0.82

$$\frac{\left(128a\,e^4x^8+16bd\,e^3x^8+8c\,d^2e^2x^8+576ad\,e^3x^6+72b\,d^2e^2x^6+36c\,d^3e\,x^6+1008a\,d^2e^2x^4+126b\,d^3e\,x^4+63c\cdots}{315\left(e\,x^2+d\right)^{\frac{9}{2}}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x)
```

```
[Out] 1/315*x*(128*a*e^4*x^8+16*b*d*e^3*x^8+8*c*d^2*e^2*x^8+576*a*d*e^3*x^6+72*b*
d^2*e^2*x^6+36*c*d^3*e*x^6+1008*a*d^2*e^2*x^4+126*b*d^3*e*x^4+63*c*d^4*x^4+
840*a*d^3*e*x^2+105*b*d^4*x^2+315*a*d^4)/(e*x^2+d)^(9/2)/d^5
```

maxima [A] time = 1.20, size = 281, normalized size = 1.70

$$-\frac{cx^3}{6\left(ex^2+d\right)^{\frac{9}{2}}e}+\frac{128\,ax}{315\,\sqrt{ex^2+d}\,d^5}+\frac{64\,ax}{315\left(ex^2+d\right)^{\frac{3}{2}}d^4}+\frac{16\,ax}{105\left(ex^2+d\right)^{\frac{5}{2}}d^3}+\frac{8\,ax}{63\left(ex^2+d\right)^{\frac{7}{2}}d^2}+\frac{ax}{9\left(ex^2+d\right)^{\frac{9}{2}}d}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/6*c*x^3/((e*x^2 + d)^(9/2)*e) + 128/315*a*x/(sqrt(e*x^2 + d)*d^5) + 64/3
15*a*x/((e*x^2 + d)^(3/2)*d^4) + 16/105*a*x/((e*x^2 + d)^(5/2)*d^3) + 8/63*
a*x/((e*x^2 + d)^(7/2)*d^2) + 1/9*a*x/((e*x^2 + d)^(9/2)*d) + 1/126*c*x/((e
*x^2 + d)^(7/2)*e^2) + 8/315*c*x/(sqrt(e*x^2 + d)*d^3*e^2) + 4/315*c*x/((e*
x^2 + d)^(3/2)*d^2*e^2) + 1/105*c*x/((e*x^2 + d)^(5/2)*d*e^2) - 1/18*c*d*x/
((e*x^2 + d)^(9/2)*e^2) - 1/9*b*x/((e*x^2 + d)^(9/2)*e) + 16/315*b*x/(sqrt(
e*x^2 + d)*d^4*e) + 8/315*b*x/((e*x^2 + d)^(3/2)*d^3*e) + 2/105*b*x/((e*x^2
 + d)^(5/2)*d^2*e) + 1/63*b*x/((e*x^2 + d)^(7/2)*d*e)
```

mupad [B] time = 4.75, size = 189, normalized size = 1.15

$$\frac{x\left(\frac{a}{9\,d}-\frac{d\left(\frac{b}{9\,d}-\frac{c}{9\,e}\right)}{e}\right)}{\left(e\,x^2+d\right)^{9/2}}-\frac{x\left(\frac{c}{7\,e^2}-\frac{-c\,d^2+b\,d\,e+8\,a\,e^2}{63\,d^2\,e^2}\right)}{\left(e\,x^2+d\right)^{7/2}}+\frac{x\left(c\,d^2+2\,b\,d\,e+16\,a\,e^2\right)}{105\,d^3\,e^2\left(e\,x^2+d\right)^{5/2}}+\frac{x\left(4\,c\,d^2+8\,b\,d\,e+64\,a\,e^2\right)}{315\,d^4\,e^2\left(e\,x^2+d\right)^{3/2}}+\frac{x\left(8\,c\cdots}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x)
```

```
[Out] (x*(a/(9*d) - (d*(b/(9*d) - c/(9*e)))/e))/(d + e*x^2)^(9/2) - (x*(c/(7*e^2)
 - (8*a*e^2 - c*d^2 + b*d*e)/(63*d^2*e^2)))/(d + e*x^2)^(7/2) + (x*(16*a*e^
2 + c*d^2 + 2*b*d*e))/(105*d^3*e^2*(d + e*x^2)^(5/2)) + (x*(64*a*e^2 + 4*c*
d^2 + 8*b*d*e))/(315*d^4*e^2*(d + e*x^2)^(3/2)) + (x*(128*a*e^2 + 8*c*d^2 +
 16*b*d*e))/(315*d^5*e^2*(d + e*x^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(11/2),x)
```

```
[Out] Timed out
```

3.285 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx$

Optimal. Leaf size=210

$$\frac{16e^3x^{11}\left(8e(10ae+bd)+3cd^2\right)}{3465d^6\left(d+ex^2\right)^{11/2}}+\frac{8e^2x^9\left(8e(10ae+bd)+3cd^2\right)}{315d^5\left(d+ex^2\right)^{11/2}}+\frac{2ex^7\left(8e(10ae+bd)+3cd^2\right)}{35d^4\left(d+ex^2\right)^{11/2}}+\frac{x^5\left(8e(10ae+bd\right.}{15d^3\left(d+ex\right.}$$

```
[Out] a*x/d/(e*x^2+d)^(11/2)+1/3*(10*a*e+b*d)*x^3/d^2/(e*x^2+d)^(11/2)+1/15*(3*c*
d^2+8*e*(10*a*e+b*d))*x^5/d^3/(e*x^2+d)^(11/2)+2/35*e*(3*c*d^2+8*e*(10*a*e+
b*d))*x^7/d^4/(e*x^2+d)^(11/2)+8/315*e^2*(3*c*d^2+8*e*(10*a*e+b*d))*x^9/d^5
/(e*x^2+d)^(11/2)+16/3465*e^3*(3*c*d^2+8*e*(10*a*e+b*d))*x^11/d^6/(e*x^2+d)
^(11/2)
```

Rubi [A] time = 0.22, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1155, 1803, 12, 271, 264}

$$\frac{16e^3x^{11}\left(8e(10ae+bd)+3cd^2\right)}{3465d^6\left(d+ex^2\right)^{11/2}}+\frac{8e^2x^9\left(8e(10ae+bd)+3cd^2\right)}{315d^5\left(d+ex^2\right)^{11/2}}+\frac{2ex^7\left(8e(10ae+bd)+3cd^2\right)}{35d^4\left(d+ex^2\right)^{11/2}}+\frac{x^5\left(8e(10ae+bd\right.}{15d^3\left(d+ex\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(11/2)) + ((b*d + 10*a*e)*x^3)/(3*d^2*(d + e*x^2)^(11/
2)) + ((3*c*d^2 + 8*e*(b*d + 10*a*e))*x^5)/(15*d^3*(d + e*x^2)^(11/2)) + (2
*e*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^7)/(35*d^4*(d + e*x^2)^(11/2)) + (8*e^2
*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^9)/(315*d^5*(d + e*x^2)^(11/2)) + (16*e^3
*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^11)/(3465*d^6*(d + e*x^2)^(11/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1155

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(a^p*x*(d + e*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{13/2}}\,dx = \frac{ax}{d\left(d+ex^2\right)^{11/2}}+\frac{\int \frac{x^2\left(10ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{13/2}}\,dx}{d}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}}+\frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}}+\frac{\int \frac{\left(3cd^2+8e(bd+10ae)\right)x^4}{\left(d+ex^2\right)^{13/2}}\,dx}{3d^2}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}}+\frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}}+\frac{1}{3}\left(3c+\frac{8e(bd+10ae)}{d^2}\right)\int \frac{x^4}{\left(d+ex^2\right)^{13/2}}\,dx$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}}+\frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}}+\frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}}+\frac{\left(2e\left(3cd^2+8e(bd+10a\right.\right.}{5d^3}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}}+\frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}}+\frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}}+\frac{2e\left(3cd^2+8e(bd+10ae\right.}{35d^4\left(d+ex^2\right)^{11/2}}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}}+\frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}}+\frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}}+\frac{2e\left(3cd^2+8e(bd+10ae\right.}{35d^4\left(d+ex^2\right)^{11/2}}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}}+\frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}}+\frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}}+\frac{2e\left(3cd^2+8e(bd+10ae\right.}{35d^4\left(d+ex^2\right)^{11/2}}$$

Mathematica [A] time = 0.14, size = 167, normalized size = 0.80

$$\frac{5a\left(693d^5x+2310d^4ex^3+3696d^3e^2x^5+3168d^2e^3x^7+1408de^4x^9+256e^5x^{11}\right)+dx^3\left(b\left(1155d^4+1848d^3ex^2+1\right.\right.}{3465d^6\left(d+ex^2\right)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

```
[Out] (5*a*(693*d^5*x + 2310*d^4*e*x^3 + 3696*d^3*e^2*x^5 + 3168*d^2*e^3*x^7 + 14
08*d*e^4*x^9 + 256*e^5*x^11) + d*x^3*(3*c*d*x^2*(231*d^3 + 198*d^2*e*x^2 +
88*d*e^2*x^4 + 16*e^3*x^6) + b*(1155*d^4 + 1848*d^3*e*x^2 + 1584*d^2*e^2*x^
4 + 704*d*e^3*x^6 + 128*e^4*x^8)))/(3465*d^6*(d + e*x^2)^(11/2))
```

fricas [A] time = 1.10, size = 224, normalized size = 1.07

$$\frac{\left(16\left(3cd^2e^3+8bde^4+80ae^5\right)x^{11}+88\left(3cd^3e^2+8bd^2e^3+80ade^4\right)x^9+198\left(3cd^4e+8bd^3e^2+80ad^2e^3\right)x^7+3\right.}{3465\left(d^6e^6x^{12}+6d^7e^5x^{10}+15d^8e^4x^8+20d^9e^3x^6+15d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(16*(3*c*d^2*e^3 + 8*b*d*e^4 + 80*a*e^5)*x^11 + 88*(3*c*d^3*e^2 + 8*
b*d^2*e^3 + 80*a*d*e^4)*x^9 + 198*(3*c*d^4*e + 8*b*d^3*e^2 + 80*a*d^2*e^3)*
x^7 + 3465*a*d^5*x + 231*(3*c*d^5 + 8*b*d^4*e + 80*a*d^3*e^2)*x^5 + 1155*(b
*d^5 + 10*a*d^4*e)*x^3)*sqrt(e*x^2 + d)/(d^6*e^6*x^12 + 6*d^7*e^5*x^10 + 15
*d^8*e^4*x^8 + 20*d^9*e^3*x^6 + 15*d^10*e^2*x^4 + 6*d^11*e*x^2 + d^12)
```

giac [A] time = 0.23, size = 189, normalized size = 0.90

$$\frac{\left(\left(\left(2\left(4x^2\left(\frac{2\left(3cd^2e^8+8bde^9+80ae^{10}\right)x^2e^{(-5)}}{d^6}+\frac{11\left(3cd^3e^7+8bd^2e^8+80ade^9\right)e^{(-5)}}{d^6}\right)+\frac{99\left(3cd^4e^6+8bd^3e^7+80ad^2e^8\right)e^{(-5)}}{d^6}\right)x^2+\frac{231\left(3cd^5 \cdots\right.}{\cdots}\right.\right.\right.}{3465\left(x^2e+d\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="giac")
```

```
[Out] 1/3465*(((2*(4*x^2*(2*(3*c*d^2*e^8 + 8*b*d*e^9 + 80*a*e^10)*x^2*e^(-5)/d^6
+ 11*(3*c*d^3*e^7 + 8*b*d^2*e^8 + 80*a*d*e^9)*e^(-5)/d^6) + 99*(3*c*d^4*e^6
 + 8*b*d^3*e^7 + 80*a*d^2*e^8)*e^(-5)/d^6)*x^2 + 231*(3*c*d^5*e^5 + 8*b*d^4
*e^6 + 80*a*d^3*e^7)*e^(-5)/d^6)*x^2 + 1155*(b*d^5*e^5 + 10*a*d^4*e^6)*e^(-
5)/d^6)*x^2 + 3465*a/d)*x/(x^2*e + d)^(11/2)
```

maple [A] time = 0.01, size = 172, normalized size = 0.82

$$\frac{\left(1280a\,e^5x^{10}+128bd\,e^4x^{10}+48c\,d^2e^3x^{10}+7040ad\,e^4x^8+704b\,d^2e^3x^8+264c\,d^3e^2x^8+15840a\,d^2e^3x^6+1584 \cdots\right.}{3465\left(e\,x^{2} \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x)
```

```
[Out] 1/3465*x*(1280*a*e^5*x^10+128*b*d*e^4*x^10+48*c*d^2*e^3*x^10+7040*a*d*e^4*x
^8+704*b*d^2*e^3*x^8+264*c*d^3*e^2*x^8+15840*a*d^2*e^3*x^6+1584*b*d^3*e^2*x
^6+594*c*d^4*e*x^6+18480*a*d^3*e^2*x^4+1848*b*d^4*e*x^4+693*c*d^5*x^4+11550
*a*d^4*e*x^2+1155*b*d^5*x^2+3465*a*d^5)/(e*x^2+d)^(11/2)/d^6
```

maxima [A] time = 1.11, size = 335, normalized size = 1.60

$$-\frac{cx^3}{8\left(ex^2+d\right)^{\frac{11}{2}}e}+\frac{256\,ax}{693\sqrt{ex^2+d}\,d^6}+\frac{128\,ax}{693\left(ex^2+d\right)^{\frac{3}{2}}d^5}+\frac{32\,ax}{231\left(ex^2+d\right)^{\frac{5}{2}}d^4}+\frac{80\,ax}{693\left(ex^2+d\right)^{\frac{7}{2}}d^3}+\frac{10\,ax}{99\left(ex^2+d\right)^{\frac{9}{2}}d^2} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/8*c*x^3/((e*x^2 + d)^(11/2)*e) + 256/693*a*x/(sqrt(e*x^2 + d)*d^6) + 128
/693*a*x/((e*x^2 + d)^(3/2)*d^5) + 32/231*a*x/((e*x^2 + d)^(5/2)*d^4) + 80/
693*a*x/((e*x^2 + d)^(7/2)*d^3) + 10/99*a*x/((e*x^2 + d)^(9/2)*d^2) + 1/11*
a*x/((e*x^2 + d)^(11/2)*d) + 1/264*c*x/((e*x^2 + d)^(9/2)*e^2) + 16/1155*c*
x/(sqrt(e*x^2 + d)*d^4*e^2) + 8/1155*c*x/((e*x^2 + d)^(3/2)*d^3*e^2) + 2/38
5*c*x/((e*x^2 + d)^(5/2)*d^2*e^2) + 1/231*c*x/((e*x^2 + d)^(7/2)*d*e^2) - 3
/88*c*d*x/((e*x^2 + d)^(11/2)*e^2) - 1/11*b*x/((e*x^2 + d)^(11/2)*e) + 128/
3465*b*x/(sqrt(e*x^2 + d)*d^5*e) + 64/3465*b*x/((e*x^2 + d)^(3/2)*d^4*e) +
16/1155*b*x/((e*x^2 + d)^(5/2)*d^3*e) + 8/693*b*x/((e*x^2 + d)^(7/2)*d^2*e)
 + 1/99*b*x/((e*x^2 + d)^(9/2)*d*e)
```

mupad [B] time = 4.76, size = 226, normalized size = 1.08

$$\frac{x\left(\frac{a}{11\,d}-\frac{d\left(\frac{b}{11\,d}-\frac{c}{11\,e}\right)}{e}\right)}{\left(e\,x^2+d\right)^{11/2}}-\frac{x\left(\frac{c}{9\,e^2}-\frac{-c\,d^2+b\,d\,e+10\,a\,e^2}{99\,d^2\,e^2}\right)}{\left(e\,x^2+d\right)^{9/2}}+\frac{x\left(3\,c\,d^2+8\,b\,d\,e+80\,a\,e^2\right)}{693\,d^3\,e^2\left(e\,x^2+d\right)^{7/2}}+\frac{x\left(6\,c\,d^2+16\,b\,d\,e+160\,a\,e^2\right)}{1155\,d^4\,e^2\left(e\,x^2+d\right)^{5/2}}+\frac{x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x)
```

```
[Out] (x*(a/(11*d) - (d*(b/(11*d) - c/(11*e)))/e))/(d + e*x^2)^(11/2) - (x*(c/(9*
e^2) - (10*a*e^2 - c*d^2 + b*d*e)/(99*d^2*e^2)))/(d + e*x^2)^(9/2) + (x*(80
*a*e^2 + 3*c*d^2 + 8*b*d*e))/(693*d^3*e^2*(d + e*x^2)^(7/2)) + (x*(160*a*e^
2 + 6*c*d^2 + 16*b*d*e))/(1155*d^4*e^2*(d + e*x^2)^(5/2)) + (x*(640*a*e^2 +
 24*c*d^2 + 64*b*d*e))/(3465*d^5*e^2*(d + e*x^2)^(3/2)) + (x*(1280*a*e^2 +
48*c*d^2 + 128*b*d*e))/(3465*d^6*e^2*(d + e*x^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(13/2),x)
```

```
[Out] Timed out
```

3.286 $\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\, dx$

Optimal. Leaf size=193

$$\frac{275}{7}\left(x^4+3x^2+2\right)^{3/2} x+\frac{1}{21}\left(757x^2+2608\right)\sqrt{x^4+3x^2+2}\, x+\frac{577\left(x^2+2\right)x}{3\sqrt{x^4+3x^2+2}}+\frac{2945\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\, F(\text{ta}}{21\sqrt{x^4+3x^2+2}}$$

```
[Out] 275/7*x*(x^4+3*x^2+2)^(3/2)+125/9*x^3*(x^4+3*x^2+2)^(3/2)+577/3*x*(x^2+2)/(
x^4+3*x^2+2)^(1/2)-577/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1
)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+29
45/21*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2)
)*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/21*x*(757*x^2+2608)
*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1206, 1679, 1176, 1189, 1099, 1135}

$$\frac{125}{9}\left(x^4+3x^2+2\right)^{3/2} x^3+\frac{275}{7}\left(x^4+3x^2+2\right)^{3/2} x+\frac{1}{21}\left(757x^2+2608\right)\sqrt{x^4+3x^2+2}\, x+\frac{577\left(x^2+2\right)x}{3\sqrt{x^4+3x^2+2}}+\frac{294}{}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (577*x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(2608 + 757*x^2)*Sqrt[2 +
3*x^2 + x^4])/21 + (275*x*(2 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(2 + 3*x^2
+ x^4)^(3/2))/9 - (577*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
E[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) + (2945*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\,dx &= \frac{125}{9}x^3\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{9}\int \sqrt{2+3x^2+x^4}\left(3087+5865x^2+2475x^4\right)dx \\
&= \frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{63}\int \left(16659+11355x^2\right) \\
&= \frac{1}{21}x\left(2608+757x^2\right)\sqrt{2+3x^2+x^4}+\frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(2+3x \right. \\
&= \frac{1}{21}x\left(2608+757x^2\right)\sqrt{2+3x^2+x^4}+\frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(2+3x \right. \\
&= \frac{577x\left(2+x^2\right)}{3\sqrt{2+3x^2+x^4}}+\frac{1}{21}x\left(2608+757x^2\right)\sqrt{2+3x^2+x^4}+\frac{275}{7}x\left(2+3x^2+x^4\right)
\end{aligned}$$

Mathematica [C] time = 0.10, size = 119, normalized size = 0.62

$$\frac{875x^{11}+7725x^9+28496x^7+57312x^5+61214x^3-5553i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-12117i\sqrt{x^2+1}}{63\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25548*x + 61214*x^3 + 57312*x^5 + 28496*x^7 + 7725*x^9 + 875*x^11 - (12117
*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (5553*
```

```
I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(63*Sqrt
[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(125\,x^6+525\,x^4+735\,x^2+343\right)\sqrt{x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

maple [C] time = 0.02, size = 172, normalized size = 0.89

$$\frac{125\sqrt{x^4+3x^2+2}\,x^7}{9}+\frac{1700\sqrt{x^4+3x^2+2}\,x^5}{21}+\frac{11446\sqrt{x^4+3x^2+2}\,x^3}{63}+\frac{4258\sqrt{x^4+3x^2+2}\,x}{21}-\frac{2945i\sqrt{2}\,\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x)
[Out] 125/9*x^7*(x^4+3*x^2+2)^(1/2)+1700/21*x^5*(x^4+3*x^2+2)^(1/2)+11446/63*x^3*
(x^4+3*x^2+2)^(1/2)+4258/21*x*(x^4+3*x^2+2)^(1/2)-2945/21*I*2^(1/2)*(2*x^2+
4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2
))+577/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Ellip
ticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(5\,x^2+7\right)}^3\,\sqrt{x^4+3\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(x^2+1\right)\left(x^2+2\right)}\,\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3, x)
```

3.287 $\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\, dx$

Optimal. Leaf size=168

$$\frac{25}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{21}x\left(114x^2+407\right)\sqrt{x^4+3x^2+2}+\frac{31x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{472\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x$$

```
[Out] 25/7*x*(x^4+3*x^2+2)^(3/2)+31*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-31*(x^2+1)^(3/2
)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)
/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+472/21*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*
EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4
+3*x^2+2)^(1/2)+1/21*x*(114*x^2+407)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1176, 1189, 1099, 1135}

$$\frac{25}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{21}x\left(114x^2+407\right)\sqrt{x^4+3x^2+2}+\frac{31x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{472\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (31*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(407 + 114*x^2)*Sqrt[2 + 3*x^2
+ x^4])/21 + (25*x*(2 + 3*x^2 + x^4)^(3/2))/7 - (31*Sqrt[2]*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (47
2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2
1*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\,dx = \frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{7}\int \left(293+190x^2\right)\sqrt{2+3x^2+x^4}\,dx$$
$$=\frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4}+\frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{105}\int \frac{4720+3}{\sqrt{2+3x^2}}$$
$$=\frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4}+\frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2}+31\int \frac{x^2}{\sqrt{2+3x^2+}}$$
$$=\frac{31x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4}+\frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2}-$$

Mathematica [C] time = 0.08, size = 114, normalized size = 0.68

$$\frac{75x^9+564x^7+1724x^5+2349x^3-293i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-651i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\right.}{21\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (1114*x + 2349*x^3 + 1724*x^5 + 564*x^7 + 75*x^9 - (651*I)*Sqrt[1 + x^2]*Sq
rt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (293*I)*Sqrt[1 + x^2]*Sqrt
[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(25\,x^4+70\,x^2+49\right)\sqrt{x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

maple [C] time = 0.01, size = 155, normalized size = 0.92

$$\frac{25\sqrt{x^4+3x^2+2}\,x^5}{7}+\frac{113\sqrt{x^4+3x^2+2}\,x^3}{7}+\frac{557\sqrt{x^4+3x^2+2}\,x}{21}-\frac{472i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}}{2}\right.}{21\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 25/7*(x^4+3*x^2+2)^(1/2)*x^5+113/7*(x^4+3*x^2+2)^(1/2)*x^3+557/21*(x^4+3*x^
2+2)^(1/2)*x-472/21*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(
1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+31/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE
(1/2*I*2^(1/2)*x,2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(5\,x^2+7\right)^2\,\sqrt{x^4+3\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\left(x^2+1\right)\left(x^2+2\right)}\,\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2, x)
```

3.288 $\int \left(7+5x^2\right)\sqrt{2+3x^2+x^4}\,dx$

Optimal. Leaf size=149

$$\frac{5x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}x\left(3x^2+10\right)\sqrt{x^4+3x^2+2}+\frac{11\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan\right.}{\sqrt{x^4+3x^2+2}}$$

```
[Out] 5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE
(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+11/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2
^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/3*x*(3*x^2+10
)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1176, 1189, 1099, 1135}

$$\frac{5x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}x\left(3x^2+10\right)\sqrt{x^4+3x^2+2}+\frac{11\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan\right.}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(10 + 3*x^2)*Sqrt[2 + 3*x^2 + x^
4])/3 - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
 1/2])/Sqrt[2 + 3*x^2 + x^4] + (11*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^
2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(7+5x^2\right)\sqrt{2+3x^2+x^4}\,dx = \frac{1}{3}x\left(10+3x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{15}\int \frac{110+75x^2}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{1}{3}x\left(10+3x^2\right)\sqrt{2+3x^2+x^4}+5\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{22}{3}\int \frac{1}{\sqrt{2+3x^2+}}$$
$$= \frac{5x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\left(10+3x^2\right)\sqrt{2+3x^2+x^4}-\frac{5\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\mathrm{ta}\right.}{\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.06, size = 109, normalized size = 0.73

$$\frac{3x^7+19x^5+36x^3-7i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-15i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+20x}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (20*x + 36*x^3 + 19*x^5 + 3*x^7 - (15*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipt
icE[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[
I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

maple [C] time = 0.01, size = 137, normalized size = 0.92

$$\sqrt{x^4+3x^2+2}\,x^3+\frac{10\sqrt{x^4+3x^2+2}\,x}{3}-\frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] (x^4+3*x^2+2)^(1/2)*x^3+10/3*(x^4+3*x^2+2)^(1/2)*x-11/3*I*2^(1/2)*(2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))
+5/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF
(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3x^2+2}\left(5x^2+7\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5x^2+7\right)\sqrt{x^4+3x^2+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(5x^2+7\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7), x)
```

3.289 $\int \sqrt{2+3x^2+x^4}\,dx$

Optimal. Leaf size=141

$$\frac{1}{3}\sqrt{x^4+3x^2+2}\,x+\frac{(x^2+2)\,x}{\sqrt{x^4+3x^2+2}}+\frac{2\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(
x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/
2)+2/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2
))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/3*x*(x^4+3*x^2+2)^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1091, 1189, 1099, 1135}

$$\frac{1}{3}\sqrt{x^4+3x^2+2}\,x+\frac{(x^2+2)\,x}{\sqrt{x^4+3x^2+2}}+\frac{2\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*Sqrt[2 + 3*x^2 + x^4])/3 - (Sqrt[2
]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3
*x^2 + x^4] + (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcT
an[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
```

```
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+3x^2+x^4}\,dx &= \frac{1}{3}x\sqrt{2+3x^2+x^4}+\frac{1}{3}\int \frac{4+3x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{2+3x^2+x^4}+\frac{4}{3}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx+\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\sqrt{2+3x^2+x^4}-\frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{2\sqrt{2}\left(1+x^2\right)}{3\sqrt{}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 102, normalized size = 0.72

$$\frac{x^5+3x^3-i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-3i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+2x}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (2*x + 3*x^3 + x^5 - (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[
x/Sqrt[2]], 2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2
]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

maple [C] time = 0.00, size = 121, normalized size = 0.86

$$\frac{\sqrt{x^4+3x^2+2}\,x}{3}-\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\;\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\;\left(-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\right.\right.}{2\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2),x)
```

[Out] 1/3*(x^4+3*x^2+2)^(1/2)*x-2/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4 + 3\,x^2 + 2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^4 + 3\,x^2 + 2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + x^4 + 2)^(1/2),x)

[Out] int((3*x^2 + x^4 + 2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4 + 3x^2 + 2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+3*x**2+2)**(1/2),x)

[Out] Integral(sqrt(x**4 + 3*x**2 + 2), x)

3.290 $\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2} \, dx$

Optimal. Leaf size=178

$$\frac{x\left(x^2+2\right)}{5\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{2x^2+2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\Pi\left(\frac{2}{7};\tan\right.}{35\sqrt{2}\sqrt{x^4+3x^2+}}$$

```
[Out] 1/5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+3/70*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Elli
pticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^
4+3*x^2+2)^(1/2)-1/5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1
/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/5*(x
^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+
2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 232, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1208, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{x\left(x^2+2\right)}{5\sqrt{x^4+3x^2+2}}+\frac{4\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{x^4+3x^2+2}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\mathrm{t}\right.}{5\sqrt{x^4+3x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] (x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) - (3*(1 +
x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(25*Sqrt[2]*Sqrt[
2 + 3*x^2 + x^4]) + (4*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
F[ArcTan[x], 1/2])/(25*Sqrt[2 + 3*x^2 + x^4]) + (3*(2 + x^2)*EllipticPi[2/7
, ArcTan[x], 1/2])/(35*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x
^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx &= -\left(\frac{1}{25}\int \frac{-8-5x^2}{\sqrt{2+3x^2+x^4}}\,dx\right)-\frac{6}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx\\
&= -\left(\frac{3}{25}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\right)+\frac{1}{5}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{3}{10}\int \frac{2+2x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+}}\\
&= \frac{x\left(2+x^2\right)}{5\sqrt{2+3x^2+x^4}}-\frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}}-\frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\right.}{25\sqrt{2}\sqrt{2+3x^2+x^4}}\\
&= \frac{x\left(2+x^2\right)}{5\sqrt{2+3x^2+x^4}}-\frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}}-\frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\right.}{25\sqrt{2}\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] time = 0.15, size = 90, normalized size = 0.51

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(21F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+35E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-6\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{175\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] ((-1/175*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(35*EllipticE[I*ArcSinh[x/Sqrt[2]],
 2] + 21*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - 6*EllipticPi[10/7, I*ArcSinh[
x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

maple [C] time = 0.04, size = 138, normalized size = 0.78

$$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{10\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{50\sqrt{x^4+3x^2+2}}+\frac{6i\sqrt{2}\,\sqrt{\frac{x^2}{2}+1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x)
```

```
[Out] -3/50*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF
(1/2*I*2^(1/2)*x,2^(1/2))-1/10*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4
+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+6/175*I*2^(1/2)*(1+1/2*x
^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7
,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7),x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7), x)
```

3.291 $\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2} dx$

Optimal. Leaf size=209

$$\frac{\sqrt{x^4+3x^2+2}\,x}{14\left(5x^2+7\right)}-\frac{\left(x^2+2\right)x}{70\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{\left(x^2+2\right.}{980\sqrt{2}\sqrt{}}$$

```
[Out] -1/70*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/1960*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1
)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1
))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/70*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE
(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+3/280*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*
2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/14*x*(x^4+3*
x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.12, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1226, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{\sqrt{x^4+3x^2+2}\,x}{14\left(5x^2+7\right)}-\frac{\left(x^2+2\right)x}{70\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{\left(x^2+2\right.}{980\sqrt{2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] -(x*(2 + x^2))/(70*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/(14*(
7 + 5*x^2)) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2
])/(35*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^
2)]*EllipticF[ArcTan[x], 1/2])/(140*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - ((2 +
x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(980*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)
]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
```

```
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1226

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx = \frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{1}{350}\int \frac{7-5x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{1}{350}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{1}{700}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{1}{280}\int \frac{2+2x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{x\left(2+x^2\right)}{70\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}}$$

$$= -\frac{x\left(2+x^2\right)}{70\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.28, size = 208, normalized size = 1.00

$$\frac{175x^5+525x^3-84i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+35i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)E\left(i\sinh^{-1}\right.}{2450\left(5x^2+7\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (350*x + 525*x^3 + 175*x^5 + (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)
*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (84*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7
 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (5*I)*x^2*Sqrt[1 + x^2]*S
qrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(2450*(7 + 5*x^2)*S
qrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{25\,x^4+70\,x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

maple [C] time = 0.02, size = 162, normalized size = 0.78

$$\frac{\sqrt{x^4+3x^2+2}\,x}{70x^2+98}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{140\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{175\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x)
```

```
[Out] 1/14*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-3/175*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/140*I*2^(1
/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/
2)*x,2^(1/2))-1/2450*I*2^(1/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^2,x)
[Out] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**2,x)
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**2, x)
```

3.292 $\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^3} dx$

Optimal. Leaf size=237

$$\frac{11\sqrt{x^4+3x^2+2}\,x}{2352\left(5x^2+7\right)}+\frac{\sqrt{x^4+3x^2+2}\,x}{28\left(5x^2+7\right)^2}-\frac{11\left(x^2+2\right)x}{11760\sqrt{x^4+3x^2+2}}+\frac{81\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{11\left(x^2+1\right)\sqrt{\frac{x^2+}{x^2+}}}{5880\sqrt{2}\sqrt{x}}$$

```
[Out] -11/11760*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1201/329280*(x^2+2)*(1/(x^2+1))^(1/
2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+
2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+11/11760*(x^2+1)^(3/2)*(1/(x^2+1))^(1
/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/
(x^4+3*x^2+2)^(1/2)+81/15680*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x
^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2
)+1/28*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+11/2352*x*(x^4+3*x^2+2)^(1/2)/(5*x
^2+7)
```

Rubi [A] time = 0.60, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1223, 1696, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{11\sqrt{x^4+3x^2+2}\,x}{2352\left(5x^2+7\right)}+\frac{\sqrt{x^4+3x^2+2}\,x}{28\left(5x^2+7\right)^2}-\frac{11\left(x^2+2\right)x}{11760\sqrt{x^4+3x^2+2}}+\frac{81\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{11\left(x^2+1\right)\sqrt{\frac{x^2+}{x^2+}}}{5880\sqrt{2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (-11*x*(2 + x^2))/(11760*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])
/(28*(7 + 5*x^2)^2) + (11*x*Sqrt[2 + 3*x^2 + x^4])/(2352*(7 + 5*x^2)) + (11
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5880*Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellipti
cF[ArcTan[x], 1/2])/(7840*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (1201*(2 + x^2)*
EllipticPi[2/7, ArcTan[x], 1/2])/(164640*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*
Sqrt[2 + 3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
```

```
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
```

```
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx = \int \left(-\frac{6}{25\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}} + \frac{1}{25\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}} + \frac{1}{25\left(7+5x^2\right)\sqrt{2+\right.}}$$

$$= \frac{1}{25}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx + \frac{1}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx - \frac{6}{25}\int \frac{}{(7+5.}$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} - \frac{x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)} + \frac{\int \frac{62+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx}{2100} - \frac{1}{700}\int \frac{74-10x^2-2}{\left(7+5x^2\right)^2\sqrt{2+3}}$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{50\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{\int \frac{2838+2310x^2+}{\left(7+5x^2\right)\sqrt{2+3x}}}{58800}$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{50\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{\left(2+x^2\right)\Pi\left(\frac{2}{7}\right.}{70\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

$$= \frac{x\left(2+x^2\right)}{420\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} - \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x\right.}{210\sqrt{2}\sqrt{2+3x^2+x^4}}$$

$$= -\frac{11x\left(2+x^2\right)}{11760\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} + \frac{11\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(t\right.}{5880\sqrt{2}\sqrt{2+3x}}$$

$$= -\frac{11x\left(2+x^2\right)}{11760\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} + \frac{11\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(t\right.}{5880\sqrt{2}\sqrt{2+3x}}$$

Mathematica [C] time = 0.35, size = 174, normalized size = 0.73

$$\frac{-434i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 385i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 1201i\sqrt{x^2+1}\sqrt{x^2+2}}{411600\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] ((14700*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (1925*x*(2 + 3*x^2 + x^4))/(7
+ 5*x^2) + (385*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2
]], 2] - (434*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]]
, 2] - (1201*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sq
rt[2]], 2])/(411600*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{125\,x^6+525\,x^4+735\,x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

maple [C] time = 0.02, size = 186, normalized size = 0.78

$$\frac{\sqrt{x^4+3x^2+2}\;x}{28\left(5x^2+7\right)^2}+\frac{11\sqrt{x^4+3x^2+2}\;x}{2352\left(5x^2+7\right)}+\frac{11 i\sqrt{2}\;\sqrt{2x^2+4}\;\sqrt{x^2+1}\;\operatorname{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{23520\sqrt{x^4+3x^2+2}}-\frac{31 i\sqrt{2}\;\sqrt{2x^2+4}\;\sqrt{x}}{58800}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x)
```

```
[Out] 1/28*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+11/2352*(x^4+3*x^2+2)^(1/2)/(5*x^2+7
)*x-31/58800*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*El
lipticF(1/2*I*2^(1/2)*x,2^(1/2))+11/23520*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-1201/411600*I
*2^(1/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2
*I*2^(1/2)*x,10/7,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^3,x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**3, x)
```

3.293 $\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2} dx$

Optimal. Leaf size=219

$$\frac{3825}{143}\left(x^4+3x^2+2\right)^{5/2}x+\frac{\left(65345x^2+208212\right)\left(x^4+3x^2+2\right)^{3/2}x}{3003}+\frac{\left(297911x^2+1032541\right)\sqrt{x^4+3x^2+2}\,x}{5005}+$$

```
[Out] 1/3003*x*(65345*x^2+208212)*(x^4+3*x^2+2)^(3/2)+3825/143*x*(x^4+3*x^2+2)^(5
/2)+125/13*x^3*(x^4+3*x^2+2)^(5/2)+20884/65*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-2
0884/65*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/
2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1171349/5005*(x^2+1
)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((
x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/5005*x*(297911*x^2+1032541)*(x^
4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1206, 1679, 1176, 1189, 1099, 1135}

$$\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3+\frac{3825}{143}\left(x^4+3x^2+2\right)^{5/2}x+\frac{\left(65345x^2+208212\right)\left(x^4+3x^2+2\right)^{3/2}x}{3003}+\frac{\left(297911x^2+10\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (20884*x*(2 + x^2))/(65*Sqrt[2 + 3*x^2 + x^4]) + (x*(1032541 + 297911*x^2)*
Sqrt[2 + 3*x^2 + x^4])/5005 + (x*(208212 + 65345*x^2)*(2 + 3*x^2 + x^4)^(3/
2))/3003 + (3825*x*(2 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(2 + 3*x^2 + x^4
)^(5/2))/13 - (20884*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[
ArcTan[x], 1/2])/(65*Sqrt[2 + 3*x^2 + x^4]) + (1171349*Sqrt[2]*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5005*Sqrt[2 + 3*x^2 + x
^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
```

```
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2} dx &= \frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2}+\frac{1}{13}\int \left(2+3x^2+x^4\right)^{3/2}\left(4459+8805x^2+3825x^4\right.\\ &= \frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2}+\frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2}+\frac{1}{143}\int \left(41399+2800\right.\\ &= \frac{x\left(208212+65345x^2\right)\left(2+3x^2+x^4\right)^{3/2}}{3003}+\frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2}+\frac{125}{13}x^3\\ &= \frac{x\left(1032541+297911x^2\right)\sqrt{2+3x^2+x^4}}{5005}+\frac{x\left(208212+65345x^2\right)\left(2+3x^2\right.}{3003}\\ &= \frac{x\left(1032541+297911x^2\right)\sqrt{2+3x^2+x^4}}{5005}+\frac{x\left(208212+65345x^2\right)\left(2+3x^2\right.}{3003}\\ &= \frac{20884x\left(2+x^2\right)}{65\sqrt{2+3x^2+x^4}}+\frac{x\left(1032541+297911x^2\right)\sqrt{2+3x^2+x^4}}{5005}+\frac{x\left(208212+\right.}{}\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(125\,x^{10} + 900\,x^{8} + 2560\,x^{6} + 3598\,x^{4} + 2499\,x^{2} + 686\right)\sqrt{x^{4} + 3\,x^{2} + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^10 + 900*x^8 + 2560*x^6 + 3598*x^4 + 2499*x^2 + 686)*sqrt(x
^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{4} + 3\,x^{2} + 2\right)^{\frac{3}{2}}\left(5\,x^{2} + 7\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

maple [C] time = 0.02, size = 206, normalized size = 0.94

$$\frac{125\sqrt{x^{4}+3x^{2}+2}\;x^{11}}{13}+\frac{12075\sqrt{x^{4}+3x^{2}+2}\;x^{9}}{143}+\frac{131810\sqrt{x^{4}+3x^{2}+2}\;x^{7}}{429}+\frac{598324\sqrt{x^{4}+3x^{2}+2}\;x^{5}}{1001}+\frac{10067}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 2262081/5005*(x^4+3*x^2+2)^(1/2)*x+10067363/15015*(x^4+3*x^2+2)^(1/2)*x^3+5
98324/1001*(x^4+3*x^2+2)^(1/2)*x^5+131810/429*(x^4+3*x^2+2)^(1/2)*x^7+125/1
3*x^11*(x^4+3*x^2+2)^(1/2)+12075/143*x^9*(x^4+3*x^2+2)^(1/2)-1171349/5005*I
*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*
2^(1/2)*x,2^(1/2))+10442/65*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*
x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,
2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{4} + 3\,x^{2} + 2\right)^{\frac{3}{2}}\left(5\,x^{2} + 7\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(5\,x^2+7\right)}^3\,{\left(x^4+3\,x^2+2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(3/2),x)
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3, x)
```

3.294 $\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=198

$$\frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2}+\frac{1}{693}x\left(2240x^2+7281\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{x\left(10643x^2+36783\right)\sqrt{x^4+3x^2+2}}{1155}+\frac{742}{15\sqrt{x^4}}$$

```
[Out] 1/693*x*(2240*x^2+7281)*(x^4+3*x^2+2)^(3/2)+25/11*x*(x^4+3*x^2+2)^(5/2)+742
/15*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-742/15*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*El
lipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3
*x^2+2)^(1/2)+13879/385*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)
^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/1
155*x*(10643*x^2+36783)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1176, 1189, 1099, 1135}

$$\frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2}+\frac{1}{693}x\left(2240x^2+7281\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{x\left(10643x^2+36783\right)\sqrt{x^4+3x^2+2}}{1155}+\frac{742}{15\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (742*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(36783 + 10643*x^2)*Sqrt[
2 + 3*x^2 + x^4])/1155 + (x*(7281 + 2240*x^2)*(2 + 3*x^2 + x^4)^(3/2))/693
+ (25*x*(2 + 3*x^2 + x^4)^(5/2))/11 - (742*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (13879*
Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(385
*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
```

```
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2} dx = \frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2} + \frac{1}{11}\int \left(489+320x^2\right)\left(2+3x^2+x^4\right)^{3/2} dx$$

$$= \frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right)^{3/2} + \frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2} + \frac{1}{231}\int \Big($$

$$= \frac{x\left(36783+10643x^2\right)\sqrt{2+3x^2+x^4}}{1155} + \frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right.$$

$$= \frac{x\left(36783+10643x^2\right)\sqrt{2+3x^2+x^4}}{1155} + \frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right.$$

$$= \frac{742x\left(2+x^2\right)}{15\sqrt{2+3x^2+x^4}} + \frac{x\left(36783+10643x^2\right)\sqrt{2+3x^2+x^4}}{1155} + \frac{1}{693}x\left(7281+2\right.$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(25\,x^8+145\,x^6+309\,x^4+287\,x^2+98\right)\sqrt{x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral((25*x^8 + 145*x^6 + 309*x^4 + 287*x^2 + 98)*sqrt(x^4 + 3*x^2 + 2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4 + 3x^2 + 2\right)^{\frac{3}{2}} \left(5x^2 + 7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

maple [C] time = 0.01, size = 189, normalized size = 0.95

$$\frac{25\sqrt{x^4+3x^2+2}\,x^9}{11}+\frac{1670\sqrt{x^4+3x^2+2}\,x^7}{99}+\frac{11492\sqrt{x^4+3x^2+2}\,x^5}{231}+\frac{258044\sqrt{x^4+3x^2+2}\,x^3}{3465}+\frac{23851\sqrt{x^4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 25/11*(x^4+3*x^2+2)^(1/2)*x^9+1670/99*(x^4+3*x^2+2)^(1/2)*x^7+11492/231*(x^
4+3*x^2+2)^(1/2)*x^5+258044/3465*(x^4+3*x^2+2)^(1/2)*x^3+23851/385*(x^4+3*x
^2+2)^(1/2)*x-13879/385*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+371/15*I*2^(1/2)*(2*x^2+4)^(1/2
)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-Ell
ipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4 + 3x^2 + 2\right)^{\frac{3}{2}} \left(5x^2 + 7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5x^2 + 7\right)^2 \left(x^4 + 3x^2 + 2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^2 + 1\right)\left(x^2 + 2\right)\right)^{\frac{3}{2}} \left(5x^2 + 7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2, x)
```

3.295 $\int \left(7+5x^2\right)\left(2+3x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=179

$$\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{105}x\left(149x^2+519\right)\sqrt{x^4+3x^2+2}+\frac{116x\left(x^2+2\right)}{15\sqrt{x^4+3x^2+2}}+\frac{197\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x}{x}}}{35\sqrt{x^4+}}$$

```
[Out] 1/63*x*(35*x^2+108)*(x^4+3*x^2+2)^(3/2)+116/15*x*(x^2+2)/(x^4+3*x^2+2)^(1/2
)-116/15*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1
/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+197/35*(x^2+1)^(3/
2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2
)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/105*x*(149*x^2+519)*(x^4+3*x^2+2)^(1
/2)
```

Rubi [A] time = 0.06, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1176, 1189, 1099, 1135}

$$\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{105}x\left(149x^2+519\right)\sqrt{x^4+3x^2+2}+\frac{116x\left(x^2+2\right)}{15\sqrt{x^4+3x^2+2}}+\frac{197\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x}{x}}}{35\sqrt{x^4+}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (116*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(519 + 149*x^2)*Sqrt[2 +
3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(2 + 3*x^2 + x^4)^(3/2))/63 - (116*Sq
rt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sq
rt[2 + 3*x^2 + x^4]) + (197*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(7+5x^2\right)\left(2+3x^2+x^4\right)^{3/2}dx &= \frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{21}\int \left(222+149x^2\right)\sqrt{2+3x^2+x^4}\\ &= \frac{1}{105}x\left(519+149x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2+x^4\right)^{3/2}\\ &= \frac{1}{105}x\left(519+149x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2+x^4\right)^{3/2}\\ &= \frac{116x\left(2+x^2\right)}{15\sqrt{2+3x^2+x^4}}+\frac{1}{105}x\left(519+149x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{63}x\left(108+35\right.\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(5\,x^6+22\,x^4+31\,x^2+14\right)\sqrt{x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral((5*x^6 + 22*x^4 + 31*x^2 + 14)*sqrt(x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

maple [C] time = 0.00, size = 172, normalized size = 0.96

$$\frac{5\sqrt{x^4+3x^2+2}\,x^7}{9}+\frac{71\sqrt{x^4+3x^2+2}\,x^5}{21}+\frac{2417\sqrt{x^4+3x^2+2}\,x^3}{315}+\frac{293\sqrt{x^4+3x^2+2}\,x}{35}-\frac{197i\sqrt{2}\,\sqrt{2x^2+4}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 5/9*(x^4+3*x^2+2)^(1/2)*x^7+71/21*(x^4+3*x^2+2)^(1/2)*x^5+2417/315*(x^4+3*x
^2+2)^(1/2)*x^3+293/35*(x^4+3*x^2+2)^(1/2)*x-197/35*I*2^(1/2)*(2*x^2+4)^(1/
2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+58/
15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1
/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4 + 3x^2 + 2\right)^{\frac{3}{2}} \left(5x^2 + 7\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5x^2 + 7\right) \left(x^4 + 3x^2 + 2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^2 + 1\right) \left(x^2 + 2\right)\right)^{\frac{3}{2}} \left(5x^2 + 7\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7), x)
```

3.296 $\int \left(2+3x^2+x^4\right)^{3/2} dx$

Optimal. Leaf size=172

$$\frac{1}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{35}x\left(9x^2+29\right)\sqrt{x^4+3x^2+2}+\frac{6x\left(x^2+2\right)}{5\sqrt{x^4+3x^2+2}}+\frac{31\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}$$

```
[Out] 1/7*x*(x^4+3*x^2+2)^(3/2)+6/5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-6/5*(x^2+1)^(3/
2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2
)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+31/35*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*
EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4
+3*x^2+2)^(1/2)+1/35*x*(9*x^2+29)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1091, 1176, 1189, 1099, 1135}

$$\frac{1}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{35}x\left(9x^2+29\right)\sqrt{x^4+3x^2+2}+\frac{6x\left(x^2+2\right)}{5\sqrt{x^4+3x^2+2}}+\frac{31\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (6*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + (x*(29 + 9*x^2)*Sqrt[2 + 3*x^2
+ x^4])/35 + (x*(2 + 3*x^2 + x^4)^(3/2))/7 - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 +
 x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + (31
*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(35
*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(2+3x^2+x^4\right)^{3/2}\,dx &= \frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{3}{7}\int \left(4+3x^2\right)\sqrt{2+3x^2+x^4}\,dx\\ &= \frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{35}\int \frac{62+42x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{6}{5}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{62}{35}\dots\\ &= \frac{6x\left(2+x^2\right)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}-\frac{6\sqrt{2}\left(1+\dots\right.}{5\dots}\end{aligned}$$

Mathematica [C] time = 0.04, size = 114, normalized size = 0.66

$$\frac{5x^9+39x^7+121x^5+165x^3-20i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-42i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\dots\right.}{35\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (78*x + 165*x^3 + 121*x^5 + 39*x^7 + 5*x^9 - (42*I)*Sqrt[1 + x^2]*Sqrt[2 +
x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (20*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2
]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(x^4+3\,x^2+2\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

maple [C] time = 0.00, size = 155, normalized size = 0.90

$$\frac{\sqrt{x^4+3x^2+2}\,x^5}{7}+\frac{24\sqrt{x^4+3x^2+2}\,x^3}{35}+\frac{39\sqrt{x^4+3x^2+2}\,x}{35}-\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right.}{35\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 1/7*(x^4+3*x^2+2)^(1/2)*x^5+24/35*(x^4+3*x^2+2)^(1/2)*x^3+39/35*(x^4+3*x^2+
2)^(1/2)*x-31/35*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2
)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+3/5*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2
*I*2^(1/2)*x,2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^4+3\,x^2+2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+3x^2+2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 2)**(3/2), x)
```

3.297 $\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx$

Optimal. Leaf size=207

$$\frac{24x\left(x^2+2\right)}{125\sqrt{x^4+3x^2+2}}+\frac{1}{75}x\left(3x^2+11\right)\sqrt{x^4+3x^2+2}+\frac{56\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{375\sqrt{x^4+3x^2+2}}-\frac{24\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}}{125\sqrt{x^4+3x^2}}$$

```
[Out] 24/125*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-9/875*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1
)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1
))^(1/2)/(x^4+3*x^2+2)^(1/2)-24/125*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellipti
cE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+
2)^(1/2)+56/375*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1
/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/75*x*(3*x
^2+11)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1208, 1176, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{24x\left(x^2+2\right)}{125\sqrt{x^4+3x^2+2}}+\frac{1}{75}x\left(3x^2+11\right)\sqrt{x^4+3x^2+2}+\frac{56\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{375\sqrt{x^4+3x^2+2}}-\frac{24\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}}{125\sqrt{x^4+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (24*x*(2 + x^2))/(125*Sqrt[2 + 3*x^2 + x^4]) + (x*(11 + 3*x^2)*Sqrt[2 + 3*x
^2 + x^4])/75 - (24*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[A
rcTan[x], 1/2])/(125*Sqrt[2 + 3*x^2 + x^4]) + (56*Sqrt[2]*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(375*Sqrt[2 + 3*x^2 + x^4]) -
 (9*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/
(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
```

```
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int \left(-8-5x^2\right)\sqrt{2+3x^2+x^4}\,dx\right) - \frac{6}{25}\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx$$
$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} - \frac{1}{375}\int \frac{-130-90x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{6}{625}\int \frac{-8-5x^2}{\sqrt{2+3x^2+x^4}}\,d$$
$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} + \frac{18}{625}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{6}{125}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,d$$

$$= \frac{24x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} - \frac{24\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(\right.}{125\sqrt{2+3x^2+x^4}}$$
$$= \frac{24x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} - \frac{24\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(\right.}{125\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.18, size = 148, normalized size = 0.71

$$\frac{525x^7+3500x^5+6825x^3-1022i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-2520i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.}{13125\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (3850*x + 6825*x^3 + 3500*x^5 + 525*x^7 - (2520*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (1022*I)*Sqrt[1 + x^2]*Sqrt[2 + x^
2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (108*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]
*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(13125*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{5x^2+7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

maple [C] time = 0.02, size = 170, normalized size = 0.82

$$\frac{\sqrt{x^4+3x^2+2}\,x^3}{25}+\frac{11\sqrt{x^4+3x^2+2}\,x}{75}-\frac{12i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{125\sqrt{x^4+3x^2+2}}-\frac{73i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2}}{1875\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x)
```

```
[Out] 1/25*(x^4+3*x^2+2)^(1/2)*x^3+11/75*(x^4+3*x^2+2)^(1/2)*x-73/1875*I*2^(1/2)*
(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x
,2^(1/2))-12/125*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2
)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-36/4375*I*2^(1/2)*(1/2*x^2+1)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+3\,x^2+2\right)^{3/2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7),x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7), x)
```

3.298 $\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=222

$$-\frac{3\sqrt{x^4+3x^2+2}\,x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+2}\,x+\frac{9\left(x^2+2\right)x}{175\sqrt{x^4+3x^2+2}}+\frac{59\left(x^2+1\right)\sqrt{\frac{x^2+2}{2x^2+2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{1050\sqrt{x^4+3x^2+2}}-\frac{9\sqrt{2}\left(x^2+1\right)}{175\sqrt{}}$$

```
[Out] 9/175*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-9/175*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*E
llipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+
3*x^2+2)^(1/2)+59/1050*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^
(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)+9/2450*(x^
2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*((
x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/75*x*(x^4+3*x^2+2)^(1/2)-3/17
5*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.44, antiderivative size = 333, normalized size of antiderivative = 1.50, number of steps used = 21, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1099, 1135, 1122, 1189, 1223, 1716, 1214, 1456, 539}

$$-\frac{3\sqrt{x^4+3x^2+2}\,x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+2}\,x+\frac{9\left(x^2+2\right)x}{175\sqrt{x^4+3x^2+2}}+\frac{44\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{1875\sqrt{x^4+3x^2+2}}+\frac{81\left(x^2+1\right)}{8750\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (9*x*(2 + x^2))/(175*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/75
- (3*x*Sqrt[2 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) - (9*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(175*Sqrt[2 + 3*x^2 + x^4]
) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(875
0*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (44*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
+ x^2)]*EllipticF[ArcTan[x], 1/2])/(1875*Sqrt[2 + 3*x^2 + x^4]) - (39*(2 +
x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(12250*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^
2)]*Sqrt[2 + 3*x^2 + x^4]) + (3*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x]
, 1/2])/(875*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx = \int \left(\frac{52}{625\sqrt{2+3x^2+x^4}}+\frac{16x^2}{125\sqrt{2+3x^2+x^4}}+\frac{x^4}{25\sqrt{2+3x^2+x^4}}+\frac{36}{625\left(7+5x^2\right)^2\sqrt{2}}\right.$$

$$= -\left(\frac{12}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx\right)+\frac{1}{25}\int \frac{x^4}{\sqrt{2+3x^2+x^4}}\,dx+\frac{36}{625}\int \frac{}{\left(7+5x^2\right.}$$

$$= \frac{16x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}}+\frac{1}{75}x\sqrt{2+3x^2+x^4}-\frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)}-\frac{16\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}}{125\sqrt{2+3x^2}}$$

$$= \frac{16x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}}+\frac{1}{75}x\sqrt{2+3x^2+x^4}-\frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)}-\frac{16\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}}{125\sqrt{2+3x^2}}$$

$$= \frac{6x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}}+\frac{1}{75}x\sqrt{2+3x^2+x^4}-\frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)}-\frac{6\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E}{125\sqrt{2+3x^2}}$$

$$= \frac{9x\left(2+x^2\right)}{175\sqrt{2+3x^2+x^4}}+\frac{1}{75}x\sqrt{2+3x^2+x^4}-\frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)}-\frac{9\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E}{175\sqrt{2+3x^2}}$$

$$= \frac{9x\left(2+x^2\right)}{175\sqrt{2+3x^2+x^4}}+\frac{1}{75}x\sqrt{2+3x^2+x^4}-\frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)}-\frac{9\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E}{175\sqrt{2+3x^2}}$$

Mathematica [C] time = 0.29, size = 213, normalized size = 0.96

$$\frac{1225x^7+5075x^5+6650x^3-182i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-945i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+\right.}{18375}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (2800*x + 6650*x^3 + 5075*x^5 + 1225*x^7 - (945*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (182*I)*Sqrt[1 + x^2]*
Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (189*I)*Sqrt
[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (135*I)
*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])
/(18375*(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{25\,x^4+70\,x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

maple [C] time = 0.02, size = 177, normalized size = 0.80

$$-\frac{3\sqrt{x^4+3x^2+2}\;x}{175\left(5x^2+7\right)}+\frac{\sqrt{x^4+3x^2+2}\;x}{75}-\frac{9i\sqrt{2}\;\sqrt{2x^2+4}\;\sqrt{x^2+1}\;\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{350\sqrt{x^4+3x^2+2}}-\frac{13i\sqrt{2}\;\sqrt{2x^2+4}\;\sqrt{x^2}}{2625\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x)
[Out] -3/175*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)*x+1/75*(x^4+3*x^2+2)^(1/2)*x-13/2625*I
*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*
2^(1/2)*x,2^(1/2))-9/350*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2
+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+9/6125*I*2^(1/2)*(1/2*x^2+1)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^4+3\,x^2+2\right)}^{3/2}}{{\left(5\,x^2+7\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^2,x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**2, x)
```

3.299 $\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=231

$$\frac{17\sqrt{x^4+3x^2+2}\,x}{9800\left(5x^2+7\right)}-\frac{3\sqrt{x^4+3x^2+2}\,x}{350\left(5x^2+7\right)^2}+\frac{3\left(x^2+2\right)x}{392\sqrt{x^4+3x^2+2}}+\frac{5\left(x^2+1\right)\sqrt{\frac{x^2+2}{2x^2+2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{784\sqrt{x^4+3x^2+2}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^2}{2x}}}{196\sqrt{x^4}}$$

```
[Out] 3/392*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+141/54880*(x^2+2)*(1/(x^2+1))^(1/2)*(x^
2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^
2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-3/196*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellip
ticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(
1/2)+5/784*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^
(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)-3/350*x*(x^4+3*x^2+2)^
(1/2)/(5*x^2+7)^2+17/9800*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.67, antiderivative size = 288, normalized size of antiderivative = 1.25, number of steps used = 27, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1099, 1135, 1223, 1696, 1716, 1189, 1214, 1456, 539}

$$\frac{17\sqrt{x^4+3x^2+2}\,x}{9800\left(5x^2+7\right)}-\frac{3\sqrt{x^4+3x^2+2}\,x}{350\left(5x^2+7\right)^2}+\frac{3\left(x^2+2\right)x}{392\sqrt{x^4+3x^2+2}}+\frac{5\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{784\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{6\sqrt{2}\left(x^2+1\right)}{875\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (3*x*(2 + x^2))/(392*Sqrt[2 + 3*x^2 + x^4]) - (3*x*Sqrt[2 + 3*x^2 + x^4])/(
350*(7 + 5*x^2)^2) + (17*x*Sqrt[2 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) - (39*
(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(24500*Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]) - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*
EllipticE[ArcTan[x], 1/2])/(875*Sqrt[2 + 3*x^2 + x^4]) + (5*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[2 + 3*x^2
 + x^4]) + (141*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(27440*Sqrt[2]*S
qrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
```

```
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx = \int \left(\frac{9}{625\sqrt{2+3x^2+x^4}} + \frac{x^2}{125\sqrt{2+3x^2+x^4}} + \frac{36}{625\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}} - \frac{\ldots}{62\ldots}\right.$$

$$= \frac{1}{125}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{9}{625}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{11}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^{\ldots}}}$$

$$= \frac{x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)} - \frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\ldots}{125\sqrt{2+3x^2\ldots}}$$

$$= \frac{x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} - \frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\ldots}{125\sqrt{2+3x\ldots}}$$

$$= \frac{x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} - \frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\ldots}{125\sqrt{2+3x\ldots}}$$

$$= \frac{6x\left(2+x^2\right)}{875\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} - \frac{6\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x\ldots}{1+x\ldots}}}{875\sqrt{2+3\ldots}}$$

$$= \frac{3x\left(2+x^2\right)}{392\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} - \frac{39\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\ldots}{24500\sqrt{2}\sqrt{2+\ldots}}$$

$$= \frac{3x\left(2+x^2\right)}{392\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} - \frac{39\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\ldots}{24500\sqrt{2}\sqrt{2+\ldots}}$$

Mathematica [C] time = 0.38, size = 174, normalized size = 0.75

$$\frac{-406i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 525i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 141i\sqrt{x^2+1}\sqrt{x^2+\ldots}}{68600\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] ((-588*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (119*x*(2 + 3*x^2 + x^4))/(7 +
5*x^2) - (525*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]]
, 2] - (406*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]],
2] + (141*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[
2]], 2])/(68600*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{125\,x^6+525\,x^4+735\,x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

maple [C] time = 0.02, size = 186, normalized size = 0.81

$$-\frac{3\sqrt{x^4+3x^2+2}\;x}{350\left(5x^2+7\right)^2}+\frac{17\sqrt{x^4+3x^2+2}\;x}{9800\left(5x^2+7\right)}-\frac{3i\sqrt{2}\;\sqrt{2x^2+4}\;\sqrt{x^2+1}\;\operatorname{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{784\sqrt{x^4+3x^2+2}}-\frac{29i\sqrt{2}\;\sqrt{2x^2+4}\;\sqrt{x^2}}{9800\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x)
```

```
[Out] -3/350*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2*x+17/9800*(x^4+3*x^2+2)^(1/2)/(5*x^2
+7)*x-29/9800*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*E
llipticF(1/2*I*2^(1/2)*x,2^(1/2))-3/784*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+141/68600*I*2^(
1/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2
^(1/2)*x,10/7,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+3\,x^2+2\right)^{3/2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^3,x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**3, x)
```

3.300 $\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}} \, dx$

Optimal. Leaf size=157

$$75\sqrt{x^4+3x^2+2}\,x+\frac{135\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}+\frac{193\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{135\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+$$

```
[Out] 135*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+193/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ell
ipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*
x^2+2)^(1/2)-135*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),
1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+75*x*(x^4+
3*x^2+2)^(1/2)+25*x^3*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1679, 1189, 1099, 1135}

$$25\sqrt{x^4+3x^2+2}\,x^3+75\sqrt{x^4+3x^2+2}\,x+\frac{135\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}+\frac{193\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{135\sqrt{2}\left(x^2+1\right)\sqrt{}}{\sqrt{x^4+}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (135*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + 75*x*Sqrt[2 + 3*x^2 + x^4] + 25*x
^3*Sqrt[2 + 3*x^2 + x^4] - (135*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]
*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (193*(1 + x^2)*Sqrt[(2
+ x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]
)
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(7+5x^2\right)^3}{\sqrt{2+3x^2+x^4}}\,dx &= 25x^3\sqrt{2+3x^2+x^4}+\frac{1}{5}\int \frac{1715+2925x^2+1125x^4}{\sqrt{2+3x^2+x^4}}\,dx\\ &= 75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}+\frac{1}{15}\int \frac{2895+2025x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= 75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}+135\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+193\int \frac{}{\sqrt{2+3}}\\ &= \frac{135x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}-\frac{135\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}}{\sqrt{2+3x^2+}}\end{aligned}$$

Mathematica [C] time = 0.10, size = 106, normalized size = 0.68

$$\frac{-58i\sqrt{x^2+1}\,\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-135i\sqrt{x^2+1}\,\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+25x\left(x^6+6x^4+11x^2\right.}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*(6 + 11*x^2 + 6*x^4 + x^6) - (135*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elli
pticE[I*ArcSinh[x/Sqrt[2]], 2] - (58*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipti
cF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{125\,x^6+525\,x^4+735\,x^2+343}{\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)/sqrt(x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^3}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

maple [C] time = 0.02, size = 138, normalized size = 0.88

$$25\sqrt{x^4+3x^2+2}\,x^3+75\sqrt{x^4+3x^2+2}\,x-\frac{193i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{135i\sqrt{2}\,\sqrt{2x^2+4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x)
[Out] 25*(x^4+3*x^2+2)^(1/2)*x^3+75*(x^4+3*x^2+2)^(1/2)*x-193/2*I*2^(1/2)*(2*x^2+
4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2
))+135/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Ellip
ticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^3}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2+7)^3}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^3}{\sqrt{(x^2+1)(x^2+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral((5*x**2 + 7)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.301 $\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=142

$$\frac{25}{3}\sqrt{x^4+3x^2+2}\,x+\frac{20\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}+\frac{97\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{20\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] 20*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+97/6*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellip
ticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^
2+2)^(1/2)-20*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2
*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+25/3*x*(x^4+3
*x^2+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1206, 1189, 1099, 1135}

$$\frac{25}{3}\sqrt{x^4+3x^2+2}\,x+\frac{20\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}+\frac{97\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{20\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (20*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (25*x*Sqrt[2 + 3*x^2 + x^4])/3 - (
20*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/S
qrt[2 + 3*x^2 + x^4] + (97*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[Ar
cTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{25}{3}x\sqrt{2+3x^2+x^4} + \frac{1}{3}\int \frac{97+60x^2}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{25}{3}x\sqrt{2+3x^2+x^4} + 20\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{97}{3}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{20x(2+x^2)}{\sqrt{2+3x^2+x^4}} + \frac{25}{3}x\sqrt{2+3x^2+x^4} - \frac{20\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{97(1+x}{3}$$

Mathematica [C] time = 0.09, size = 104, normalized size = 0.73

$$\frac{-37i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 60i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 25x(x^4+3x^2+2)}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*(2 + 3*x^2 + x^4) - (60*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*Ar
cSinh[x/Sqrt[2]], 2] - (37*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSi
nh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{25x^4+70x^2+49}{\sqrt{x^4+3x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)/sqrt(x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^2}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

maple [C] time = 0.01, size = 121, normalized size = 0.85

$$\frac{25\sqrt{x^4+3x^2+2}\,x}{3} - \frac{97i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2}, \sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}} + \frac{10i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(\frac{i}{}\right.\right.}{\sqrt{x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 25/3*(x^4+3*x^2+2)^(1/2)*x-97/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^
4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+10*I*2^(1/2)*(2*x^2+4)^
(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))
-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^2}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.302 $\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=121

$$\frac{5x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{7\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}\,\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] 5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+7/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellipti
cF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+
2)^(1/2)-5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^
(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1189, 1099, 1135}

$$\frac{5x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{7\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}\,\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (7*(1 + x^2)
*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x
^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx = 5\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + 7\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{5x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}} - \frac{5\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{7\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\right.}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.06, size = 69, normalized size = 0.57

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(2F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+5E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(5*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + 2
*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

maple [C] time = 0.00, size = 106, normalized size = 0.88

$$-\frac{7i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\;\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\;\left(-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)+\text{Ellipti}\right.}{2\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 5/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(
1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-7/2*I*2^(1/2)*
(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x
,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+7}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral((5*x**2 + 7)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.303 $\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=48

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] 1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1099}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sq
rt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx = \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 1.04

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[
2 + 3*x^2 + x^4]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{x^4+3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4 + 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

maple [C] time = 0.00, size = 46, normalized size = 0.96

$$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\;\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(
1/2*I*2^(1/2)*x,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4 + 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^4 + 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^2 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 3*x**2 + 2), x)
```

3.304 $\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=106

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+2)\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] -5/28*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/
7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/4*(x^2
+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*
((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1214, 1099, 1456, 539}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+2)\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*
Sqrt[2 + 3*x^2 + x^4]) - (5*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(14*
Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
```

```
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{5}{4}\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{\left(5\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}\,(7+5x^2)}\,dx}{4\sqrt{2+3x^2+x^4}}$$

$$= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{5(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.10, size = 55, normalized size = 0.52

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\Big|2\right)}{7\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
[Out] ((-1/7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]]
, 2])/Sqrt[2 + 3*x^2 + x^4]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+2}}{5x^6+22x^4+31x^2+14}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^6 + 22*x^4 + 31*x^2 + 14), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3x^2+2}\,(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

maple [C] time = 0.01, size = 47, normalized size = 0.44

$$-\frac{i\sqrt{2}\sqrt{\frac{x^2}{2}+1}\sqrt{x^2+1}\ \mathrm{EllipticPi}\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{7\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -1/7*I*2^(1/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
Pi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4 + 3\,x^2 + 2}\,\left(5\,x^2 + 7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(5\,x^2 + 7\right)\,\sqrt{x^4 + 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(x^2 + 1\right)\left(x^2 + 2\right)}\,\left(5x^2 + 7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)), x)
```

3.305 $\int \frac{1}{(7+5x^2)^2 \sqrt{2+3x^2+x^4}} \, dx$

Optimal. Leaf size=209

$$-\frac{25\sqrt{x^4+3x^2+2}\,x}{84\left(5x^2+7\right)}+\frac{5\left(x^2+2\right)x}{84\sqrt{x^4+3x^2+2}}+\frac{9\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{56\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{65}{117}$$

```
[Out] 5/84*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-65/2352*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1
)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1
))^(1/2)/(x^4+3*x^2+2)^(1/2)-5/84*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE
(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+9/112*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*
2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-25/84*x*(x^4+3
*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.19, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1223, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$-\frac{25\sqrt{x^4+3x^2+2}\,x}{84\left(5x^2+7\right)}+\frac{5\left(x^2+2\right)x}{84\sqrt{x^4+3x^2+2}}+\frac{9\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{56\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{65}{117}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (5*x*(2 + x^2))/(84*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4])/(
84*(7 + 5*x^2)) - (5*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(42*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (9*(1 + x^2)*Sqrt[(2 + x^2)/(
1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(56*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (
65*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(1176*Sqrt[2]*Sqrt[(2 + x^2)/
(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
```

```
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}}\,dx = -\frac{25x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)} + \frac{1}{84}\int \frac{62+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)} - \frac{\int \frac{-175-125x^2}{\sqrt{2+3x^2+x^4}}\,dx}{2100} + \frac{13}{84}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)} + \frac{5}{84}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{13}{168}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{5x\left(2+x^2\right)}{84\sqrt{2+3x^2+x^4}} - \frac{25x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)} - \frac{5\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{2+3x^2+x^4}} +$$

$$= \frac{5x\left(2+x^2\right)}{84\sqrt{2+3x^2+x^4}} - \frac{25x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)} - \frac{5\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{2+3x^2+x^4}} +$$

Mathematica [C] time = 0.26, size = 208, normalized size = 1.00

$$\frac{-175x^5-525x^3-14i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-35i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)E\left(i\sinh^{-1}\right.}{588\left(5x^2+7\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (-350*x - 525*x^3 - 175*x^5 - (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2
)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(
7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (91*I)*Sqrt[1 + x^2]*Sqrt[2
 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (65*I)*x^2*Sqrt[1 + x^2
]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(588*(7 + 5*x^2)
*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{25\,x^8+145\,x^6+309\,x^4+287\,x^2+98}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^8 + 145*x^6 + 309*x^4 + 287*x^2 + 98),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

maple [C] time = 0.02, size = 162, normalized size = 0.78

$$-\frac{25\sqrt{x^4+3x^2+2}\,x}{84\left(5x^2+7\right)}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{168\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}}{2}\right.}{84\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -25/84*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)*x-1/84*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-5/168*I*2^(
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1
/2)*x,2^(1/2))-13/588*I*2^(1/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(5\,x^2+7\right)^2\,\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2), x)
```

3.306 $\int \frac{1}{(7+5x^2)^3 \sqrt{2+3x^2+x^4}} \, dx$

Optimal. Leaf size=237

$$-\frac{325\sqrt{x^4+3x^2+2}\,x}{4704\left(5x^2+7\right)}-\frac{25\sqrt{x^4+3x^2+2}\,x}{168\left(5x^2+7\right)^2}+\frac{65\left(x^2+2\right)x}{4704\sqrt{x^4+3x^2+2}}+\frac{631\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{9408\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{65\left(x^2+1\right)}{2352\cdot}$$

```
[Out] 65/4704*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-2525/131712*(x^2+2)*(1/(x^2+1))^(1/2)
*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)
/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-65/4704*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)
*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^
4+3*x^2+2)^(1/2)+631/18816*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2
+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-
25/168*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2-325/4704*x*(x^4+3*x^2+2)^(1/2)/(5*
x^2+7)
```

Rubi [A] time = 0.25, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1223, 1696, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$-\frac{325\sqrt{x^4+3x^2+2}\,x}{4704\left(5x^2+7\right)}-\frac{25\sqrt{x^4+3x^2+2}\,x}{168\left(5x^2+7\right)^2}+\frac{65\left(x^2+2\right)x}{4704\sqrt{x^4+3x^2+2}}+\frac{631\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{9408\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{65\left(x^2+1\right)}{2352\cdot}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (65*x*(2 + x^2))/(4704*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4]
)/(168*(7 + 5*x^2)^2) - (325*x*Sqrt[2 + 3*x^2 + x^4])/(4704*(7 + 5*x^2)) -
(65*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(2352*Sq
rt[2]*Sqrt[2 + 3*x^2 + x^4]) + (631*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(9408*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x
^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(65856*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2
)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
```

```
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
```

```
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}}\,dx = -\frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2} + \frac{1}{168}\int \frac{74-10x^2-25x^4}{\left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2} - \frac{325x\sqrt{2+3x^2+x^4}}{4704\left(7+5x^2\right)} + \frac{\int \frac{2838+2310x^2+975x^4}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx}{14112}$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2} - \frac{325x\sqrt{2+3x^2+x^4}}{4704\left(7+5x^2\right)} - \frac{\int \frac{-4725-4875x^2}{\sqrt{2+3x^2+x^4}}\,dx}{352800} + \frac{505\int \frac{}{(7+5x}}{4}$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2} - \frac{325x\sqrt{2+3x^2+x^4}}{4704\left(7+5x^2\right)} + \frac{3}{224}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx + \frac{65}{}$$

$$= \frac{65x\left(2+x^2\right)}{4704\sqrt{2+3x^2+x^4}} - \frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2} - \frac{325x\sqrt{2+3x^2+x^4}}{4704\left(7+5x^2\right)} - \frac{65\left(1+x^2\right.}{2352}$$

$$= \frac{65x\left(2+x^2\right)}{4704\sqrt{2+3x^2+x^4}} - \frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2} - \frac{325x\sqrt{2+3x^2+x^4}}{4704\left(7+5x^2\right)} - \frac{65\left(1+x^2\right.}{2352}$$

Mathematica [C] time = 0.34, size = 186, normalized size = 0.78

$$\frac{14i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)^2 F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 455i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)^2 E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 50}{32928\left(5x^2+7\right)^2\sqrt{x^4+3.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (-175*x*(238 + 487*x^2 + 314*x^4 + 65*x^6) - (455*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*(7 + 5*x^2)^2*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (14*I)*Sqrt[1 + x^
2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (505*I)
*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticPi[10/7, I*ArcSinh[x/Sqr
t[2]], 2])/(32928*(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+2}}{125x^{10}+900x^8+2560x^6+3598x^4+2499x^2+686}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^10 + 900*x^8 + 2560*x^6 + 3598*x^4 +
2499*x^2 + 686), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

maple [C] time = 0.02, size = 186, normalized size = 0.78

$$-\frac{25\sqrt{x^4+3x^2+2}\,x}{168\left(5x^2+7\right)^2}-\frac{325\sqrt{x^4+3x^2+2}\,x}{4704\left(5x^2+7\right)}-\frac{65i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9408\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x)
[Out] -25/168*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2*x-325/4704*(x^4+3*x^2+2)^(1/2)/(5*x
^2+7)*x+1/4704*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*
EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-65/9408*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-505/32928*I*
2^(1/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*
I*2^(1/2)*x,10/7,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(5\,x^2+7\right)^3\,\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}\,\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3), x)
```

3.307 $\int \frac{\left(7+5x^2\right)^5}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=189

$$\frac{5000}{3}\sqrt{x^4+3x^2+2}\,x+\frac{7679\left(x^2+2\right)x}{2\sqrt{x^4+3x^2+2}}-\frac{\left(179x^2+115\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{15383\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{7679\left(x^2+1\right)}{\sqrt{2}\sqrt{}}$$

```
[Out] 7679/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/2*x*(179*x^2+115)/(x^4+3*x^2+2)^(1/2
)-7679/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1
/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+15383/6*(x^2+1)^(3
/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+
2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+5000/3*x*(x^4+3*x^2+2)^(1/2)+625*x^3*
(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1679, 1189, 1099, 1135}

$$625\sqrt{x^4+3x^2+2}\,x^3+\frac{5000}{3}\sqrt{x^4+3x^2+2}\,x+\frac{7679\left(x^2+2\right)x}{2\sqrt{x^4+3x^2+2}}-\frac{\left(179x^2+115\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{15383\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}\right.}{3\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (7679*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) - (x*(115 + 179*x^2))/(2*Sqrt[
2 + 3*x^2 + x^4]) + (5000*x*Sqrt[2 + 3*x^2 + x^4])/3 + 625*x^3*Sqrt[2 + 3*x
^2 + x^4] - (7679*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (15383*(1 + x^2)*Sqrt[(2 + x^2)/(1
+ x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(7+5x^2\right)^5}{\left(2+3x^2+x^4\right)^{3/2}}\,dx &= -\frac{x\left(115+179x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{-16922-35179x^2-25000x^4-6250x^6}{\sqrt{2+3x^2+x^4}}\,dx\\
&= -\frac{x\left(115+179x^2\right)}{2\sqrt{2+3x^2+x^4}}+625x^3\sqrt{2+3x^2+x^4}-\frac{1}{10}\int \frac{-84610-138395x^2-50000x^4}{\sqrt{2+3x^2+x^4}}\,d\\
&= -\frac{x\left(115+179x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{5000}{3}x\sqrt{2+3x^2+x^4}+625x^3\sqrt{2+3x^2+x^4}-\frac{1}{30}\int \frac{-15383}{\sqrt{2+}}\\
&= -\frac{x\left(115+179x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{5000}{3}x\sqrt{2+3x^2+x^4}+625x^3\sqrt{2+3x^2+x^4}+\frac{7679}{2}\int \frac{}{\sqrt{2+}}\\
&= \frac{7679x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{x\left(115+179x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{5000}{3}x\sqrt{2+3x^2+x^4}+625x^3\sqrt{2+3x^2+x^4}\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3125\,x^{10}+21875\,x^{8}+61250\,x^{6}+85750\,x^{4}+60025\,x^{2}+16807\right)\sqrt{x^{4}+3\,x^{2}+2}}{x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*x^2 + 16807
)*sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)
```

maple [C] time = 0.04, size = 274, normalized size = 1.45

$$625\sqrt{x^4+3x^2+2}\;x^3+\frac{5000\sqrt{x^4+3x^2+2}\;x}{3}-\frac{15383i\sqrt{2}\;\sqrt{2x^2+4}\;\sqrt{x^2+1}\;\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}}-\frac{6250\left(\frac{17}{2}x^3+\right.}{\sqrt{x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -6250*(17/2*x^3+9*x)/(x^4+3*x^2+2)^(1/2)+625*(x^4+3*x^2+2)^(1/2)*x^3+5000/3
*(x^4+3*x^2+2)^(1/2)*x+7679/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+
3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*
x,2^(1/2)))-15383/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(
1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-43750*(-9/2*x^3-5*x)/(x^4+3*x^2+2)^
(1/2)-122500*(5/2*x^3+3*x)/(x^4+3*x^2+2)^(1/2)-171500*(-3/2*x^3-2*x)/(x^4+3
*x^2+2)^(1/2)-120050*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-33614*(-3/4*x^3-5/4*x)
/(x^4+3*x^2+2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(x^4+3\,x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^5}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**5/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

3.308 $\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{625}{3}\sqrt{x^4+3x^2+2}\,x+\frac{637\left(x^2+2\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{\left(113x^2+145\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{1067\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{637\left(x^2+1\right)}{\sqrt{2}\sqrt{}}$$

```
[Out] 637/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(113*x^2+145)/(x^4+3*x^2+2)^(1/2)
-637/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2
))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1067/3*(x^2+1)^(3/2)
*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/
(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/3*x*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1679, 1189, 1099, 1135}

$$\frac{625}{3}\sqrt{x^4+3x^2+2}\,x+\frac{637\left(x^2+2\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{\left(113x^2+145\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{1067\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{637\left(x^2+1\right)}{\sqrt{2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (637*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(145 + 113*x^2))/(2*Sqrt[2
 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/3 - (637*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4
]) + (1067*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x],
 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(7+5x^2\right)^4}{\left(2+3x^2+x^4\right)^{3/2}}\,dx &= \frac{x\left(145+113x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{-2256-3137x^2-1250x^4}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x\left(145+113x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{625}{3}x\sqrt{2+3x^2+x^4}-\frac{1}{6}\int \frac{-4268-1911x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x\left(145+113x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{625}{3}x\sqrt{2+3x^2+x^4}+\frac{637}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{2134}{3}\int \frac{}{\sqrt{2}}\\ &= \frac{637x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{x\left(145+113x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{625}{3}x\sqrt{2+3x^2+x^4}-\frac{637\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E}{\sqrt{2}\sqrt{2+3x^2}}\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(625\,x^8+3500\,x^6+7350\,x^4+6860\,x^2+2401\right)\sqrt{x^4+3\,x^2+2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(x^4 + 3*x^2
 + 2)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^4}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

maple [C] time = 0.01, size = 234, normalized size = 1.38

$$\frac{625\sqrt{x^4+3x^2+2}\,x}{3}-\frac{1067i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{1250\left(-\frac{9}{2}x^3-5x\right)}{\sqrt{x^4+3x^2+2}}+\frac{637i\sqrt{2}\,\sqrt{2x^2+4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -1250*(-9/2*x^3-5*x)/(x^4+3*x^2+2)^(1/2)+625/3*(x^4+3*x^2+2)^(1/2)*x+637/4*
I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*
I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-1067/3*I*2^(1/2)*(
2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,
2^(1/2))-7000*(5/2*x^3+3*x)/(x^4+3*x^2+2)^(1/2)-14700*(-3/2*x^3-2*x)/(x^4+3
*x^2+2)^(1/2)-13720*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-4802*(-3/4*x^3-5/4*x)/(
x^4+3*x^2+2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^4}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+7\right)^4}{\left(x^4+3\,x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^4}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**4/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

3.309 $\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=149

$$\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}}+\frac{261x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{169(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{261(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] 1/2*x*(-11*x^2+5)/(x^4+3*x^2+2)^(1/2)+261/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-2
61/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))
*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+169/2*(x^2+1)^(3/2)*(1
/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^
2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1205, 1189, 1099, 1135}

$$\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}}+\frac{261x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{169(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{261(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(5 - 11*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (261*x*(2 + x^2))/(2*Sqrt[2 +
3*x^2 + x^4]) - (261*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (169*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(7+5x^2\right)^3}{\left(2+3x^2+x^4\right)^{3/2}}\,dx &= \frac{x\left(5-11x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{-338-261x^2}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{x\left(5-11x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{261}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+169\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{x\left(5-11x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{261x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{261\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{169\left(1\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(125\,x^6+525\,x^4+735\,x^2+343\right)\sqrt{x^4+3\,x^2+2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6
*x^6 + 13*x^4 + 12*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

maple [C] time = 0.01, size = 196, normalized size = 1.32

$$-\frac{169i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{250\left(\frac{5}{2}x^3+3x\right)}{\sqrt{x^4+3x^2+2}}+\frac{261i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(-\operatorname{EllipticE}\left(\frac{i}{\ }\right.\right.}{4\sqrt{x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -250*(5/2*x^3+3*x)/(x^4+3*x^2+2)^(1/2)-169/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+261/4*I*2^
(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^
(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-1050*(-3/2*x^3-2*x)/(x
^4+3*x^2+2)^(1/2)-1470*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-686*(-3/4*x^3-5/4*x)
/(x^4+3*x^2+2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(x^4+3\,x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^3}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

3.310 $\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{17x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(17x^2+25\right)}{2\sqrt{x^4+3x^2+2}}+\frac{6\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{17\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] -17/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(17*x^2+25)/(x^4+3*x^2+2)^(1/2)+1
7/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*
2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+6*(x^2+1)^(3/2)*(1/(x^2
+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))
^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1205, 1189, 1099, 1135}

$$-\frac{17x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(17x^2+25\right)}{2\sqrt{x^4+3x^2+2}}+\frac{6\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{17\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (-17*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(25 + 17*x^2))/(2*Sqrt[2 +
 3*x^2 + x^4]) + (17*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x
^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx &= \frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{-24+17x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{17}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+12\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\ &= -\frac{17x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{17(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{6\sqrt{2}\,(1+}{}\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(25x^4+70x^2+49)\sqrt{x^4+3x^2+2}}{x^8+6x^6+13x^4+12x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6*x^6 + 13*x^4
 + 12*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^2}{(x^4+3x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

maple [C] time = 0.01, size = 173, normalized size = 1.16

$$-\frac{6i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{50\left(-\frac{3}{2}x^3-2x\right)}{\sqrt{x^4+3x^2+2}}-\frac{17i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(-\operatorname{EllipticE}\left(\frac{i\sqrt{}}{}\right.\right.}{4\sqrt{x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -50*(-3/2*x^3-2*x)/(x^4+3*x^2+2)^(1/2)-6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-17/4*I*2^(1/2)
*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)
*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-140*(x^3+3/2*x)/(x^4+3*x^2+
2)^(1/2)-98*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(x^4+3\,x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^2}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

3.311 $\int \frac{7+5x^2}{\left(2+3x^2+x^4\right)^{3/2}} dx$

Optimal. Leaf size=145

$$-\frac{x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(x^2+5\right)}{2\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] -1/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(x^2+5)/(x^4+3*x^2+2)^(1/2)+1/2*(x
^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2
)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/2*(x^2+1)^(3/2)*(1/(x^2+1))
^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/
2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1178, 1189, 1099, 1135}

$$-\frac{x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(x^2+5\right)}{2\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + x^2))/(2*Sqrt[2 + 3*x^2
 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(
Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellip
ticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{7+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx &= \frac{x\left(5+x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{-2+x^2}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{x\left(5+x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\
&= -\frac{x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{x\left(5+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{\left(1+x^2\right)\ldots}{\sqrt{2}\ldots}
\end{aligned}
$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)}{x^8+6\,x^6+13\,x^4+12\,x^2+4},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 +
 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{5\,x^2+7}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

maple [C] time = 0.01, size = 150, normalized size = 1.03

$$
-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{10\left(x^3+\frac{3}{2}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\ldots\right.\right.}{4\sqrt{x^4+3x^2+\ldots}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -10*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-1/4*I*2^(1/2)*(
2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x
,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-14*(-3/4*x^3-5/4*x)/(x^4+3*x^
2+2)^(1/2)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^2+7}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^2+7}{\left(x^4+3\,x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+7}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

3.312 $\int \frac{1}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{3x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(3x^2+5\right)}{2\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] -3/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(3*x^2+5)/(x^4+3*x^2+2)^(1/2)+3/2*
(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1
/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-(x^2+1)^(3/2)*(1/(x^2+1))^(
1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)
/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1092, 1189, 1099, 1135}

$$-\frac{3x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(3x^2+5\right)}{2\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (-3*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 3*x^2))/(2*Sqrt[2 + 3*
x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1
/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(2+3x^2+x^4\right)^{3/2}}\,dx = \frac{x\left(5+3x^2\right)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{4+3x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x\left(5+3x^2\right)}{2\sqrt{2+3x^2+x^4}} - \frac{3}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx - 2\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{3x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}} + \frac{x\left(5+3x^2\right)}{2\sqrt{2+3x^2+x^4}} + \frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}\left(1+x^2\right.}{\sqrt{}}$$

Mathematica [C] time = 0.04, size = 99, normalized size = 0.66

$$\frac{3x^3 + i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 3i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 5x}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (5*x + 3*x^3 + (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt
[2]], 2] + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]
)/(2*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

maple [C] time = 0.00, size = 129, normalized size = 0.87

$$\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\ \text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2}, \sqrt{2}\right)}{\sqrt{x^4+3x^2+2}} - \frac{2\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}} - \frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2}, \sqrt{2}\right.\right.}{4\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -2*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)+I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-3/4*I*2^(1/2)*(
2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x
,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^4+3\,x^2+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int(1/(3*x^2 + x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{4}+3x^{2}+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 2)**(-3/2), x)
```

3.313 $\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{x}{6\sqrt{x^4+3x^2+2}}-\frac{9(x^2+1)\sqrt{\frac{x^2+2}{2x^2+2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+3x^2+2}}+\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{125(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\Pi\left(\ldots\right.}{84\sqrt{2}\sqrt{x^4+3x\ldots}}$$

```
[Out] 1/6*x/(x^4+3*x^2+2)^(1/2)+125/168*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticP
i(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*x
^2+2)^(1/2)+1/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1
/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-9/4*(x^2+1)
^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(2
*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 207, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1221, 1178, 1189, 1099, 1135, 1214, 1456, 539}

$$-\frac{x(x^2+2)}{3\sqrt{x^4+3x^2+2}}+\frac{x(2x^2+5)}{6\sqrt{x^4+3x^2+2}}-\frac{9(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{125(\ldots}{84\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 2*x^2))/(6*Sqrt[2 + 3*x^
2 + x^4]) + (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) - (9*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]
*EllipticF[ArcTan[x], 1/2])/(4*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (125*(2 + x
^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(84*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*
Sqrt[2 + 3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx = -\left(\frac{1}{6}\int \frac{-8-5x^2}{(2+3x^2+x^4)^{3/2}}\,dx\right) - \frac{25}{6}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} + \frac{1}{12}\int \frac{-2-4x^2}{\sqrt{2+3x^2+x^4}}\,dx - \frac{25}{12}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx + \frac{12}{24}$$
$$= \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} - \frac{25(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{1}{6}\int \frac{1}{\sqrt{2+3x^2+x^4}}$$
$$= -\frac{x(2+x^2)}{3\sqrt{2+3x^2+x^4}} + \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} + \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}} -$$

Mathematica [C] time = 0.17, size = 138, normalized size = 0.80

$$\frac{14x^3 - 7i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 14i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 25i\sqrt{x^2+1}\sqrt{x^2+2}}{42\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (35*x + 14*x^3 + (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/S
qrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[
2]], 2] + (25*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/S
qrt[2]], 2])/(42*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+2}}{5x^{10}+37x^8+107x^6+151x^4+104x^2+28}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^10 + 37*x^8 + 107*x^6 + 151*x^4 + 104*x
^2 + 28), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

maple [C] time = 0.02, size = 161, normalized size = 0.93

$$\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\ \text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2}, \sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}} - \frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2}, \sqrt{2}\right)}{12\sqrt{x^4+3x^2+2}} + \frac{25i\sqrt{2}\sqrt{\frac{x^2}{2}+1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -2*(-1/6*x^3-5/12*x)/(x^4+3*x^2+2)^(1/2)-1/12*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^
2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/6*I*2^(
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1
/2)*x,2^(1/2))+25/42*I*2^(1/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(5\,x^2+7\right)\,\left(x^4+3\,x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)), x)
```

3.314 $\int \frac{1}{\left(7+5x^2\right)^2\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=235

$$\frac{625\sqrt{x^4+3x^2+2}\,x}{504\left(5x^2+7\right)}-\frac{31\left(x^2+2\right)x}{56\sqrt{x^4+3x^2+2}}+\frac{\left(11x^2+20\right)x}{36\sqrt{x^4+3x^2+2}}-\frac{463\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{336\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{31\left(x^2+1\right)\sqrt{\frac{x^2}{x^2}}}{28\sqrt{2}\sqrt{x^4}}$$

```
[Out] -31/56*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/36*x*(11*x^2+20)/(x^4+3*x^2+2)^(1/2)
+375/1568*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2
),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+31/5
6*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^
(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-463/672*(x^2+1)^(3/2)*(1/
(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2
+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/504*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.43, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1178, 1189, 1099, 1135, 1223, 1716, 1214, 1456, 539}

$$\frac{625\sqrt{x^4+3x^2+2}\,x}{504\left(5x^2+7\right)}-\frac{31\left(x^2+2\right)x}{56\sqrt{x^4+3x^2+2}}+\frac{\left(11x^2+20\right)x}{36\sqrt{x^4+3x^2+2}}-\frac{463\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{336\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{31\left(x^2+1\right)\sqrt{\frac{x^2}{x^2}}}{28\sqrt{2}\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-31*x*(2 + x^2))/(56*Sqrt[2 + 3*x^2 + x^4]) + (x*(20 + 11*x^2))/(36*Sqrt[2
 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(504*(7 + 5*x^2)) + (31*(1
 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(28*Sqrt[2]*Sq
rt[2 + 3*x^2 + x^4]) - (463*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[A
rcTan[x], 1/2])/(336*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (375*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 +
3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
```

```
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^2\left(2+3x^2+x^4\right)^{3/2}}\,dx = \int\left(\frac{14+5x^2}{36\left(2+3x^2+x^4\right)^{3/2}} - \frac{25}{6\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}} - \frac{25}{36\left(7+5x^2\right)\sqrt{2+\right.$$

$$= \frac{1}{36}\int\frac{14+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx - \frac{25}{36}\int\frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx - \frac{25}{6}\int\frac{}{(7}$$

$$= \frac{x\left(20+11x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\left(7+5x^2\right)} - \frac{1}{72}\int\frac{26+22x^2}{\sqrt{2+3x^2+x^4}}\,dx - \frac{25}{504}$$

$$= \frac{x\left(20+11x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\left(7+5x^2\right)} - \frac{25\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right.}{72\sqrt{2}\sqrt{2+3x^2+x^4}}$$

$$= -\frac{11x\left(2+x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{x\left(20+11x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\left(7+5x^2\right)} + \frac{11\left(1+x^2\right.}{18\sqrt{}}$$

$$= -\frac{31x\left(2+x^2\right)}{56\sqrt{2+3x^2+x^4}} + \frac{x\left(20+11x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\left(7+5x^2\right)} + \frac{31\left(1+x^2\right.}{28\sqrt{}}$$

$$= -\frac{31x\left(2+x^2\right)}{56\sqrt{2+3x^2+x^4}} + \frac{x\left(20+11x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\left(7+5x^2\right)} + \frac{31\left(1+x^2\right.}{28\sqrt{}}$$

Mathematica [C] time = 0.28, size = 208, normalized size = 0.89

$$\frac{3255x^5+10157x^3+182i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+651i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)E\left(i\,\mathrm{s}\right.}{1176\left(5x\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (7490*x + 10157*x^3 + 3255*x^5 + (651*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5
*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (182*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (1575*I)*Sqrt[1 + x^2]
*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (1125*I)*x^2*Sqr
t[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(1176*(
7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{25\,x^{12}+220\,x^{10}+794\,x^8+1504\,x^6+1577\,x^4+868\,x^2+196}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^12 + 220*x^10 + 794*x^8 + 1504*x^6 + 1
577*x^4 + 868*x^2 + 196), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

maple [C] time = 0.02, size = 185, normalized size = 0.79

$$\frac{625\sqrt{x^4+3x^2+2}\,x}{504\left(5x^2+7\right)}+\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{112\sqrt{x^4+3x^2+2}}+\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\right.}{168\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -2*(-11/72*x^3-5/18*x)/(x^4+3*x^2+2)^(1/2)+625/504*(x^4+3*x^2+2)^(1/2)/(5*x
^2+7)*x+13/168*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*
EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+31/112*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+75/392*I*2^(1
/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^
(1/2)*x,10/7,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(5\,x^2+7\right)}^2\,{\left(x^4+3\,x^2+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2), x)
```

3.315 $\int \frac{1}{(7+5x^2)^3 (2+3x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=263

$$\frac{41875\sqrt{x^4+3x^2+2}\,x}{84672(5x^2+7)}+\frac{625\sqrt{x^4+3x^2+2}\,x}{1008(5x^2+7)^2}-\frac{5797(x^2+2)x}{28224\sqrt{x^4+3x^2+2}}+\frac{(23x^2+50)x}{216\sqrt{x^4+3x^2+2}}-\frac{49907(x^2+1)\sqrt{\frac{x^2}{x^2}}}{56448\sqrt{2}\sqrt{x^4}}$$

```
[Out] -5797/28224*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/216*x*(23*x^2+50)/(x^4+3*x^2+2)
^(1/2)+192625/790272*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(
x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+5797/28224*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2)
,1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-49907/112
896*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/1008*x*(x^4+3*x^2+2
)^(1/2)/(5*x^2+7)^2+41875/84672*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.76, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1228, 1178, 1189, 1099, 1135, 1223, 1696, 1716, 1214, 1456, 539}

$$\frac{41875\sqrt{x^4+3x^2+2}\,x}{84672(5x^2+7)}+\frac{625\sqrt{x^4+3x^2+2}\,x}{1008(5x^2+7)^2}-\frac{5797(x^2+2)x}{28224\sqrt{x^4+3x^2+2}}+\frac{(23x^2+50)x}{216\sqrt{x^4+3x^2+2}}-\frac{49907(x^2+1)\sqrt{\frac{x^2}{x^2}}}{56448\sqrt{2}\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-5797*x*(2 + x^2))/(28224*Sqrt[2 + 3*x^2 + x^4]) + (x*(50 + 23*x^2))/(216*
Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(1008*(7 + 5*x^2)^2)
 + (41875*x*Sqrt[2 + 3*x^2 + x^4])/(84672*(7 + 5*x^2)) + (5797*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(14112*Sqrt[2]*Sqrt[2 +
3*x^2 + x^4]) - (49907*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan
[x], 1/2])/(56448*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (192625*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(395136*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2
 + 3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^3\left(2+3x^2+x^4\right)^{3/2}}\,dx = \int\left(-\frac{-62-35x^2}{216\left(2+3x^2+x^4\right)^{3/2}} - \frac{25}{6\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}} - \frac{25}{36\left(7+5x^2\right)^2\sqrt{}}\right.$$

$$= -\left(\frac{1}{216}\int \frac{-62-35x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx\right) - \frac{25}{36}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx - \frac{175}{216}$$

$$= \frac{x\left(50+23x^2\right)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} + \frac{625x\sqrt{2+3x^2+x^4}}{3024\left(7+5x^2\right)} + \frac{1}{432}\int$$

$$= \frac{x\left(50+23x^2\right)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} + \frac{41875x\sqrt{2+3x^2+x^4}}{84672\left(7+5x^2\right)} - \frac{175(}{}$$

$$= -\frac{23x\left(2+x^2\right)}{216\sqrt{2+3x^2+x^4}} + \frac{x\left(50+23x^2\right)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} + \frac{41875x}{846}$$

$$= -\frac{149x\left(2+x^2\right)}{1008\sqrt{2+3x^2+x^4}} + \frac{x\left(50+23x^2\right)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} + \frac{41875}{846}$$

$$= -\frac{5797x\left(2+x^2\right)}{28224\sqrt{2+3x^2+x^4}} + \frac{x\left(50+23x^2\right)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} + \frac{4187}{8}$$

$$= -\frac{5797x\left(2+x^2\right)}{28224\sqrt{2+3x^2+x^4}} + \frac{x\left(50+23x^2\right)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} + \frac{4187}{8}$$

Mathematica [C] time = 0.50, size = 159, normalized size = 0.60

$$\frac{-742i\sqrt{x^2+1}\sqrt{x^2+2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 40579i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 38525i\sqrt{x^2+1}\sqrt{x^2}}{197568\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] ((7*x*(550550 + 1089803*x^2 + 698290*x^4 + 144925*x^6))/(7 + 5*x^2)^2 + (40
579*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (74
2*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (3852
5*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])
/(197568*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+2}}{125x^{14}+1275x^{12}+5510x^{10}+13078x^8+18413x^6+15379x^4+7056x^2+1372}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^14 + 1275*x^12 + 5510*x^10 + 13078*x^
8 + 18413*x^6 + 15379*x^4 + 7056*x^2 + 1372), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4 + 3\,x^2 + 2\right)^{\frac{3}{2}}\left(5\,x^2 + 7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

maple [C] time = 0.02, size = 209, normalized size = 0.79

$$\frac{625\sqrt{x^4+3x^2+2}\;x}{1008\left(5x^2+7\right)^2}+\frac{41875\sqrt{x^4+3x^2+2}\;x}{84672\left(5x^2+7\right)}+\frac{5797i\sqrt{2}\;\sqrt{2x^2+4}\;\sqrt{x^2+1}\;\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{56448\sqrt{x^4+3x^2+2}}-\frac{53i\sqrt{2}\;\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -2*(-23/432*x^3-25/216*x)/(x^4+3*x^2+2)^(1/2)+625/1008*(x^4+3*x^2+2)^(1/2)/
(5*x^2+7)^2*x+41875/84672*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)*x-53/28224*I*2^(1/2
)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)
*x,2^(1/2))+5797/56448*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+38525/197568*I*2^(1/2)*(1/2*x^2+
1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,
2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4 + 3\,x^2 + 2\right)^{\frac{3}{2}}\left(5\,x^2 + 7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(5\,x^2 + 7\right)^3\left(x^4 + 3\,x^2 + 2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3), x)
```

3.316 $\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\, dx$

Optimal. Leaf size=116

$$-\frac{116100}{77}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{231}\left(717372x^2+177953\right)\sqrt{-x^4+x^2+2}\,x-\frac{625}{11}\left(-x^4+x^2+2\right)^{3/2}x^5-\frac{14500}{33}\left(-\right.$$

```
[Out] -116100/77*x*(-x^4+x^2+2)^(3/2)-14500/33*x^3*(-x^4+x^2+2)^(3/2)-625/11*x^5*
(-x^4+x^2+2)^(3/2)+3764813/231*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-539419/77
*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/231*x*(717372*x^2+177953)*(-x^4+x^2+2
)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{625}{11}\left(-x^4+x^2+2\right)^{3/2}x^5-\frac{14500}{33}\left(-x^4+x^2+2\right)^{3/2}x^3-\frac{116100}{77}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{231}\left(717372x^2+177953\right.$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(177953 + 717372*x^2)*Sqrt[2 + x^2 - x^4])/231 - (116100*x*(2 + x^2 - x^
4)^(3/2))/77 - (14500*x^3*(2 + x^2 - x^4)^(3/2))/33 - (625*x^5*(2 + x^2 - x
^4)^(3/2))/11 + (3764813*EllipticE[ArcSin[x/Sqrt[2]], -2])/231 - (539419*El
lipticF[ArcSin[x/Sqrt[2]], -2])/77
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\,dx &= -\frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2}-\frac{1}{11}\int \sqrt{2+x^2-x^4}\left(-26411-75460x^2-87100x^4-\right.\\ &= -\frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2}-\frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2}+\frac{1}{99}\int \sqrt{2+x^2-x^4}\left(237\right.\\ &= -\frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2}-\frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2}-\frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2}\\ &= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4}-\frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2}-\frac{14500}{33}x\\ &= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4}-\frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2}-\frac{14500}{33}x\\ &= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4}-\frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2}-\frac{14500}{33}x\\ &= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4}-\frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2}-\frac{14500}{33}x\end{aligned}$$

Mathematica [C] time = 0.13, size = 112, normalized size = 0.97

$$\frac{-13125x^{13}-75250x^{11}-105925x^9+231228x^7+1125819x^5-186503x^3-4838091i\sqrt{-2x^4+2x^2+4}\,F\left(i\sinh^{-1}\right.}{231\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-1037294*x - 186503*x^3 + 1125819*x^5 + 231228*x^7 - 105925*x^9 - 75250*x^
11 - 13125*x^13 + (3764813*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x
], -1/2] - (4838091*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2
])/(231*Sqrt[2 + x^2 - x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(625\,x^{8}+3500\,x^{6}+7350\,x^{4}+6860\,x^{2}+2401\right)\sqrt{-x^{4}+x^{2}+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(-x^4 + x^2
+ 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^{4}+x^{2}+2}\,\left(5\,x^{2}+7\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)
```

maple [A] time = 0.02, size = 193, normalized size = 1.66

$$\frac{625\sqrt{-x^{4}+x^{2}+2}\;x^{9}}{11}+\frac{12625\sqrt{-x^{4}+x^{2}+2}\;x^{7}}{33}+\frac{20050\sqrt{-x^{4}+x^{2}+2}\;x^{5}}{21}+\frac{166072\sqrt{-x^{4}+x^{2}+2}\;x^{3}}{231}-\frac{51864}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 166072/231*x^3*(-x^4+x^2+2)^(1/2)-518647/231*x*(-x^4+x^2+2)^(1/2)+20050/21*
x^5*(-x^4+x^2+2)^(1/2)+12625/33*x^7*(-x^4+x^2+2)^(1/2)+625/11*x^9*(-x^4+x^2
+2)^(1/2)-3764813/462*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(
1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2))
)+1073278/231*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ell
ipticF(1/2*2^(1/2)*x,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^{4}+x^{2}+2}\,\left(5\,x^{2}+7\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(5\,x^2+7\right)}^4\,\sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4*(-x**4+x**2+2)**(1/2),x)
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**4, x)
```

3.317 $\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\, dx$

Optimal. Leaf size=95

$$-\frac{1825}{21}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{63}\left(14691x^2+5956\right)\sqrt{-x^4+x^2+2}\,x-\frac{125}{9}\left(-x^4+x^2+2\right)^{3/2}x^3-\frac{8735}{21}F\left(\sin^{-1}\left(\right.\right.$$

```
[Out] -1825/21*x*(-x^4+x^2+2)^(3/2)-125/9*x^3*(-x^4+x^2+2)^(3/2)+79411/63*Ellipti
cE(1/2*x*2^(1/2),I*2^(1/2))-8735/21*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/63
*x*(14691*x^2+5956)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{125}{9}\left(-x^4+x^2+2\right)^{3/2}x^3-\frac{1825}{21}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{63}\left(14691x^2+5956\right)\sqrt{-x^4+x^2+2}\,x-\frac{8735}{21}F\left(\sin^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(5956 + 14691*x^2)*Sqrt[2 + x^2 - x^4])/63 - (1825*x*(2 + x^2 - x^4)^(3/
2))/21 - (125*x^3*(2 + x^2 - x^4)^(3/2))/9 + (79411*EllipticE[ArcSin[x/Sqrt
[2]], -2])/63 - (8735*EllipticF[ArcSin[x/Sqrt[2]], -2])/21
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx = -\frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2}-\frac{1}{9}\int \left(-3087-7365x^2-5475x^4\right)\sqrt{2+x^2-x^4}\,dx$$
$$= -\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2}-\frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2}+\frac{1}{63}\int \left(32559+73455x^2\right)\sqrt{}$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4}-\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2}-\frac{125}{9}x^3\left(2+x^2-\right.$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4}-\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2}-\frac{125}{9}x^3\left(2+x^2-\right.$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4}-\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2}-\frac{125}{9}x^3\left(2+x^2-\right.$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4}-\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2}-\frac{125}{9}x^3\left(2+x^2-\right.$$

Mathematica [C] time = 0.10, size = 107, normalized size = 1.13

$$\frac{-875x^{11}-3725x^9-1116x^7+21660x^5+9938x^3-106014i\sqrt{-2x^4+2x^2+4}\,F\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)+79411i\sqrt{-2x^4}}{63\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-9988*x + 9938*x^3 + 21660*x^5 - 1116*x^7 - 3725*x^9 - 875*x^11 + (79411*I
)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (106014*I)*Sqrt[4
 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(63*Sqrt[2 + x^2 - x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(125\,x^6+525\,x^4+735\,x^2+343\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(-x^4 + x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+x^2+2}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

maple [B] time = 0.01, size = 176, normalized size = 1.85

$$\frac{125\sqrt{-x^4+x^2+2}\,x^7}{9}+\frac{4600\sqrt{-x^4+x^2+2}\,x^5}{63}+\frac{7466\sqrt{-x^4+x^2+2}\,x^3}{63}-\frac{4994\sqrt{-x^4+x^2+2}\,x}{63}+\frac{26603\sqrt{2}\ \sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x)
[Out] 125/9*(-x^4+x^2+2)^(1/2)*x^7+4600/63*(-x^4+x^2+2)^(1/2)*x^5+7466/63*(-x^4+x
^2+2)^(1/2)*x^3-4994/63*(-x^4+x^2+2)^(1/2)*x+26603/63*2^(1/2)*(-2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-794
11/126*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF
(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+x^2+2}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5x^2+7\right)^3\sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(-x**4+x**2+2)**(1/2),x)
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3, x)
```

3.318 $\int (7+5x^2)^2 \sqrt{2+x^2-x^4}\, dx$

Optimal. Leaf size=74

$$-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{21}x\left(354x^2+275\right)\sqrt{-x^4+x^2+2}-\frac{79}{7}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -25/7*x*(-x^4+x^2+2)^(3/2)+2045/21*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-79/7*
EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/21*x*(354*x^2+275)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1206, 1176, 1180, 524, 424, 419}

$$-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{21}x\left(354x^2+275\right)\sqrt{-x^4+x^2+2}-\frac{79}{7}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(275 + 354*x^2)*Sqrt[2 + x^2 - x^4])/21 - (25*x*(2 + x^2 - x^4)^(3/2))/7
 + (2045*EllipticE[ArcSin[x/Sqrt[2]], -2])/21 - (79*EllipticF[ArcSin[x/Sqrt
[2]], -2])/7
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\, dx = -\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}-\frac{1}{7}\int \left(-393-590x^2\right)\sqrt{2+x^2-x^4}\, dx$$
$$= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{1}{105}\int \frac{9040+10}{\sqrt{2+x^2}}$$
$$= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{2}{105}\int \frac{9040+1}{\sqrt{4-2x^2}}$$
$$= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}-\frac{158}{7}\int \frac{1}{\sqrt{4-2x^2}}$$
$$= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{}}\right.\right.$$

Mathematica [C] time = 0.09, size = 102, normalized size = 1.38

$$\frac{-75x^9-204x^7+304x^5+683x^3-2949i\sqrt{-2x^4+2x^2+4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+2045i\sqrt{-2x^4+2x^2+4}\,E\left(i\,\mathrm{si}\right.}{21\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (250*x + 683*x^3 + 304*x^5 - 204*x^7 - 75*x^9 + (2045*I)*Sqrt[4 + 2*x^2 - 2
*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2949*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Elli
pticF[I*ArcSinh[x], -1/2])/(21*Sqrt[2 + x^2 - x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(25\,x^4+70\,x^2+49\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(-x^4 + x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

maple [B] time = 0.01, size = 159, normalized size = 2.15

$$\frac{25\sqrt{-x^4+x^2+2}\,x^5}{7}+\frac{93\sqrt{-x^4+x^2+2}\,x^3}{7}+\frac{125\sqrt{-x^4+x^2+2}\,x}{21}+\frac{904\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},\right.}{21\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 25/7*(-x^4+x^2+2)^(1/2)*x^5+93/7*(-x^4+x^2+2)^(1/2)*x^3+125/21*(-x^4+x^2+2)
^(1/2)*x+904/21*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*E
llipticF(1/2*2^(1/2)*x,I*2^(1/2))-2045/42*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*
2^(1/2)*x,I*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)^2\,\sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\,\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2, x)
```

3.319 $\int \left(7+5x^2\right)\sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=46

$$x\sqrt{-x^4+x^2+2}\,\left(x^2+2\right)+3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 7*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+3*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+x
*(x^2+2)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1176, 1180, 524, 424, 419}

$$x\sqrt{-x^4+x^2+2}\,\left(x^2+2\right)+3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] x*(2 + x^2)*Sqrt[2 + x^2 - x^4] + 7*EllipticE[ArcSin[x/Sqrt[2]], -2] + 3*El
lipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
```

```
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx &= x(2+x^2)\sqrt{2+x^2-x^4}-\frac{1}{15}\int \frac{-150-105x^2}{\sqrt{2+x^2-x^4}}\,dx\\ &= x(2+x^2)\sqrt{2+x^2-x^4}-\frac{2}{15}\int \frac{-150-105x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\ &= x(2+x^2)\sqrt{2+x^2-x^4}+6\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+7\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\ &= x(2+x^2)\sqrt{2+x^2-x^4}+7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] time = 0.08, size = 94, normalized size = 2.04

$$\frac{-x^7-x^5+4x^3-12i\sqrt{-2x^4+2x^2+4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+7i\sqrt{-2x^4+2x^2+4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+4x}{\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
[Out] (4*x + 4*x^3 - x^5 - x^7 + (7*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSin
h[x], -1/2] - (12*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])
/Sqrt[2 + x^2 - x^4]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^4+x^2+2}\,(5x^2+7),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+x^2+2}\,(5x^2+7)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

maple [B] time = 0.01, size = 141, normalized size = 3.07

$$\sqrt{-x^4+x^2+2}\,x^3+2\sqrt{-x^4+x^2+2}\,x+\frac{5\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{\sqrt{-x^4+x^2+2}}-\frac{7\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(-x^4+x^2+2)^(1/2),x)
```

```
[Out] (-x^4+x^2+2)^(1/2)*x^3+2*(-x^4+x^2+2)^(1/2)*x+5*2^(1/2)*(-2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-7/2*2^(1/
2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)
*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(5\,x^2+7\right)\,\sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\,\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7), x)
```

3.320 $\int \sqrt{2+x^2-x^4}\, dx$

Optimal. Leaf size=44

$$\frac{1}{3}\sqrt{-x^4+x^2+2}\,x+F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 1/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1
/3*x*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1091, 1180, 524, 424, 419}

$$\frac{1}{3}\sqrt{-x^4+x^2+2}\,x+F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/3 + EllipticE[ArcSin[x/Sqrt[2]], -2]/3 + EllipticF[
ArcSin[x/Sqrt[2]], -2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+x^2-x^4}\,dx &= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}\int \frac{4+x^2}{\sqrt{2+x^2-x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{2}{3}\int \frac{4+x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\ &= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+2\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\ &= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] time = 0.05, size = 90, normalized size = 2.05

$$\frac{-x^5+x^3-3i\sqrt{-2x^4+2x^2+4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+i\sqrt{-2x^4+2x^2+4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+2x}{3\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (2*x + x^3 - x^5 + I*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2]
- (3*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(3*Sqrt[2 +
x^2 - x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2), x)
```

maple [B] time = 0.00, size = 125, normalized size = 2.84

$$\frac{\sqrt{-x^4+x^2+2}\;x}{3}+\frac{2\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{3\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\;\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}}{2}\right.\right.}{6\sqrt{-x^4+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2),x)
```

```
[Out] 1/3*(-x^4+x^2+2)^(1/2)*x+2/3*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-1/6*2^(1/2)*(-2*x^2+4)^(1/2)
```

```
*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-Ellip
ticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4 + x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-x^4 + x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(1/2),x)
[Out] int((x^2 - x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4 + x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2),x)
[Out] Integral(sqrt(-x**4 + x**2 + 2), x)
```

3.321 $\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2} dx$

Optimal. Leaf size=46

$$\frac{17}{25}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{5}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -1/5*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+17/25*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))-34/175*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Rubi [A] time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1208, 1180, 524, 424, 419, 1212, 537}

$$\frac{17}{25}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{5}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
```

```
[Out] -EllipticE[ArcSin[x/Sqrt[2]], -2]/5 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])
/25 - (34*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/175
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
```

```
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx &= -\left(\frac{1}{25}\int \frac{-12+5x^2}{\sqrt{2+x^2-x^4}}\,dx\right)-\frac{34}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx\\
&= -\left(\frac{2}{25}\int \frac{-12+5x^2}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\right)-\frac{68}{25}\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}\,\left(7+5x^2\right)}\,dx\\
&= -\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{5}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{34}{25}\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\
&= -\frac{1}{5}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{17}{25}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [C] time = 0.13, size = 51, normalized size = 1.11

$$-\frac{1}{175}i\sqrt{2}\left(7F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+35E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-17\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
```

```
[Out] (-1/175*I)*Sqrt[2]*(35*EllipticE[I*ArcSinh[x], -1/2] + 7*EllipticF[I*ArcSin
h[x], -1/2] - 17*EllipticPi[5/7, I*ArcSinh[x], -1/2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

maple [B] time = 0.02, size = 141, normalized size = 3.07

$$-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticE}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)}{10\sqrt{-x^4+x^2+2}}+\frac{17\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)}{50\sqrt{-x^4+x^2+2}}-\frac{34\sqrt{2}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7),x)
```

```
[Out] 17/50*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1
/2*2^(1/2)*x,I*2^(1/2))-1/10*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))-34/175*2^(1/2)*(1-1/2*x^2)^(
1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7),x)
```

```
[Out] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7), x)
```

3.322 $\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{-x^4+x^2+2}\,x}{14\left(5x^2+7\right)}-\frac{6}{175}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{70}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{99\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450}$$

```
[Out] 1/70*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-6/175*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))+99/2450*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/14*x*(-x^4+x^2+2)
^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1226, 1180, 524, 424, 419, 1212, 537}

$$\frac{\sqrt{-x^4+x^2+2}\,x}{14\left(5x^2+7\right)}-\frac{6}{175}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{70}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{99\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(14*(7 + 5*x^2)) + EllipticE[ArcSin[x/Sqrt[2]], -2]
/70 - (6*EllipticF[ArcSin[x/Sqrt[2]], -2])/175 + (99*EllipticPi[-10/7, ArcS
in[x/Sqrt[2]], -2])/2450
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^2}\,dx = \frac{x\sqrt{2+x^2-x^4}}{14\left(7+5x^2\right)} - \frac{1}{350}\int \frac{7-5x^2}{\sqrt{2+x^2-x^4}}\,dx + \frac{99}{350}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx$$

$$= \frac{x\sqrt{2+x^2-x^4}}{14\left(7+5x^2\right)} - \frac{1}{175}\int \frac{7-5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx + \frac{99}{175}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right.}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{14\left(7+5x^2\right)} + \frac{99\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450} + \frac{1}{70}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx - \frac{12}{175}\int \frac{}{\sqrt{4-2x}}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{14\left(7+5x^2\right)} + \frac{1}{70}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{6}{175}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{99\Pi\left(-\frac{10}{7};\sin\right.}{245}$$

Mathematica [C] time = 0.28, size = 196, normalized size = 2.65

$$\frac{-350x^5+350x^3-21i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+70i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}\,E\left(i\right.}{4900\left(5x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (700*x + 350*x^3 - 350*x^5 + (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (21*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 -
x^4]*EllipticF[I*ArcSinh[x], -1/2] - (693*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*El
lipticPi[5/7, I*ArcSinh[x], -1/2] - (495*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]
*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(4900*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]
)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{25\,x^4+70\,x^2+49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

maple [B] time = 0.02, size = 165, normalized size = 2.23

$$\frac{\sqrt{-x^4+x^2+2}\,x}{70x^2+98}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\;\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{140\sqrt{-x^4+x^2+2}}-\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{}\right.}{175\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x)
```

```
[Out] 1/14*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-3/175*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+1/140*2^(1/2)*(
-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*
2^(1/2))+99/2450*2^(1/2)*(-1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2
)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^2,x)
```

```
[Out] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**2, x)
```

3.323 $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=102

$$-\frac{31\sqrt{-x^4+x^2+2}\,x}{13328(5x^2+7)}+\frac{\sqrt{-x^4+x^2+2}\,x}{28(5x^2+7)^2}-\frac{269F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{166600}-\frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640}+\frac{16601\Pi\left(-\frac{10}{7};\sin^{-1}\left(\right.\right.}{2332400}$$

```
[Out] -31/66640*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-269/166600*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))+16601/2332400*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/2
8*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-31/13328*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.41, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1223, 1696, 1716, 1180, 524, 424, 419, 1212, 537}

$$-\frac{31\sqrt{-x^4+x^2+2}\,x}{13328(5x^2+7)}+\frac{\sqrt{-x^4+x^2+2}\,x}{28(5x^2+7)^2}-\frac{269F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{166600}-\frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640}+\frac{16601\Pi\left(-\frac{10}{7};\sin^{-1}\left(\right.\right.}{2332400}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(28*(7 + 5*x^2)^2) - (31*x*Sqrt[2 + x^2 - x^4])/(13
328*(7 + 5*x^2)) - (31*EllipticE[ArcSin[x/Sqrt[2]], -2])/66640 - (269*Ellip
ticF[ArcSin[x/Sqrt[2]], -2])/166600 + (16601*EllipticPi[-10/7, ArcSin[x/Sqr
t[2]], -2])/2332400
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^3}dx = \int \left(-\frac{34}{25\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}} + \frac{19}{25\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}} - \frac{1}{25\left(7+5x^2\right)\sqrt{2+x^2-\right.}}$$

$$= -\left(\frac{1}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}dx\right) + \frac{19}{25}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}dx - \frac{34}{25}\int \frac{}{(7+5}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{19x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)} - \frac{1}{700}\int \frac{186-190x^2+25x^4}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}dx + \frac{19\int \frac{118-70x^2}{(7+5x^2)\sqrt{2}}}{1190}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} - \frac{1}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{\int \frac{37698-32690x^2-125}{(7+5x^2)\sqrt{2+x^2-x}}}{333200}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} - \frac{1}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{\int \frac{75775+62625x^2}{\sqrt{2+x^2-x^4}}dx}{8330000}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} + \frac{2697\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{83300} + \frac{\int \frac{75775+62625x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}dx}{4165000}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} - \frac{19E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2380} - \frac{19F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5950} + \frac{1}{}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} - \frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640} - \frac{269F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{166600} +$$

Mathematica [C] time = 0.36, size = 244, normalized size = 2.39

$$\frac{54250x^7 - 144900x^5 - 17850x^3 + 7021i\sqrt{2}\left(5x^2+7\right)^2\sqrt{-x^4+x^2+2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) - 2170i\sqrt{2}\left(5x^2+7\right)^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (181300*x - 17850*x^3 - 144900*x^5 + 54250*x^7 - (2170*I)*Sqrt[2]*(7 + 5*x^
2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (7021*I)*Sqrt[2]*(
7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (813449*I)
*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (1162070
*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (
415025*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/
2])/(4664800*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{125\,x^6+525\,x^4+735\,x^2+343}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

maple [A] time = 0.02, size = 189, normalized size = 1.85

$$\frac{\sqrt{-x^4+x^2+2}\;x}{28\left(5x^2+7\right)^2}-\frac{31\sqrt{-x^4+x^2+2}\;x}{13328\left(5x^2+7\right)}-\frac{31\sqrt{2}\;\sqrt{-2x^2+4}\;\sqrt{x^2+1}\;\operatorname{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{133280\sqrt{-x^4+x^2+2}}-\frac{269\sqrt{2}\;\sqrt{-2x^2+4}}{3332}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x)
```

```
[Out] 1/28*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-31/13328*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
*x-269/333200*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ell
ipticF(1/2*2^(1/2)*x,I*2^(1/2))-31/133280*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))+16601/2332400*2
^(1/2)*(-1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2
^(1/2)*x,-10/7,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^3,x)
```

```
[Out] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**3, x)
```

3.324 $\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2}\,dx$

Optimal. Leaf size=142

$$-\frac{132300}{143}\left(-x^4+x^2+2\right)^{5/2}x-\frac{\left(69817-1581440x^2\right)\left(-x^4+x^2+2\right)^{3/2}x}{1001}+\frac{3\left(7837383x^2+2193559\right)\sqrt{-x^4+x^2+}}{5005}$$

```
[Out] -1/1001*x*(-1581440*x^2+69817)*(-x^4+x^2+2)^(3/2)-132300/143*x*(-x^4+x^2+2)
^(5/2)-11750/39*x^3*(-x^4+x^2+2)^(5/2)-125/3*x^5*(-x^4+x^2+2)^(5/2)+1241414
22/5005*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-50794416/5005*EllipticF(1/2*x*2^
(1/2),I*2^(1/2))+3/5005*x*(7837383*x^2+2193559)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{125}{3}\left(-x^4+x^2+2\right)^{5/2}x^5-\frac{11750}{39}\left(-x^4+x^2+2\right)^{5/2}x^3-\frac{132300}{143}\left(-x^4+x^2+2\right)^{5/2}x-\frac{\left(69817-1581440x^2\right)\left(-x^4+\right.}{1001}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (3*x*(2193559 + 7837383*x^2)*Sqrt[2 + x^2 - x^4])/5005 - (x*(69817 - 158144
0*x^2)*(2 + x^2 - x^4)^(3/2))/1001 - (132300*x*(2 + x^2 - x^4)^(5/2))/143 -
 (11750*x^3*(2 + x^2 - x^4)^(5/2))/39 - (125*x^5*(2 + x^2 - x^4)^(5/2))/3 +
 (124141422*EllipticE[ArcSin[x/Sqrt[2]], -2])/5005 - (50794416*EllipticF[Ar
cSin[x/Sqrt[2]], -2])/5005
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2}\,dx &= -\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2}-\frac{1}{15}\int \left(2+x^2-x^4\right)^{3/2}\left(-36015-102900x^2-11\right.\\
&= -\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2}-\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2}+\frac{1}{195}\int \left(2+x^2-x^4\right.\\
&= -\frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2}-\frac{125}{3}x^5\left(2+x^2-x\right.\\
&= -\frac{x\left(69817-1581440x^2\right)\left(2+x^2-x^4\right)^{3/2}}{1001}-\frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{11}{3}\\
&= \frac{3x\left(2193559+7837383x^2\right)\sqrt{2+x^2-x^4}}{5005}-\frac{x\left(69817-1581440x^2\right)\left(2+x\right.}{1001}\\
&= \frac{3x\left(2193559+7837383x^2\right)\sqrt{2+x^2-x^4}}{5005}-\frac{x\left(69817-1581440x^2\right)\left(2+x\right.}{1001}\\
&= \frac{3x\left(2193559+7837383x^2\right)\sqrt{2+x^2-x^4}}{5005}-\frac{x\left(69817-1581440x^2\right)\left(2+x\right.}{1001}\\
&= \frac{3x\left(2193559+7837383x^2\right)\sqrt{2+x^2-x^4}}{5005}-\frac{x\left(69817-1581440x^2\right)\left(2+x\right.}{1001}
\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(625\,x^{12}+2875\,x^{10}+2600\,x^{8}-7490\,x^{6}-19159\,x^{4}-16121\,x^{2}-4802\right)\sqrt{-x^{4}+x^{2}+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(625*x^12 + 2875*x^10 + 2600*x^8 - 7490*x^6 - 19159*x^4 - 16121*x
^2 - 4802)*sqrt(-x^4 + x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

maple [A] time = 0.02, size = 227, normalized size = 1.60

$$-\frac{125\sqrt{-x^{4}+x^{2}+2}\;x^{13}}{3}-\frac{8500\sqrt{-x^{4}+x^{2}+2}\;x^{11}}{39}-\frac{84775\sqrt{-x^{4}+x^{2}+2}\;x^{9}}{429}+\frac{432290\sqrt{-x^{4}+x^{2}+2}\;x^{7}}{429}+\frac{833561}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 43271392/15015*(-x^4+x^2+2)^(1/2)*x^3-12639493/5005*(-x^4+x^2+2)^(1/2)*x+83
3561/273*(-x^4+x^2+2)^(1/2)*x^5+432290/429*(-x^4+x^2+2)^(1/2)*x^7-84775/429
*(-x^4+x^2+2)^(1/2)*x^9-8500/39*x^11*(-x^4+x^2+2)^(1/2)-125/3*x^13*(-x^4+x^
2+2)^(1/2)-62070711/5005*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2
)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/
2)))+36673503/5005*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2
)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(5\,x^{2}+7\right)^{4}\,\left(-x^{4}+x^{2}+2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4*(-x**4+x**2+2)**(3/2),x)
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**4, x)
```

3.325 $\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} dx$

Optimal. Leaf size=121

$$-\frac{7825}{143}\left(-x^4+x^2+2\right)^{5/2}x+\frac{\left(374045x^2+33792\right)\left(-x^4+x^2+2\right)^{3/2}x}{3003}+\frac{\left(5712051x^2+2512273\right)\sqrt{-x^4+x^2+2}\,x}{15015}-$$

```
[Out] 1/3003*x*(374045*x^2+33792)*(-x^4+x^2+2)^(3/2)-7825/143*x*(-x^4+x^2+2)^(5/2
)-125/13*x^3*(-x^4+x^2+2)^(5/2)+31072528/15015*EllipticE(1/2*x*2^(1/2),I*2^
(1/2))-3199778/5005*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/15015*x*(5712051*x
^2+2512273)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{125}{13}\left(-x^4+x^2+2\right)^{5/2}x^3-\frac{7825}{143}\left(-x^4+x^2+2\right)^{5/2}x+\frac{\left(374045x^2+33792\right)\left(-x^4+x^2+2\right)^{3/2}x}{3003}+\frac{\left(5712051x^2+25\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(2512273 + 5712051*x^2)*Sqrt[2 + x^2 - x^4])/15015 + (x*(33792 + 374045*
x^2)*(2 + x^2 - x^4)^(3/2))/3003 - (7825*x*(2 + x^2 - x^4)^(5/2))/143 - (12
5*x^3*(2 + x^2 - x^4)^(5/2))/13 + (31072528*EllipticE[ArcSin[x/Sqrt[2]], -2
])/15015 - (3199778*EllipticF[ArcSin[x/Sqrt[2]], -2])/5005
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(7+5x^2\right)^3\left(2+x^2-x^4\right)^{3/2}\,dx &= -\frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2}-\frac{1}{13}\int\left(-4459-10305x^2-7825x^4\right)\left(2+x^2-x^4\right.\\
&= -\frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2}+\frac{1}{143}\int\left(64699+1603\right.\\
&= \frac{x\left(33792+374045x^2\right)\left(2+x^2-x^4\right)^{3/2}}{3003}-\frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{125}{13}x^3\\
&= \frac{x\left(2512273+5712051x^2\right)\sqrt{2+x^2-x^4}}{15015}+\frac{x\left(33792+374045x^2\right)\left(2+x^2\right.}{3003}\\
&= \frac{x\left(2512273+5712051x^2\right)\sqrt{2+x^2-x^4}}{15015}+\frac{x\left(33792+374045x^2\right)\left(2+x^2\right.}{3003}\\
&= \frac{x\left(2512273+5712051x^2\right)\sqrt{2+x^2-x^4}}{15015}+\frac{x\left(33792+374045x^2\right)\left(2+x^2\right.}{3003}\\
&= \frac{x\left(2512273+5712051x^2\right)\sqrt{2+x^2-x^4}}{15015}+\frac{x\left(33792+374045x^2\right)\left(2+x^2\right.}{3003}
\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

[Out] $Aborted

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(125\,x^{10}+400\,x^{8}-40\,x^{6}-1442\,x^{4}-1813\,x^{2}-686\right)\sqrt{-x^{4}+x^{2}+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(125*x^10 + 400*x^8 - 40*x^6 - 1442*x^4 - 1813*x^2 - 686)*sqrt(-x
^4 + x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

maple [A] time = 0.01, size = 210, normalized size = 1.74

$$-\frac{125\sqrt{-x^{4}+x^{2}+2}\;x^{11}}{13}-\frac{5075\sqrt{-x^{4}+x^{2}+2}\;x^{9}}{143}+\frac{5890\sqrt{-x^{4}+x^{2}+2}\;x^{7}}{429}+\frac{65248\sqrt{-x^{4}+x^{2}+2}\;x^{5}}{273}+\frac{5757461\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 5757461/15015*(-x^4+x^2+2)^(1/2)*x^3-436307/15015*(-x^4+x^2+2)^(1/2)*x+6524
8/273*(-x^4+x^2+2)^(1/2)*x^5+5890/429*(-x^4+x^2+2)^(1/2)*x^7-5075/143*(-x^4
+x^2+2)^(1/2)*x^9-125/13*(-x^4+x^2+2)^(1/2)*x^11-15536264/15015*2^(1/2)*(-2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2
^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+10736597/15015*2^(1/2)*(-2*x^2+
4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(5\,x^2+7\right)}^3\,{\left(-x^4+x^2+2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(-x**4+x**2+2)**(3/2),x)
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3, x)
```

3.326 $\int \left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2} \, dx$

Optimal. Leaf size=100

$$-\frac{25}{11}x\left(-x^4+x^2+2\right)^{5/2}+\frac{1}{99}x\left(920x^2+363\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{495}x\left(14889x^2+11497\right)\sqrt{-x^4+x^2+2}-\frac{3392}{165}F$$

```
[Out] 1/99*x*(920*x^2+363)*(-x^4+x^2+2)^(3/2)-25/11*x*(-x^4+x^2+2)^(5/2)+85942/49
5*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-3392/165*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))+1/495*x*(14889*x^2+11497)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1206, 1176, 1180, 524, 424, 419}

$$-\frac{25}{11}x\left(-x^4+x^2+2\right)^{5/2}+\frac{1}{99}x\left(920x^2+363\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{495}x\left(14889x^2+11497\right)\sqrt{-x^4+x^2+2}-\frac{3392}{165}F$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(11497 + 14889*x^2)*Sqrt[2 + x^2 - x^4])/495 + (x*(363 + 920*x^2)*(2 + x
^2 - x^4)^(3/2))/99 - (25*x*(2 + x^2 - x^4)^(5/2))/11 + (85942*EllipticE[Ar
cSin[x/Sqrt[2]], -2])/495 - (3392*EllipticF[ArcSin[x/Sqrt[2]], -2])/165
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2}\,dx = -\frac{25}{11}x\left(2+x^2-x^4\right)^{5/2} - \frac{1}{11}\int \left(-589-920x^2\right)\left(2+x^2-x^4\right)^{3/2}\,dx$$

$$= \frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2} - \frac{25}{11}x\left(2+x^2-x^4\right)^{5/2} + \frac{1}{231}\int \left(2304\right.$$

$$= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)$$

$$= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)$$

$$= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)$$

$$= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(25\,x^8+45\,x^6-71\,x^4-189\,x^2-98\right)\sqrt{-x^4+x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral(-(25*x^8 + 45*x^6 - 71*x^4 - 189*x^2 - 98)*sqrt(-x^4 + x^2 + 2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

maple [B] time = 0.01, size = 193, normalized size = 1.93

$$-\frac{25\sqrt{-x^4+x^2+2}\,x^9}{11}-\frac{470\sqrt{-x^4+x^2+2}\,x^7}{99}+\frac{112\sqrt{-x^4+x^2+2}\,x^5}{9}+\frac{21404\sqrt{-x^4+x^2+2}\,x^3}{495}+\frac{10627\sqrt{-x^4+x}}{495}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] -25/11*(-x^4+x^2+2)^(1/2)*x^9-470/99*(-x^4+x^2+2)^(1/2)*x^7+112/9*(-x^4+x^2
+2)^(1/2)*x^5+21404/495*(-x^4+x^2+2)^(1/2)*x^3+10627/495*(-x^4+x^2+2)^(1/2)
*x+37883/495*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elli
pticF(1/2*2^(1/2)*x,I*2^(1/2))-42971/495*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2
^(1/2)*x,I*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)^2\,\left(-x^4+x^2+2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2, x)
```

3.327 $\int \left(7+5x^2\right)\left(2+x^2-x^4\right)^{3/2} dx$

Optimal. Leaf size=81

$$\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{315}x\left(669x^2+1087\right)\sqrt{-x^4+x^2+2}+\frac{418}{105}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{4432}{315}E\left(\right.$$

```
[Out] 1/63*x*(35*x^2+48)*(-x^4+x^2+2)^(3/2)+4432/315*EllipticE(1/2*x*2^(1/2),I*2^
(1/2))+418/105*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/315*x*(669*x^2+1087)*(-
x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1176, 1180, 524, 424, 419}

$$\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{315}x\left(669x^2+1087\right)\sqrt{-x^4+x^2+2}+\frac{418}{105}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{4432}{315}E\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(1087 + 669*x^2)*Sqrt[2 + x^2 - x^4])/315 + (x*(48 + 35*x^2)*(2 + x^2 -
x^4)^(3/2))/63 + (4432*EllipticE[ArcSin[x/Sqrt[2]], -2])/315 + (418*Ellipti
cF[ArcSin[x/Sqrt[2]], -2])/105
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \left(7+5x^2\right)\left(2+x^2-x^4\right)^{3/2}\,dx = \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}-\frac{1}{21}\int \left(-262-223x^2\right)\sqrt{2+x^2-x^4}\,dx$$
$$= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}+\frac{1}{315}$$
$$= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}+\frac{2}{315}$$
$$= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}+\frac{836}{105}$$
$$= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}+\frac{4432}{315}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(5\,x^6+2\,x^4-17\,x^2-14\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral(-(5*x^6 + 2*x^4 - 17*x^2 - 14)*sqrt(-x^4 + x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

maple [B] time = 0.01, size = 176, normalized size = 2.17

$$-\frac{5\sqrt{-x^4+x^2+2}\,x^7}{9}-\frac{13\sqrt{-x^4+x^2+2}\,x^5}{63}+\frac{1259\sqrt{-x^4+x^2+2}\,x^3}{315}+\frac{1567\sqrt{-x^4+x^2+2}\,x}{315}+\frac{2843\sqrt{2}\,\sqrt{-2x^2+4}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] -5/9*(-x^4+x^2+2)^(1/2)*x^7-13/63*(-x^4+x^2+2)^(1/2)*x^5+1259/315*(-x^4+x^2
+2)^(1/2)*x^3+1567/315*(-x^4+x^2+2)^(1/2)*x+2843/315*2^(1/2)*(-2*x^2+4)^(1/
2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-2216
/315*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1
/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)\,{\left(-x^4+x^2+2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7), x)
```

3.328 $\int \left(2+x^2-x^4\right)^{3/2} \, dx$

Optimal. Leaf size=74

$$\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{35}x\left(3x^2+19\right)\sqrt{-x^4+x^2+2}+\frac{48}{35}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 1/7*x*(-x^4+x^2+2)^(3/2)+34/35*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+48/35*Ell
ipticF(1/2*x*2^(1/2),I*2^(1/2))+1/35*x*(3*x^2+19)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1091, 1176, 1180, 524, 424, 419}

$$\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{35}x\left(3x^2+19\right)\sqrt{-x^4+x^2+2}+\frac{48}{35}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(19 + 3*x^2)*Sqrt[2 + x^2 - x^4])/35 + (x*(2 + x^2 - x^4)^(3/2))/7 + (34
*EllipticE[ArcSin[x/Sqrt[2]], -2])/35 + (48*EllipticF[ArcSin[x/Sqrt[2]], -2
])/35
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
```

```
b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(2+x^2-x^4\right)^{3/2} dx &= \frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{3}{7}\int \left(4+x^2\right)\sqrt{2+x^2-x^4}\,dx\\ &= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}-\frac{1}{35}\int \frac{-82-34x^2}{\sqrt{2+x^2-x^4}}\,dx\\ &= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}-\frac{2}{35}\int \frac{-82-34x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\ &= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{34}{35}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{96}{35}\int \frac{}{\sqrt{}}\\ &= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{48}{35}F\end{aligned}$$

Mathematica [C] time = 0.05, size = 102, normalized size = 1.38

$$\frac{5x^9-13x^7-31x^5+45x^3-75i\sqrt{-2x^4+2x^2+4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+34i\sqrt{-2x^4+2x^2+4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{35\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (58*x + 45*x^3 - 31*x^5 - 13*x^7 + 5*x^9 + (34*I)*Sqrt[4 + 2*x^2 - 2*x^4]*E
llipticE[I*ArcSinh[x], -1/2] - (75*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*A
rcSinh[x], -1/2])/(35*Sqrt[2 + x^2 - x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-x^4+x^2+2\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

maple [B] time = 0.00, size = 159, normalized size = 2.15

$$-\frac{\sqrt{-x^4+x^2+2}\,x^5}{7}+\frac{8\sqrt{-x^4+x^2+2}\,x^3}{35}+\frac{29\sqrt{-x^4+x^2+2}\,x}{35}+\frac{41\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{35\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2),x)
```

```
[Out] -1/7*(-x^4+x^2+2)^(1/2)*x^5+8/35*(-x^4+x^2+2)^(1/2)*x^3+29/35*(-x^4+x^2+2)^
(1/2)*x+41/35*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ell
ipticF(1/2*2^(1/2)*x,I*2^(1/2))-17/35*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2
)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1
/2)*x,I*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-x^4+x^2+2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(-x^4+x^2+2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((x^2 - x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-x^4+x^2+2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**4 + x**2 + 2)**(3/2), x)
```

3.329 $\int \frac{\left(2+x^2-x^4\right)^{3/2}}{7+5x^2} dx$

Optimal. Leaf size=72

$$\frac{1}{75}x\sqrt{-x^4+x^2+2}\left(13-3x^2\right)-\frac{178}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1156\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{4375}$$

```
[Out] 92/375*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-178/625*EllipticF(1/2*x*2^(1/2),I
*2^(1/2))+1156/4375*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/75*x*(-3*x^
2+13)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1208, 1176, 1180, 524, 424, 419, 1212, 537}

$$\frac{1}{75}x\sqrt{-x^4+x^2+2}\left(13-3x^2\right)-\frac{178}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1156\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (x*(13 - 3*x^2)*Sqrt[2 + x^2 - x^4])/75 + (92*EllipticE[ArcSin[x/Sqrt[2]],
-2])/375 - (178*EllipticF[ArcSin[x/Sqrt[2]], -2])/625 + (1156*EllipticPi[-1
0/7, ArcSin[x/Sqrt[2]], -2])/4375
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+x^2-x^4\right)^{3/2}}{7+5x^2}\,dx &= -\left(\frac{1}{25}\int \left(-12+5x^2\right)\sqrt{2+x^2-x^4}\,dx\right)-\frac{34}{25}\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx\\
&= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{375}\int \frac{230-10x^2}{\sqrt{2+x^2-x^4}}\,dx+\frac{34}{625}\int \frac{-12+5x^2}{\sqrt{2+x^2-x^4}}\,dx+\\
&= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{2}{375}\int \frac{230-10x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{68}{625}\int \frac{-12+5x^2}{\sqrt{4-2x^2}\sqrt{2+}}\\
&= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{1156\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375}-\frac{2}{75}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\\
&= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{178}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+
\end{aligned}$$

Mathematica [C] time = 0.20, size = 130, normalized size = 1.81

$$\frac{525x^7-2800x^5+1225x^3-2961i\sqrt{-2x^4+2x^2+4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+3220i\sqrt{-2x^4+2x^2+4}\,E\left(i\sinh^{-1}(x)\middle|-\right.}{13125\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (4550*x + 1225*x^3 - 2800*x^5 + 525*x^7 + (3220*I)*Sqrt[4 + 2*x^2 - 2*x^4]*
EllipticE[I*ArcSinh[x], -1/2] - (2961*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[
I*ArcSinh[x], -1/2] - (1734*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticPi[5/7, I*Ar
cSinh[x], -1/2])/(13125*Sqrt[2 + x^2 - x^4])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{5\,x^2+7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

maple [B] time = 0.02, size = 173, normalized size = 2.40

$$-\frac{\sqrt{-x^4+x^2+2}\,x^3}{25}+\frac{13\sqrt{-x^4+x^2+2}\,x}{75}+\frac{46\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)}{375\sqrt{-x^4+x^2+2}}-\frac{89\sqrt{2}\,\sqrt{-2x^2+}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7),x)
```

```
[Out] -1/25*(-x^4+x^2+2)^(1/2)*x^3+13/75*(-x^4+x^2+2)^(1/2)*x-89/625*2^(1/2)*(-2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(
1/2))+46/375*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elli
pticE(1/2*2^(1/2)*x,I*2^(1/2))+1156/4375*2^(1/2)*(-1/2*x^2+1)^(1/2)*(x^2+1)
^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(-x^4+x^2+2\right)}^{3/2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7),x)
```

```
[Out] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7), x)
```

3.330 $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{17\sqrt{-x^4+x^2+2}\,x}{175\,(5x^2+7)}-\frac{1}{75}\sqrt{-x^4+x^2+2}\,x+\frac{458}{875}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1241\Pi\left(-\frac{10}{7};\right.}{}$$

```
[Out] -97/525*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+458/875*EllipticF(1/2*x*2^(1/2),
I*2^(1/2))-1241/6125*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-1/75*x*(-x^4
+x^2+2)^(1/2)-17/175*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.32, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {1228, 1095, 419, 1132, 493, 424, 1122, 1180, 1223, 1716, 524, 1212, 537}

$$-\frac{17\sqrt{-x^4+x^2+2}\,x}{175\,(5x^2+7)}-\frac{1}{75}\sqrt{-x^4+x^2+2}\,x+\frac{458}{875}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1241\Pi\left(-\frac{10}{7};\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] -(x*Sqrt[2 + x^2 - x^4])/75 - (17*x*Sqrt[2 + x^2 - x^4])/(175*(7 + 5*x^2))
- (97*EllipticE[ArcSin[x/Sqrt[2]], -2])/525 + (458*EllipticF[ArcSin[x/Sqrt[
2]], -2])/875 - (1241*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/6125
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1132

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt[
-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] &
& LtQ[c, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
```

```
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx = \int \left(\frac{212}{625\sqrt{2+x^2-x^4}} - \frac{24x^2}{125\sqrt{2+x^2-x^4}} + \frac{x^4}{25\sqrt{2+x^2-x^4}} + \frac{1156}{625\left(7+5x^2\right)^2\sqrt{2+}}\right.$$

$$= \frac{1}{25}\int \frac{x^4}{\sqrt{2+x^2-x^4}}\,dx - \frac{24}{125}\int \frac{x^2}{\sqrt{2+x^2-x^4}}\,dx + \frac{212}{625}\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx + \frac{1156}{625}$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} + \frac{17\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{4375} + \frac{1}{75}\int \frac{2+2x^2}{\sqrt{2+x^2-}}$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} + \frac{212}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{1292\Pi\left(-\frac{10}{7};\text{s}\right.}{43}$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} - \frac{62}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{332}{625}F\left(\sin^{-1}\left(\right.\right.$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} - \frac{62}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{332}{625}F\left(\sin^{-1}\left(\right.\right.$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} - \frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{458}{875}F\left(\sin^{-1}\left(\right.\right.$$

Mathematica [C] time = 0.31, size = 201, normalized size = 2.16

$$\frac{2450x^7+4550x^5-11900x^3+567i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-6790i\sqrt{2}\left(5x^2+7\right)\sqrt{-}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (-14000*x - 11900*x^3 + 4550*x^5 + 2450*x^7 - (6790*I)*Sqrt[2]*(7 + 5*x^2)*
Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (567*I)*Sqrt[2]*(7 + 5*
x^2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (26061*I)*Sqrt[2]*
Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (18615*I)*Sqrt[2]
*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(36750*(7 + 5
*x^2)*Sqrt[2 + x^2 - x^4])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{25\,x^4+70\,x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

maple [B] time = 0.02, size = 180, normalized size = 1.94

$$-\frac{17\sqrt{-x^4+x^2+2}\;x}{175\left(5x^2+7\right)}-\frac{\sqrt{-x^4+x^2+2}\;x}{75}-\frac{97\sqrt{2}\;\sqrt{-2x^2+4}\;\sqrt{x^2+1}\;\text{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{1050\sqrt{-x^4+x^2+2}}+\frac{229\sqrt{2}\;\sqrt{-2x^2+4}}{875}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x)
[Out] -17/175*(-x^4+x^2+2)^(1/2)/(5*x^2+7)*x-1/75*(-x^4+x^2+2)^(1/2)*x+229/875*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/
2)*x,I*2^(1/2))-97/1050*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))-1241/6125*2^(1/2)*(-1/2*x^2+1)^(1
/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/
2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(-x^4+x^2+2\right)^{3/2}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^2,x)
[Out] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**2,x)
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**2, x)
```

3.331 $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=102

$$\frac{563\sqrt{-x^4+x^2+2}\,x}{9800(5x^2+7)}-\frac{17\sqrt{-x^4+x^2+2}\,x}{350(5x^2+7)^2}-\frac{1251F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24500}+\frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}+\frac{9879\Pi\left(-\frac{10}{7};\sin^{-1}\ldots\right)}{34300\ldots}$$

```
[Out] 191/9800*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1251/24500*EllipticF(1/2*x*2^(1
/2),I*2^(1/2))+9879/343000*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-17/350
*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+563/9800*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.50, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {1228, 1095, 419, 1132, 493, 424, 1223, 1696, 1716, 1180, 524, 1212, 537}

$$\frac{563\sqrt{-x^4+x^2+2}\,x}{9800(5x^2+7)}-\frac{17\sqrt{-x^4+x^2+2}\,x}{350(5x^2+7)^2}-\frac{1251F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24500}+\frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}+\frac{9879\Pi\left(-\frac{10}{7};\sin^{-1}\ldots\right)}{34300\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (-17*x*Sqrt[2 + x^2 - x^4])/(350*(7 + 5*x^2)^2) + (563*x*Sqrt[2 + x^2 - x^4
])/(9800*(7 + 5*x^2)) + (191*EllipticE[ArcSin[x/Sqrt[2]], -2])/9800 - (1251
*EllipticF[ArcSin[x/Sqrt[2]], -2])/24500 + (9879*EllipticPi[-10/7, ArcSin[x
/Sqrt[2]], -2])/343000
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1132

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt[
-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] &
& LtQ[c, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
```

```
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5x^2\right)^3}dx = \int \left(-\frac{31}{625\sqrt{2+x^2-x^4}} + \frac{x^2}{125\sqrt{2+x^2-x^4}} + \frac{1156}{625\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}} - \frac{}{625\left(7+5\right.}\right.$$

$$= \frac{1}{125}\int \frac{x^2}{\sqrt{2+x^2-x^4}}dx - \frac{31}{625}\int \frac{1}{\sqrt{2+x^2-x^4}}dx + \frac{429}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}dx$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{19x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} + \frac{17\int \frac{186-190x^2+25x^4}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}dx}{8750} - \frac{19\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x}}}{4375}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} - \frac{31}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{429\Pi\left(-\frac{10}{7};\sin\right.}{4375}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{1}{125}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{36}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{}}\right.\right.$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{1}{125}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{36}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{}}\right.\right.$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{26}{875}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{214F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.}{4375}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800} - \frac{1251F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.}{24500}$$

Mathematica [C] time = 0.41, size = 244, normalized size = 2.39

$$-197050x^7 - 45500x^5 + 636650x^3 - 2541i\sqrt{2}\left(5x^2+7\right)^2\sqrt{-x^4+x^2+2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) + 13370i\sqrt{2}\left(5x^2+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (485100*x + 636650*x^3 - 45500*x^5 - 197050*x^7 + (13370*I)*Sqrt[2]*(7 + 5*
x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2541*I)*Sqrt[2]
*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (484071*
I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (69153
0*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] -
(246975*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1
/2])/(686000*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{125\,x^6+525\,x^4+735\,x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

maple [A] time = 0.02, size = 189, normalized size = 1.85

$$-\frac{17\sqrt{-x^4+x^2+2}\,x}{350\left(5x^2+7\right)^2}+\frac{563\sqrt{-x^4+x^2+2}\,x}{9800\left(5x^2+7\right)}+\frac{191\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{19600\sqrt{-x^4+x^2+2}}-\frac{1251\sqrt{2}\,\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x)
```

```
[Out] -17/350*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2*x+563/9800*(-x^4+x^2+2)^(1/2)/(5*x^2
+7)*x-1251/49000*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*
EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+191/19600*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))+9879/343000*
2^(1/2)*(-1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*
2^(1/2)*x,-10/7,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(-x^4+x^2+2\right)^{3/2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^3,x)
[Out] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**3,x)
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**3, x)
```

3.332 $\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=65

$$-\frac{625}{3}\sqrt{-x^4+x^2+2}\,x-25\sqrt{-x^4+x^2+2}\,x^3-542F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 3905/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-542*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))-625/3*x*(-x^4+x^2+2)^(1/2)-25*x^3*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1206, 1679, 1180, 524, 424, 419}

$$-25\sqrt{-x^4+x^2+2}\,x^3-\frac{625}{3}\sqrt{-x^4+x^2+2}\,x-542F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-625*x*Sqrt[2 + x^2 - x^4])/3 - 25*x^3*Sqrt[2 + x^2 - x^4] + (3905*Ellipti
cE[ArcSin[x/Sqrt[2]], -2])/3 - 542*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
```

```
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(7+5x^2\right)^3}{\sqrt{2+x^2-x^4}}\,dx &= -25x^3\sqrt{2+x^2-x^4}-\frac{1}{5}\int \frac{-1715-4425x^2-3125x^4}{\sqrt{2+x^2-x^4}}\,dx\\
&= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{1}{15}\int \frac{11395+19525x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{2}{15}\int \frac{11395+19525x^2}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\
&= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}-1084\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx+\frac{3905}{3}\int \frac{\sqrt{2}}{\sqrt{4}}\\
&= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-542F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.
\end{aligned}$$

Mathematica [C] time = 0.11, size = 97, normalized size = 1.49

$$\frac{150x^7+1100x^5-1550x^3-10089i\sqrt{-2x^4+2x^2+4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+7810i\sqrt{-2x^4+2x^2+4}\,E\left(i\sinh^{-1}(x)\right|}{6\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-2500*x - 1550*x^3 + 1100*x^5 + 150*x^7 + (7810*I)*Sqrt[4 + 2*x^2 - 2*x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (10089*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Elliptic
F[I*ArcSinh[x], -1/2])/(6*Sqrt[2 + x^2 - x^4])
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(125\,x^6+525\,x^4+735\,x^2+343\right)\sqrt{-x^4+x^2+2}}{x^4-x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(-x^4 + x^2 + 2)/(x^4 - x
^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

maple [B] time = 0.02, size = 142, normalized size = 2.18

$$-25\sqrt{-x^4+x^2+2}\,x^3-\frac{625\sqrt{-x^4+x^2+2}\,x}{3}+\frac{2279\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{3905\sqrt{2}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -25*(-x^4+x^2+2)^(1/2)*x^3-625/3*(-x^4+x^2+2)^(1/2)*x+2279/6*2^(1/2)*(-2*x^
2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/
2))-3905/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(Ellip
ticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(5\,x^2+7\right)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^3}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

3.333 $\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=46

$$-\frac{25}{3}\sqrt{-x^4+x^2+2}\,x-21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 260/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-21*EllipticF(1/2*x*2^(1/2),I*2^(1/
2))-25/3*x*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1180, 524, 424, 419}

$$-\frac{25}{3}\sqrt{-x^4+x^2+2}\,x-21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/3 + (260*EllipticE[ArcSin[x/Sqrt[2]], -2])/3 -
21*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
```

```
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(7+5x^2\right)^2}{\sqrt{2+x^2-x^4}}\,dx &= -\frac{25}{3}x\sqrt{2+x^2-x^4}-\frac{1}{3}\int \frac{-197-260x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= -\frac{25}{3}x\sqrt{2+x^2-x^4}-\frac{2}{3}\int \frac{-197-260x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= -\frac{25}{3}x\sqrt{2+x^2-x^4}-42\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{260}{3}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&= -\frac{25}{3}x\sqrt{2+x^2-x^4}+\frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] time = 0.10, size = 92, normalized size = 2.00

$$\frac{50x^5-50x^3-717i\sqrt{-2x^4+2x^2+4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+520i\sqrt{-2x^4+2x^2+4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-100x}{6\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-100*x - 50*x^3 + 50*x^5 + (520*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*Arc
Sinh[x], -1/2] - (717*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1
/2])/(6*Sqrt[2 + x^2 - x^4])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(25\,x^4+70\,x^2+49\right)\sqrt{-x^4+x^2+2}}{x^4-x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(25*x^4 + 70*x^2 + 49)*sqrt(-x^4 + x^2 + 2)/(x^4 - x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

maple [B] time = 0.01, size = 125, normalized size = 2.72

$$-\frac{25\sqrt{-x^4+x^2+2}\;x}{3}+\frac{197\sqrt{2}\;\sqrt{-2x^2+4}\;\sqrt{x^2+1}\;\;\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{130\sqrt{2}\;\sqrt{-2x^2+4}\;\sqrt{x^2+1}\;\;\left(-\mathrm{Ell}\right.}{3\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -25/3*(-x^4+x^2+2)^(1/2)*x+197/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x
^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-130/3*2^(1/2)*(-2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))
-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^2}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

3.334 $\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=25

$$2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 5*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+2*EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1180, 524, 424, 419}

$$2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] 5*EllipticE[ArcSin[x/Sqrt[2]], -2] + 2*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = 2\int \frac{7+5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= 4\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx + 5\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx$$
$$= 5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + 2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Mathematica [C] time = 0.06, size = 34, normalized size = 1.36

$$\frac{i\left(10E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-17F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (I*(10*EllipticE[I*ArcSinh[x], -1/2] - 17*EllipticF[I*ArcSinh[x], -1/2]))/S
qrt[2]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+x^2+2}\,(5\,x^2+7)}{x^4-x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)/(x^4 - x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

maple [B] time = 0.00, size = 110, normalized size = 4.40

$$\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)}{2\sqrt{-x^4+x^2+2}} - \frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\;\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)+\mathrm{EllipticF}\right.}{2\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -5/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1
/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+7/2*2^(1/2)*(-2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+7}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
[Out] Integral((5*x**2 + 7)/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

3.335 $\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1095, 419}

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x^2 - x^4],x]
[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx &= 2\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\ &= F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 19, normalized size = 1.90

$$-\frac{iF\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x^2 - x^4],x]
[Out] ((-I)*EllipticF[I*ArcSinh[x], -1/2])/Sqrt[2]
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+x^2+2}}{x^4-x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(x^4 - x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

maple [B] time = 0.00, size = 47, normalized size = 4.70

$$\frac{\sqrt{2}\, \sqrt{-2x^2 + 4}\, \sqrt{x^2 + 1}\, \operatorname{EllipticF}\left(\frac{\sqrt{2}\, x}{2}, i\sqrt{2}\right)}{2\sqrt{-x^4 + x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2
*2^(1/2)*x,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(x^2 - x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + x**2 + 2), x)
```

3.336 $\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=17

$$\frac{1}{7}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 1/7*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1212, 537}

$$\frac{1}{7}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2]/7
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx &= 2\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\,(7+5x^2)}\,dx\\
&= \frac{1}{7}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] time = 0.10, size = 24, normalized size = 1.41

$$-\frac{i\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] ((-1/7*I)*EllipticPi[5/7, I*ArcSinh[x], -1/2])/Sqrt[2]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+x^2+2}}{5\,x^6+2\,x^4-17\,x^2-14},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(5*x^6 + 2*x^4 - 17*x^2 - 14), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

maple [B] time = 0.02, size = 48, normalized size = 2.82

$$\frac{\sqrt{2}\,\sqrt{-\frac{x^2}{2}+1}\,\sqrt{x^2+1}\,\;\text{EllipticPi}\left(\frac{\sqrt{2}\,x}{2},-\frac{10}{7},i\sqrt{2}\right)}{7\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x)
[Out] 1/7*2^(1/2)*(-1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(
1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(5\,x^2+7\right)\,\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\,\left(5x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)), x)
```

3.337 $\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=74

$$-\frac{25\sqrt{-x^4+x^2+2}\,x}{476\left(5x^2+7\right)}-\frac{1}{238}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{5}{476}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{167\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332}$$

```
[Out] -5/476*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1/238*EllipticF(1/2*x*2^(1/2),I*2
^(1/2))+167/3332*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-25/476*x*(-x^4+x
^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1223, 1716, 1180, 524, 424, 419, 1212, 537}

$$-\frac{25\sqrt{-x^4+x^2+2}\,x}{476\left(5x^2+7\right)}-\frac{1}{238}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{5}{476}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{167\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(476*(7 + 5*x^2)) - (5*EllipticE[ArcSin[x/Sqrt[
2]], -2])/476 - EllipticF[ArcSin[x/Sqrt[2]], -2]/238 + (167*EllipticPi[-10/
7, ArcSin[x/Sqrt[2]], -2])/3332
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx = -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)}+\frac{1}{476}\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)}-\frac{\int \frac{175+125x^2}{\sqrt{2+x^2-x^4}}\,dx}{11900}+\frac{167}{476}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)}-\frac{\int \frac{175+125x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{5950}+\frac{167}{238}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+\right.}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)}+\frac{167\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332}-\frac{1}{119}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+}}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)}-\frac{5}{476}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{238}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Mathematica [C] time = 0.29, size = 196, normalized size = 2.65

$$\frac{350x^5-350x^3+119i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-70i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}\,E\left(i\,\mathrm{s}\right.}{6664\left(5x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-700*x - 350*x^3 + 350*x^5 - (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4
]*EllipticE[I*ArcSinh[x], -1/2] + (119*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2
- x^4]*EllipticF[I*ArcSinh[x], -1/2] - (1169*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]
*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (835*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x
^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(6664*(7 + 5*x^2)*Sqrt[2 + x^2 - x
^4])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-x^4+x^2+2}}{25\,x^8+45\,x^6-71\,x^4-189\,x^2-98},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(25*x^8 + 45*x^6 - 71*x^4 - 189*x^2 - 98), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

maple [B] time = 0.02, size = 165, normalized size = 2.23

$$-\frac{25\sqrt{-x^4+x^2+2}\,x}{476\left(5x^2+7\right)}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{952\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2}\right.}{476\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -25/476*(-x^4+x^2+2)^(1/2)/(5*x^2+7)*x-1/476*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-5/952*2^(1/2
)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x
,I*2^(1/2))+167/3332*2^(1/2)*(-1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^
(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(5\,x^2+7\right)^2\,\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\,\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2), x)
```

3.338 $\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=102

$$-\frac{12525\sqrt{-x^4+x^2+2}\,x}{453152\left(5x^2+7\right)}-\frac{25\sqrt{-x^4+x^2+2}\,x}{952\left(5x^2+7\right)^2}-\frac{263F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{226576}-\frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152}+\frac{58915\Pi\left(-\frac{10}{7}\right.}{3}$$

```
[Out] -2505/453152*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-263/226576*EllipticF(1/2*x*
2^(1/2),I*2^(1/2))+58915/3172064*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-
25/952*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-12525/453152*x*(-x^4+x^2+2)^(1/2)/(
5*x^2+7)
```

Rubi [A] time = 0.19, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1223, 1696, 1716, 1180, 524, 424, 419, 1212, 537}

$$-\frac{12525\sqrt{-x^4+x^2+2}\,x}{453152\left(5x^2+7\right)}-\frac{25\sqrt{-x^4+x^2+2}\,x}{952\left(5x^2+7\right)^2}-\frac{263F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{226576}-\frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152}+\frac{58915\Pi\left(-\frac{10}{7}\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(952*(7 + 5*x^2)^2) - (12525*x*Sqrt[2 + x^2 - x
^4])/(453152*(7 + 5*x^2)) - (2505*EllipticE[ArcSin[x/Sqrt[2]], -2])/453152
- (263*EllipticF[ArcSin[x/Sqrt[2]], -2])/226576 + (58915*EllipticPi[-10/7,
ArcSin[x/Sqrt[2]], -2])/3172064
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx = -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}+\frac{1}{952}\int \frac{186-190x^2+25x^4}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)}+\frac{\int \frac{37698-32690x^2-12525x^4}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx}{453152}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)}-\frac{\int \frac{75775+62625x^2}{\sqrt{2+x^2-x^4}}\,dx}{11328800}+\frac{58915\int \frac{}{(7+5}}{45}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)}-\frac{\int \frac{75775+62625x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{5664400}+\frac{58915\int \frac{}{\sqrt{4}}}{}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)}+\frac{58915\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3172064}-$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)}-\frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152}-\frac{263F}{}$$

Mathematica [C] time = 0.42, size = 108, normalized size = 1.06

$$\frac{\frac{350x(2505x^6+1478x^4-8993x^2-7966)}{(5x^2+7)^2\sqrt{-x^4+x^2+2}}+56287i\sqrt{2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-35070i\sqrt{2}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-58915i\sqrt{2}\,\Pi\left(\frac{5}{7};\right.}{6344128}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] ((350*x*(-7966 - 8993*x^2 + 1478*x^4 + 2505*x^6))/((7 + 5*x^2)^2*Sqrt[2 + x
^2 - x^4]) - (35070*I)*Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] + (56287*I)*Sq
rt[2]*EllipticF[I*ArcSinh[x], -1/2] - (58915*I)*Sqrt[2]*EllipticPi[5/7, I*A
rcSinh[x], -1/2])/6344128
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+x^2+2}}{125\,x^{10}+400\,x^8-40\,x^6-1442\,x^4-1813\,x^2-686}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(125*x^10 + 400*x^8 - 40*x^6 - 1442*x^4 - 18
13*x^2 - 686), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\,(5\,x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)

maple [A] time = 0.02, size = 189, normalized size = 1.85

$$-\frac{25\sqrt{-x^4+x^2+2}\,x}{952\left(5x^2+7\right)^2}-\frac{12525\sqrt{-x^4+x^2+2}\,x}{453152\left(5x^2+7\right)}-\frac{2505\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{906304\sqrt{-x^4+x^2+2}}-\frac{263\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x)

[Out] -25/952*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2*x-12525/453152*(-x^4+x^2+2)^(1/2)/(5*x^2+7)*x-263/453152*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-2505/906304*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))+58915/3172064*2^(1/2)*(-1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(5\,x^2+7\right)^3\,\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2)),x)

[Out] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\,\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)

[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3), x)

3.339 $\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{27500}{3}\sqrt{-x^4+x^2+2}\,x+\frac{(1419793x^2+1419985)\,x}{18\sqrt{-x^4+x^2+2}}+625\sqrt{-x^4+x^2+2}\,x^3+\frac{627857}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{348229}{18}$$

```
[Out] -3482293/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+627857/6*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))+1/18*x*(1419793*x^2+1419985)/(-x^4+x^2+2)^(1/2)+27500/3*x*(
-x^4+x^2+2)^(1/2)+625*x^3*(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1205, 1679, 1180, 524, 424, 419}

$$625\sqrt{-x^4+x^2+2}\,x^3+\frac{27500}{3}\sqrt{-x^4+x^2+2}\,x+\frac{(1419793x^2+1419985)\,x}{18\sqrt{-x^4+x^2+2}}+\frac{627857}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{348229}{18}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(1419985 + 1419793*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (27500*x*Sqrt[2 + x^
2 - x^4])/3 + 625*x^3*Sqrt[2 + x^2 - x^4] - (3482293*EllipticE[ArcSin[x/Sqr
t[2]], -2])/18 + (627857*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(7+5x^2\right)^5}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{x\left(1419985+1419793x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{18}\int \frac{1268722+3084793x^2+450000x^4+56250x^6}{\sqrt{2+x^2-x^4}}\,dx\\ &= \frac{x\left(1419985+1419793x^2\right)}{18\sqrt{2+x^2-x^4}}+625x^3\sqrt{2+x^2-x^4}+\frac{1}{90}\int \frac{-6343610-15761465x^2-}{\sqrt{2+x^2-x^4}}\\ &= \frac{x\left(1419985+1419793x^2\right)}{18\sqrt{2+x^2-x^4}}+\frac{27500}{3}x\sqrt{2+x^2-x^4}+625x^3\sqrt{2+x^2-x^4}-\frac{1}{270}\int \frac{23}{}\\ &= \frac{x\left(1419985+1419793x^2\right)}{18\sqrt{2+x^2-x^4}}+\frac{27500}{3}x\sqrt{2+x^2-x^4}+625x^3\sqrt{2+x^2-x^4}-\frac{1}{135}\int \frac{23}{}\\ &= \frac{x\left(1419985+1419793x^2\right)}{18\sqrt{2+x^2-x^4}}+\frac{27500}{3}x\sqrt{2+x^2-x^4}+625x^3\sqrt{2+x^2-x^4}-\frac{3482293}{18}\\ &= \frac{x\left(1419985+1419793x^2\right)}{18\sqrt{2+x^2-x^4}}+\frac{27500}{3}x\sqrt{2+x^2-x^4}+625x^3\sqrt{2+x^2-x^4}-\frac{3482293}{18}\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3125\,x^{10}+21875\,x^{8}+61250\,x^{6}+85750\,x^{4}+60025\,x^{2}+16807\right)\sqrt{-x^{4}+x^{2}+2}}{x^{8}-2\,x^{6}-3\,x^{4}+4\,x^{2}+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*x^2 + 16807
)*sqrt(-x^4 + x^2 + 2)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

maple [B] time = 0.04, size = 280, normalized size = 3.01

$$625\sqrt{-x^4+x^2+2}\;x^3+\frac{27500\sqrt{-x^4+x^2+2}\;x}{3}-\frac{799361\sqrt{2}\;\sqrt{-2x^2+4}\;\sqrt{x^2+1}\;\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{\frac{53125}{9}x^3}{\sqrt{-x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 6250*(17/18*x^3+7/9*x)/(-x^4+x^2+2)^(1/2)+625*(-x^4+x^2+2)^(1/2)*x^3+27500/
3*(-x^4+x^2+2)^(1/2)*x-799361/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x
^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+3482293/36*2^(1/2)*(-2*x
^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(
1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+43750*(7/18*x^3+5/9*x)/(-x^4+x^2+
2)^(1/2)+122500*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^(1/2)+171500*(1/18*x^3+2/9*x)
/(-x^4+x^2+2)^(1/2)+120050*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+33614*(5/36*
x-1/36*x^3)/(-x^4+x^2+2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^5/(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^5/(x^2 - x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^5}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**5/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

3.340 $\int \frac{\left(7+5x^2\right)^4}{\left(2+x^2-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{625}{3}\sqrt{-x^4+x^2+2}\,x+\frac{\left(83489x^2+83585\right)x}{18\sqrt{-x^4+x^2+2}}+\frac{31921}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{165239}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -165239/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+31921/6*EllipticF(1/2*x*2^(1/
2),I*2^(1/2))+1/18*x*(83489*x^2+83585)/(-x^4+x^2+2)^(1/2)+625/3*x*(-x^4+x^2
+2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1205, 1679, 1180, 524, 424, 419}

$$\frac{625}{3}\sqrt{-x^4+x^2+2}\,x+\frac{\left(83489x^2+83585\right)x}{18\sqrt{-x^4+x^2+2}}+\frac{31921}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{165239}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(83585 + 83489*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^
4])/3 - (165239*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (31921*EllipticF[Arc
Sin[x/Sqrt[2]], -2])/6
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx &= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}}-\frac{1}{18}\int \frac{61976+157739x^2+11250x^4}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}}+\frac{625}{3}x\sqrt{2+x^2-x^4}+\frac{1}{54}\int \frac{-208428-495717x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}}+\frac{625}{3}x\sqrt{2+x^2-x^4}+\frac{1}{27}\int \frac{-208428-495717x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}}+\frac{625}{3}x\sqrt{2+x^2-x^4}-\frac{165239}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{31921}{3}\int \frac{}{\sqrt{}}\\
&= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}}+\frac{625}{3}x\sqrt{2+x^2-x^4}-\frac{165239}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{31921}{6}F
\end{aligned}
$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(625\,x^8+3500\,x^6+7350\,x^4+6860\,x^2+2401\right)\sqrt{-x^4+x^2+2}}{x^8-2\,x^6-3\,x^4+4\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(-x^4 + x^2
+ 2)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^4}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

maple [B] time = 0.01, size = 240, normalized size = 3.24

$$\frac{625\sqrt{-x^4+x^2+2}\,x}{3}-\frac{17369\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{\frac{4375}{9}x^3+\frac{6250}{9}x}{\sqrt{-x^4+x^2+2}}+\frac{165239\sqrt{2}\,\sqrt{-2x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 1250*(7/18*x^3+5/9*x)/(-x^4+x^2+2)^(1/2)+625/3*(-x^4+x^2+2)^(1/2)*x-17369/9
*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^
(1/2)*x,I*2^(1/2))+165239/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2
^(1/2)))+7000*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^(1/2)+14700*(1/18*x^3+2/9*x)/(-
x^4+x^2+2)^(1/2)+13720*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+4802*(-1/36*x^3+
5/36*x)/(-x^4+x^2+2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^4}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2+7)^4}{(-x^4+x^2+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4/(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^4/(x^2 - x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^4}{\left(-(x^2-2)(x^2+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**4/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

3.341 $\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}} \, dx$

Optimal. Leaf size=55

$$\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}+\frac{1763}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{7147}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -7147/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1763/6*EllipticF(1/2*x*2^(1/2),
I*2^(1/2))+1/18*x*(4897*x^2+4945)/(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1180, 524, 424, 419}

$$\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}+\frac{1763}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{7147}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(4945 + 4897*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (7147*EllipticE[ArcSin[x/S
qrt[2]], -2])/18 + (1763*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
```

```
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx &= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{1858+7147x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9}\int \frac{1858+7147x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{7147}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{1763}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{7147}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1763}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(125\,x^6+525\,x^4+735\,x^2+343)\sqrt{-x^4+x^2+2}}{x^8-2\,x^6-3\,x^4+4\,x^2+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(-x^4 + x^2 + 2)/(x^8 - 2*
x^6 - 3*x^4 + 4*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x^2+7)^3}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

maple [B] time = 0.01, size = 202, normalized size = 3.67

$$-\frac{929\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{\frac{625}{9}x^3+\frac{250}{9}x}{\sqrt{-x^4+x^2+2}}+\frac{7147\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(-\operatorname{EllipticI}\right.}{36\sqrt{-x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 250*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^(1/2)-929/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^
2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+7147/36*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1
/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+1050*(1/18*x^3+2/9*x)/
(-x^4+x^2+2)^(1/2)+1470*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+686*(-1/36*x^3+
5/36*x)/(-x^4+x^2+2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^3}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

3.342 $\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}+\frac{139}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{281}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -281/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+139/6*EllipticF(1/2*x*2^(1/2),I*
2^(1/2))+1/18*x*(281*x^2+305)/(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1180, 524, 424, 419}

$$\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}+\frac{139}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{281}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(305 + 281*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (281*EllipticE[ArcSin[x/Sqrt
[2]], -2])/18 + (139*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
```

```
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(7+5x^2\right)^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{x\left(305+281x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{18}\int \frac{-136+281x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x\left(305+281x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{9}\int \frac{-136+281x^2}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(305+281x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{281}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{139}{3}\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(305+281x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{281}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{139}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(25\,x^4+70\,x^2+49\right)\sqrt{-x^4+x^2+2}}{x^8-2\,x^6-3\,x^4+4\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(-x^4 + x^2 + 2)/(x^8 - 2*x^6 - 3*x^4 +
 4*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

maple [B] time = 0.01, size = 179, normalized size = 3.25

$$\frac{34\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{\frac{25}{9}x^3+\frac{100}{9}x}{\sqrt{-x^4+x^2+2}}+\frac{281\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(-\,\mathrm{EllipticE}\right.}{36\sqrt{-x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 50*(1/18*x^3+2/9*x)/(-x^4+x^2+2)^(1/2)+34/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+281/36*2^(1/2
)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*
x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+140*(1/9*x^3-1/18*x)/(-x^4
+x^2+2)^(1/2)+98*(-1/36*x^3+5/36*x)/(-x^4+x^2+2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^2}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

3.343 $\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{x\left(13x^2+25\right)}{18\sqrt{-x^4+x^2+2}}+\frac{17}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{13}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -13/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+17/6*EllipticF(1/2*x*2^(1/2),I*2^
(1/2))+1/18*x*(13*x^2+25)/(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1178, 1180, 524, 424, 419}

$$\frac{x\left(13x^2+25\right)}{18\sqrt{-x^4+x^2+2}}+\frac{17}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{13}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(25 + 13*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (13*EllipticE[ArcSin[x/Sqrt[2]
], -2])/18 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{x\left(25+13x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{18}\int \frac{-38+13x^2}{\sqrt{2+x^2-x^4}}\,dx\\ &= \frac{x\left(25+13x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{9}\int \frac{-38+13x^2}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\ &= \frac{x\left(25+13x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{13}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{17}{3}\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\ &= \frac{x\left(25+13x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{13}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{17}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)}{x^8-2\,x^6-3\,x^4+4\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^2+7}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

maple [B] time = 0.01, size = 156, normalized size = 2.84

$$\frac{19\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{\frac{10}{9}x^3-\frac{5}{9}x}{\sqrt{-x^4+x^2+2}}+\frac{13\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(-\operatorname{EllipticE}\left(\right.\right.}{36\sqrt{-x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 10*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+19/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+13/36*2^(1/2
)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*
x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+14*(-1/36*x^3+5/36*x)/(-x^
4+x^2+2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+7}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{5x^2+7}{\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+7}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

3.344 $\int \frac{1}{\left(2+x^2-x^4\right)^{3/2}} \, dx$

Optimal. Leaf size=55

$$\frac{x\left(5-x^2\right)}{18\sqrt{-x^4+x^2+2}}+\frac{1}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 1/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1/6*EllipticF(1/2*x*2^(1/2),I*2^(1/
2))+1/18*x*(-x^2+5)/(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1092, 1180, 524, 424, 419}

$$\frac{x\left(5-x^2\right)}{18\sqrt{-x^4+x^2+2}}+\frac{1}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(-3/2),x]
```

```
[Out] (x*(5 - x^2))/(18*Sqrt[2 + x^2 - x^4]) + EllipticE[ArcSin[x/Sqrt[2]], -2]/1
8 + EllipticF[ArcSin[x/Sqrt[2]], -2]/6
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
```

```
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{x\left(5-x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{18}\int \frac{-4-x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x\left(5-x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{9}\int \frac{-4-x^2}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(5-x^2\right)}{18\sqrt{2+x^2-x^4}}+\frac{1}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{1}{3}\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(5-x^2\right)}{18\sqrt{2+x^2-x^4}}+\frac{1}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(2 + x^2 - x^4)^(-3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{x^8-2\,x^6-3\,x^4+4\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + x^2 + 2)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(-3/2), x)
```

maple [B] time = 0.00, size = 133, normalized size = 2.42

$$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\ \text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{-\frac{1}{18}x^3+\frac{5}{18}x}{\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\ \left(-\text{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{\right.\right.}{36\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 2*(-1/36*x^3+5/36*x)/(-x^4+x^2+2)^(1/2)+1/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-1/36*2^(1/2)*
(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,
I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int(1/(x^2 - x^4 + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**4 + x**2 + 2)**(-3/2), x)
```

3.345 $\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}} \, dx$

Optimal. Leaf size=72

$$\frac{x(35-16x^2)}{306\sqrt{-x^4+x^2+2}}+\frac{1}{102}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{8}{153}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 8/153*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1/102*EllipticF(1/2*x*2^(1/2),I*2^
(1/2))-25/238*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/306*x*(-16*x^2+35
)/(-x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1221, 1178, 1180, 524, 424, 419, 1212, 537}

$$\frac{x(35-16x^2)}{306\sqrt{-x^4+x^2+2}}+\frac{1}{102}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{8}{153}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(35 - 16*x^2))/(306*Sqrt[2 + x^2 - x^4]) + (8*EllipticE[ArcSin[x/Sqrt[2]
], -2])/153 + EllipticF[ArcSin[x/Sqrt[2]], -2]/102 - (25*EllipticPi[-10/7,
ArcSin[x/Sqrt[2]], -2])/238
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(7+5x^2\right)\left(2+x^2-x^4\right)^{3/2}}\,dx &= -\left(\frac{1}{34}\int \frac{-12+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx\right)-\frac{25}{34}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x\left(35-16x^2\right)}{306\sqrt{2+x^2-x^4}}+\frac{1}{612}\int \frac{38+32x^2}{\sqrt{2+x^2-x^4}}\,dx-\frac{25}{17}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\,(} \\
&= \frac{x\left(35-16x^2\right)}{306\sqrt{2+x^2-x^4}}-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{306}\int \frac{38+32}{\sqrt{4-2x^2}\sqrt{2}} \\
&= \frac{x\left(35-16x^2\right)}{306\sqrt{2+x^2-x^4}}-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{51}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+}} \\
&= \frac{x\left(35-16x^2\right)}{306\sqrt{2+x^2-x^4}}+\frac{8}{153}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{102}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-
\end{aligned}$$

Mathematica [C] time = 0.22, size = 101, normalized size = 1.40

$$\frac{\frac{490x}{\sqrt{-x^4+x^2+2}}-\frac{224x^3}{\sqrt{-x^4+x^2+2}}-357i\sqrt{2}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+224i\sqrt{2}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+225i\sqrt{2}\Pi\left(\frac{5}{7};i\sinh^{-1}(x\right.}{4284}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] ((490*x)/Sqrt[2 + x^2 - x^4] - (224*x^3)/Sqrt[2 + x^2 - x^4] + (224*I)*Sqrt
[2]*EllipticE[I*ArcSinh[x], -1/2] - (357*I)*Sqrt[2]*EllipticF[I*ArcSinh[x],
 -1/2] + (225*I)*Sqrt[2]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/4284
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{5\,x^{10}-3\,x^8-29\,x^6-x^4+48\,x^2+28},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(5*x^10 - 3*x^8 - 29*x^6 - x^4 + 48*x^2 + 28)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

maple [B] time = 0.02, size = 164, normalized size = 2.28

$$\frac{4\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{153\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{204\sqrt{-x^4+x^2+2}}-\frac{25\sqrt{2}\,\sqrt{-\frac{x^2}{2}+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 2*(-4/153*x^3+35/612*x)/(-x^4+x^2+2)^(1/2)+1/204*2^(1/2)*(-2*x^2+4)^(1/2)*(
x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+4/153*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/
2)*x,I*2^(1/2))-25/238*2^(1/2)*(-1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2
)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(5\,x^2+7\right)\,{\left(-x^4+x^2+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)), x)
```

3.346 $\int \frac{1}{(7+5x^2)^2 (2+x^2-x^4)^{3/2}} dx$

Optimal. Leaf size=100

$$\frac{625\sqrt{-x^4+x^2+2}\,x}{16184\left(5x^2+7\right)}+\frac{\left(580-287x^2\right)x}{10404\sqrt{-x^4+x^2+2}}+\frac{89F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24276}+\frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656}-\frac{10825\Pi\left(-\frac{10}{7};\text{s}\right.}{113}$$

```
[Out] 5143/145656*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+89/24276*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))-10825/113288*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/10
404*x*(-287*x^2+580)/(-x^4+x^2+2)^(1/2)+625/16184*x*(-x^4+x^2+2)^(1/2)/(5*x
^2+7)
```

Rubi [A] time = 0.30, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1178, 1180, 524, 424, 419, 1223, 1716, 1212, 537}

$$\frac{625\sqrt{-x^4+x^2+2}\,x}{16184\left(5x^2+7\right)}+\frac{\left(580-287x^2\right)x}{10404\sqrt{-x^4+x^2+2}}+\frac{89F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24276}+\frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656}-\frac{10825\Pi\left(-\frac{10}{7};\text{s}\right.}{113}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(580 - 287*x^2))/(10404*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^4
])/(16184*(7 + 5*x^2)) + (5143*EllipticE[ArcSin[x/Sqrt[2]], -2])/145656 + (
89*EllipticF[ArcSin[x/Sqrt[2]], -2])/24276 - (10825*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/113288
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^2 (2+x^2-x^4)^{3/2}}\,dx = \int \left(\frac{194-95x^2}{1156(2+x^2-x^4)^{3/2}} - \frac{25}{34(7+5x^2)^2\sqrt{2+x^2-x^4}} - \frac{475}{1156(7+5x^2)\sqrt{2}} \right.$$

$$= \frac{\int \frac{194-95x^2}{(2+x^2-x^4)^{3/2}}dx}{1156} - \frac{475\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}dx}{1156} - \frac{25}{34}\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}$$

$$= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} - \frac{\int \frac{-586-574x^2}{\sqrt{2+x^2-x^4}}dx}{20808} - \frac{25\int \frac{118-70x^2}{(7+5x^2)\sqrt{2}}}{1618}$$

$$= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} - \frac{475\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{8092} +$$

$$= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} - \frac{475\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{8092} +$$

$$= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} + \frac{287E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{10404} + \frac{F\left(\mathrm{si}\right.}{}$$

$$= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} + \frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656} + \frac{89F}{}$$

Mathematica [C] time = 0.33, size = 196, normalized size = 1.96

$$\frac{-360010x^5+253386x^3-111741i\sqrt{2}\,(5x^2+7)\sqrt{-x^4+x^2+2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+72002i\sqrt{2}\,(5x^2+7)\sqrt{-x^4}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (953260*x + 253386*x^3 - 360010*x^5 + (72002*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2
+ x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (111741*I)*Sqrt[2]*(7 + 5*x^2)
*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (681975*I)*Sqrt[2]*Sqr
t[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (487125*I)*Sqrt[2]*x
^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(2039184*(7 + 5
*x^2)*Sqrt[2 + x^2 - x^4])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{25\,x^{12}+20\,x^{10}-166\,x^8-208\,x^6+233\,x^4+476\,x^2+196}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(25*x^12 + 20*x^10 - 166*x^8 - 208*x^6 + 233*
x^4 + 476*x^2 + 196), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

maple [B] time = 0.02, size = 188, normalized size = 1.88

$$\frac{625\sqrt{-x^4+x^2+2}\,x}{16184\left(5x^2+7\right)}+\frac{5143\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{291312\sqrt{-x^4+x^2+2}}+\frac{89\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{Ellipt}}{48552\sqrt{-x^4+x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 2*(-287/20808*x^3+145/5202*x)/(-x^4+x^2+2)^(1/2)+625/16184*(-x^4+x^2+2)^(1/
2)/(5*x^2+7)*x+89/48552*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+5143/291312*2^(1/2)*(-2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))-108
25/113288*2^(1/2)*(-1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ellip
ticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(5\,x^2+7\right)^2\,\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2), x)
```

3.347 $\int \frac{1}{\left(7+5x^2\right)^3\left(2+x^2-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{645625\sqrt{-x^4+x^2+2}\,x}{15407168\left(5x^2+7\right)}+\frac{625\sqrt{-x^4+x^2+2}\,x}{32368\left(5x^2+7\right)^2}+\frac{\left(9830-4909x^2\right)x}{353736\sqrt{-x^4+x^2+2}}+\frac{60409F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{23110752}+\frac{3086453E\left(\right.}{13}$$

```
[Out] 3086453/138664512*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+60409/23110752*Ellipti
cF(1/2*x*2^(1/2),I*2^(1/2))-6898575/107850176*EllipticPi(1/2*x*2^(1/2),-10/
7,I*2^(1/2))+1/353736*x*(-4909*x^2+9830)/(-x^4+x^2+2)^(1/2)+625/32368*x*(-x
^4+x^2+2)^(1/2)/(5*x^2+7)^2+645625/15407168*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] time = 0.57, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1228, 1178, 1180, 524, 424, 419, 1223, 1696, 1716, 1212, 537}

$$\frac{645625\sqrt{-x^4+x^2+2}\,x}{15407168\left(5x^2+7\right)}+\frac{625\sqrt{-x^4+x^2+2}\,x}{32368\left(5x^2+7\right)^2}+\frac{\left(9830-4909x^2\right)x}{353736\sqrt{-x^4+x^2+2}}+\frac{60409F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{23110752}+\frac{3086453E\left(\right.}{13}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(9830 - 4909*x^2))/(353736*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 -
x^4])/(32368*(7 + 5*x^2)^2) + (645625*x*Sqrt[2 + x^2 - x^4])/(15407168*(7 +
 5*x^2)) + (3086453*EllipticE[ArcSin[x/Sqrt[2]], -2])/138664512 + (60409*El
lipticF[ArcSin[x/Sqrt[2]], -2])/23110752 - (6898575*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/107850176
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^3\left(2+x^2-x^4\right)^{3/2}}\,dx = \int \left(-\frac{-3278+1635x^2}{39304\left(2+x^2-x^4\right)^{3/2}}-\frac{25}{34\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}}-\frac{47}{1156\left(7+5x^2\right)^2}\right.$$

$$= -\frac{\int \frac{-3278+1635x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx}{39304}-\frac{8175\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{39304}-\frac{475\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx}{1156}-$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2}+\frac{11875x\sqrt{2+x^2-x^4}}{550256\left(7+5x^2\right)}+\frac{\int \frac{9842+}{\sqrt{2+}}}{70}$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2}+\frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)}-\frac{8175}{}$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2}+\frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)}-\frac{8175}{}$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2}+\frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)}+\frac{4909}{}$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2}+\frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)}+\frac{9010}{}$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2}+\frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)}+\frac{3086}{}$$

Mathematica [C] time = 0.40, size = 244, normalized size = 1.91

$$\frac{-1080258550x^7-737347940x^5+3876617542x^3-67352691i\sqrt{2}\left(5x^2+7\right)^2\sqrt{-x^4+x^2+2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (3857257460*x + 3876617542*x^3 - 737347940*x^5 - 1080258550*x^7 + (43210342
*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2]
 - (67352691*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSi
nh[x], -1/2] + (3042271575*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I
*ArcSinh[x], -1/2] + (4346102250*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*Ellipti
cPi[5/7, I*ArcSinh[x], -1/2] + (1552179375*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^
```

```
4]*EllipticPi[5/7, I*ArcSinh[x], -1/2]])/(1941303168*(7 + 5*x^2)^2*Sqrt[2 +
x^2 - x^4])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{125\,x^{14}+275\,x^{12}-690\,x^{10}-2202\,x^8-291\,x^6+4011\,x^4+4312\,x^2+1372},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(125*x^14 + 275*x^12 - 690*x^10 - 2202*x^8 -
291*x^6 + 4011*x^4 + 4312*x^2 + 1372), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

maple [A] time = 0.02, size = 212, normalized size = 1.66

$$\frac{625\sqrt{-x^4+x^2+2}\;x}{32368\left(5x^2+7\right)^2}+\frac{645625\sqrt{-x^4+x^2+2}\;x}{15407168\left(5x^2+7\right)}+\frac{3086453\sqrt{2}\;\sqrt{-2x^2+4}\;\sqrt{x^2+1}\;\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{277329024\sqrt{-x^4+x^2+2}}+\frac{60}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 2*(-4909/707472*x^3+4915/353736*x)/(-x^4+x^2+2)^(1/2)+625/32368*(-x^4+x^2+2
)^(1/2)/(5*x^2+7)^2*x+645625/15407168*(-x^4+x^2+2)^(1/2)/(5*x^2+7)*x+60409/
46221504*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elliptic
F(1/2*2^(1/2)*x,I*2^(1/2))+3086453/277329024*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))-6898575/1078
50176*2^(1/2)*(-1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticP
i(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(5\,x^2+7\right)^3\,\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3), x)
```

3.348 $\int \left(7+5x^2\right)^4 \sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=242

$$\frac{3050}{11}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{33}\left(4516x^2+18727\right)\sqrt{x^4+3x^2+4}\,x+\frac{51665\sqrt{x^4+3x^2+4}\,x}{33\left(x^2+2\right)}+\frac{33159\left(x^2+2\right)\sqrt{\frac{x^4}{(}}}{11\sqrt{2}\sqrt{}}$$

```
[Out] 3050/11*x*(x^4+3*x^2+4)^(3/2)+23500/99*x^3*(x^4+3*x^2+4)^(3/2)+625/11*x^5*(
x^4+3*x^2+4)^(3/2)+51665/33*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/33*x*(4516*x^2+
18727)*(x^4+3*x^2+4)^(1/2)+33159/22*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2
)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2)))
,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-5
1665/33*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2
^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5+\frac{23500}{99}\left(x^4+3x^2+4\right)^{3/2}x^3+\frac{3050}{11}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{33}\left(4516x^2+18727\right)\sqrt{x^4+}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (51665*x*Sqrt[4 + 3*x^2 + x^4])/(33*(2 + x^2)) + (x*(18727 + 4516*x^2)*Sqrt
[4 + 3*x^2 + x^4])/33 + (3050*x*(4 + 3*x^2 + x^4)^(3/2))/11 + (23500*x^3*(4
 + 3*x^2 + x^4)^(3/2))/99 + (625*x^5*(4 + 3*x^2 + x^4)^(3/2))/11 - (51665*S
qrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/S
qrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4]) + (33159*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(11*Sqrt[2]*Sqrt
[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
```

```
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(7+5x^2\right)^4 \sqrt{4+3x^2+x^4}\,dx = \frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2} + \frac{1}{11}\int \sqrt{4+3x^2+x^4}\,\left(26411+75460x^2+68350x^4\right.$$

$$= \frac{23500}{99}x^3\left(4+3x^2+x^4\right)^{3/2} + \frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2} + \frac{1}{99}\int \sqrt{4+3x^2+x^4}\,($$

$$= \frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2} + \frac{23500}{99}x^3\left(4+3x^2+x^4\right)^{3/2} + \frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3}$$

$$= \frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4} + \frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2} + \frac{23500}{99}x^3($$

$$= \frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4} + \frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2} + \frac{23500}{99}x^3($$

$$= \frac{51665x\sqrt{4+3x^2+x^4}}{33\left(2+x^2\right)} + \frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4} + \frac{3050}{11}x\left(4+\right.$$

Mathematica [C] time = 0.59, size = 354, normalized size = 1.46

$$\frac{3\sqrt{2}\left(51665\sqrt{7}-36253i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-154995\sqrt{2}\left(\sqrt{7}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(663924 + 1257535*x^2 + 1217475*x^4 + 7127
48*x^6 + 264075*x^8 + 57250*x^10 + 5625*x^12) - 154995*Sqrt[2]*(3*I + Sqrt[
7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt
[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-36253*I + 51665*Sq
rt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt
[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(396*Sqrt[(-I)/(-3*I + Sqrt[
7])]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(625\,x^8+3500\,x^6+7350\,x^4+6860\,x^2+2401\right)\sqrt{x^4+3\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(x^4 + 3*x^2
 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

maple [C] time = 0.17, size = 292, normalized size = 1.21

$$\frac{625\sqrt{x^4+3x^2+4}\;x^9}{11}+\frac{40375\sqrt{x^4+3x^2+4}\;x^7}{99}+\frac{3650\sqrt{x^4+3x^2+4}\;x^5}{3}+\frac{189898\sqrt{x^4+3x^2+4}\;x^3}{99}+\frac{55327\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 625/11*x^9*(x^4+3*x^2+4)^(1/2)+40375/99*x^7*(x^4+3*x^2+4)^(1/2)+189898/99*x
^3*(x^4+3*x^2+4)^(1/2)+55327/33*x*(x^4+3*x^2+4)^(1/2)+382496/33/(-6+2*I*7^(
1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)
^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*
I*7^(1/2))^(1/2))-1653280/33/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))
*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/
2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-El
```

```
lipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+3650/3*x^5
*(x^4+3*x^2+4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3\,x^2+4}\,\left(5\,x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(5\,x^2+7\right)^4\,\sqrt{x^4+3\,x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\,\left(5x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(1/2),x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**4, x)
```

3.349 $\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\, dx$

Optimal. Leaf size=221

$$\frac{275}{7}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{21}\left(407x^2+1708\right)\sqrt{x^4+3x^2+4}\,x+\frac{4717\sqrt{x^4+3x^2+4}\,x}{21\left(x^2+2\right)}+\frac{1301\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{3\sqrt{2}\sqrt{x^4+3}}$$

```
[Out] 275/7*x*(x^4+3*x^2+4)^(3/2)+125/9*x^3*(x^4+3*x^2+4)^(3/2)+4717/21*x*(x^4+3*
x^2+4)^(1/2)/(x^2+2)+1/21*x*(407*x^2+1708)*(x^4+3*x^2+4)^(1/2)+1301/6*(x^2+
2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Elli
pticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(
1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-4717/21*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2
)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1
/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1
/2)
```

Rubi [A] time = 0.11, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3+\frac{275}{7}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{21}\left(407x^2+1708\right)\sqrt{x^4+3x^2+4}\,x+\frac{4717\sqrt{x^4+3x^2+4}\,x}{21\left(x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4717*x*Sqrt[4 + 3*x^2 + x^4])/(21*(2 + x^2)) + (x*(1708 + 407*x^2)*Sqrt[4
+ 3*x^2 + x^4])/21 + (275*x*(4 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(4 + 3*x^
2 + x^4)^(3/2))/9 - (4717*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(21*Sqrt[4 + 3*x^2 + x^4]) + (130
1*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2
]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
```

```
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\, dx &= \frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{9}\int \sqrt{4+3x^2+x^4}\left(3087+5115x^2+2475x^4\right) dx \\ &= \frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{63}\int \left(11709+6105x^2\right) \\ &= \frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4}+\frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(4+3x\right. \\ &= \frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4}+\frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(4+3x\right. \\ &= \frac{4717x\sqrt{4+3x^2+x^4}}{21\left(2+x^2\right)}+\frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4}+\frac{275}{7}x\left(4+3x^2\right.\end{aligned}$$

Mathematica [C] time = 0.50, size = 349, normalized size = 1.58

$$3\sqrt{2}\left(4717\sqrt{7}-3409i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-14151\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\cdots}$$

252

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(60096 + 93656*x^2 + 71862*x^4 + 30946*x^6
 + 7725*x^8 + 875*x^10) - 14151*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7
] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sq
rt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7
])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-3409*I + 4717*Sqrt[7])*Sqrt[(-3*I + Sqrt[
7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + S
qrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[
7])/(3*I + Sqrt[7])])/(252*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4
])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(125\,x^{6}+525\,x^{4}+735\,x^{2}+343\right)\sqrt{x^{4}+3\,x^{2}+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^{4}+3\,x^{2}+4}\left(5\,x^{2}+7\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

maple [C] time = 0.01, size = 275, normalized size = 1.24

$$\frac{125\sqrt{x^{4}+3x^{2}+4}\;x^{7}}{9}+\frac{1700\sqrt{x^{4}+3x^{2}+4}\;x^{5}}{21}+\frac{12146\sqrt{x^{4}+3x^{2}+4}\;x^{3}}{63}+\frac{5008\sqrt{x^{4}+3x^{2}+4}\;x}{21}+\frac{35120\sqrt{-\left(-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 125/9*(x^4+3*x^2+4)^(1/2)*x^7+1700/21*(x^4+3*x^2+4)^(1/2)*x^5+12146/63*(x^4
+3*x^2+4)^(1/2)*x^3+5008/21*(x^4+3*x^2+4)^(1/2)*x+35120/21/(-6+2*I*7^(1/2))
^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1
/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7
^(1/2))^(1/2))-150944/21/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+
1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)
+3)*(EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-Elli
pticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^{4}+3\,x^{2}+4}\left(5\,x^{2}+7\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(5\,x^2+7\right)^3\,\sqrt{x^4+3\,x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\,\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(1/2),x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3, x)
```

3.350 $\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\, dx$

Optimal. Leaf size=198

$$\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{7}x\left(38x^2+119\right)\sqrt{x^4+3x^2+4}+\frac{319x\sqrt{x^4+3x^2+4}}{7\left(x^2+2\right)}+\frac{81\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan\right.}{\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] 25/7*x*(x^4+3*x^2+4)^(3/2)+319/7*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/7*x*(38*x^
2+119)*(x^4+3*x^2+4)^(1/2)+81/2*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1
/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4
*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-319/7
*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2))
)*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4
)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1176, 1197, 1103, 1195}

$$\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{7}x\left(38x^2+119\right)\sqrt{x^4+3x^2+4}+\frac{319x\sqrt{x^4+3x^2+4}}{7\left(x^2+2\right)}+\frac{81\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan\right.}{\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (319*x*Sqrt[4 + 3*x^2 + x^4])/(7*(2 + x^2)) + (x*(119 + 38*x^2)*Sqrt[4 + 3*
x^2 + x^4])/7 + (25*x*(4 + 3*x^2 + x^4)^(3/2))/7 - (319*Sqrt[2]*(2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(7*
Sqrt[4 + 3*x^2 + x^4]) + (81*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*
EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\, dx = \frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2} + \frac{1}{7}\int \left(243+190x^2\right)\sqrt{4+3x^2+x^4}\, dx$$
$$= \frac{1}{7}x\left(119+38x^2\right)\sqrt{4+3x^2+x^4} + \frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2} + \frac{1}{105}\int \frac{7440+478}{\sqrt{4+3x^2+}}$$
$$= \frac{1}{7}x\left(119+38x^2\right)\sqrt{4+3x^2+x^4} + \frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2} - \frac{638}{7}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+}}$$
$$= \frac{319x\sqrt{4+3x^2+x^4}}{7\left(2+x^2\right)} + \frac{1}{7}x\left(119+38x^2\right)\sqrt{4+3x^2+x^4} + \frac{25}{7}x\left(4+3x^2+x^4\right)^{3/}$$

Mathematica [C] time = 0.47, size = 343, normalized size = 1.73

$$\frac{\sqrt{2}\left(319\sqrt{7}-35i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right) - 319\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}}{\sqrt{7}-3i}}}{28\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(876 + 1109*x^2 + 658*x^4 + 188*x^6 + 25*x
^8) - 319*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I +
 Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*Ar
cSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] +
Sqrt[2]*(-35*I + 319*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(28*S
qrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(25\,x^4+70\,x^2+49\right)\sqrt{x^4+3\,x^2+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3\,x^2+4}\,\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

maple [C] time = 0.01, size = 258, normalized size = 1.30

$$\frac{25\sqrt{x^4+3x^2+4}\,x^5}{7}+\frac{113\sqrt{x^4+3x^2+4}\,x^3}{7}+\frac{219\sqrt{x^4+3x^2+4}\,x}{7}+\frac{1984\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right.}}{7\sqrt{-6+2i\sqrt{7}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25/7*(x^4+3*x^2+4)^(1/2)*x^5+113/7*(x^4+3*x^2+4)^(1/2)*x^3+219/7*(x^4+3*x^2
+4)^(1/2)*x+1984/7/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/
2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-
6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-10208/7/(-6+2*I*7^(1/2))^
(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/
2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*
x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+
6*I*7^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3\,x^2+4}\,\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)^2\,\sqrt{x^4+3\,x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\,\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2, x)
```

3.351 $\int \left(7+5x^2\right)\sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=177

$$\frac{1}{3}\left(3x^2+10\right)\sqrt{x^4+3x^2+4}\,x+\frac{9\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{49\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{9\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4}{(}}}{\sqrt{x}}$$

```
[Out] 9*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/3*x*(3*x^2+10)*(x^4+3*x^2+4)^(1/2)+49/6*(
x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*
EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^
2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-9*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2
))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2
)
```

Rubi [A] time = 0.05, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1176, 1197, 1103, 1195}

$$\frac{1}{3}\left(3x^2+10\right)\sqrt{x^4+3x^2+4}\,x+\frac{9\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{49\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{9\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4}{(}}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (x*(10 + 3*x^2)*Sqrt[4 + 3*x^2 + x^
4])/3 - (9*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (49*(2 + x^2)*Sqrt[(4 +
3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \left(7+5x^2\right)\sqrt{4+3x^2+x^4}\,dx = \frac{1}{3}x\left(10+3x^2\right)\sqrt{4+3x^2+x^4} + \frac{1}{15}\int \frac{220+135x^2}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{1}{3}x\left(10+3x^2\right)\sqrt{4+3x^2+x^4} - 18\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx + \frac{98}{3}\int \frac{1}{\sqrt{4+3x^2+\cdots}}$$
$$= \frac{9x\sqrt{4+3x^2+x^4}}{2+x^2} + \frac{1}{3}x\left(10+3x^2\right)\sqrt{4+3x^2+x^4} - \frac{9\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x\cdots}{\left(2+x^2\right)^2}}}{\sqrt{4+3x\cdots}}$$

Mathematica [C] time = 0.43, size = 338, normalized size = 1.91

$$\frac{\sqrt{2}\left(27\sqrt{7}-7i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-27\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-\cdots}{\sqrt{7}-3i}}\cdots}{12\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(40 + 42*x^2 + 19*x^4 + 3*x^6) - 27*Sqrt[2
]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[
(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I
)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I +
27*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I +
 Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*
I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-3*I + S
qrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

maple [C] time = 0.01, size = 240, normalized size = 1.36

$$\sqrt{x^4+3x^2+4}\,x^3+\frac{10\sqrt{x^4+3x^2+4}\,x}{3}+\frac{176\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4}\right.}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] (x^4+3*x^2+4)^(1/2)*x^3+10/3*(x^4+3*x^2+4)^(1/2)*x+176/3/(-6+2*I*7^(1/2))^(
1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2
)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(
1/2))^(1/2))-288/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)
*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(Ell
ipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/
4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)\sqrt{x^4+3\,x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\left(5x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7), x)
```

3.352 $\int \sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=169

$$\frac{\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{1}{3}\sqrt{x^4+3x^2+4}\,x+\frac{7\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,E\left(2\text{t}\right.}{\sqrt{x^4+3x^2+4}}$$

```
[Out] 1/3*x*(x^4+3*x^2+4)^(1/2)+x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+7/6*(x^2+2)*(cos(2*
arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(
2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/
2)/(x^4+3*x^2+4)^(1/2)-(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2
*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))
*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1091, 1197, 1103, 1195}

$$\frac{\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{1}{3}\sqrt{x^4+3x^2+4}\,x+\frac{7\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,E\left(2\text{t}\right.}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/3 + (x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (Sqrt[2
]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2
]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (7*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^
4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \sqrt{4+3x^2+x^4}\,dx = \frac{1}{3}x\sqrt{4+3x^2+x^4} + \frac{1}{3}\int \frac{8+3x^2}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{1}{3}x\sqrt{4+3x^2+x^4} - 2\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx + \frac{14}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{1}{3}x\sqrt{4+3x^2+x^4} + \frac{x\sqrt{4+3x^2+x^4}}{2+x^2} - \frac{\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}} + \frac{7(}{}$$

Mathematica [C] time = 0.35, size = 331, normalized size = 1.96

$$\frac{\sqrt{2}\left(3\sqrt{7}-7i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right) - 3\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{}}{12\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 3*Sqrt[2]*(3*I + Sqrt[
7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt
[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqr
t[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x
], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4
 + 3*x^2 + x^4])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{x^4+3x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

maple [C] time = 0.00, size = 224, normalized size = 1.33

$$\frac{\sqrt{x^{4}+3x^{2}+4}\,x}{3}+\frac{32\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^{2}+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^{4}+3x^{2}+4}}-\frac{32\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 1/3*(x^4+3*x^2+4)^(1/2)*x+32/3/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2)
)*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*Elli
pticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-32/(-6+2*I*7^
(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2
+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*(-6+2*I*7^(1/2))
^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,
1/4*(2+6*I*7^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{4}+3\,x^{2}+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^{4}+3\,x^{2}+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int((3*x^2 + x^4 + 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{4}+3x^{2}+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt(x**4 + 3*x**2 + 4), x)
```

3.353 $\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$

Optimal. Leaf size=322

$$\frac{\sqrt{x^4+3x^2+4}\,x}{5\left(x^2+2\right)}+\frac{1}{5}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)-\frac{11\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{75\sqrt{x^4+3x^2+4}}+\frac{9\left(x^2+2\right)\sqrt{\frac{x}{}}}{25\sqrt{}}$$

```
[Out] 1/175*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/5*x*(x^4+3*x
^2+4)^(1/2)/(x^2+2)+1/30*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos
(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2
))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+187/1050*(x^
2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*El
lipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x
^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-1/5*(x^2+2)*(cos(2*arctan(1/2*x*
2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*
x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2
+4)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1208, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{x^4+3x^2+4}\,x}{5\left(x^2+2\right)}+\frac{1}{5}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)-\frac{11\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{75\sqrt{x^4+3x^2+4}}+\frac{9\left(x^2+2\right)\sqrt{\frac{x}{}}}{25\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/(5*(2 + x^2)) + (Sqrt[11/35]*ArcTan[(2*Sqrt[11/35
]*x)/Sqrt[4 + 3*x^2 + x^4]])/5 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5*Sqrt[4 + 3*x^2 + x^4])
 + (9*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sq
rt[2]], 1/8])/(25*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (11*Sqrt[2]*(2 + x^2)*Sq
rt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(75*
Sqrt[4 + 3*x^2 + x^4]) + (187*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(525*Sqrt[2]*Sqrt[4 + 3*x^2
+ x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int \frac{-8-5x^2}{\sqrt{4+3x^2+x^4}}\,dx\right)+\frac{44}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\left(\frac{2}{5}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\right)-\frac{44}{75}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\frac{18}{25}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\ldots$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{5\left(2+x^2\right)}+\frac{1}{5}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)-\frac{\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,E\left(2\tan\ldots\right.}{5\sqrt{4+3x^2+x^4}}$$

Mathematica [C] time = 0.25, size = 283, normalized size = 0.88

$$-\frac{\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\left(-35\sqrt{7}+7i\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+35\left(\sqrt{7}+3i\right)E\left(i\sinh^{-1}\left(\sqrt{-\ldots}\right.\right.\right.}{350\sqrt{2}\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] -1/350*(Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I +
Sqrt[7])]*(35*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[
7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I - 35*Sqrt[7])*EllipticF[I*
ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
+ (88*I)*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(Sqrt[2]*Sqrt[(-I)/(-3*I +
Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

maple [C] time = 0.12, size = 386, normalized size = 1.20

$$\frac{32\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\,\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{5\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(i\sqrt{7}+3\right)}+\frac{32\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\,\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+\cdots}}{25\sqrt{-6+2i\sqrt{7}}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x)
```

```
[Out] 32/25/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2
+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2)
)^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-32/5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2
-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)
^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1
/2))^(1/2))+32/5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)
*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*Elli
pticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+44/175/(-3/8+
1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x
^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)
*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2)
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7),x)
```

```
[Out] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7), x)
```

3.354 $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=284

$$-\frac{\sqrt{x^4+3x^2+4}\,x}{70\left(x^2+2\right)}+\frac{\sqrt{x^4+3x^2+4}\,x}{14\left(5x^2+7\right)}+\frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{280\sqrt{385}}-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{\left(x^2+2\right)\ \cdots}{3\cdots}$$

```
[Out] 51/107800*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/70*x*(x^
4+3*x^2+4)^(1/2)/(x^2+2)+1/14*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+1/70*(x^2+2)*
(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellipti
cE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)
^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-1/70*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)
^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),
1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+28
9/19600*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2
^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1226, 1197, 1103, 1195, 1216, 1706}

$$-\frac{\sqrt{x^4+3x^2+4}\,x}{70\left(x^2+2\right)}+\frac{\sqrt{x^4+3x^2+4}\,x}{14\left(5x^2+7\right)}+\frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{280\sqrt{385}}-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{\left(x^2+2\right)\ \cdots}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] -(x*Sqrt[4 + 3*x^2 + x^4])/(70*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4])/(14*(
7 + 5*x^2)) + (51*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(280*Sqr
t[385]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan
[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4
+ 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2
]*Sqrt[4 + 3*x^2 + x^4]) + (289*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(9800*Sqrt[2]*Sqrt[4 + 3*x
^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1226

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx = \frac{x\sqrt{4+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{1}{350}\int \frac{7-5x^2}{\sqrt{4+3x^2+x^4}}\,dx + \frac{51}{350}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{14\left(7+5x^2\right)} - \frac{3}{350}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{1}{35}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx - \frac{17}{350}\int \frac{}{\sqrt{4}}$$

$$= -\frac{x\sqrt{4+3x^2+x^4}}{70\left(2+x^2\right)} + \frac{x\sqrt{4+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)}{280\sqrt{385}} + \frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,E}{35\sqrt{2}\sqrt{4+}}$$

Mathematica [C] time = 0.77, size = 481, normalized size = 1.69

$$-98i\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right) - 102i\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}\sqrt{1+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I + Sqrt[7])*(
7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])] - EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]) - (98*I)*(7 + 5*x^2)*Sqrt[2 - ((4*
I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I
*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
 - (102*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*
I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[
(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(9800*Sqrt[(
-I)/(-3*I + Sqrt[7])]*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}}{25\,x^4+70\,x^2+49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^4 + 70*x^2 + 49), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+4}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

maple [C] time = 0.02, size = 410, normalized size = 1.44

$$\frac{\sqrt{x^4+3x^2+4}\,x}{70x^2+98}-\frac{16\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\,\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(i\sqrt{7}+3\right)}+\frac{2\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x)
```

```
[Out] 1/14*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+2/25/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1
/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)
^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+16
/35/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8
*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*(-6
+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-16/35/(-6+2*I*7^(1/2))^(1/
2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/
(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/
4*(2+6*I*7^(1/2))^(1/2))+51/2450/(-3/8+1/8*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*
7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2
```

```
)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1
/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 + 3\,x^2 + 4}}{\left(5\,x^2 + 7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4 + 3\,x^2 + 4}}{\left(5\,x^2 + 7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^2,x)
[Out] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2 - x + 2\right)\left(x^2 + x + 2\right)}}{\left(5x^2 + 7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**2,x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**2, x)
```

3.355 $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3} \, dx$

Optimal. Leaf size=312

$$-\frac{139\sqrt{x^4+3x^2+4}\,x}{86240\left(x^2+2\right)}+\frac{139\sqrt{x^4+3x^2+4}\,x}{17248\left(5x^2+7\right)}+\frac{\sqrt{x^4+3x^2+4}\,x}{28\left(5x^2+7\right)^2}+\frac{14999\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{344960\sqrt{385}}-\frac{23\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{2940\sqrt{2}\sqrt{x^4}}$$

```
[Out] 14999/132809600*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-139/
86240*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/28*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+
139/17248*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+139/86240*(x^2+2)*(cos(2*arctan(1
/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan
(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+
3*x^2+4)^(1/2)-23/5880*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2
*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))
*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+254983/7244160
0*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+
4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1228, 1223, 1696, 1714, 1195, 1708, 1103, 1706, 1216}

$$-\frac{139\sqrt{x^4+3x^2+4}\,x}{86240\left(x^2+2\right)}+\frac{139\sqrt{x^4+3x^2+4}\,x}{17248\left(5x^2+7\right)}+\frac{\sqrt{x^4+3x^2+4}\,x}{28\left(5x^2+7\right)^2}+\frac{14999\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{344960\sqrt{385}}-\frac{23\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{2940\sqrt{2}\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (-139*x*Sqrt[4 + 3*x^2 + x^4])/(86240*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4]
)/(28*(7 + 5*x^2)^2) + (139*x*Sqrt[4 + 3*x^2 + x^4])/(17248*(7 + 5*x^2)) +
(14999*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(344960*Sqrt[385])
+ (139*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/S
qrt[2]], 1/8])/(43120*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (23*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2940*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4]) + (254983*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(36220800*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
```

```
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx = \int \left(\frac{44}{25\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}}+\frac{1}{25\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}+\frac{1}{25\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\right.$$

$$=\frac{1}{25}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx+\frac{1}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx+\frac{44}{25}\int \frac{}{\left(7+5\right.}$$

$$=\frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)}-\frac{\int \frac{12+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{15400}-\frac{1}{700}\int \frac{-76-10x^2-}{\left(7+5x^2\right)^2\sqrt{4+3}}$$

$$=\frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)}+\frac{\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)}{20\sqrt{385}}-\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,F\left(2\right.}{150\sqrt{2}\sqrt{4+3x^2}}$$

$$=-\frac{x\sqrt{4+3x^2+x^4}}{3080\left(2+x^2\right)}+\frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)}+\frac{\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)}{20\sqrt{385}}+\frac{\left(2\right.}{}$$

$$=-\frac{139x\sqrt{4+3x^2+x^4}}{86240\left(2+x^2\right)}+\frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)}+\frac{653\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right.}{12320\sqrt{385}}$$

$$=-\frac{139x\sqrt{4+3x^2+x^4}}{86240\left(2+x^2\right)}+\frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)}+\frac{14999\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+}}\right.}{344960\sqrt{385}}$$

Mathematica [C] time = 0.68, size = 308, normalized size = 0.99

$$\frac{\frac{700x\left(695x^2+1589\right)\left(x^4+3x^2+4\right)}{\left(5x^2+7\right)^2}+i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\left(-9597+4865i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right.\right.\right.}{1207360}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] ((700*x*(1589 + 695*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*I
)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(4865*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9597 + (4865*I)*Sqrt[7])
*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I
 + Sqrt[7])] - 29998*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*
I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(12073600*Sqrt[
4 + 3*x^2 + x^4])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}}{125\,x^6+525\,x^4+735\,x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+4}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

maple [C] time = 0.03, size = 434, normalized size = 1.39

$$\frac{\sqrt{x^4+3x^2+4}\;x}{28\left(5x^2+7\right)^2}+\frac{139\sqrt{x^4+3x^2+4}\;x}{17248\left(5x^2+7\right)}-\frac{139\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\;\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\;\mathrm{EllipticE}\left(\frac{\sqrt{-6+2i\sqrt{7}}\;x}{4},\frac{\sqrt{2+\cdots}}{\cdots}\right.}{2695\sqrt{-6+2i\sqrt{7}}\;\sqrt{x^4+3x^2+4}\;\left(i\sqrt{7}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x)
```

```
[Out] 1/28*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+139/17248*(x^4+3*x^2+4)^(1/2)/(5*x^2
+7)*x-51/15400/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(
3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*
I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+139/2695/(-6+2*I*7^(1/2))^(1/
2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/
(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/
4*(2+6*I*7^(1/2))^(1/2))-139/2695/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(
1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(
I*7^(1/2)+3)*EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/
2))+14999/3018400/(-3/8+1/8*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^
(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-
3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(
1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+4}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4 + 3\,x^2 + 4}}{\left(5\,x^2 + 7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^3,x)
[Out] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2 - x + 2\right)\left(x^2 + x + 2\right)}}{\left(5x^2 + 7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**3,x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**3, x)
```

3.356 $\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2} \, dx$

Optimal. Leaf size=268

$$\frac{92150}{429}\left(x^4+3x^2+4\right)^{5/2}x+\frac{\left(131080x^2+452001\right)\left(x^4+3x^2+4\right)^{3/2}x}{1287}+\frac{7\left(174989x^2+661429\right)\sqrt{x^4+3x^2+4}\,x}{2145}$$

```
[Out] 1/1287*x*(131080*x^2+452001)*(x^4+3*x^2+4)^(3/2)+92150/429*x*(x^4+3*x^2+4)^
(5/2)+2250/13*x^3*(x^4+3*x^2+4)^(5/2)+125/3*x^5*(x^4+3*x^2+4)^(5/2)+1266508
6/2145*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+7/2145*x*(174989*x^2+661429)*(x^4+3*x^
2+4)^(1/2)-12665086/2145*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos
(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2
))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+2383556/429*
(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))
*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)
^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5+\frac{2250}{13}\left(x^4+3x^2+4\right)^{5/2}x^3+\frac{92150}{429}\left(x^4+3x^2+4\right)^{5/2}x+\frac{\left(131080x^2+452001\right)\left(x^4+3\right.}{1287}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (12665086*x*Sqrt[4 + 3*x^2 + x^4])/(2145*(2 + x^2)) + (7*x*(661429 + 174989
*x^2)*Sqrt[4 + 3*x^2 + x^4])/2145 + (x*(452001 + 131080*x^2)*(4 + 3*x^2 + x
^4)^(3/2))/1287 + (92150*x*(4 + 3*x^2 + x^4)^(5/2))/429 + (2250*x^3*(4 + 3*
x^2 + x^4)^(5/2))/13 + (125*x^5*(4 + 3*x^2 + x^4)^(5/2))/3 - (12665086*Sqrt
[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt
[2]], 1/8])/(2145*Sqrt[4 + 3*x^2 + x^4]) + (2383556*Sqrt[2]*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(429*Sq
rt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2} dx = \frac{125}{3}x^5 \left(4+3x^2+x^4\right)^{5/2} + \frac{1}{15}\int \left(4+3x^2+x^4\right)^{3/2} \left(36015+102900x^2+977\right.$$

$$= \frac{2250}{13}x^3 \left(4+3x^2+x^4\right)^{5/2} + \frac{125}{3}x^5 \left(4+3x^2+x^4\right)^{5/2} + \frac{1}{195}\int \left(4+3x^2+x^4\right.$$

$$= \frac{92150}{429}x \left(4+3x^2+x^4\right)^{5/2} + \frac{2250}{13}x^3 \left(4+3x^2+x^4\right)^{5/2} + \frac{125}{3}x^5 \left(4+3x^2+x^4\right.$$

$$= \frac{x\left(452001+131080x^2\right)\left(4+3x^2+x^4\right)^{3/2}}{1287} + \frac{92150}{429}x \left(4+3x^2+x^4\right)^{5/2} + \frac{225}{13}$$

$$= \frac{7x\left(661429+174989x^2\right)\sqrt{4+3x^2+x^4}}{2145} + \frac{x\left(452001+131080x^2\right)\left(4+3x^2\right.}{1287}$$

$$= \frac{7x\left(661429+174989x^2\right)\sqrt{4+3x^2+x^4}}{2145} + \frac{x\left(452001+131080x^2\right)\left(4+3x^2\right.}{1287}$$

$$= \frac{12665086x\sqrt{4+3x^2+x^4}}{2145\left(2+x^2\right)} + \frac{7x\left(661429+174989x^2\right)\sqrt{4+3x^2+x^4}}{2145} + \frac{x(}{}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(625\,x^{12} + 5375\,x^{10} + 20350\,x^{8} + 42910\,x^{6} + 52381\,x^{4} + 34643\,x^{2} + 9604\right)\sqrt{x^{4} + 3\,x^{2} + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^12 + 5375*x^10 + 20350*x^8 + 42910*x^6 + 52381*x^4 + 34643*
x^2 + 9604)*sqrt(x^4 + 3*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{4} + 3\,x^{2} + 4\right)^{\frac{3}{2}} \left(5\,x^{2} + 7\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

maple [C] time = 0.04, size = 326, normalized size = 1.22

$$\frac{125\sqrt{x^4+3x^2+4}\;x^{13}}{3}+\frac{5500\sqrt{x^4+3x^2+4}\;x^{11}}{13}+\frac{841525\sqrt{x^4+3x^2+4}\;x^{9}}{429}+\frac{6863530\sqrt{x^4+3x^2+4}\;x^{7}}{1287}+356$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 15015343/2145*(x^4+3*x^2+4)^(1/2)*x+64070384/6435*(x^4+3*x^2+4)^(1/2)*x^3+6
863530/1287*(x^4+3*x^2+4)^(1/2)*x^7+356027/39*(x^4+3*x^2+4)^(1/2)*x^5+5500/
13*x^11*(x^4+3*x^2+4)^(1/2)+125/3*x^13*(x^4+3*x^2+4)^(1/2)+841525/429*(x^4+
3*x^2+4)^(1/2)*x^9-405282752/2145/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1
/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(
I*7^(1/2)+3)*(EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1
/2))-EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))+893
63792/2145/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3
/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^
(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{4} + 3\,x^{2} + 4\right)^{\frac{3}{2}} \left(5\,x^{2} + 7\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(5\,x^2+7\right)^4\,{\left(x^4+3\,x^2+4\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^{2} - x + 2\right) \left(x^{2} + x + 2\right)\right)^{\frac{3}{2}} \left(5 x^{2} + 7\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(3/2),x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**4, x)
```

3.357 $\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=247

$$\frac{3825}{143}\left(x^4+3x^2+4\right)^{5/2}x+\frac{\left(15365x^2+53504\right)\left(x^4+3x^2+4\right)^{3/2}x}{1001}+\frac{\left(435441x^2+1653701\right)\sqrt{x^4+3x^2+4}\,x}{5005}+4$$

```
[Out] 1/1001*x*(15365*x^2+53504)*(x^4+3*x^2+4)^(3/2)+3825/143*x*(x^4+3*x^2+4)^(5/
2)+125/13*x^3*(x^4+3*x^2+4)^(5/2)+4525662/5005*x*(x^4+3*x^2+4)^(1/2)/(x^2+2
)+1/5005*x*(435441*x^2+1653701)*(x^4+3*x^2+4)^(1/2)-4525662/5005*(x^2+2)*(c
os(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE
(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2
)^(1/2)/(x^4+3*x^2+4)^(1/2)+121826/143*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2
))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2
)
```

Rubi [A] time = 0.13, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3+\frac{3825}{143}\left(x^4+3x^2+4\right)^{5/2}x+\frac{\left(15365x^2+53504\right)\left(x^4+3x^2+4\right)^{3/2}x}{1001}+\frac{\left(435441x^2+165\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4525662*x*Sqrt[4 + 3*x^2 + x^4])/(5005*(2 + x^2)) + (x*(1653701 + 435441*x
^2)*Sqrt[4 + 3*x^2 + x^4])/5005 + (x*(53504 + 15365*x^2)*(4 + 3*x^2 + x^4)^
(3/2))/1001 + (3825*x*(4 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(4 + 3*x^2 +
x^4)^(5/2))/13 - (4525662*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5005*Sqrt[4 + 3*x^2 + x^4]) + (1
21826*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Arc
Tan[x/Sqrt[2]], 1/8])/(143*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
```

```
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2} dx = \frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2}+\frac{1}{13}\int \left(4+3x^2+x^4\right)^{3/2}\left(4459+8055x^2+3825x^4\right.$$

$$= \frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2}+\frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2}+\frac{1}{143}\int \left(33749+1975\right.$$

$$= \frac{x\left(53504+15365x^2\right)\left(4+3x^2+x^4\right)^{3/2}}{1001}+\frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2}+\frac{125}{13}x^3$$

$$= \frac{x\left(1653701+435441x^2\right)\sqrt{4+3x^2+x^4}}{5005}+\frac{x\left(53504+15365x^2\right)\left(4+3x^2+\right.}{1001}$$

$$= \frac{x\left(1653701+435441x^2\right)\sqrt{4+3x^2+x^4}}{5005}+\frac{x\left(53504+15365x^2\right)\left(4+3x^2+\right.}{1001}$$

$$= \frac{4525662x\sqrt{4+3x^2+x^4}}{5005\left(2+x^2\right)}+\frac{x\left(1653701+435441x^2\right)\sqrt{4+3x^2+x^4}}{5005}+\frac{x\left(5\right.}{}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(125\,x^{10} + 900\,x^{8} + 2810\,x^{6} + 4648\,x^{4} + 3969\,x^{2} + 1372\right)\sqrt{x^{4} + 3\,x^{2} + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^10 + 900*x^8 + 2810*x^6 + 4648*x^4 + 3969*x^2 + 1372)*sqrt(
x^4 + 3*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{4} + 3\,x^{2} + 4\right)^{\frac{3}{2}} \left(5\,x^{2} + 7\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

maple [C] time = 0.01, size = 309, normalized size = 1.25

$$\frac{125\sqrt{x^{4}+3x^{2}+4}\;x^{11}}{13}+\frac{12075\sqrt{x^{4}+3x^{2}+4}\;x^{9}}{143}+\frac{48520\sqrt{x^{4}+3x^{2}+4}\;x^{7}}{143}+\frac{71434\sqrt{x^{4}+3x^{2}+4}\;x^{5}}{91}+\frac{552830}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 4865781/5005*(x^4+3*x^2+4)^(1/2)*x+5528301/5005*(x^4+3*x^2+4)^(1/2)*x^3+485
20/143*(x^4+3*x^2+4)^(1/2)*x^7+71434/91*(x^4+3*x^2+4)^(1/2)*x^5+125/13*(x^4
+3*x^2+4)^(1/2)*x^11+12075/143*(x^4+3*x^2+4)^(1/2)*x^9-144821184/5005/(-6+2
*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2)
)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*(-6+2*I*7^(
1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*I*7^(1/2))^(1/
2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))+32017264/5005/(-6+2*I*7^(1/2))^(1/2)*(-(-3
/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x
^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{4} + 3\,x^{2} + 4\right)^{\frac{3}{2}} \left(5\,x^{2} + 7\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(5\,x^2+7\right)}^3\,{\left(x^4+3\,x^2+4\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^2 - x + 2\right)\left(x^2 + x + 2\right)\right)^{\frac{3}{2}}\left(5x^2 + 7\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3, x)
```

3.358 $\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2} dx$

Optimal. Leaf size=226

$$\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}+\frac{1}{693}x\left(2240x^2+6831\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{x\left(18253x^2+64533\right)\sqrt{x^4+3x^2+4}}{1155}+\frac{17534}{1}$$

```
[Out] 1/693*x*(2240*x^2+6831)*(x^4+3*x^2+4)^(3/2)+25/11*x*(x^4+3*x^2+4)^(5/2)+175
346/1155*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/1155*x*(18253*x^2+64533)*(x^4+3*x^
2+4)^(1/2)-175346/1155*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2
*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))
*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+4628/33*(x^2+2
)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellip
ticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1
/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1176, 1197, 1103, 1195}

$$\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}+\frac{1}{693}x\left(2240x^2+6831\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{x\left(18253x^2+64533\right)\sqrt{x^4+3x^2+4}}{1155}+\frac{17534}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (175346*x*Sqrt[4 + 3*x^2 + x^4])/(1155*(2 + x^2)) + (x*(64533 + 18253*x^2)*
Sqrt[4 + 3*x^2 + x^4])/1155 + (x*(6831 + 2240*x^2)*(4 + 3*x^2 + x^4)^(3/2))
/693 + (25*x*(4 + 3*x^2 + x^4)^(5/2))/11 - (175346*Sqrt[2]*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(1155*Sq
rt[4 + 3*x^2 + x^4]) + (4628*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
```

```
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2}\,dx &= \frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2}+\frac{1}{11}\int \left(439+320x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx\\
&= \frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x^4\right)^{3/2}+\frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2}+\frac{1}{231}\int (\\
&= \frac{x\left(64533+18253x^2\right)\sqrt{4+3x^2+x^4}}{1155}+\frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x\right.\\
&= \frac{x\left(64533+18253x^2\right)\sqrt{4+3x^2+x^4}}{1155}+\frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x\right.\\
&= \frac{175346x\sqrt{4+3x^2+x^4}}{1155\left(2+x^2\right)}+\frac{x\left(64533+18253x^2\right)\sqrt{4+3x^2+x^4}}{1155}+\frac{1}{693}x\left(68\right.
\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(25\,x^8+145\,x^6+359\,x^4+427\,x^2+196\right)\sqrt{x^4+3\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^8 + 145*x^6 + 359*x^4 + 427*x^2 + 196)*sqrt(x^4 + 3*x^2 + 4)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

maple [C] time = 0.01, size = 292, normalized size = 1.29

$$\frac{25\sqrt{x^4+3x^2+4}\;x^9}{11}+\frac{1670\sqrt{x^4+3x^2+4}\;x^7}{99}+\frac{1222\sqrt{x^4+3x^2+4}\;x^5}{21}+\frac{391024\sqrt{x^4+3x^2+4}\;x^3}{3465}+\frac{50691\sqrt{x^4}}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 25/11*(x^4+3*x^2+4)^(1/2)*x^9+1670/99*(x^4+3*x^2+4)^(1/2)*x^7+1222/21*(x^4+
3*x^2+4)^(1/2)*x^5+391024/3465*(x^4+3*x^2+4)^(1/2)*x^3+50691/385*(x^4+3*x^2
+4)^(1/2)*x+396304/385/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)
^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/
4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-5611072/1155/(-6+2*I*
7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x
^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*(-6+2*I*7^(1/2
))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*
x,1/4*(2+6*I*7^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(5\,x^2+7\right)}^2\,{\left(x^4+3\,x^2+4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2, x)
```

3.359 $\int \left(7+5x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=207

$$\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{105}x\left(289x^2+1029\right)\sqrt{x^4+3x^2+4}+\frac{2798x\sqrt{x^4+3x^2+4}}{105\left(x^2+2\right)}+\frac{74\sqrt{2}\left(x^2+2\right.}{}$$

```
[Out] 1/63*x*(35*x^2+108)*(x^4+3*x^2+4)^(3/2)+2798/105*x*(x^4+3*x^2+4)^(1/2)/(x^2
+2)+1/105*x*(289*x^2+1029)*(x^4+3*x^2+4)^(1/2)-2798/105*(x^2+2)*(cos(2*arct
an(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*ar
ctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)+74/3*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(
2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2)
)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1176, 1197, 1103, 1195}

$$\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{105}x\left(289x^2+1029\right)\sqrt{x^4+3x^2+4}+\frac{2798x\sqrt{x^4+3x^2+4}}{105\left(x^2+2\right)}+\frac{74\sqrt{2}\left(x^2+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2798*x*Sqrt[4 + 3*x^2 + x^4])/(105*(2 + x^2)) + (x*(1029 + 289*x^2)*Sqrt[4
 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(4 + 3*x^2 + x^4)^(3/2))/63 - (279
8*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[
x/Sqrt[2]], 1/8])/(105*Sqrt[4 + 3*x^2 + x^4]) + (74*Sqrt[2]*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt
[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
```

```
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \left(7+5x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{21}\int \left(444+289x^2\right)\sqrt{4+3x^2+x^4}$$

$$= \frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/}$$

$$= \frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/}$$

$$= \frac{2798x\sqrt{4+3x^2+x^4}}{105\left(2+x^2\right)}+\frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{63}x\left(108\right.$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(5\,x^6+22\,x^4+41\,x^2+28\right)\sqrt{x^4+3\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
[Out] integral((5*x^6 + 22*x^4 + 41*x^2 + 28)*sqrt(x^4 + 3*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

maple [C] time = 0.01, size = 275, normalized size = 1.33

$$\frac{5\sqrt{x^4+3x^2+4}\,x^7}{9}+\frac{71\sqrt{x^4+3x^2+4}\,x^5}{21}+\frac{3187\sqrt{x^4+3x^2+4}\,x^3}{315}+\frac{583\sqrt{x^4+3x^2+4}\,x}{35}+\frac{6352\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 5/9*(x^4+3*x^2+4)^(1/2)*x^7+71/21*(x^4+3*x^2+4)^(1/2)*x^5+3187/315*(x^4+3*x
^2+4)^(1/2)*x^3+583/35*(x^4+3*x^2+4)^(1/2)*x+6352/35/(-6+2*I*7^(1/2))^(1/2)
*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x
^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2)
)^(1/2))-89536/105/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/
2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(E
llipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(
1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(5\,x^2+7\right)\,\left(x^4+3\,x^2+4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7), x)
```

3.360 $\int \left(4+3x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=198

$$\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{35}x\left(9x^2+49\right)\sqrt{x^4+3x^2+4}+\frac{138x\sqrt{x^4+3x^2+4}}{35\left(x^2+2\right)}+\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan\right.}{\sqrt{x^4+3x^2+4}}$$

```
[Out] 1/7*x*(x^4+3*x^2+4)^(3/2)+138/35*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/35*x*(9*x^
2+49)*(x^4+3*x^2+4)^(1/2)-138/35*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(
1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/
4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+4*(x
^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*E
llipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2
)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1091, 1176, 1197, 1103, 1195}

$$\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{35}x\left(9x^2+49\right)\sqrt{x^4+3x^2+4}+\frac{138x\sqrt{x^4+3x^2+4}}{35\left(x^2+2\right)}+\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan\right.}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (138*x*Sqrt[4 + 3*x^2 + x^4])/(35*(2 + x^2)) + (x*(49 + 9*x^2)*Sqrt[4 + 3*x
^2 + x^4])/35 + (x*(4 + 3*x^2 + x^4)^(3/2))/7 - (138*Sqrt[2]*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sq
rt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\int \left(4+3x^2+x^4\right)^{3/2}\,dx &= \frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{3}{7}\int \left(8+3x^2\right)\sqrt{4+3x^2+x^4}\,dx\\ &= \frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{35}\int \frac{284+138x^2}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}-\frac{276}{35}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+1\\ &= \frac{138x\sqrt{4+3x^2+x^4}}{35\left(2+x^2\right)}+\frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}-\frac{138\sqrt{2}}{}\end{aligned}$$

Mathematica [C] time = 0.42, size = 343, normalized size = 1.73

$$\frac{\sqrt{2}\left(69\sqrt{7}-77i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-69\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}}{70\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(276 + 303*x^2 + 161*x^4 + 39*x^6 + 5*x^8)
 - 69*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt
[2]*(-77*I + 69*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])
]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqr
t[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(70*Sqrt[(
-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(x^4+3\,x^2+4\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

maple [C] time = 0.00, size = 258, normalized size = 1.30

$$\frac{\sqrt{x^4+3x^2+4}\;x^5}{7}+\frac{24\sqrt{x^4+3x^2+4}\;x^3}{35}+\frac{69\sqrt{x^4+3x^2+4}\;x}{35}+\frac{1136\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\;\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}}{35\sqrt{-6+2i\sqrt{7}}\;\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 1/7*(x^4+3*x^2+4)^(1/2)*x^5+24/35*(x^4+3*x^2+4)^(1/2)*x^3+69/35*(x^4+3*x^2+
4)^(1/2)*x+1136/35/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/
2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-
6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-4416/35/(-6+2*I*7^(1/2))^
(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/
2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*
x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+
6*I*7^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x^4+3\,x^2+4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((3*x^2 + x^4 + 4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+3x^2+4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 4)**(3/2), x)
```

3.361 $\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2} dx$

Optimal. Leaf size=284

$$\frac{1}{75}(3x^2+11)\sqrt{x^4+3x^2+4}\,x+\frac{94\sqrt{x^4+3x^2+4}\,x}{125(x^2+2)}+\frac{44}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)+\frac{54\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+}{(x^2+2)}}}{125\sqrt{x^4+}}$$

```
[Out] 44/4375*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+94/125*x*(x^
4+3*x^2+4)^(1/2)/(x^2+2)+1/75*x*(3*x^2+11)*(x^4+3*x^2+4)^(1/2)-94/125*(x^2+
2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Elli
pticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2
+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+54/125*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)
))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/
2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/
2)+4114/13125*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1
/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*
((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1208, 1176, 1197, 1103, 1195, 1216, 1706}

$$\frac{1}{75}(3x^2+11)\sqrt{x^4+3x^2+4}\,x+\frac{94\sqrt{x^4+3x^2+4}\,x}{125(x^2+2)}+\frac{44}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)+\frac{54\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+}{(x^2+2)}}}{125\sqrt{x^4+}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (94*x*Sqrt[4 + 3*x^2 + x^4])/(125*(2 + x^2)) + (x*(11 + 3*x^2)*Sqrt[4 + 3*x
^2 + x^4])/75 + (44*Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x
^4]])/125 - (94*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellip
ticE[2*ArcTan[x/Sqrt[2]], 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (54*Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (4114*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(13125*
Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int \left(-8-5x^2\right)\sqrt{4+3x^2+x^4}\,dx\right)+\frac{44}{25}\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$$

$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}-\frac{1}{375}\int \frac{-260-150x^2}{\sqrt{4+3x^2+x^4}}\,dx-\frac{44}{625}\int \frac{-8-5x^2}{\sqrt{4+3x^2+x^4}}\,d$$

$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}-\frac{88}{125}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-\frac{4}{5}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-$$

$$= \frac{94x\sqrt{4+3x^2+x^4}}{125\left(2+x^2\right)}+\frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}+\frac{44}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x}}\right.$$

Mathematica [C] time = 0.72, size = 477, normalized size = 1.68

$$7\sqrt{2}\left(705\sqrt{7}-241i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-4935\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2}{\sqrt{}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (350*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(44 + 45*x^2 + 20*x^4 + 3*x^6) - 4935*Sq
rt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*S
qrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*Sqrt[2]*(-
241*I + 705*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sq
rt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-
2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (5808*I)*Sqrt
[2]*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh
[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(26250
*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{5\,x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

maple [C] time = 0.02, size = 418, normalized size = 1.47

$$\frac{\sqrt{x^4+3x^2+4}\,x^3}{25}+\frac{11\sqrt{x^4+3x^2+4}\,x}{75}+\frac{3008\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\,\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2}}{}\right.}{125\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(i\sqrt{7}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x)
```

```
[Out] 1/25*(x^4+3*x^2+4)^(1/2)*x^3+11/75*(x^4+3*x^2+4)^(1/2)*x+9424/1875/(-6+2*I*
7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x
^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*
(2+6*I*7^(1/2))^(1/2))-3008/125/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/
2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*
7^(1/2)+3)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)
)+3008/125/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*
x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(
1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+1936/4375/(-3/8+1/8
*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2
)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,
-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+3\,x^2+4\right)^{3/2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7),x)
```

```
[Out] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7), x)
```

3.362 $\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2} dx$

Optimal. Leaf size=305

$$\frac{4\sqrt{x^4+3x^2+4}\,x}{175\left(x^2+2\right)}+\frac{22\sqrt{x^4+3x^2+4}\,x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+4}\,x+\frac{13}{350}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)+\frac{4\sqrt{2}\left(x^2+2\right)}{175}$$

```
[Out] 13/12250*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/75*x*(x^4
+3*x^2+4)^(1/2)+4/175*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+22/175*x*(x^4+3*x^2+4)^
(1/2)/(5*x^2+7)+2431/73500*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/c
os(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,
1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-4/
175*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/
2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^
2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+4/175*(x^2+2)*(cos(2*arctan(1/2*x
*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2
*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^
2+4)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 372, normalized size of antiderivative = 1.22, number of steps used = 19, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1228, 1103, 1139, 1195, 1122, 1197, 1223, 1714, 1708, 1706, 1216}

$$\frac{4\sqrt{x^4+3x^2+4}\,x}{175\left(x^2+2\right)}+\frac{22\sqrt{x^4+3x^2+4}\,x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+4}\,x+\frac{13}{350}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)+\frac{4\sqrt{2}\left(x^2+2\right)}{175}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/75 + (4*x*Sqrt[4 + 3*x^2 + x^4])/(175*(2 + x^2))
+ (22*x*Sqrt[4 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) + (13*Sqrt[11/35]*ArcTan[(
2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/350 - (4*Sqrt[2]*(2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt
[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (6919
*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[
x/Sqrt[2]], 1/8])/(183750*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (187*Sqrt[2]*(2
+ x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sq
rt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
```

```
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
```

```
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx = \int \left(\frac{152}{625\sqrt{4+3x^2+x^4}} + \frac{16x^2}{125\sqrt{4+3x^2+x^4}} + \frac{x^4}{25\sqrt{4+3x^2+x^4}} + \frac{1936}{625\left(7+5x^2\right)^2\sqrt{4\ldots}}\right.$$

$$= \frac{1}{25}\int \frac{x^4}{\sqrt{4+3x^2+x^4}}\,dx + \frac{16}{125}\int \frac{x^2}{\sqrt{4+3x^2+x^4}}\,dx + \frac{88}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+\ldots}}$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4} + \frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)} + \frac{38\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\ldots\right.}{625\sqrt{4+3x^2+x^4}}$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4} + \frac{16x\sqrt{4+3x^2+x^4}}{125\left(2+x^2\right)} + \frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)} + \frac{2}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{\ldots}{\sqrt{4\ldots}}\right.$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4} + \frac{4x\sqrt{4+3x^2+x^4}}{175\left(2+x^2\right)} + \frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)} + \frac{2}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\ldots}{\sqrt{4\ldots}}\right.$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4} + \frac{4x\sqrt{4+3x^2+x^4}}{175\left(2+x^2\right)} + \frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)} + \frac{13}{350}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\ldots}{\sqrt{4\ldots}}\right.$$

Mathematica [C] time = 0.57, size = 309, normalized size = 1.01

$$\frac{\frac{175x(7x^2+23)(x^4+3x^2+4)}{5x^2+7} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(7\left(158+15i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-}{3i+}\right.\right.}{18375\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] ((175*x*(23 + 7*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2) - I*Sqrt[6 + (2*I)*Sqrt
[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqr
t[7])]*(105*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(158 + (15*I)*Sqrt[7])*Elliptic
F[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7
])] + 429*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(18375*Sqrt[4 + 3*x^2 + x^
4])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{25\,x^4+70\,x^2+49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

maple [C] time = 0.03, size = 425, normalized size = 1.39

$$\frac{22\sqrt{x^4+3x^2+4}\;x}{175\left(5x^2+7\right)}+\frac{\sqrt{x^4+3x^2+4}\;x}{75}+\frac{128\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\;\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\;\mathrm{EllipticE}\left(\frac{\sqrt{-6+2i\sqrt{7}}\;x}{4}, \frac{\sqrt{2+6}}{4}\right.}{175\sqrt{-6+2i\sqrt{7}}\;\sqrt{x^4+3x^2+4}\;\left(i\sqrt{7}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x)
```

```
[Out] 22/175*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)*x+1/75*(x^4+3*x^2+4)^(1/2)*x+232/375/(
-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^
(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)
*x,1/4*(2+6*I*7^(1/2))^(1/2))-128/175/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I
*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/
```

```
2)/(I*7^(1/2)+3)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))
^(1/2))+128/175/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*
(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*Ellip
ticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+286/6125/(-3/8
+1/8*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^
(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2
)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2
))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+3x^2+4\right)^{3/2}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^2,x)
[Out] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**2,x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**2, x)
```

3.363 $\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx$

Optimal. Leaf size=440

$$\frac{9\sqrt{x^4+3x^2+4}\,x}{1960\left(x^2+2\right)}+\frac{167\sqrt{x^4+3x^2+4}\,x}{9800\left(5x^2+7\right)}+\frac{11\sqrt{x^4+3x^2+4}\,x}{175\left(5x^2+7\right)^2}+\frac{1347\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{7840\sqrt{385}}-\frac{22\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x}{(x^2+}}}{13125\sqrt{x^4}}$$

```
[Out] 1347/3018400*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+9/1960*
x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+11/175*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+167/
9800*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-9/1960*(x^2+2)*(cos(2*arctan(1/2*x*2^(
1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2
^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)
^(1/2)-3/490*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/
2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x
^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+7633/548800*(x^2+2)*(cos
(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(
sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^
(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1103, 1139, 1195, 1223, 1696, 1714, 1708, 1706, 1216}

$$\frac{9\sqrt{x^4+3x^2+4}\,x}{1960\left(x^2+2\right)}+\frac{167\sqrt{x^4+3x^2+4}\,x}{9800\left(5x^2+7\right)}+\frac{11\sqrt{x^4+3x^2+4}\,x}{175\left(5x^2+7\right)^2}+\frac{1347\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{7840\sqrt{385}}-\frac{22\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x}{(x^2+}}}{13125\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(1960*(2 + x^2)) + (11*x*Sqrt[4 + 3*x^2 + x^4])
/(175*(7 + 5*x^2)^2) + (167*x*Sqrt[4 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) + (
1347*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(7840*Sqrt[385]) + (1
11*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[
2]], 1/8])/(24500*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (6*Sqrt[2]*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(875*
Sqrt[4 + 3*x^2 + x^4]) - (817*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(91875*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
 - (22*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Ar
cTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4]) + (7633*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8
])/(274400*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
```

```
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx = \int \left(\frac{9}{625\sqrt{4+3x^2+x^4}} + \frac{x^2}{125\sqrt{4+3x^2+x^4}} + \frac{1936}{625\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}} + \frac{\ }{62\ldots}\right.$$

$$= \frac{1}{125}\int \frac{x^2}{\sqrt{4+3x^2+x^4}}\,dx + \frac{9}{625}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{88}{625}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3\ldots}}$$

$$= \frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2} + \frac{x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)} + \frac{9\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1250\sqrt{2}\sqrt{4+3x^2+x^4}} \ldots$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{125\left(2+x^2\right)} + \frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2} + \frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)} + \frac{89\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\ldots\right.}{500\sqrt{385}}$$

$$= \frac{6x\sqrt{4+3x^2+x^4}}{875\left(2+x^2\right)} + \frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2} + \frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)} + \frac{89\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+\ldots}}\right.}{500\sqrt{385}}$$

$$= \frac{9x\sqrt{4+3x^2+x^4}}{1960\left(2+x^2\right)} + \frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2} + \frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)} + \frac{3}{175}\sqrt{\frac{11}{35}}\tan^{-1}\left(\ldots\right.$$

$$= \frac{9x\sqrt{4+3x^2+x^4}}{1960\left(2+x^2\right)} + \frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2} + \frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)} + \frac{3}{175}\sqrt{\frac{11}{35}}\tan^{-1}\left(\ldots\right.$$

Mathematica [C] time = 0.69, size = 309, normalized size = 0.70

$$\frac{\frac{140x(167x^2+357)(x^4+3x^2+4)}{(5x^2+7)^2} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(7\left(103+45i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)\middle|\frac{3i-}{3i+}\right.\right.}{274400\sqrt{x^{}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] ((140*x*(357 + 167*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 - I*Sqrt[6 + (2*I)
*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
+ Sqrt[7])]*(315*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(103 + (45*I)*Sqrt[7])*Ell
ipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + S
qrt[7])] + 2694*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-
3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(274400*Sqrt[4 + 3*x
^2 + x^4])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{125\,x^6+525\,x^4+735\,x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

maple [C] time = 0.03, size = 434, normalized size = 0.99

$$\frac{11\sqrt{x^4+3x^2+4}\,x}{175\left(5x^2+7\right)^2}+\frac{167\sqrt{x^4+3x^2+4}\,x}{9800\left(5x^2+7\right)}+\frac{36\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\,\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\,\operatorname{EllipticE}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i}}{4}\right.}{245\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(i\sqrt{7}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x)
```

```
[Out] 11/175*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2*x+167/9800*(x^4+3*x^2+4)^(1/2)/(5*x^
2+7)*x+17/350/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3
/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I
*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-36/245/(-6+2*I*7^(1/2))^(1/2)*
(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^
```

```
4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(
2+6*I*7^(1/2))^(1/2))+36/245/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*
x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(
1/2)+3)*EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+1
347/68600/(-3/8+1/8*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3
/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*
I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3
/8+1/8*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+3\,x^2+4\right)^{3/2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^3,x)
[Out] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**3,x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**3, x)
```

3.364 $\int \frac{\left(7+5x^2\right)^3}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=187

$$-\frac{15\sqrt{x^4+3x^2+4}\,x}{x^2+2}+75\sqrt{x^4+3x^2+4}\,x+\frac{13\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{15\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{\sqrt{x^4+3x^2}}$$

```
[Out] 75*x*(x^4+3*x^2+4)^(1/2)+25*x^3*(x^4+3*x^2+4)^(1/2)-15*x*(x^4+3*x^2+4)^(1/2
)/(x^2+2)+13/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(
1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3
*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+15*(x^2+2)*(cos(2*arct
an(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*ar
ctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1206, 1679, 1197, 1103, 1195}

$$25\sqrt{x^4+3x^2+4}\,x^3-\frac{15\sqrt{x^4+3x^2+4}\,x}{x^2+2}+75\sqrt{x^4+3x^2+4}\,x+\frac{13\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{15\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] 75*x*Sqrt[4 + 3*x^2 + x^4] + 25*x^3*Sqrt[4 + 3*x^2 + x^4] - (15*x*Sqrt[4 +
3*x^2 + x^4])/(2 + x^2) + (15*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
 x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (13*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx &= 25x^3\sqrt{4+3x^2+x^4}+\frac{1}{5}\int \frac{1715+2175x^2+1125x^4}{\sqrt{4+3x^2+x^4}}\,dx\\ &= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}+\frac{1}{15}\int \frac{645-225x^2}{\sqrt{4+3x^2+x^4}}\,dx\\ &= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}+13\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+30\int \frac{1-\cdots}{\sqrt{4+3x^2\cdots}}\\ &= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}-\frac{15x\sqrt{4+3x^2+x^4}}{2+x^2}+\frac{15\sqrt{2}\,(2+x^2)\sqrt{\frac{4+\cdots}{(2\cdots}}}{\sqrt{4+\cdots}}\end{aligned}$$

Mathematica [C] time = 0.48, size = 337, normalized size = 1.80

$$\frac{-\sqrt{2}\left(15\sqrt{7}+131i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+15\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\cdots}{\sqrt{7}\cdots}}\cdots}{4\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (100*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(12 + 13*x^2 + 6*x^4 + x^6) + 15*Sqrt[2]
*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(
3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)
/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(131*I +
15*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I +
 Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*
I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(4*Sqrt[(-I)/(-3*I + Sq
rt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{125\,x^6+525\,x^4+735\,x^2+343}{\sqrt{x^4+3\,x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)/sqrt(x^4 + 3*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

maple [C] time = 0.03, size = 241, normalized size = 1.29

$$25\sqrt{x^4+3x^2+4}\,x^3+75\sqrt{x^4+3x^2+4}\,x+\frac{172\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6+2i\sqrt{7}}}{4}\right.}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25*(x^4+3*x^2+4)^(1/2)*x^3+75*(x^4+3*x^2+4)^(1/2)*x+172/(-6+2*I*7^(1/2))^(1
/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)
/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1
/2))^(1/2))+480/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*
(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(Elli
pticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4
*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+7\right)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^3}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

3.365 $\int \frac{\left(7+5x^2\right)^2}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=170

$$\frac{20\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{25}{3}\sqrt{x^4+3x^2+4}\,x+\frac{167\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{20\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{\sqrt{x^4+3x^2}}$$

```
[Out] 25/3*x*(x^4+3*x^2+4)^(1/2)+20*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+167/12*(x^2+2)*
(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellipti
cF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2
)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-20*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(
1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/
4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1206, 1197, 1103, 1195}

$$\frac{20\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{25}{3}\sqrt{x^4+3x^2+4}\,x+\frac{167\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{20\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{\sqrt{x^4+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*Sqrt[4 + 3*x^2 + x^4])/3 + (20*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (
20*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan
[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (167*(2 + x^2)*Sqrt[(4 + 3*x^2 +
 x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(6*Sqrt[2]*Sqrt[4 +
 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx &= \frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{1}{3}\int \frac{47+60x^2}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{25}{3}x\sqrt{4+3x^2+x^4}-40\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{167}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{20x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{20\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right.}{\sqrt{4+3x^2+x^4}}\end{aligned}$$

Mathematica [C] time = 0.43, size = 331, normalized size = 1.95

$$\frac{\sqrt{2}\left(30\sqrt{7}+43i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-30\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{\frac{-2ix^2+\sqrt{7}}{\sqrt{7}-3}}}{6\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (50*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 30*Sqrt[2]*(3*I + Sqr
t[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[
7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(43*I + 30*Sqrt[7])*
Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (
2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])
]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(6*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt
[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{25\,x^4+70\,x^2+49}{\sqrt{x^4+3\,x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)/sqrt(x^4 + 3*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

maple [C] time = 0.01, size = 224, normalized size = 1.32

$$\frac{25\sqrt{x^4+3x^2+4}\,x}{3}+\frac{188\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{640\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x)
[Out] 25/3*(x^4+3*x^2+4)^(1/2)*x+188/3/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/
2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*El
lipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-640/(-6+2*I
*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*
x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*(-6+2*I*7^(1/
2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)
*x,1/4*(2+6*I*7^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^2}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

3.366 $\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=151

$$\frac{5\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{17\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{5\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

```
[Out] 5*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+17/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^
2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))
),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-
5*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+
4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1197, 1103, 1195}

$$\frac{5\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{17\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{5\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (5*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 +
 x^4] + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTa
n[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = -\left(10\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\right)+17\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{5x\sqrt{4+3x^2+x^4}}{2+x^2} - \frac{5\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}} + \frac{17\left(2+x^2\right)\sqrt{\frac{4+3x^2+}{(2+x^2)}}}{2\sqrt{2}\sqrt{4}}$$

Mathematica [C] time = 0.18, size = 214, normalized size = 1.42

$$\frac{\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\left(5\sqrt{7}+i\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-5\left(\sqrt{7}+3i\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\right.\right.}{2\sqrt{2}\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7]
)]*(-5*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (I + 5*Sqrt[7])*EllipticF[I*ArcSinh[Sq
rt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(2*Sqrt[
2]*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

maple [C] time = 0.00, size = 209, normalized size = 1.38

$$\frac{28\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} - \frac{160\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] -160/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8
*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*(
-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*I*7^
(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))+28/(-6+2*I*7^(1/2))^(1/2)*(-(-3/
8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^
2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+7}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
[Out] Integral((5*x**2 + 7)/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

3.367 $\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=64

$$\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] 1/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/
2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^
2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1103}

$$\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]
], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx=\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] time = 0.05, size = 142, normalized size = 2.22

$$-\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}\,x\right)|\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt[7])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7])]*x], (-3 - I*Sqrt[7])/(-3 + I
*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^(-1)]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{x^4+3\,x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(x^4 + 3*x^2 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

maple [C] time = 0.00, size = 85, normalized size = 1.33

$$\frac{4\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+4)^(1/2),x)
[Out] 4/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*
7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1
/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int(1/(3*x^2 + x^4 + 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 3*x**2 + 4), x)
```

3.368 $\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=168

$$\frac{1}{4}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\tan\right.}{84\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] 1/308*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/12*(x^2+2)*(
cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Elliptic
F(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)
*2^(1/2)/(x^4+3*x^2+4)^(1/2)+17/168*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2
)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))
),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)
^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1216, 1103, 1706}

$$\frac{1}{4}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\tan\right.}{84\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/4 - ((2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(
6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(84*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
```

```
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\right)+\frac{10}{3}\int \frac{1+\frac{x^2}{2}}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{1}{4}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)-\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{4+3x^2+x^4}}+ \text{—}\ 17$$

Mathematica [C] time = 0.14, size = 159, normalized size = 0.95

$$-\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,\Pi\left(-\frac{5}{14}\left(-3-i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}\,x\right)|\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{7\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] ((-1/7*I)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt[
7])]*EllipticPi[(-5*(-3 - I*Sqrt[7]))/14, I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7]
)]*x], (-3 - I*Sqrt[7])/(-3 + I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^
(-1)]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}}{5\,x^6+22\,x^4+41\,x^2+28},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^6 + 22*x^4 + 41*x^2 + 28), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

maple [C] time = 0.02, size = 107, normalized size = 0.64

$$\frac{\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\ \text{EllipticPi}\left(\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,x,-\frac{5}{7\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)},\frac{\sqrt{-\frac{3}{8}-\frac{i\sqrt{7}}{8}}}{\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}}\right)}{7\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\sqrt{x^4+3x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 1/7/(-3/8+1/8*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2
+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1
/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8
*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(5\,x^2+7\right)\,\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\,\left(5x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)), x)
```

3.369 $\int \frac{1}{(7+5x^2)^2 \sqrt{4+3x^2+x^4}} \, dx$

Optimal. Leaf size=286

$$-\frac{5\sqrt{x^4+3x^2+4}\,x}{616(x^2+2)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{616(5x^2+7)}+\frac{37\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{2464}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+4}}+\ldots 5$$

```
[Out] 37/189728*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-5/616*x*(x
^4+3*x^2+4)^(1/2)/(x^2+2)+25/616*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+5/616*(x^2
+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ell
ipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^
2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-1/84*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2
))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2
)+629/103488*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/
2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*(
(x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1223, 1714, 1195, 1708, 1103, 1706}

$$-\frac{5\sqrt{x^4+3x^2+4}\,x}{616(x^2+2)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{616(5x^2+7)}+\frac{37\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{2464}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+4}}+\ldots 5$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (-5*x*Sqrt[4 + 3*x^2 + x^4])/(616*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 + x^4])
/(616*(7 + 5*x^2)) + (37*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2
 + x^4]])/2464 + (5*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE
[2*ArcTan[x/Sqrt[2]], 1/8])/(308*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2
)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/
(42*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (629*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(51744*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx = \frac{25x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)} - \frac{1}{616}\int \frac{12+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{25x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)} - \frac{\int \frac{410+425x^2}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{3080} + \frac{5}{308}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{5x\sqrt{4+3x^2+x^4}}{616\left(2+x^2\right)} + \frac{25x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)} + \frac{5\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{308\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$= -\frac{5x\sqrt{4+3x^2+x^4}}{616\left(2+x^2\right)} + \frac{25x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)} + \frac{37\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)}{2464} + \frac{5\left(2\right.}{}$$

Mathematica [C] time = 0.78, size = 481, normalized size = 1.68

$$\frac{98i\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right) - 74i\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I + Sqrt[7])*(
7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])] - EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]) + (98*I)*(7 + 5*x^2)*Sqrt[2 - ((4*
I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I
*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
 - (74*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(17248*Sqrt[(
-I)/(-3*I + Sqrt[7])]*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}}{25\,x^8+145\,x^6+359\,x^4+427\,x^2+196},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^8 + 145*x^6 + 359*x^4 + 427*x^2 + 196)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

maple [C] time = 0.02, size = 410, normalized size = 1.43

$$\frac{25\sqrt{x^4+3x^2+4}\,x}{616\left(5x^2+7\right)}-\frac{20\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\,\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\,\operatorname{EllipticE}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(i\sqrt{7}+3\right)}-\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25/616*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)*x-1/22/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2
-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+
4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+
20/77/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1
/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*(
-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-20/77/(-6+2*I*7^(1/2))^(
1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2
)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,
1/4*(2+6*I*7^(1/2))^(1/2))+37/4312/(-3/8+1/8*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*
I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1
/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8
-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(5\,x^2+7\right)^2\,\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\,\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2), x)
```

3.370 $\int \frac{1}{\left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}} \, dx$

Optimal. Leaf size=314

$$-\frac{555\sqrt{x^4+3x^2+4}\,x}{758912\left(x^2+2\right)}+\frac{2775\sqrt{x^4+3x^2+4}\,x}{758912\left(5x^2+7\right)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{1232\left(5x^2+7\right)^2}-\frac{3285\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{3035648}-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3}{(x^2}}}{8624\sqrt{}}$$

```
[Out] -3285/233744896*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-555/
758912*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+25/1232*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7
)^2+2775/758912*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+555/758912*(x^2+2)*(cos(2*a
rctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2
*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2
)/(x^4+3*x^2+4)^(1/2)-1/17248*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2
)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2
^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-18615/4
2499072*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2
^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1223, 1696, 1714, 1195, 1708, 1103, 1706}

$$-\frac{555\sqrt{x^4+3x^2+4}\,x}{758912\left(x^2+2\right)}+\frac{2775\sqrt{x^4+3x^2+4}\,x}{758912\left(5x^2+7\right)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{1232\left(5x^2+7\right)^2}-\frac{3285\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{3035648}-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3}{(x^2}}}{8624\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (-555*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 +
x^4])/(1232*(7 + 5*x^2)^2) + (2775*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(7 + 5*
x^2)) - (3285*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/3
035648 + (555*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*Arc
Tan[x/Sqrt[2]], 1/8])/(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(86
24*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (18615*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)
/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(21249536*Sqrt[
2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}}\,dx = \frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2} - \frac{\int \frac{-76-10x^2-25x^4}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx}{1232}$$

$$= \frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\left(7+5x^2\right)} + \frac{\int \frac{-4412-4690x^2-2775x^4}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{758912}$$

$$= \frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\left(7+5x^2\right)} + \frac{\int \frac{-60910-31775x^2}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{3794560} + \frac{555\int}{3}$$

$$= -\frac{555x\sqrt{4+3x^2+x^4}}{758912\left(2+x^2\right)} + \frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\left(7+5x^2\right)} + \frac{555\left(2\right.}{}$$

$$= -\frac{555x\sqrt{4+3x^2+x^4}}{758912\left(2+x^2\right)} + \frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\left(7+5x^2\right)} - \frac{3285\sqrt{}}{}$$

Mathematica [C] time = 0.89, size = 308, normalized size = 0.98

$$\frac{\frac{700x\left(555x^2+1393\right)\left(x^4+3x^2+4\right)}{\left(5x^2+7\right)^2} + i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\left(-9401+3885i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)\right.\right.}{21249536}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] ((700*x*(1393 + 555*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*I
)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(3885*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9401 + (3885*I)*Sqrt[7])
*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I
 + Sqrt[7])] + 6570*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I
)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(21249536*Sqrt[4
 + 3*x^2 + x^4])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}}{125\,x^{10}+900\,x^8+2810\,x^6+4648\,x^4+3969\,x^2+1372}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^10 + 900*x^8 + 2810*x^6 + 4648*x^4 +
3969*x^2 + 1372), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

maple [C] time = 0.02, size = 434, normalized size = 1.38

$$\frac{25\sqrt{x^4+3x^2+4}\,x}{1232\left(5x^2+7\right)^2}+\frac{2775\sqrt{x^4+3x^2+4}\,x}{758912\left(5x^2+7\right)}-\frac{555\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\,\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\right.}{23716\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(i\sqrt{7}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25/1232*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2*x+2775/758912*(x^4+3*x^2+4)^(1/2)/(
5*x^2+7)*x-23/27104/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1
/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(
-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+555/23716/(-6+2*I*7^(1/2
))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^
(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2
)*x,1/4*(2+6*I*7^(1/2))^(1/2))-555/23716/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/
8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^
(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/
2))^(1/2))-3285/5312384/(-3/8+1/8*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x
^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipti
cPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1
/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+4}\,\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(5\,x^2+7\right)^3\,\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\,\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3), x)
```

3.371 $\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{220779\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}+\frac{5000}{3}\sqrt{x^4+3x^2+4}\,x+\frac{\left(45779x^2+99493\right)x}{28\sqrt{x^4+3x^2+4}}-\frac{130729\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{12\sqrt{2}\sqrt{x^4+3x^2+4}}$$

[Out] 1/28*x*(45779*x^2+99493)/(x^4+3*x^2+4)^(1/2)+5000/3*x*(x^4+3*x^2+4)^(1/2)+625*x^3*(x^4+3*x^2+4)^(1/2)-220779/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+220779/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-130729/24*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)

Rubi [A] time = 0.12, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1679, 1197, 1103, 1195}

$$625\sqrt{x^4+3x^2+4}\,x^3-\frac{220779\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}+\frac{5000}{3}\sqrt{x^4+3x^2+4}\,x+\frac{\left(45779x^2+99493\right)x}{28\sqrt{x^4+3x^2+4}}-\frac{130729\left(x^2+2\right)}{12\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(99493 + 45779*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (5000*x*Sqrt[4 + 3*x^2
 + x^4])/3 + 625*x^3*Sqrt[4 + 3*x^2 + x^4] - (220779*x*Sqrt[4 + 3*x^2 + x^4
])/(28*(2 + x^2)) + (220779*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*E
llipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (1
30729*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sq
rt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(7+5x^2\right)^5}{\left(4+3x^2+x^4\right)^{3/2}}\,dx = \frac{x\left(99493+45779x^2\right)}{28\sqrt{4+3x^2+x^4}} + \frac{1}{28}\int \frac{18156+269221x^2+350000x^4+87500x^6}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x\left(99493+45779x^2\right)}{28\sqrt{4+3x^2+x^4}} + 625x^3\sqrt{4+3x^2+x^4} + \frac{1}{140}\int \frac{90780+296105x^2+70000}{\sqrt{4+3x^2+x^4}}$$

$$= \frac{x\left(99493+45779x^2\right)}{28\sqrt{4+3x^2+x^4}} + \frac{5000}{3}x\sqrt{4+3x^2+x^4} + 625x^3\sqrt{4+3x^2+x^4} + \frac{1}{420}\int \frac{-25}{}$$

$$= \frac{x\left(99493+45779x^2\right)}{28\sqrt{4+3x^2+x^4}} + \frac{5000}{3}x\sqrt{4+3x^2+x^4} + 625x^3\sqrt{4+3x^2+x^4} + \frac{220779}{14}\int$$

$$= \frac{x\left(99493+45779x^2\right)}{28\sqrt{4+3x^2+x^4}} + \frac{5000}{3}x\sqrt{4+3x^2+x^4} + 625x^3\sqrt{4+3x^2+x^4} - \frac{220779x\sqrt{}}{28\,(2}$$

Mathematica [C] time = 0.52, size = 339, normalized size = 1.55

$$\frac{-\sqrt{2}\left(662337\sqrt{7}+975947i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+662337\sqrt{2}\left(\sqrt{7}\right.}{336}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(858479 + 767337*x^2 + 297500*x^4 + 52500*
x^6) + 662337*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3
*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[
I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])
] - Sqrt[2]*(975947*I + 662337*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(
-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*Elliptic
F[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7
])])/(336*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3125\,x^{10}+21875\,x^{8}+61250\,x^{6}+85750\,x^{4}+60025\,x^{2}+16807\right)\sqrt{x^{4}+3\,x^{2}+4}}{x^{8}+6\,x^{6}+17\,x^{4}+24\,x^{2}+16},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*x^2 + 16807
)*sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^{2}+7\right)^{5}}{\left(x^{4}+3\,x^{2}+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

maple [C] time = 0.05, size = 379, normalized size = 1.73

$$625\sqrt{x^{4}+3x^{2}+4}\;x^{3}+\frac{5000\sqrt{x^{4}+3x^{2}+4}\;x}{3}-\frac{505532\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}+1}\;\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^{2}+1}\;\text{EllipticF}\left(\cdots\right.}{21\sqrt{-6+2i\sqrt{7}}\;\sqrt{x^{4}+3x^{2}+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -6250*(31/14*x^3+18/7*x)/(x^4+3*x^2+4)^(1/2)+625*(x^4+3*x^2+4)^(1/2)*x^3+50
00/3*(x^4+3*x^2+4)^(1/2)*x-505532/21/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7
^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2
)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+1766232
/7/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I
*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*(-6
+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*I*7^(1
/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))-43750*(-9/14*x^3+2/7*x)/(x^4+3*x^2
+4)^(1/2)-122500*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)-171500*(3/14*x^3+4/7
*x)/(x^4+3*x^2+4)^(1/2)-120050*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-33614*
(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^{2}+7\right)^{5}}{\left(x^{4}+3\,x^{2}+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(x^4+3\,x^2+4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^5}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**5/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral((5*x**2 + 7)**5/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

3.372 $\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=200

$$\frac{14523\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}+\frac{625}{3}\sqrt{x^4+3x^2+4}\,x+\frac{(2719-4023x^2)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{4243\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}-$$

```
[Out] 1/28*x*(-4023*x^2+2719)/(x^4+3*x^2+4)^(1/2)+625/3*x*(x^4+3*x^2+4)^(1/2)+145
23/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-14523/28*(x^2+2)*(cos(2*arctan(1/2*x*2^
(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*
2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4
)^(1/2)+4243/24*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan
(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1679, 1197, 1103, 1195}

$$\frac{14523\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}+\frac{625}{3}\sqrt{x^4+3x^2+4}\,x+\frac{(2719-4023x^2)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{4243\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(2719 - 4023*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (625*x*Sqrt[4 + 3*x^2 +
x^4])/3 + (14523*x*Sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)) - (14523*(2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(
14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (4243*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^
2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
```

```
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx &= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int \frac{14088+49523x^2+17500x^4}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{625}{3}x\sqrt{4+3x^2+x^4}+\frac{1}{84}\int \frac{-27736+43569x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{625}{3}x\sqrt{4+3x^2+x^4}+\frac{4243}{6}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx-\frac{14523}{14}\int\\
&= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{625}{3}x\sqrt{4+3x^2+x^4}+\frac{14523x\sqrt{4+3x^2+x^4}}{28(2+x^2)}-\frac{14523(2+x}{1}
\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(625\,x^8+3500\,x^6+7350\,x^4+6860\,x^2+2401)\sqrt{x^4+3\,x^2+4}}{x^8+6\,x^6+17\,x^4+24\,x^2+16},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(x^4 + 3*x^2
 + 4)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^4}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

maple [C] time = 0.01, size = 339, normalized size = 1.70

$$\frac{625\sqrt{x^4+3x^2+4}\,x}{3}-\frac{27736\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2+1}\,\,\mathrm{EllipticF}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -1250*(-9/14*x^3+2/7*x)/(x^4+3*x^2+4)^(1/2)+625/3*(x^4+3*x^2+4)^(1/2)*x-277
36/21/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/
8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2)
)^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-116184/7/(-6+2*I*7^(1/2))^(1/2)*(-(-3/
8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^
2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I
*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))
^(1/2)))-7000*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)-14700*(3/14*x^3+4/7*x)/
(x^4+3*x^2+4)^(1/2)-13720*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-4802*(3/56*
x^3+1/56*x)/(x^4+3*x^2+4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^4}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(5\,x^2+7\right)}^4}{{\left(x^4+3\,x^2+4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^4}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral((5*x**2 + 7)**4/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

3.373 $\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}} dx$

Optimal. Leaf size=181

$$\frac{4449\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}-\frac{\left(949x^2+2323\right)x}{28\sqrt{x^4+3x^2+4}}+\frac{973\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{4449\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{14\sqrt{2}\sqrt{x^4+}}$$

```
[Out] -1/28*x*(949*x^2+2323)/(x^4+3*x^2+4)^(1/2)+4449/28*x*(x^4+3*x^2+4)^(1/2)/(x
^2+2)-4449/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1
/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*
((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+973/8*(x^2+2)*(cos(2*ar
ctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*
arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)
/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1205, 1197, 1103, 1195}

$$\frac{4449\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}-\frac{\left(949x^2+2323\right)x}{28\sqrt{x^4+3x^2+4}}+\frac{973\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{4449\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{14\sqrt{2}\sqrt{x^4+}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x*(2323 + 949*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (4449*x*Sqrt[4 + 3*x^2 +
 x^4])/(28*(2 + x^2)) - (4449*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) +
(973*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqr
t[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
```

```
c/a]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(7+5x^2\right)^3}{\left(4+3x^2+x^4\right)^{3/2}}\,dx &= -\frac{x\left(2323+949x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int \frac{4724+4449x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x\left(2323+949x^2\right)}{28\sqrt{4+3x^2+x^4}}-\frac{4449}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{973}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x\left(2323+949x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{4449x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}-\frac{4449\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{14\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(125\,x^6+525\,x^4+735\,x^2+343\right)\sqrt{x^4+3\,x^2+4}}{x^8+6\,x^6+17\,x^4+24\,x^2+16},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6
*x^6 + 17*x^4 + 24*x^2 + 16), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

maple [C] time = 0.01, size = 301, normalized size = 1.66

$$\frac{4724\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{250\left(-\frac{1}{14}x^3-\frac{6}{7}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{35592}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x)
[Out] -250*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)+4724/7/(-6+2*I*7^(1/2))^(1/2)*(-
(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+
3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(
1/2))-35592/7/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-
(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(Ellipt
icF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(
-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))-1050*(3/14*x^3+4/7*x)/(
x^4+3*x^2+4)^(1/2)-1470*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-686*(3/56*x^3
+1/56*x)/(x^4+3*x^2+4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(x^4+3\,x^2+4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^3}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral((5*x**2 + 7)**3/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

3.374 $\int \frac{\left(7+5x^2\right)^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{113\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}-\frac{\left(9-113x^2\right)x}{28\sqrt{x^4+3x^2+4}}+\frac{9\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{113\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\cdots}{14\sqrt{2}\sqrt{x^4+\cdots}}$$

```
[Out] -1/28*x*(-113*x^2+9)/(x^4+3*x^2+4)^(1/2)-113/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+
2)+113/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x
*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^
4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+9/8*(x^2+2)*(cos(2*arctan(1
/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan
(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+
3*x^2+4)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1205, 1197, 1103, 1195}

$$-\frac{113\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}-\frac{\left(9-113x^2\right)x}{28\sqrt{x^4+3x^2+4}}+\frac{9\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{113\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\cdots}{14\sqrt{2}\sqrt{x^4+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x*(9 - 113*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (113*x*Sqrt[4 + 3*x^2 + x^4
])/(28*(2 + x^2)) + (113*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Elli
pticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (9*(2
 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
```

```
c/a]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(7+5x^2\right)^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx &= -\frac{x\left(9-113x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int \frac{352-113x^2}{\sqrt{4+3x^2+x^4}}\,dx \\
&= -\frac{x\left(9-113x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{9}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\frac{113}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx \\
&= -\frac{x\left(9-113x^2\right)}{28\sqrt{4+3x^2+x^4}}-\frac{113x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}+\frac{113\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right.}{14\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(25\,x^4+70\,x^2+49\right)\sqrt{x^4+3\,x^2+4}}{x^8+6\,x^6+17\,x^4+24\,x^2+16}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6*x^6 + 17*x^4
 + 24*x^2 + 16), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

maple [C] time = 0.01, size = 278, normalized size = 1.54

$$\frac{352\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{-6+2i\sqrt{7}}\ x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\ \sqrt{x^4+3x^2+4}}-\frac{50\left(\frac{3}{14}x^3+\frac{4}{7}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{904\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -50*(3/14*x^3+4/7*x)/(x^4+3*x^2+4)^(1/2)+352/7/(-6+2*I*7^(1/2))^(1/2)*(-(-3
/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x
^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2
))+904/7/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8
-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1
/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*
I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))-140*(-1/7*x^3-3/14*x)/(x^4+3
*x^2+4)^(1/2)-98*(3/56*x^3+1/56*x)/(x^4+3*x^2+4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(x^4+3\,x^2+4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^2}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

3.375 $\int \frac{7+5x^2}{\left(4+3x^2+x^4\right)^{3/2}} dx$

Optimal. Leaf size=181

$$-\frac{19\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}+\frac{\left(19x^2+53\right)x}{28\sqrt{x^4+3x^2+4}}-\frac{3\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{19\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan\right.}{14\sqrt{2}\sqrt{x^4+3x^2+}}$$

```
[Out] 1/28*x*(19*x^2+53)/(x^4+3*x^2+4)^(1/2)-19/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+
19/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(
1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*
x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-3/8*(x^2+2)*(cos(2*arctan(1/2*x
*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2
*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^
2+4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1178, 1197, 1103, 1195}

$$-\frac{19\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}+\frac{\left(19x^2+53\right)x}{28\sqrt{x^4+3x^2+4}}-\frac{3\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{19\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan\right.}{14\sqrt{2}\sqrt{x^4+3x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(53 + 19*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (19*x*Sqrt[4 + 3*x^2 + x^4])
/(28*(2 + x^2)) + (19*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipti
cE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3*(2 +
x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8
])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
```

```
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{7+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx &= \frac{x\left(53+19x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int \frac{-4-19x^2}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{x\left(53+19x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{19}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-\frac{3}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{x\left(53+19x^2\right)}{28\sqrt{4+3x^2+x^4}}-\frac{19x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}+\frac{19\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right.}{14\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)}{x^8+6\,x^6+17\,x^4+24\,x^2+16},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 +
 16), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^2+7}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

maple [C] time = 0.01, size = 255, normalized size = 1.41

$$-\frac{4\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{10\left(-\frac{1}{7}x^3-\frac{3}{14}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{152\sqrt{-\left(-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -10*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-4/7/(-6+2*I*7^(1/2))^(1/2)*(-(-3/
8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^
2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)
)+152/7/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-
1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/
4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*I
*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))-14*(3/56*x^3+1/56*x)/(x^4+3*x
^2+4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^2+7}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^2+7}{\left(x^4+3\,x^2+4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+7}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

3.376 $\int \frac{1}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{3\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}-\frac{\left(3x^2+1\right)x}{28\sqrt{x^4+3x^2+4}}+\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{3\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,E\left(2\tan\right.}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] -1/28*x*(3*x^2+1)/(x^4+3*x^2+4)^(1/2)+3/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-3/
28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2
)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2
+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+1/8*(x^2+2)*(cos(2*arctan(1/2*x*2^
(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*
2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4
)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1092, 1197, 1103, 1195}

$$\frac{3\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}-\frac{\left(3x^2+1\right)x}{28\sqrt{x^4+3x^2+4}}+\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{3\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,E\left(2\tan\right.}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] -(x*(1 + 3*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (3*x*Sqrt[4 + 3*x^2 + x^4])/(
28*(2 + x^2)) - (3*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + ((2 + x^2)*
Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4
*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(4+3x^2+x^4\right)^{3/2}}\,dx &= -\frac{x\left(1+3x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int \frac{8+3x^2}{\sqrt{4+3x^2+x^4}}\,dx\\ &= -\frac{x\left(1+3x^2\right)}{28\sqrt{4+3x^2+x^4}}-\frac{3}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\ &= -\frac{x\left(1+3x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{3x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}-\frac{3\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}+\end{aligned}$$

Mathematica [C] time = 0.35, size = 328, normalized size = 1.81

$$\frac{-4\sqrt{-\frac{i}{\sqrt{7}-3i}}\,x\left(3x^2+1\right)+\sqrt{2}\left(3\sqrt{7}-7i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-3\sqrt{2}\,(}{112\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (-4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(1 + 3*x^2) - 3*Sqrt[2]*(3*I + Sqrt[7])*S
qrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2
*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]
*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqrt[(-3
*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2
)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3
*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3
*x^2 + x^4])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}}{x^8+6\,x^6+17\,x^4+24\,x^2+16},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

maple [C] time = 0.00, size = 232, normalized size = 1.28

$$\frac{8\sqrt{-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{2\left(\frac{3}{56}x^3+\frac{1}{56}x\right)}{\sqrt{x^4+3x^2+4}}-24\sqrt{-\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -2*(3/56*x^3+1/56*x)/(x^4+3*x^2+4)^(1/2)+8/7/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8
+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2
+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))
-24/7/(-6+2*I*7^(1/2))^(1/2)*(-(-3/8+1/8*I*7^(1/2))*x^2+1)^(1/2)*(-(-3/8-1/
8*I*7^(1/2))*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*
(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*(-6+2*I*7
^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^4+3\,x^2+4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int(1/(3*x^2 + x^4 + 4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 4)**(-3/2), x)
```

3.377 $\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=284

$$\frac{\sqrt{x^4+3x^2+4}\,x}{77\left(x^2+2\right)}-\frac{\left(4x^2+13\right)x}{308\sqrt{x^4+3x^2+4}}+\frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] 25/13552*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/308*x*(4*
x^2+13)/(x^4+3*x^2+4)^(1/2)+1/77*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-1/24*(x^2+2)
*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellipt
icF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/
2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+425/7392*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)
))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1
/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^
2+4)^(1/2)-1/77*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan
(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2
)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1221, 1178, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{x^4+3x^2+4}\,x}{77\left(x^2+2\right)}-\frac{\left(4x^2+13\right)x}{308\sqrt{x^4+3x^2+4}}+\frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(13 + 4*x^2))/(308*Sqrt[4 + 3*x^2 + x^4]) + (x*Sqrt[4 + 3*x^2 + x^4])/(
77*(2 + x^2)) + (25*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^
4]])/176 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE
[2*ArcTan[x/Sqrt[2]], 1/8])/(77*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[
2]*Sqrt[4 + 3*x^2 + x^4]) + (425*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)
^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(3696*Sqrt[2]*Sqrt[4 + 3*
x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx = \frac{1}{44}\int \frac{-8-5x^2}{(4+3x^2+x^4)^{3/2}}\,dx + \frac{25}{44}\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}} + \frac{\int \frac{-4+16x^2}{\sqrt{4+3x^2+x^4}}\,dx}{1232} - \frac{25}{132}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{125}{66}\int$$

$$= -\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}} + \frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right) - \frac{25(2+x^2)\sqrt{\frac{4+}{(2}}}{264\sqrt{2}}$$

$$= -\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}} + \frac{x\sqrt{4+3x^2+x^4}}{77(2+x^2)} + \frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)$$

Mathematica [C] time = 0.55, size = 483, normalized size = 1.70

$$\frac{-8\sqrt{-\frac{i}{\sqrt{7}-3i}}\,x^3 + \sqrt{2}\,(2\sqrt{7}+7i)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right) - 2\sqrt{2}\,(\sqrt{7}+3i}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-26*Sqrt[(-I)/(-3*I + Sqrt[7])]*x - 8*Sqrt[(-I)/(-3*I + Sqrt[7])]*x^3 - 2*
Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]
*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt
[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(7
*I + 2*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3
*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/
(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (25*I)*Sqrt[2]*Sqr
t[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(616*Sqrt[(-I
)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{x^4+3x^2+4}}{5x^{10}+37x^8+127x^6+239x^4+248x^2+112}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^10 + 37*x^8 + 127*x^6 + 239*x^4 + 248*x
^2 + 112), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

maple [C] time = 0.02, size = 409, normalized size = 1.44

$$\frac{32\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\,\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\ \mathrm{EllipticE}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(i\sqrt{7}+3\right)}-\frac{\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\,\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+}}{77\sqrt{-6+2i\sqrt{7}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -2*(1/154*x^3+13/616*x)/(x^4+3*x^2+4)^(1/2)-1/77/(-6+2*I*7^(1/2))^(1/2)*(3/
8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3
*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1
/2))-32/77/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*
x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(
1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+32/77/(-6+2*I*7^(1/
2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)
^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*(-6+2*I*7^(1/2))^(1/
2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+25/308/(-3/8+1/8*I*7^(1/2))^(1/2)*(3/8*x^2-
1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4
)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(
-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(5\,x^2+7\right)\,{\left(x^4+3\,x^2+4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)), x)
```

3.378 $\int \frac{1}{(7+5x^2)^2 (4+3x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=312

$$-\frac{199\sqrt{x^4+3x^2+4}\,x}{27104\,(x^2+2)}+\frac{625\sqrt{x^4+3x^2+4}\,x}{27104\,(5x^2+7)}+\frac{(37x^2+24)\,x}{13552\sqrt{x^4+3x^2+4}}+\frac{575\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{108416}-\frac{2\sqrt{2}\,(x^2+2)}{23}$$

```
[Out] 575/8348032*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/13552*
x*(37*x^2+24)/(x^4+3*x^2+4)^(1/2)-199/27104*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+6
25/27104*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+199/27104*(x^2+2)*(cos(2*arctan(1/
2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(
1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3
*x^2+4)^(1/2)+9775/4553472*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/c
os(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,
1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-2/
231*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/
2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^
2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1178, 1197, 1103, 1195, 1223, 1714, 1708, 1706, 1216}

$$-\frac{199\sqrt{x^4+3x^2+4}\,x}{27104\,(x^2+2)}+\frac{625\sqrt{x^4+3x^2+4}\,x}{27104\,(5x^2+7)}+\frac{(37x^2+24)\,x}{13552\sqrt{x^4+3x^2+4}}+\frac{575\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{108416}-\frac{2\sqrt{2}\,(x^2+2)}{23}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (x*(24 + 37*x^2))/(13552*Sqrt[4 + 3*x^2 + x^4]) - (199*x*Sqrt[4 + 3*x^2 + x
^4])/(27104*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(27104*(7 + 5*x^2))
+ (575*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/108416 +
 (199*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sq
rt[2]], 1/8])/(13552*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (2*Sqrt[2]*(2 + x^2)*
Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2
31*Sqrt[4 + 3*x^2 + x^4]) + (9775*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(2276736*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
```

```
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^2\left(4+3x^2+x^4\right)^{3/2}}\,dx = \int\left(\frac{-36+5x^2}{1936\left(4+3x^2+x^4\right)^{3/2}}+\frac{25}{44\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}-\frac{\ldots}{1936\left(7+5x^2\ldots\right.}\right.$$

$$=\frac{\int\frac{-36+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx}{1936}-\frac{25\int\frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{1936}+\frac{25}{44}\int\frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2\ldots}}$$

$$=\frac{x\left(24+37x^2\right)}{13552\sqrt{4+3x^2+x^4}}+\frac{625x\sqrt{4+3x^2+x^4}}{27104\left(7+5x^2\right)}+\frac{\int\frac{-348-148x^2}{\sqrt{4+3x^2+x^4}}\,dx}{54208}-\frac{25\int\frac{12\ldots}{(7+5x\ldots}}{2\ldots}$$

$$=\frac{x\left(24+37x^2\right)}{13552\sqrt{4+3x^2+x^4}}+\frac{625x\sqrt{4+3x^2+x^4}}{27104\left(7+5x^2\right)}-\frac{25\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{7744}+\ldots$$

$$=\frac{x\left(24+37x^2\right)}{13552\sqrt{4+3x^2+x^4}}-\frac{199x\sqrt{4+3x^2+x^4}}{27104\left(2+x^2\right)}+\frac{625x\sqrt{4+3x^2+x^4}}{27104\left(7+5x^2\right)}-\frac{25\sqrt{\ldots}}{\ldots}$$

$$=\frac{x\left(24+37x^2\right)}{13552\sqrt{4+3x^2+x^4}}-\frac{199x\sqrt{4+3x^2+x^4}}{27104\left(2+x^2\right)}+\frac{625x\sqrt{4+3x^2+x^4}}{27104\left(7+5x^2\right)}+\frac{575\sqrt{\ldots}}{\ldots}$$

Mathematica [C] time = 0.60, size = 311, normalized size = 1.00

$$\frac{28x\left(995x^4+2633x^2+2836\right)+i\sqrt{6+2i\sqrt{7}}\left(5x^2+7\right)\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(7\left(101+199i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\ldots\right.\right.\right.}{758912\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (28*x*(2836 + 2633*x^2 + 995*x^4) + I*Sqrt[6 + (2*I)*Sqrt[7]]*(7 + 5*x^2)*S
qrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]
*(1393*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(101 + (199*I)*Sqrt[7])*EllipticF[I*
ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
- 1150*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqr
t[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(758912*(7 + 5*x^2)*Sqrt[4 +
3*x^2 + x^4])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+4}}{25x^{12}+220x^{10}+894x^8+2084x^6+2913x^4+2296x^2+784}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^12 + 220*x^10 + 894*x^8 + 2084*x^6 + 2
913*x^4 + 2296*x^2 + 784), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3x^2+4\right)^{\frac{3}{2}}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

maple [C] time = 0.03, size = 433, normalized size = 1.39

$$\frac{625\sqrt{x^4+3x^2+4}\,x}{27104\left(5x^2+7\right)}-\frac{199\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\,\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\,\text{EllipticE}\left(\frac{\sqrt{-6+2i\sqrt{7}}\,x}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{847\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(i\sqrt{7}+3\right)}-\frac{349\sqrt{\frac{3x^2}{8}-\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -2*(-37/27104*x^3-3/3388*x)/(x^4+3*x^2+4)^(1/2)+625/27104*(x^4+3*x^2+4)^(1/
2)/(5*x^2+7)*x-349/6776/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1
)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1
/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+199/847/(-6+2*I*7^(1
/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1
)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1
/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))-199/847/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/
8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^
(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/
2))^(1/2))+575/189728/(-3/8+1/8*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2
+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticP
i((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2
))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(5\,x^2+7\right)^2\,\left(x^4+3\,x^2+4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2), x)
```

3.379 $\int \frac{1}{\left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2}} \, dx$

Optimal. Leaf size=340

$$-\frac{18159\sqrt{x^4+3x^2+4}\,x}{33392128\left(x^2+2\right)}+\frac{51875\sqrt{x^4+3x^2+4}\,x}{33392128\left(5x^2+7\right)}+\frac{625\sqrt{x^4+3x^2+4}\,x}{54208\left(5x^2+7\right)^2}+\frac{\left(139x^2+548\right)x}{596288\sqrt{x^4+3x^2+4}}-\frac{529425\sqrt{\frac{5}{77}}\ \cdots}{133\cdots}$$

```
[Out] -529425/10284775424*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+
1/596288*x*(139*x^2+548)/(x^4+3*x^2+4)^(1/2)-18159/33392128*x*(x^4+3*x^2+4)
^(1/2)/(x^2+2)+625/54208*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+51875/33392128*x
*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+18159/33392128*(x^2+2)*(cos(2*arctan(1/2*x*2
^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x
*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+
4)^(1/2)+843/758912*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*ar
ctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((
x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-3000075/186995916
8*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+
4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1228, 1178, 1197, 1103, 1195, 1223, 1696, 1714, 1708, 1706, 1216}

$$-\frac{18159\sqrt{x^4+3x^2+4}\,x}{33392128\left(x^2+2\right)}+\frac{51875\sqrt{x^4+3x^2+4}\,x}{33392128\left(5x^2+7\right)}+\frac{625\sqrt{x^4+3x^2+4}\,x}{54208\left(5x^2+7\right)^2}+\frac{\left(139x^2+548\right)x}{596288\sqrt{x^4+3x^2+4}}-\frac{529425\sqrt{\frac{5}{77}}\ \cdots}{133\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (x*(548 + 139*x^2))/(596288*Sqrt[4 + 3*x^2 + x^4]) - (18159*x*Sqrt[4 + 3*x^
2 + x^4])/(33392128*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(54208*(7 +
5*x^2)^2) + (51875*x*Sqrt[4 + 3*x^2 + x^4])/(33392128*(7 + 5*x^2)) - (52942
5*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/133568512 + (
18159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sq
rt[2]], 1/8])/(16696064*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (843*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3794
56*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3000075*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^
4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(934979584*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
```

```
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
```

```
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}}\,dx = \int \left(\frac{388+215x^2}{85184 (4+3x^2+x^4)^{3/2}} + \frac{25}{44 (7+5x^2)^3 \sqrt{4+3x^2+x^4}} - \frac{}{1936 (7+5x}\right.$$

$$= \frac{\int \frac{388+215x^2}{(4+3x^2+x^4)^{3/2}}\,dx}{85184} - \frac{1075 \int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{85184} - \frac{25 \int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx}{1936}$$

$$= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{54208 (7+5x^2)^2} - \frac{625x\sqrt{4+3x^2+x^4}}{1192576 (7+5x^2)} + \frac{\int}{}$$

$$= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{54208 (7+5x^2)^2} + \frac{51875x\sqrt{4+3x^2+x^4}}{33392128 (7+5x^2)} - \frac{1}{}$$

$$= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{153x\sqrt{4+3x^2+x^4}}{1192576 (2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208 (7+5x^2)^2} + \frac{518}{33}$$

$$= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{18159x\sqrt{4+3x^2+x^4}}{33392128 (2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208 (7+5x^2)^2} + \frac{5}{}$$

$$= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{18159x\sqrt{4+3x^2+x^4}}{33392128 (2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208 (7+5x^2)^2} + \frac{5}{}$$

Mathematica [C] time = 0.75, size = 320, normalized size = 0.94

$$\frac{28x(453975x^6+2838330x^4+5811451x^2+4496212) + 3i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}(5x^2+7)^2\left(7i\left(6\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (28*x*(4496212 + 5811451*x^2 + 2838330*x^4 + 453975*x^6) + (3*I)*Sqrt[6 + (
2*I)*Sqrt[7]]*(7 + 5*x^2)^2*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 +
 ((2*I)*x^2)/(3*I + Sqrt[7])]*(42371*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sq
rt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I)*(2
3633*I + 6053*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 352950*EllipticPi[(5*(3 + I*Sqrt[7]))/
14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt
[7])]))/(934979584*(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+4}}{125x^{14}+1275x^{12}+6010x^{10}+16678x^8+29153x^6+31871x^4+19992x^2+5488}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^14 + 1275*x^12 + 6010*x^10 + 16678*x^
8 + 29153*x^6 + 31871*x^4 + 19992*x^2 + 5488), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

maple [C] time = 0.03, size = 457, normalized size = 1.34

$$\frac{625\sqrt{x^4+3x^2+4}\;x}{54208\left(5x^2+7\right)^2}+\frac{51875\sqrt{x^4+3x^2+4}\;x}{33392128\left(5x^2+7\right)}-\frac{18159\sqrt{\frac{3x^2}{8}-\frac{i\sqrt{7}\,x^2}{8}+1}\;\sqrt{\frac{3x^2}{8}+\frac{i\sqrt{7}\,x^2}{8}+1}\;\mathrm{EllipticE}\left(\frac{\sqrt{-6+2i\sqrt{}}}{4}\right.}{1043504\sqrt{-6+2i\sqrt{7}}\;\sqrt{x^4+3x^2+4}\;\left(i\sqrt{7}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 625/54208*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2*x+51875/33392128*(x^4+3*x^2+4)^(1
/2)/(5*x^2+7)*x-2*(-139/1192576*x^3-137/298144*x)/(x^4+3*x^2+4)^(1/2)+1173/
1192576/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2
+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*(-6+2*I*7^(1/
2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^(1/2))+18159/1043504/(-6+2*I*7^(1/2))^(1/2)
*(3/8*x^2-1/8*I*7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x
^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*
(2+6*I*7^(1/2))^(1/2))-18159/1043504/(-6+2*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*
7^(1/2)*x^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2
)/(I*7^(1/2)+3)*EllipticE(1/4*(-6+2*I*7^(1/2))^(1/2)*x,1/4*(2+6*I*7^(1/2))^
(1/2))-529425/233744896/(-3/8+1/8*I*7^(1/2))^(1/2)*(3/8*x^2-1/8*I*7^(1/2)*x
^2+1)^(1/2)*(3/8*x^2+1/8*I*7^(1/2)*x^2+1)^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipti
cPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1
/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(5\,x^2+7\right)}^3\,{\left(x^4+3\,x^2+4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2 - x + 2\right)\left(x^2 + x + 2\right)\right)^{\frac{3}{2}}\left(5x^2 + 7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3), x)
```

3.380 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=467

$$\frac{ex\sqrt{a+bx^2+cx^4}\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(-3ce(3ae+10bd)\right.}{15c^{11/4}\sqrt{a+}}$$

```
[Out] 1/15*e^2*(-4*b*e+15*c*d)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/5*e^3*x^3*(c*x^4+b*x
^2+a)^(1/2)/c+1/15*e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d))*x*(c*x^4+b
*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+x^2*c^(1/2))-1/15*a^(1/4)*e*(45*c^2*d^2+8*b^
2*e^2-3*c*e*(3*a*e+10*b*d))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/
2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)+1/30*a^(1/4)*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellip
ticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(
1/2)+x^2*c^(1/2))*(e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d))+(4*a*b*e^3
-15*a*c*d*e^2+15*c^2*d^3)*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^
(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1206, 1679, 1197, 1103, 1195}

$$\frac{ex\sqrt{a+bx^2+cx^4}\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(e\left(-3ce(3ae+10bd\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e^2*(15*c*d - 4*b*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (e^3*x^3*Sqrt[a
 + b*x^2 + c*x^4])/(5*c) + (e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a
*e))*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(
1/4)*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e))*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]])/(15*c^(11/4)*Sqrt[a +
 b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a*b*e^
3))/Sqrt[a] + e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)))*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4
)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
```

```
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\int \frac{5cd^3+3e\left(5cd^2-ae^2\right)x^2+e^2(15cd-4be)x^4}{\sqrt{a+bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\int \frac{15c^2d^3-15acde^2+4abe^3+e\left(45c^2d^2+8\cdots\right.}{\sqrt{a+bx^2+cx^4}}}{15c^2}$$

$$= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{\left(\sqrt{a}\,e\left(45c^2d^2+8b^2e^2-3ce(1\cdots\right.\right.}{15c\cdots}$$

$$= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{e\left(45c^2d^2+8b^2e^2-3ce(10bd\cdots\right.}{15c^{5/2}\left(\sqrt{a}+\cdots\right.}$$

Mathematica [C] time = 2.87, size = 584, normalized size = 1.25

$$\frac{ie\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\cdots}\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4)*(-4*b*e + 3*
c*(5*d + e*x^2)) + I*(-b + Sqrt[b^2 - 4*a*c])*e*(45*c^2*d^2 + 8*b^2*e^2 - 3
*c*e*(10*b*d + 3*a*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(30*c^3*d^3 + 8*b^2*(-b + Sq
rt[b^2 - 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*Sqrt[b^2 - 4*a*c]*d - 2*a*e)
+ c*e^2*(30*b^2*d - 30*b*Sqrt[b^2 - 4*a*c]*d + 17*a*b*e - 9*a*Sqrt[b^2 - 4*
a*c]*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
a + b*x^2 + c*x^4])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(c*x^4 + b*x^2 + a
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^3}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [B] time = 0.02, size = 1186, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] e^3*(1/5/c*x^3*(c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(c*x^4+b*x^2+a)^(1/2)+1/1
5*b/c^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b
+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/2*(-3/5*a/c+8/15*b^2/c^2)*a*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2
)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c
+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2
*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))))+3
*d*e^2*(1/3*(c*x^4+b*x^2+a)^(1/2)/c*x-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+1
/3*b/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/
```

```
2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*
b/c-4)^(1/2))))-3/2*d^2*e*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(
-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/
2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4
*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))+1/4*d^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^3}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 + c*x**4), x)
```

3.381 $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=356

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\left(3cd^2-ae^2\right)}{\sqrt{a}}+2e(3cd-be)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2ex\sqrt{a+bx^2}}{3c^{3/2}\left(\sqrt{\right.}\right.}$$

```
[Out] 1/3*e^2*x*(c*x^4+b*x^2+a)^(1/2)/c+2/3*e*(-b*e+3*c*d)*x*(c*x^4+b*x^2+a)^(1/2
)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-2/3*a^(1/4)*e*(-b*e+3*c*d)*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^
2+a)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b
/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*e*(-b*e+3*c*d)+(-a*e^2+3*
c*d^2)*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(
7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1206, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\left(3cd^2-ae^2\right)}{\sqrt{a}}+2e(3cd-be)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2ex\sqrt{a+bx^2}}{3c^{3/2}\left(\sqrt{\right.}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + (2*e*(3*c*d - b*e)*x*Sqrt[a + b*x^2
 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*e*(3*c*d - b*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3
*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*e*(3*c*d - b*e) + (Sqrt[c]*
(3*c*d^2 - a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c} + \frac{\int \frac{3cd^2-ae^2+2e(3cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c}$$

$$= \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c} - \frac{\left(2\sqrt{a}\,e(3cd-be)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}} + \frac{\left(3cd^2-ae^2+\frac{2\sqrt{a}\,e(3cd-be)}{\sqrt{c}}\right)\int}{3c}$$

$$= \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c} + \frac{2e(3cd-be)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{2\sqrt[4]{a}\,e(3cd-be)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{}}{3c^{7/4}}$$

Mathematica [C] time = 1.62, size = 488, normalized size = 1.37

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(ce\left(-3d\sqrt{b^2-4ac}+ae+3bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-3c^2d^2\right)F\left(i\sinh^{-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4) - I*(-b + Sq
rt[b^2 - 4*a*c])*e*(-3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-3*c^2*d^2 + b*(
-b + Sqrt[b^2 - 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[b^2 - 4*a*c]*d + a*e))*Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2
*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])])/(6*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 +
c*x^4])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^2x^4+2\,dex^2+d^2}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [B] time = 0.01, size = 756, normalized size = 2.12

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Elli}\right.}{\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] e^2*(1/3*(c*x^4+b*x^2+a)^(1/2)/c*x-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1
/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+1/3*
b/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))
/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c
-4)^(1/2))))-d*e*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*
c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-Ellip
ticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)
^(1/2))/a*b/c-4)^(1/2)))+1/4*d^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+
4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 + c*x**4), x)
```

3.382 $\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{c^{3/4}\sqrt{a+}}$$

```
[Out] e*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x
^4+b*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))
*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{c^{3/4}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(\sqrt{a}\,e\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.26, size = 302, normalized size = 1.07

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+e\left(\sqrt{b\right.\right.}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] + (-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ex^2+d}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.00, size = 362, normalized size = 1.28

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Elli}\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*e*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*
b/c-4)^(1/2)))+1/4*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.383 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=401

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}d\sqrt{a+bx^2+cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\tan^{-1}\left(\frac{x\sqrt{ae^2-b}}{\sqrt{d}\sqrt{e}\sqrt{a}}\right)}{2\sqrt{d}\sqrt{ae^2-bde}}$$

```
[Out] 1/2*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2
))*e^(1/2)/d^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(
1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(-e*a^(1/2)+d
*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)-1/4*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4
)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*(2-b/a^
(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((c*x^4
+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(c*x^4+b*
x^2+a)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1216, 1103, 1706}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}d\sqrt{a+bx^2+cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\tan^{-1}\left(\frac{x\sqrt{ae^2-b}}{\sqrt{d}\sqrt{e}\sqrt{a}}\right)}{2\sqrt{d}\sqrt{ae^2-bde}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b
*x^2 + c*x^4])])/(2*Sqrt[d]*Sqrt[c*d^2 - b*d*e + a*e^2]) + (c^(1/4)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)
*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (a^(3/4)*((Sqrt[c]*d)/S
qrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e
), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*
d*(c*d^2 - a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{c}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e} - \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e}$$

$$= \frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\,\sqrt{cd^2-bde+ae^2}} + \frac{\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{2\sqrt[4]{a}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.22, size = 214, normalized size = 0.53

$$-\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\,d\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\,\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*d*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 45.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{cex^6+(cd+be)x^4+(bd+ae)x^2+ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x
^2 + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

maple [A] time = 0.04, size = 200, normalized size = 0.50

$$\frac{\sqrt{2}\,\sqrt{\frac{b\,x^2}{2a}-\frac{\sqrt{-4ac+b^2}\,x^2}{2a}+1}\,\sqrt{\frac{b\,x^2}{2a}+\frac{\sqrt{-4ac+b^2}\,x^2}{2a}+1}\,\mathrm{EllipticPi}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},-\frac{2ae}{\left(-b+\sqrt{-4ac+b^2}\right)d},\frac{\sqrt{-\frac{b+\sqrt{-4ac+b^2}}{2a}}}{\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}\right)}{\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2/a*x^2*(-
4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/
(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*x,-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\,\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.384 $\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=718

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\,ex\sqrt{a+bx^2+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(ae^2-bde+cd^2\right)}+\frac{\sqrt[4]{a}\,\sqrt[4]{c}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\right.}{2d\sqrt{a+bx^2+cx^4}\,\left(ae^2-\right.}$$

```
[Out] 1/4*(3*c*d^2-e*(-a*e+2*b*d))*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(
1/2)/(c*x^4+b*x^2+a)^(1/2))*e^(1/2)/d^(3/2)/(a*e^2-b*d*e+c*d^2)^(3/2)+1/2*e
^2*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1/2*e*x*c^(1/2)*
(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(a^(1/2)+x^2*c^(1/2))+1/2*a^(1/
4)*c^(1/4)*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
F(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d/(-
e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)-1/8*(3*c*d^2-e*(-a*e+2*b*d))*(co
s(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e
/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^
(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/
c^(1/4)/d^2/(a*e^2-b*d*e+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2
)
```

Rubi [A] time = 1.08, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1223, 1714, 1195, 1708, 1103, 1706}

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\,ex\sqrt{a+bx^2+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{e}\left(3cd^2-e(2bd-ae)\right)\tan^{-1}\left(\frac{x\sqrt{ae^2-bde}}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+b}}\right.}{4d^{3/2}\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)*(Sqrt[a
] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a
*e^2)*(d + e*x^2)) + (Sqrt[e]*(3*c*d^2 - e*(2*b*d - a*e))*ArcTan[(Sqrt[c*d^
2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3/2
)*(c*d^2 - b*d*e + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e + a*
e^2)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*
Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 - e*(2*b*d - a
*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcT
an[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*a^(1/4)*c^(1/4)*d
^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{-2cd^2+e(2bd-ae)+2cdex^2+ce^2x^4}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{2d\left(cd^2-bde+ae^2\right)}$$

$$= \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{\sqrt{a}\,c^{3/2}de^2+ce\left(-2cd^2+e(2bd-ae)\right)+\left(2c^2de^2-ce^2\left(cd-\right.\right.}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}}{2cde\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{\sqrt{c}\,ex\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + -$$

$$= -\frac{\sqrt{c}\,ex\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + -$$

Mathematica [C] time = 1.86, size = 1069, normalized size = 1.49

$$2i\sqrt{2}\,c\sqrt{\frac{2cx^2+b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(ex^2+d\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)d^2-6i\sqrt{2}\,c\sqrt{\frac{2cx^2}{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e^2*x*(a + b*x^2 + c*x^4) + I*Sqrt[2]*
(b - Sqrt[b^2 - 4*a*c])*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*(E
llipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
]) + (2*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (6*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])] + (4*I)*Sqrt[2]*b*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh
[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])] - (2*I)*Sqrt[2]*a*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(
d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])])/(8*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*(c*d^3 + d*e*(-(b*d) + a*
e))*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 125.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{ce^2x^8+\left(2\,cde+be^2\right)x^6+\left(cd^2+2\,bde+ae^2\right)x^4+ad^2+\left(bd^2+2\,ade\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c*e^2*x^8 + (2*c*d*e + b*e^2)*x^6 + (c*d^
2 + 2*b*d*e + a*e^2)*x^4 + a*d^2 + (b*d^2 + 2*a*d*e)*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

maple [A] time = 0.04, size = 1279, normalized size = 1.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*e^2*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1/8*c/(a*e^
2-b*d*e+c*d^2)*2^(1/2)/(-1/a*b+(-4*a*c+b^2)^(1/2)/a)^(1/2)*(4+2/a*b*x^2-2*(
-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)*(4+2/a*b*x^2+2*(-4*a*c+b^2)^(1/2)/a*x^2)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+1/4*c*e/(a*e^2-b*d*e+
c*d^2)/d*a*2^(1/2)/(-1/a*b+(-4*a*c+b^2)^(1/2)/a)^(1/2)*(4+2/a*b*x^2-2*(-4*a
*c+b^2)^(1/2)/a*x^2)^(1/2)*(4+2/a*b*x^2+2*(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/
4*c*e/(a*e^2-b*d*e+c*d^2)/d*a*2^(1/2)/(-1/a*b+(-4*a*c+b^2)^(1/2)/a)^(1/2)*(
4+2/a*b*x^2-2*(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)*(4+2/a*b*x^2+2*(-4*a*c+b^2)^(
1/2)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/
2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))
/a*b/c-4)^(1/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1/2)/(-1/a*b+(-4*a*c+b^
2)^(1/2)/a)^(1/2)*(1/2/a*b*x^2-1/2*(-4*a*c+b^2)^(1/2)/a*x^2+1)^(1/2)*(1/2/a
*b*x^2+1/2*(-4*a*c+b^2)^(1/2)/a*x^2+1)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Elliptic
Pi(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,-2/(-b+(-4*a*c+b^2)^(1/2
))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2))*a-1/(a*e^2-b*d*e+c*d^2)/d*e*2^(1/2)/(-1/a*b+(-4*a*c+b^2)^(1/
2)/a)^(1/2)*(1/2/a*b*x^2-1/2*(-4*a*c+b^2)^(1/2)/a*x^2+1)^(1/2)*(1/2/a*b*x^2
+1/2*(-4*a*c+b^2)^(1/2)/a*x^2+1)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,-2/(-b+(-4*a*c+b^2)^(1/2))*a/d
*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2))*b+3/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(-1/a*b+(-4*a*c+b^2)^(1/2)/a)^(1
/2)*(1/2/a*b*x^2-1/2*(-4*a*c+b^2)^(1/2)/a*x^2+1)^(1/2)*(1/2/a*b*x^2+1/2*(-4
*a*c+b^2)^(1/2)/a*x^2+1)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2
*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,x^2+d\right)^2\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 + c*x**4)), x)
```

3.385 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=553

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\,\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\,\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\left(3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)E\left(\sin\right.}{30\sqrt{2}\,c^{7/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -1/15*e^2*(4*b*e+15*c*d)*x*(-c*x^4+b*x^2+a)^(1/2)/c^2-1/5*e^3*x^3*(-c*x^4+b
*x^2+a)^(1/2)/c-1/60*e*(45*c^2*d^2+8*b^2*e^2+3*c*e*(3*a*e+10*b*d))*Elliptic
E(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(
4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(
1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^
(1/2)/c^(7/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/60*EllipticF(x*2^(1/2)*c^(1/
2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2))
)^(1/2))*(e*(45*c^2*d^2+8*b^2*e^2+3*c*e*(3*a*e+10*b*d))+2*c*(4*a*b*e^3+15*a
*c*d*e^2+15*c^2*d^3)/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^
2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4
*a*c+b^2)^(1/2)))^(1/2)/c^(7/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 1.28, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1206, 1679, 1202, 524, 424, 419}

$$\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\,\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\,\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{2c\left(4abe^3+15acde^2+15c^2d^3\right)}{b-\sqrt{4ac+b^2}}+e\left(3ce(3ae+10bd)+\right.\right.}{30\sqrt{2}\,c^{7/2}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -(e^2*(15*c*d + 4*b*e)*x*Sqrt[a + b*x^2 - c*x^4])/(15*c^2) - (e^3*x^3*Sqrt[
a + b*x^2 - c*x^4])/(5*c) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^
2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 -
c*x^4]) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*((2*c*(15*c^
2*d^3 + 15*a*c*d*e^2 + 4*a*b*e^3))/(b - Sqrt[b^2 + 4*a*c]) + e*(45*c^2*d^2
+ 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e)))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sq
rt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\int \frac{-5cd^3-3e(5cd^2+ae^2)x^2-e^2(15cd+4be)x^4}{\sqrt{a+bx^2-cx^4}}\,dx}{5c}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} + \frac{\int \frac{15c^2d^3+15acde^2+4abe^3+e(45c^2d^2+}{\sqrt{a+bx^2-cx}}}{15c^2}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} + \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2c}{b+\sqrt{b^2}}}\right.}{}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\left(\left(b-\sqrt{b^2+4ac}\right)e\left(45c^2d^2+\right.\right.}{}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+}}}{}$$

Mathematica [C] time = 2.47, size = 596, normalized size = 1.08

$$\frac{-i\sqrt{2}e\left(\sqrt{4ac+b^2}-b\right)\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}\left(3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)E\left(i\sinh^{-1}\left(\sqrt{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-4*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(a + b*x^2 - c*x^4)*(4*b*e +
 3*c*(5*d + e*x^2)) - I*Sqrt[2]*(-b + Sqrt[b^2 + 4*a*c])*e*(45*c^2*d^2 + 8*
b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b
 + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b
^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))
]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-30*c^3
*d^3 + 8*b^2*(-b + Sqrt[b^2 + 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*Sqrt[b^2
 + 4*a*c]*d - 2*a*e) + c*e^2*(-30*b^2*d + 30*b*Sqrt[b^2 + 4*a*c]*d - 17*a*b
*e + 9*a*Sqrt[b^2 + 4*a*c]*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2
+ 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x
], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(60*c^3*Sqrt[-(c/(b +
Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3\right)\sqrt{-cx^4+bx^2+a}}{cx^4-bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(-c*x^4 + b*x^2 +
 a)/(c*x^4 - b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

maple [B] time = 0.02, size = 1195, normalized size = 2.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x)
[Out] e^3*(-1/5/c*x^3*(-c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(-c*x^4+b*x^2+a)^(1/2)+
1/15*b/c^2*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a*c+b^2)^(
1/2))/a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x^4+b*x^2+
a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(
b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/2*(3/5*a/c+8/15*b^2/c^2)*a*2^(1/2)/(
(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(
2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2
)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*
(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(4*a*c+b^2
)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))))+3*d*e^2
*(-1/3*(-c*x^4+b*x^2+a)^(1/2)/c*x+1/12/c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/
a)^(1/2)*(-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))
/a*x^2+4)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^
2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/3*b/c*
a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2
+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b
+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)
*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b
+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)
)))-3/2*d^2*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a*c+b^2
)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x^4+b*x
^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(
1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/
2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/
a*b/c-4)^(1/2)))+1/4*d^3*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(
4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-
c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)
*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^3}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^3/(a + b*x^2 - c*x^4)^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 - c*x**4), x)
```

3.386 $\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=454

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,(be+3cd)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+}}{b-\sqrt{b^2+}}\right.}{3\sqrt{2}\,c^{5/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -1/3*e^2*x*(-c*x^4+b*x^2+a)^(1/2)/c-1/6*e*(b*e+3*c*d)*EllipticE(x*2^(1/2)*c
^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1
/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*
(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(5/2)
*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/6*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b
^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(3*c^
2*d^2+b*e^2*(b-(4*a*c+b^2)^(1/2))+c*e*(3*b*d+a*e-3*d*(4*a*c+b^2)^(1/2)))*(1
-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^
2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(5/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1206, 1202, 524, 424, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)+3c\right.}{3\sqrt{2}\,c^{5/2}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b +
Sqrt[b^2 + 4*a*c]]*e*(3*c*d + b*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c
])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b
^2 + 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt
[b^2 + 4*a*c]]*(3*c^2*d^2 + b*(b - Sqrt[b^2 + 4*a*c])*e^2 + c*e*(3*b*d - 3*
Sqrt[b^2 + 4*a*c]*d + a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqr
t[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*
a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2-cx^4}}\,dx &= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}-\frac{\int \frac{-3cd^2-ae^2-2e(3cd+be)x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{3c}\\
&= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}-\frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{-3cd^2-ae^2-2e(3cd+be)x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,d}{3c\sqrt{a+bx^2-cx^4}}\\
&= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}-\frac{\left(\left(b-\sqrt{b^2+4ac}\right)e(3cd+be)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int}{3c^2\sqrt{a+bx^2-cx^4}}\\
&= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}-\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e(3cd+be)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\,\sqrt{}}{3\sqrt{2}\,c^{5/2}\sqrt{a+bx^2-}}\end{aligned}$$

Mathematica [C] time = 1.39, size = 503, normalized size = 1.11

$$\frac{i\sqrt{2}\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}\left(-ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(\sqrt{4ac+b^2}-b\right)-3c^2d^2\right)F\left(i\sinh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(-a - b*x^2 + c*x^4) - I*Sqrt
[2]*(-b + Sqrt[b^2 + 4*a*c])*e*(3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 + 4*a*c] -
2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(
-b + Sqrt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2
```

```
 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[
2]*(-3*c^2*d^2 + b*(-b + Sqrt[b^2 + 4*a*c])*e^2 - c*e*(3*b*d - 3*Sqrt[b^2 +
 4*a*c]*d + a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 + 4*
a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[
b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*c^2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a
*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(e^2x^4+2\,dex^2+d^2\right)\sqrt{-cx^4+bx^2+a}}{cx^4-bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-c*x^4 + b*x^2 + a)/(c*x^4 - b*x
^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.01, size = 761, normalized size = 1.68

$$-\frac{\sqrt{2}\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Ellipti}\right.}{\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}\,\left(b+\sqrt{4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] e^2*(-1/3*(-c*x^4+b*x^2+a)^(1/2)/c*x+1/12/c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2
))/a)^(1/2)*(-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/
2))/a*x^2+4)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(4*a*c
+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/3*b
/c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a*c+b^2)^(1/2))/a*
x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1
/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*(
(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1
/2))))-d*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a*c+b^2)^(
1/2))/a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x^4+b*x^2+
a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2
))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2
^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b
```

```
/c-4)^(1/2)))+1/4*d^2*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a
*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x
^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,
1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(a + b*x^2 - c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 - c*x**4), x)
```

3.387 $\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=385

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] 1/4*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)
^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d+e*(b-(4*a*c+b^2)^(1/2)))*(1-2*
c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(
b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)-1/4*e*El
lipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2)
)/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+
b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1
/2)))^(1/2)/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1202, 524, 424, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^2)/
(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ellipt
icE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2
+ 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]])/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x
^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Sqr
t[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 +
 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]]
, (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt
[a + b*x^2 - c*x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{d+ex^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\left(\left(b-\sqrt{b^2+4ac}\right)\left(-\frac{2cd}{b-\sqrt{b^2+4ac}}-e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{\cdots}}}{2c\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{b+\sqrt{b^2+\cdots}}}\right.\right.}{2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.26, size = 293, normalized size = 0.76

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+e\left(\cdots\right.\right.}{2\sqrt{2}\,c\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(
b + Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sq
rt[b^2 + 4*a*c])] + (2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b
- Sqrt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[
a + b*x^2 - c*x^4])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-cx^4+bx^2+a}\left(ex^2+d\right)}{cx^4-bx^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)/(c*x^4 - b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.01, size = 364, normalized size = 0.95

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,x}{2}, \frac{\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Ellipti}\right.}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}\,\left(b+\sqrt{4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x)
[Out] -1/2*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a*c+b^2)^(1/2)
)/a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x^4+b*x^2+a)^(
1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a
)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/
2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4
)^(1/2)))+1/4*d*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a*c+b^2
)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x^4+b*x
^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-
2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{\sqrt{-c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*x^2 - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)/(a + b*x^2 - c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d + e*x**2)/sqrt(a + b*x**2 - c*x**4), x)
```

3.388 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=197

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\,d\sqrt{a+bx^2-cx^4}}$$

```
[Out] 1/2*EllipticPi(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),-1/2*e*(b+(4*a
*c+b^2)^(1/2))/c/d,((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(1-
2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2
/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1220, 537}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\,d\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sq
rt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-((b + Sqrt[b^2 + 4*a*
c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b
 + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[a +
 b*x^2 - c*x^4])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d+ex^2)}dx}{\sqrt{a+bx^2-cx^4}}$$

$$= \frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\right.\right.}{\sqrt{2}\sqrt{c}\,d\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.23, size = 205, normalized size = 1.04

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|-\frac{b+\sqrt{b^2+4ac}}{\sqrt{b^2+4ac}-b}\right)}{\sqrt{2}\,d\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2 + 4*a*c])*e)/(c*d), I*Ar
cSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*
c])/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]
*d*Sqrt[a + b*x^2 - c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}\,(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

maple [A] time = 0.04, size = 201, normalized size = 1.02

$$\frac{\sqrt{2}\sqrt{\frac{b\,x^2}{2a}-\frac{\sqrt{4ac+b^2}\,x^2}{2a}+1}\sqrt{\frac{b\,x^2}{2a}+\frac{\sqrt{4ac+b^2}\,x^2}{2a}+1}\ \mathrm{EllipticPi}\left(\frac{\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,x}{2},-\frac{2ae}{\left(-b+\sqrt{4ac+b^2}\right)d},\frac{\sqrt{-\frac{b+\sqrt{4ac+b^2}}{2a}}\sqrt{2}}{\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}\right.}{\sqrt{-\frac{b}{a}+\frac{\sqrt{4ac+b^2}}{a}}\sqrt{-c\,x^4+b\,x^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(-1/a*b+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*x^2-1/2/a*x^2*(
4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(
-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/
2)*x,-2/(-b+(4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2
^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(e\,x^2+d\right)\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 - c*x^4)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)*(a + b*x^2 - c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 - c*x**4)), x)
```

3.389 $\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+bx^2-cx^4}} \, dx$

Optimal. Leaf size=718

$$-\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4}}{b-\sqrt{b^2+4}}\right.}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}$$

```
[Out] -1/2*e^2*x*(-c*x^4+b*x^2+a)^(1/2)/d/(-a*e^2+b*d*e+c*d^2)/(e*x^2+d)+1/4*(3*c
*d^2+e*(-a*e+2*b*d))*EllipticPi(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/
2),-1/2*e*(b+(4*a*c+b^2)^(1/2))/c/d,((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(
1/2)))^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))
^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d^2/(c*d^2+e*(-a*e+b*d))*2^(
1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)-1/8*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a
*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(
2*c*d+e*(b-(4*a*c+b^2)^(1/2)))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(
4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d/(c*d^2+e*
(-a*e+b*d))*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/8*e*EllipticE(x*2^(1/2
)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)
^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/
2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d/(c
*d^2+e*(-a*e+b*d))*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 1.02, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1223, 1716, 1202, 524, 424, 419, 1220, 537}

$$-\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4}}{b-\sqrt{b^2+4}}\right.}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(2*d*(c*d^2 + e*(b*d - a*e))*(d + e*x^2))
+ ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^2)
/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ellip
ticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2
 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d -
 a*e))*Sqrt[a + b*x^2 - c*x^4]) - (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b
- Sqrt[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1
- (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]
)])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4]) +
 (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(3*c*d^2 + e*(2*b*d - a*e))*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]
)/(2*Sqrt[2]*Sqrt[c]*d^2*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+e(bd-ae)\right)\left(d+ex^2\right)}+\frac{\int \frac{2cd^2+e(2bd-ae)-2cdex^2-ce^2x^4}{\left(d+ex^2\right)\sqrt{a+bx^2-cx^4}}\,dx}{2d\left(cd^2+e(bd-ae)\right)}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+e(bd-ae)\right)\left(d+ex^2\right)}-\frac{\int \frac{cde^2+ce^3x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{2de^2\left(cd^2+e(bd-ae)\right)}+\frac{\left(3cd^2+e(2b\right.}{2a}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+e(bd-ae)\right)\left(d+ex^2\right)}-\frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int -}{2de^2\left(cd^2+e(bd-ae)\right)\sqrt{}}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+e(bd-ae)\right)\left(d+ex^2\right)}+\frac{\sqrt{b+\sqrt{b^2+4ac}}\left(3cd^2+e(2bd-ae)\right)}{2}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+e(bd-ae)\right)\left(d+ex^2\right)}+\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\sqrt{1}}{4\sqrt{2}\sqrt{c}\,d}$$

Mathematica [C] time = 5.53, size = 464, normalized size = 0.65

$$-\frac{\sqrt{a+bx^2-cx^4}\left(4de^2x+\frac{i\left(d+ex^2\right)\sqrt{\frac{4cx^2}{\sqrt{4ac+b^2}-b}+2}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(2\left(e(ae-2bd)-3cd^2\right)\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right.\right.\right.}{}\right.}{8d^2\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -1/8*(Sqrt[a + b*x^2 - c*x^4]*(4*d*e^2*x + (I*Sqrt[2 + (4*c*x^2)/(-b + Sqrt
[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(d + e*x^2)*((-
b + Sqrt[b^2 + 4*a*c])*d*e*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b
^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + d*(2*
c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b +
Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] +
 2*(-3*c*d^2 + e*(-2*b*d + a*e))*EllipticPi[-1/2*((b + Sqrt[b^2 + 4*a*c])*e
)/(c*d), I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt
[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c])
)]*(-a - b*x^2 + c*x^4))))/(d^2*(c*d^2 + e*(b*d - a*e))*(d + e*x^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

maple [B] time = 0.04, size = 1293, normalized size = 1.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x)
[Out] 1/2*e^2/(a*e^2-b*d*e-c*d^2)/d*x*(-c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)+1/8*c/(a*e
^2-b*d*e-c*d^2)*2^(1/2)/(-1/a*b+(4*a*c+b^2)^(1/2)/a)^(1/2)*(4+2/a*b*x^2-2*(
4*a*c+b^2)^(1/2)/a*x^2)^(1/2)*(4+2/a*b*x^2+2*(4*a*c+b^2)^(1/2)/a*x^2)^(1/2)
/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1
/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/4*c*e/(a*e^2-b*d*e-c*
d^2)/d*a*2^(1/2)/(-1/a*b+(4*a*c+b^2)^(1/2)/a)^(1/2)*(4+2/a*b*x^2-2*(4*a*c+b
^2)^(1/2)/a*x^2)^(1/2)*(4+2/a*b*x^2+2*(4*a*c+b^2)^(1/2)/a*x^2)^(1/2)/(-c*x^
4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^
2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+1/4*c*e/
(a*e^2-b*d*e-c*d^2)/d*a*2^(1/2)/(-1/a*b+(4*a*c+b^2)^(1/2)/a)^(1/2)*(4+2/a*b
*x^2-2*(4*a*c+b^2)^(1/2)/a*x^2)^(1/2)*(4+2/a*b*x^2+2*(4*a*c+b^2)^(1/2)/a*x^
2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticE(1/2*2^(1/2)
*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^
(1/2))+1/2/(a*e^2-b*d*e-c*d^2)/d^2*e^2*2^(1/2)/(-1/a*b+(4*a*c+b^2)^(1/2)/a)
^(1/2)*(1/2/a*b*x^2-1/2*(4*a*c+b^2)^(1/2)/a*x^2+1)^(1/2)*(1/2/a*b*x^2+1/2*(
4*a*c+b^2)^(1/2)/a*x^2+1)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*2^(1/
2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,-2/(-b+(4*a*c+b^2)^(1/2))*a/d*e,(-1/2
*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*a
-1/(a*e^2-b*d*e-c*d^2)/d*e*2^(1/2)/(-1/a*b+(4*a*c+b^2)^(1/2)/a)^(1/2)*(1/2/
a*b*x^2-1/2*(4*a*c+b^2)^(1/2)/a*x^2+1)^(1/2)*(1/2/a*b*x^2+1/2*(4*a*c+b^2)^(
1/2)/a*x^2+1)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*2^(1/2)*((-b+(4*a
*c+b^2)^(1/2))/a)^(1/2)*x,-2/(-b+(4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(4*a*c+b
^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*b-3/2/(a*e^2-
b*d*e-c*d^2)*2^(1/2)/(-1/a*b+(4*a*c+b^2)^(1/2)/a)^(1/2)*(1/2/a*b*x^2-1/2*(4
*a*c+b^2)^(1/2)/a*x^2+1)^(1/2)*(1/2/a*b*x^2+1/2*(4*a*c+b^2)^(1/2)/a*x^2+1)^
(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))
/a)^(1/2)*x,-2/(-b+(4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^
(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,x^2+d\right)^2\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 - c*x^4)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)^2*(a + b*x^2 - c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex^2\right)^2 \sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 - c*x**4)), x)
```

3.390 $\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=479

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}+\frac{d\sqrt{\sqrt{4ac+b^2}+b}\,\Big(}{\sqrt{2}}$$

```
[Out] 1/2*e*x*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))/c/(c*x^4+b*
x^2-a)^(1/2)+1/2*d*(1/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2/(
b+(4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2
))^(1/2)/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/(b-(
4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b+(4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*a*c+b^
2)^(1/2)))/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)-1/4*e*(1/(1+2*c*x^2/(b+
(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)*Elliptic
E(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(b+(4*a*c+b^2)^(
1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2
/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))*(b+(4*a*c+b^2)^(1/2))^(1/2)/c
^(3/2)*2^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))/(1+
2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1202, 531, 418, 492, 411}

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}+\frac{d\sqrt{\sqrt{4ac+b^2}+b}\,\Big(}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

```
[Out] ((b - Sqrt[b^2 + 4*a*c])*e*x*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])))/(2*c*
Sqrt[-a + b*x^2 + c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*e*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*EllipticE[ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^
2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]
))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sq
rt[b + Sqrt[b^2 + 4*a*c]]*d*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*Ellipti
cF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b -
 Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x
^2 + c*x^4])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{d+ex^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{\sqrt{-a+bx^2+cx^4}}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{\sqrt{-a+bx^2+cx^4}} + \frac{\left(e\sqrt{1+\frac{2}{b-\sqrt{}}}\right.}{}$$

$$= \frac{\left(b-\sqrt{b^2+4ac}\right)ex\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)}{2c\sqrt{-a+bx^2+cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\,d\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)F\left(\tan^{-1}\right.}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+}}$$

$$= \frac{\left(b-\sqrt{b^2+4ac}\right)ex\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)}{2c\sqrt{-a+bx^2+cx^4}} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\left(1+\frac{2}{b-\sqrt{}}\right.}{2\sqrt{2}\,c^{3/2}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4}}}}}$$

Mathematica [C] time = 0.30, size = 304, normalized size = 0.63

$$\frac{i\sqrt{\frac{\sqrt{4ac+b^2}+b+2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\left(\left(e\left(b-\sqrt{4ac+b^2}\right)-2cd\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+e\left(\right.}{2\sqrt{2}\,c\sqrt{\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 + 4*
a*c])/(b - Sqrt[b^2 + 4*a*c])] + (-2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 + 4*a*c]
)]*Sqrt[-a + b*x^2 + c*x^4])
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ex^2 + d}{\sqrt{cx^4 + bx^2 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

maple [A] time = 0.03, size = 355, normalized size = 0.74

$$\frac{\sqrt{\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,x}{2}, \frac{\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{EllipticF}\left(\cdots\right.\right.}{\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+b\,x^2-a}\,\left(b+\sqrt{4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] e*a/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)/(b+(4*a
*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-2
*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*x*(-2*(-b+(4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))+1/2*d/(-2*(-
b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2
*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*EllipticF(1/2*x*(
-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(b*x^2 - a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(b*x^2 - a + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 + c*x**4), x)
```

3.391 $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=204

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,\Pi\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

```
[Out] 1/2*EllipticPi(x*2^(1/2)*c^(1/2)/(-b+(4*a*c+b^2)^(1/2))^(1/2),1/2*e*(b-(4*a
*c+b^2)^(1/2))/c/d,((b-(4*a*c+b^2)^(1/2))/(b+(4*a*c+b^2)^(1/2)))^(1/2))*(1+
2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(-b+(4*a*c+b^2)^(1/2))^(1/2)*(1+2*c*x^
2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1220, 537}

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,\Pi\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[((b - Sqrt[b^2 + 4*a*
c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (
b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[-a
 + b*x^2 + c*x^4])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d+ex^2)}dx}{\sqrt{-a+bx^2+cx^4}}$$
$$= \frac{\sqrt{-b+\sqrt{b^2+4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\Pi\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\right.}{\sqrt{2}\sqrt{c}\,d\sqrt{-a+bx^2+cx^4}}$$

Mathematica [C] time = 0.22, size = 216, normalized size = 1.06

$$-\frac{i\sqrt{\frac{\sqrt{4ac+b^2}+b+2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\,\Pi\left(\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\,d\sqrt{\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*EllipticPi[((b + Sqrt[b^2 + 4*a*c])*
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt
[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*
a*c])]*d*Sqrt[-a + b*x^2 + c*x^4])
```

fricas [F] time = 76.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2-a}}{cex^6+(cd+be)x^4+(bd-ae)x^2-ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 - a)/(c*e*x^6 + (c*d + b*e)*x^4 + (b*d - a*e)*x
^2 - a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2-a}\,(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

maple [A] time = 0.03, size = 198, normalized size = 0.97

$$\frac{\sqrt{-\frac{bx^2}{2a}+\frac{\sqrt{4ac+b^2}\,x^2}{2a}+1}\sqrt{-\frac{bx^2}{2a}-\frac{\sqrt{4ac+b^2}\,x^2}{2a}+1}\;\mathrm{EllipticPi}\left(\sqrt{-\frac{-b+\sqrt{4ac+b^2}}{2a}}\,x, \frac{2ae}{\left(-b+\sqrt{4ac+b^2}\right)d}, \frac{\sqrt{2}\sqrt{\frac{b+\sqrt{4ac+b^2}}{a}}}{2\sqrt{-\frac{-b+\sqrt{4ac+b^2}}{2a}}}\right)}{\sqrt{\frac{b}{2a}-\frac{\sqrt{4ac+b^2}}{2a}}\sqrt{cx^4+bx^2-a}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/d/(1/2/a*b-1/2*(4*a*c+b^2)^(1/2)/a)^(1/2)*(1-1/2/a*b*x^2+1/2*(4*a*c+b^2)^
(1/2)/a*x^2)^(1/2)*(1-1/2/a*b*x^2-1/2*(4*a*c+b^2)^(1/2)/a*x^2)^(1/2)/(c*x^4
+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,2/(-b+(4
*a*c+b^2)^(1/2))*a/d*e,1/2*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)^(1/2)/(-1/2*(-
b+(4*a*c+b^2)^(1/2))/a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2 - a}\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,x^2 + d\right)\,\sqrt{c\,x^4 + b\,x^2 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)*(b*x^2 - a + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex^2\right)\sqrt{-a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 + c*x**4)), x)
```

3.392 $\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=293

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\cdots}{c^{3/4}\sqrt{-a+\cdots}}$$

```
[Out] -e*x*(-c*x^4+b*x^2-a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*
arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipt
icE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1
/2)+x^2*c^(1/2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-
c*x^4+b*x^2-a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/
2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-c*x^4+b*x^2-a
)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\cdots}{c^{3/4}\sqrt{-a+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

```
[Out] -((e*x*Sqrt[-a + b*x^2 - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^(1
/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4
])/(c^(3/4)*Sqrt[-a + b*x^2 - c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*
c^(3/4)*Sqrt[-a + b*x^2 - c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx = -\frac{\left(\sqrt{a}\,e\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx$$

$$= -\frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\,\sqrt{}\right.\right.}{c^{3/4}\sqrt{-a+bx^2-cx^4}}$$

Mathematica [C] time = 0.31, size = 295, normalized size = 1.01

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}\left(\left(e\left(b-\sqrt{b^2-4ac}\right)+2cd\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+e\left(\right.\right.}{2\sqrt{2}\,c\sqrt{-\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{-a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])] + (2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[
-a + b*x^2 - c*x^4])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-cx^4+bx^2-a}\left(ex^2+d\right)}{cx^4-bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)/(c*x^4 - b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{-cx^4+bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

maple [A] time = 0.04, size = 357, normalized size = 1.22

$$\frac{\sqrt{\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{EllipticF}\right.}{\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-c\,x^4+b\,x^2-a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] e*a/(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)/(b+
(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*x*(-2*(-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))+1/2*d
/(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)*Ellipt
icF(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(2*(b+(-4*a*c+b^2)^(1/2)
)/a*b/c-4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{-cx^4+bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{\sqrt{-c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(b*x^2 - a - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(b*x^2 - a - c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 - c*x**4), x)
```

3.393 $\int \frac{1}{\left(d+ex^2\right)\sqrt{-a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=412

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{4\sqrt[4]{c}d\sqrt{-a+bx^2-cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\tan^{-1}\left(\frac{x\sqrt{-e(ae}}{\sqrt{d}\sqrt{e}\sqrt{-}}\right.}{2\sqrt{d}\sqrt{-e(ae+b}}$$

```
[Out] 1/2*arctan(x*(-a*e^2-b*d*e-c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(-c*x^4+b*x^2-a)^(1
/2))*e^(1/2)/d^(1/2)/(-a*e^2-b*d*e-c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*
c^(1/2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(-e*a^(1/2
)+d*c^(1/2))/(-c*x^4+b*x^2-a)^(1/2)-1/4*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^
(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*(2+
b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((c
*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(-c*x
^4+b*x^2-a)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1216, 1103, 1706}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{4\sqrt[4]{c}d\sqrt{-a+bx^2-cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\tan^{-1}\left(\frac{x\sqrt{-e(ae}}{\sqrt{d}\sqrt{e}\sqrt{-}}\right.}{2\sqrt{d}\sqrt{-e(ae+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - e*(b*d + a*e)]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a
 + b*x^2 - c*x^4])])/(2*Sqrt[d]*Sqrt[-(c*d^2) - e*(b*d + a*e)]) + (c^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*
a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a + b*x^2 - c*x^4]) - (a^(3/4)*((Sqrt
[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqr
t[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(4*
c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[-a + b*x^2 - c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{-a+bx^2-cx^4}}\,dx = \frac{\sqrt{c}\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e} - \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e}$$

$$= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{-cd^2-e(bd+ae)}\,x}{\sqrt{d}\sqrt{e}\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2-e(bd+ae)}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\right.}{2\sqrt[4]{a}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{-a+b}}$$

Mathematica [C] time = 0.22, size = 207, normalized size = 0.50

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|-\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{2}\,d\sqrt{-\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{-a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2 - 4*a*c])*e)/(c*d), I*Ar
cSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], -((b + Sqrt[b^2 - 4*a*
c])/(-b + Sqrt[b^2 - 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]
*d*Sqrt[-a + b*x^2 - c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4+bx^2-a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

maple [A] time = 0.03, size = 199, normalized size = 0.48

$$\frac{\sqrt{-\frac{b\,x^{2}}{2a}+\frac{\sqrt{-4ac+b^{2}}\,x^{2}}{2a}+1}\,\sqrt{-\frac{b\,x^{2}}{2a}-\frac{\sqrt{-4ac+b^{2}}\,x^{2}}{2a}+1}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-b+\sqrt{-4ac+b^{2}}}{2a}}\,x,\frac{2ae}{\left(-b+\sqrt{-4ac+b^{2}}\right)d},\frac{\sqrt{2}\,\sqrt{\frac{b+\sqrt{-4ac+b^{2}}}{a}}}{2\sqrt{-\frac{-b+\sqrt{-4ac+b^{2}}}{2a}}}\right)}{\sqrt{\frac{b}{2a}-\frac{\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{-c\,x^{4}+b\,x^{2}-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/d/(1/2/a*b-1/2*(-4*a*c+b^2)^(1/2)/a)^(1/2)*(1-1/2/a*b*x^2+1/2*(-4*a*c+b^2
)^(1/2)/a*x^2)^(1/2)*(1-1/2/a*b*x^2-1/2*(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)/(-c
*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,2/(
-b+(-4*a*c+b^2)^(1/2))*a/d*e,1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(
-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^{4}+bx^{2}-a}\left(ex^{2}+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)\,\sqrt{-c\,x^{4}+b\,x^{2}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(b*x^2 - a - c*x^4)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)*(b*x^2 - a - c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^{2}\right)\sqrt{-a+bx^{2}-cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 - c*x**4)), x)
```

3.394 $\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=229

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(5d^3-10de^2+8e^3)\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{3e(x^2+2)\,x\,(5d^2-10de+6e^2)}{5\sqrt{x^4+3x^2+2}}-\frac{3\sqrt{2}\,e\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(\ldots}{5\sqrt{\ldots}}$$

```
[Out] 3/5*e*(5*d^2-10*d*e+6*e^2)*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/10*(5*d^3-10*d*e
^2+8*e^3)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(
1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-3/5*e*(5*d^2-10*d
*e+6*e^2)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(
1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/5*(5*d-4*e)*e^2
*x*(x^4+3*x^2+2)^(1/2)+1/5*e^3*x^3*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1206, 1679, 1189, 1099, 1135}

$$\frac{3e(x^2+2)\,x\,(5d^2-10de+6e^2)}{5\sqrt{x^4+3x^2+2}}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(5d^3-10de^2+8e^3)\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{3\sqrt{2}\,e\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(\ldots}{5\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (3*e*(5*d^2 - 10*d*e + 6*e^2)*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*
d - 4*e)*e^2*x*Sqrt[2 + 3*x^2 + x^4])/5 + (e^3*x^3*Sqrt[2 + 3*x^2 + x^4])/5
 - (3*Sqrt[2]*e*(5*d^2 - 10*d*e + 6*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*d^3 - 10*d*e^2
 + 8*e^3)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5
*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{2+3x^2+x^4}}\,dx = \frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}+\frac{1}{5}\int \frac{5d^3+3e\left(5d^2-2e^2\right)x^2+3(5d-4e)e^2x^4}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4}+\frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}+\frac{1}{15}\int \frac{3\left(5d^3-10de^2+8e^3\right)+9e}{\sqrt{2+3x^2}}$$

$$= \frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4}+\frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}+\frac{1}{5}\left(3e\left(5d^2-10de+6e^2\right)\right)\int \frac{}{\sqrt{2}}$$

$$= \frac{3e\left(5d^2-10de+6e^2\right)x\left(2+x^2\right)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4}+\frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}$$

Mathematica [C] time = 0.23, size = 154, normalized size = 0.67

$$\frac{-3ie\sqrt{x^2+1}\,\sqrt{x^2+2}\,\left(5d^2-10de+6e^2\right)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-5i\sqrt{x^2+1}\,\sqrt{x^2+2}\,\left(d^3-3d^2e+4de^2-2e^3\right)F\left(i\right.}{5\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e^2*x*(2 + 3*x^2 + x^4)*(5*d + e*(-4 + x^2)) - (3*I)*e*(5*d^2 - 10*d*e + 6
*e^2)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (5*I
)*(d^3 - 3*d^2*e + 4*d*e^2 - 2*e^3)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I
*ArcSinh[x/Sqrt[2]], 2])/(5*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}{\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(x^4 + 3*x^2 + 2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

maple [C] time = 0.01, size = 380, normalized size = 1.66

$$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,d^3\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)+\mathrm{Ellipt}\right.}{2\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] e^3*(1/5*(x^4+3*x^2+2)^(1/2)*x^3-4/5*(x^4+3*x^2+2)^(1/2)*x-4/5*I*2^(1/2)*(2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2
^(1/2))+9/5*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(El
lipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2))))+3*d*e
^2*(1/3*(x^4+3*x^2+2)^(1/2)*x+1/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(
x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-I*2^(1/2)*(2*x^2+4)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-
EllipticE(1/2*I*2^(1/2)*x,2^(1/2))))+3/2*I*d^2*e*2^(1/2)*(2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-Ellipti
cE(1/2*I*2^(1/2)*x,2^(1/2)))-1/2*I*d^3*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2
)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((d + e*x^2)^3/(3*x^2 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral((d + e*x**2)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.395 $\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=168

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(3d^2-2e^2)\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{2ex(x^2+2)(d-e)}{\sqrt{x^4+3x^2+2}}-\frac{2\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(d-e)E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+$$

```
[Out] 2*(d-e)*e*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/6*(3*d^2-2*e^2)*(x^2+1)^(3/2)*(1/
(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/
2)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-2*(d-e)*e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*El
lipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3
*x^2+2)^(1/2)+1/3*e^2*x*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1206, 1189, 1099, 1135}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(3d^2-2e^2)\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{2ex(x^2+2)(d-e)}{\sqrt{x^4+3x^2+2}}-\frac{2\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(d-e)E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (2*(d - e)*e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (e^2*x*Sqrt[2 + 3*x^2 + x
^4])/3 - (2*Sqrt[2]*(d - e)*e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE
[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + ((3*d^2 - 2*e^2)*(1 + x^2)*Sqrt[(
2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 +
x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{1}{3}e^2x\sqrt{2+3x^2+x^4} + \frac{1}{3}\int \frac{3d^2-2e^2+6(d-e)ex^2}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{1}{3}e^2x\sqrt{2+3x^2+x^4} + (2(d-e)e)\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{1}{3}\left(3d^2-2e^2\right)\int \frac{1}{\sqrt{2+3x^2+x^4}$$
$$= \frac{2(d-e)ex\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}} + \frac{1}{3}e^2x\sqrt{2+3x^2+x^4} - \frac{2\sqrt{2}\,(d-e)e\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.16, size = 127, normalized size = 0.76

$$\frac{-i\sqrt{x^2+1}\,\sqrt{x^2+2}\,\left(3d^2-6de+4e^2\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-6ie\sqrt{x^2+1}\,\sqrt{x^2+2}\,(d-e)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+e^2}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e^2*x*(2 + 3*x^2 + x^4) - (6*I)*(d - e)*e*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elli
pticE[I*ArcSinh[x/Sqrt[2]], 2] - I*(3*d^2 - 6*d*e + 4*e^2)*Sqrt[1 + x^2]*Sq
rt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^2x^4+2\,dex^2+d^2}{\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

maple [C] time = 0.01, size = 235, normalized size = 1.40

$$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,d^2\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)+\mathrm{EllipticF}\right.}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] e^2*(1/3*(x^4+3*x^2+2)^(1/2)*x+1/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/
(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-I*2^(1/2)*(2*x^2+4)^
(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))
-EllipticE(1/2*I*2^(1/2)*x,2^(1/2))))+I*d*e*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/
2*I*2^(1/2)*x,2^(1/2)))-1/2*I*d^2*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^
4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(3*x^2 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.396 $\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=122

$$\frac{d\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{ex\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}e\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] e*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*d*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellip
ticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*x^
2+2)^(1/2)-e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*
2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1189, 1099, 1135}

$$\frac{d\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{ex\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}e\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*e*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (d*(1 + x^2)
*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x
^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = d\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx + e\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{ex\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}e\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{d\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\right.}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.07, size = 73, normalized size = 0.60

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left((d-e)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+eE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(e*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (
d - e)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ex^2+d}{\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

maple [C] time = 0.00, size = 108, normalized size = 0.89

$$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,d\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)+\mathrm{Elliptic}\right.}{2\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 1/2*I*e*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Elliptic
F(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-1/2*I*d*2^(1
/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/
2)*x,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(3*x^2 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.397 $\int \frac{1}{\left(d+ex^2\right)\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=124

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}\,(d-e)}-\frac{e\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,\Pi\left(1-\frac{e}{d};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\,d\sqrt{x^4+3x^2+2}\,(d-e)}$$

```
[Out] 1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
((x^2+2)/(x^2+1))^(1/2)/(d-e)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-1/2*e*(x^2+1)^(3/
2)*(1/(x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-e/d,1/2*2^(1/2))*((x^2+2)
/(x^2+1))^(1/2)/d/(d-e)*2^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1214, 1099, 1456, 539}

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}\,(d-e)}-\frac{e\left(x^2+2\right)\Pi\left(1-\frac{e}{d};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\,d\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}\,(d-e)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*(d
 - e)*Sqrt[2 + 3*x^2 + x^4]) - (e*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x],
1/2])/(Sqrt[2]*d*(d - e)*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
```

```
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{2+3x^2+x^4}}\,dx = \frac{\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{d-e} - \frac{e\int \frac{2+2x^2}{\left(d+ex^2\right)\sqrt{2+3x^2+x^4}}\,dx}{2(d-e)}$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,(d-e)\sqrt{2+3x^2+x^4}} - \frac{\left(e\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}\left(d+ex^2\right)}\,dx}{2(d-e)\sqrt{2+3x^2+x^4}}$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,(d-e)\sqrt{2+3x^2+x^4}} - \frac{e\left(2+x^2\right)\Pi\left(1-\frac{e}{d};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,d(d-e)\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.11, size = 59, normalized size = 0.48

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\,\Pi\left(\frac{2e}{d};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{d\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[(2*e)/d, I*ArcSinh[x/Sqrt[2]],
 2])/(d*Sqrt[2 + 3*x^2 + x^4])
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{ex^6+(d+3\,e)x^4+(3\,d+2\,e)x^2+2\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(e*x^6 + (d + 3*e)*x^4 + (3*d + 2*e)*x^2 + 2
*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)), x)
```

maple [C] time = 0.02, size = 55, normalized size = 0.44

$$-\frac{i\sqrt{2}\,\sqrt{\frac{x^2}{2}+1}\,\sqrt{x^2+1}\ \mathrm{EllipticPi}\left(\frac{i\sqrt{2}\,x}{2},\frac{2e}{d},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -I/d*2^(1/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi
(1/2*I*2^(1/2)*x,2*e/d,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4 + 3\,x^2 + 2}\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(e\,x^2 + d\right)\,\sqrt{x^4 + 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(3*x^2 + x^4 + 2)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(x^2 + 1\right)\left(x^2 + 2\right)}\,\left(d + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)), x)
```

3.398 $\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{2+3x^2+x^4}} \, dx$

Optimal. Leaf size=316

$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}-\frac{ex\left(x^2+2\right)}{2d\sqrt{x^4+3x^2+2}\left(d^2-3de+2e^2\right)}-\frac{e\left(x^2+2\right)\left(3d^2-6de+2e^2\right)\Pi\left(1-\frac{e}{d};\tan^{-1}(x\right.}{2\sqrt{2}\,d^2\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}\,(d-2e)(d-e)}$$

```
[Out] -1/2*e*x*(x^2+2)/d/(d^2-3*d*e+2*e^2)/(x^4+3*x^2+2)^(1/2)-1/4*e*(3*d^2-6*d*e
+2*e^2)*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),
1-e/d,1/2*2^(1/2))/d^2/(d-2*e)/(d-e)^2*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4
+3*x^2+2)^(1/2)+1/2*e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(
1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)/d/(d-2*e)/(d-e)*2^(1/2)/(x^4+3*x^
2+2)^(1/2)+1/2*(2*d-e)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^
(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/d/(d-e)^2/(x^4+3*x^2+2)^(1/2)+
1/2*e^2*x*(x^4+3*x^2+2)^(1/2)/d/(d^2-3*d*e+2*e^2)/(e*x^2+d)
```

Rubi [A] time = 0.33, antiderivative size = 399, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1223, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}-\frac{ex\left(x^2+2\right)}{2d\sqrt{x^4+3x^2+2}\left(d^2-3de+2e^2\right)}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\left(3d^2-6de+2e^2\right)F\left(\tan^{-1}(x)\right.}{2\sqrt{2}\,d\sqrt{x^4+3x^2+2}\,(d-2e)(d-e)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] -(e*x*(2 + x^2))/(2*d*(d^2 - 3*d*e + 2*e^2)*Sqrt[2 + 3*x^2 + x^4]) + (e^2*x
*Sqrt[2 + 3*x^2 + x^4])/(2*d*(d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (e*(1 + x
^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*d*(d - 2*
e)*(d - e)*Sqrt[2 + 3*x^2 + x^4]) - ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*El
lipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*(d - 2*e)*(d - e)*Sqrt[2 + 3*x^2 + x^4]
) + ((3*d^2 - 6*d*e + 2*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[
ArcTan[x], 1/2])/(2*Sqrt[2]*d*(d - 2*e)*(d - e)^2*Sqrt[2 + 3*x^2 + x^4]) -
(e*(3*d^2 - 6*d*e + 2*e^2)*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x], 1/2])/(
2*Sqrt[2]*d^2*(d - 2*e)*(d - e)^2*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2
+ x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{2+3x^2+x^4}}\,dx = \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} - \frac{\int \frac{-2\left(d^2-3de+e^2\right)+2dex^2+e^2x^4}{\left(d+ex^2\right)\sqrt{2+3x^2+x^4}}\,dx}{2d(d-2e)(d-e)}$$

$$= \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} + \frac{\int \frac{-de^2-e^3x^2}{\sqrt{2+3x^2+x^4}}\,dx}{2d(d-2e)(d-e)e^2} + \frac{\left(3d^2-6de+2e^2\right)\int \frac{}{(d+}}{2d(d-2e)(d}$$

$$= \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} - \frac{\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{2(d-2e)(d-e)} + \frac{\left(3d^2-6de+2e^2\right)\int \frac{1}{\sqrt{2+3x^2}}}{2d(d-2e)(d-e)^2}$$

$$= -\frac{ex\left(2+x^2\right)}{2d\left(d^2-3de+2e^2\right)\sqrt{2+3x^2+x^4}} + \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} + \frac{e\left(1+\right.}{\sqrt{2}\,d(}$$

$$= -\frac{ex\left(2+x^2\right)}{2d\left(d^2-3de+2e^2\right)\sqrt{2+3x^2+x^4}} + \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} + \frac{e\left(1+\right.}{\sqrt{2}\,d(}$$

Mathematica [C] time = 0.60, size = 175, normalized size = 0.55

$$\frac{\frac{e^2x\left(x^4+3x^2+2\right)}{\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} + \frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(\left(-3d^2+6de-2e^2\right)\Pi\left(\frac{2e}{d};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+d(d-e)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+deE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{d(d-2e)(d-e)}}{2d\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((e^2*x*(2 + 3*x^2 + x^4))/((d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (I*Sqrt[1
+ x^2]*Sqrt[2 + x^2]*(d*e*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + d*(d - e)*El
lipticF[I*ArcSinh[x/Sqrt[2]], 2] + (-3*d^2 + 6*d*e - 2*e^2)*EllipticPi[(2*e
)/d, I*ArcSinh[x/Sqrt[2]], 2]))/(d*(d - 2*e)*(d - e)))/(2*d*Sqrt[2 + 3*x^2
+ x^4])
```

fricas [F] time = 1.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{e^2x^8+\left(2\,de+3\,e^2\right)x^6+\left(d^2+6\,de+2\,e^2\right)x^4+\left(3\,d^2+4\,de\right)x^2+2\,d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(e^2*x^8 + (2*d*e + 3*e^2)*x^6 + (d^2 + 6*d*
e + 2*e^2)*x^4 + (3*d^2 + 4*d*e)*x^2 + 2*d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)

maple [C] time = 0.03, size = 443, normalized size = 1.40

$$\frac{\sqrt{x^4+3x^2+2}\,e^2x}{2\left(d^2-3de+2e^2\right)\left(e\,x^2+d\right)d}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,e\,\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4\left(d^2-3de+2e^2\right)\sqrt{x^4+3x^2+2}\,d}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,e\,\mathrm{Elli}}{4\left(d^2-3de+2e^2\right)\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x)

[Out] 1/2*e^2*x*(x^4+3*x^2+2)^(1/2)/d/(d^2-3*d*e+2*e^2)/(e*x^2+d)+1/4*I/(d^2-3*d*e+2*e^2)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-1/4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-3/2*I/(d^2-3*d*e+2*e^2)*2^(1/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,2/d*e,2^(1/2))+3*I/(d^2-3*d*e+2*e^2)/d*e*2^(1/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,2/d*e,2^(1/2))-I/(d^2-3*d*e+2*e^2)/d^2*e^2*2^(1/2)*(1/2*x^2+1)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,2/d*e,2^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,x^2+d\right)^2\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e*x^2)^2*(3*x^2 + x^4 + 2)^(1/2)),x)

[Out] int(1/((d + e*x^2)^2*(3*x^2 + x^4 + 2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)

[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)**2), x)

3.399 $\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\left(c+ex^2\right)^q \left(a+bx^4+cx^2\right)^p,x\right)$$

```
[Out] Unintegrable((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]
[Out] Defer[Int][(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

Rubi steps

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \,dx=\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \,dx$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]
[Out] Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

fricas [A] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+cx^2+a\right)^p \left(ex^2+c\right)^q,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+cx^2+a\right)^p \left(ex^2+c\right)^q \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+c\right)^q \left(b\,x^4+c\,x^2+a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
[Out] int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + cx^2 + a\right)^p \left(ex^2 + c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(e\,x^2 + c\right)^q \left(b\,x^4 + c\,x^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + e*x^2)^q*(a + b*x^4 + c*x^2)^p,x)
[Out] int((c + e*x^2)^q*(a + b*x^4 + c*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**q*(b*x**4+c*x**2+a)**p,x)
[Out] Timed out
```

3.400 $\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx$

Optimal. Leaf size=498

$$\frac{ex^3\left(-3be\left(ae(4p+5)+c^2\left(8p^2+26p+21\right)\right)+3b^2c^2\left(16p^2+48p+35\right)+c^2e^2\left(4p^2+16p+15\right)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)}{3b^2(4p+5)(4p+7)}$$

```
[Out] -c*e^2*(e*(5+2*p)-3*b*(7+4*p))*x*(b*x^4+c*x^2+a)^(1+p)/b^2/(16*p^2+48*p+35)
+e^3*x^3*(b*x^4+c*x^2+a)^(1+p)/b/(7+4*p)+c*(a*e^3*(5+2*p)-3*a*b*e^2*(7+4*p)
+b^2*c^2*(16*p^2+48*p+35))*x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*
x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/b^2/(5+4*p)/(7+
4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c^2)^(1/
2)))^p)+1/3*e*(c^2*e^2*(4*p^2+16*p+15)+3*b^2*c^2*(16*p^2+48*p+35)-3*b*e*(a*
e*(5+4*p)+c^2*(8*p^2+26*p+21)))*x^3*(b*x^4+c*x^2+a)^p*AppellF1(3/2,-p,-p,5/
2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/b^2/(5+4
*p)/(7+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c
^2)^(1/2)))^p)
```

Rubi **[A]** time = 0.81, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1206, 1679, 1203, 1105, 429, 1141, 510}

$$\frac{cx\left(-3abe^2(4p+7)+ae^3(2p+5)+b^2c^2\left(16p^2+48p+35\right)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}}{b^2(4p+5)(4p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (c*e^2*(21*b - 5*e + 12*b*p - 2*e*p)*x*(a + c*x^2 + b*x^4)^(1 + p))/(b^2*(5
 + 4*p)*(7 + 4*p)) + (e^3*x^3*(a + c*x^2 + b*x^4)^(1 + p))/(b*(7 + 4*p)) +
(c*(a*e^3*(5 + 2*p) - 3*a*b*e^2*(7 + 4*p) + b^2*c^2*(35 + 48*p + 16*p^2))*x
*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a
*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(b^2*(5 + 4*p)*(7 + 4*p)*
(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b
+ c^2]))^p) + (e*(c^2*e^2*(15 + 16*p + 4*p^2) + 3*b^2*c^2*(35 + 48*p + 16*p
^2) - 3*b*e*(a*e*(5 + 4*p) + c^2*(21 + 26*p + 8*p^2)))*x^3*(a + c*x^2 + b*x
^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b
*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*b^2*(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (
2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1
+ (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx = \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\int \left(a+cx^2+bx^4\right)^p \left(bc^3(7+4p)-3e\left(ae^2-bc^2(7\right.\right.}{b(7+4}$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} +$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} +$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} +$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} +$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} +$$

Mathematica [A] time = 0.51, size = 373, normalized size = 0.75

$$\frac{1}{35}x\left(\frac{-\sqrt{c^2-4ab}+2bx^2+c}{c-\sqrt{c^2-4ab}}\right)^{-p}\left(\frac{\sqrt{c^2-4ab}+2bx^2+c}{\sqrt{c^2-4ab}+c}\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(ex^2\left(ex^2\left(5ex^2F_1\left(\frac{7}{2};-p,-p;\frac{9}{2};-\frac{}{c}\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(35*c^3*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c +
 Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*(35*c^2*
AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(
-c + Sqrt[-4*a*b + c^2])] + e*x^2*(21*c*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^
2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + 5*e*x^2
*AppellF1[7/2, -p, -p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/
(-c + Sqrt[-4*a*b + c^2])]))))/(35*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c -
 Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*
b + c^2]))^p)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^6+3\,ce^2x^4+3\,c^2ex^2+c^3\right)\left(bx^4+cx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + c*x^2 + a)^p,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+c\right)^3\left(bx^4+cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+c\right)^{3} \left(b\,x^{4}+c\,x^{2}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
[Out] int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^{2}+c\right)^{3}\left(bx^{4}+cx^{2}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x^{2}+c\right)^{3} \left(b\,x^{4}+c\,x^{2}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + e*x^2)^3*(a + b*x^4 + c*x^2)^p,x)
[Out] int((c + e*x^2)^3*(a + b*x^4 + c*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**3*(b*x**4+c*x**2+a)**p,x)
[Out] Timed out
```

3.401 $\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx$

Optimal. Leaf size=358

$$-\frac{x\left(ae^2-bc^2(4p+5)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4}}\right.}{b(4p+5)}$$

```
[Out] e^2*x*(b*x^4+c*x^2+a)^(1+p)/b/(5+4*p)-(a*e^2-b*c^2*(5+4*p))*x*(b*x^4+c*x^2+
a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4
*a*b+c^2)^(1/2)))/b/(5+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*
x^2/(c+(-4*a*b+c^2)^(1/2)))^p)+1/3*c*e*(8*b*p-2*e*p+10*b-3*e)*x^3*(b*x^4+c*
x^2+a)^p*AppellF1(3/2,-p,-p,5/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c
+(-4*a*b+c^2)^(1/2)))/b/(5+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+
2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)
```

Rubi [A] time = 0.36, antiderivative size = 345, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1206, 1203, 1105, 429, 1141, 510}

$$x\left(c^2-\frac{ae^2}{4bp+5b}\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4a}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (e^2*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c^2 - (a*e^2)/(5*b +
4*b*p))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c -
Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (
c*e*(2 - (e*(3 + 2*p))/(b*(5 + 4*p)))*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/
2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-
4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2
)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (
2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1
+ (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \left(c+ex^2\right)^2\left(a+cx^2+bx^4\right)^p\,dx=\frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)}+\frac{\int\left(-ae^2+bc^2(5+4p)+ce(10b-3e+8bp-2ep\right.}{b(5+4p)}$$

$$=\frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)}+\frac{\int\left(-ae^2\left(1-\frac{bc^2(5+4p)}{ae^2}\right)\left(a+cx^2+bx^4\right)^p-ce(-10\right.}{b(5+4}$$

$$=\frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)}+\left(ce\left(2-\frac{e(3+2p)}{b(5+4p)}\right)\right)\int x^2\left(a+cx^2+bx^4\right)^p\,dx-$$

$$=\frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)}+\left(ce\left(2-\frac{e(3+2p)}{b(5+4p)}\right)\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1-\right.\right.$$

$$=\frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)}+\left(c^2-\frac{ae^2}{5b+4bp}\right)x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\right.$$

Mathematica [A] time = 0.37, size = 303, normalized size = 0.85

$$\frac{1}{15}x\left(\frac{-\sqrt{c^2-4ab}+2bx^2+c}{c-\sqrt{c^2-4ab}}\right)^{-p}\left(\frac{\sqrt{c^2-4ab}+2bx^2+c}{\sqrt{c^2-4ab}+c}\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(ex^2\left(3ex^2F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{}{c+}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(15*c^2*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c +
 Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*(10*c*Ap
pellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c
 + Sqrt[-4*a*b + c^2])] + 3*e*x^2*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^2)/(c
+ Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])))/(15*((c - Sq
```

```
rt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b +
 c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^4 + 2\,cex^2 + c^2\right)\left(bx^4 + cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + c*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2 + c\right)^2\left(bx^4 + cx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(e\,x^2 + c\right)^2 \left(b\,x^4 + c\,x^2 + a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)
[Out] int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2 + c\right)^2\left(bx^4 + cx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x^2 + c\right)^2 \left(b\,x^4 + c\,x^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + e*x^2)^2*(a + b*x^4 + c*x^2)^p,x)
[Out] int((c + e*x^2)^2*(a + b*x^4 + c*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**2*(b*x**4+c*x**2+a)**p,x)
[Out] Timed out
```

3.402 $\int \left(c+ex^2\right) \left(a+cx^2+bx^4\right)^p \, dx$

Optimal. Leaf size=274

$$\frac{1}{3}ex^3\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{\cdots}{c+\sqrt{\cdots}}\right.$$

```
[Out] c*x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)
),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/(
(1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)+1/3*e*x^3*(b*x^4+c*x^2+a)^p*AppellF1(
3/2,-p,-p,5/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2
)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2
)))^p)
```

Rubi [A] time = 0.22, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1203, 1105, 429, 1141, 510}

$$\frac{1}{3}ex^3\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{\cdots}{c+\sqrt{\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (c*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[
-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c -
Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (e*x^3
*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a
*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sq
rt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (
2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1
+ (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
```

```
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\int \left(c+ex^2\right)\left(a+cx^2+bx^4\right)^p\,dx &= \int \left(c\left(a+cx^2+bx^4\right)^p+ex^2\left(a+cx^2+bx^4\right)^p\right)dx\\ &= c\int \left(a+cx^2+bx^4\right)^p\,dx+e\int x^2\left(a+cx^2+bx^4\right)^p\,dx\\ &= \left(c\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\right)\int \Bigg(1+\\ &= cx\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p F_1\Bigg(\frac{1}{2};-\end{aligned}$$

Mathematica [A] time = 0.25, size = 232, normalized size = 0.85

$$\frac{1}{3}x\left(\frac{-\sqrt{c^2-4ab}+2bx^2+c}{c-\sqrt{c^2-4ab}}\right)^{-p}\left(\frac{\sqrt{c^2-4ab}+2bx^2+c}{\sqrt{c^2-4ab}+c}\right)^{-p}\left(a+bx^4+cx^2\right)^p\Bigg(ex^2F_1\Bigg(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(3*c*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sq
rt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*AppellF1[3/
2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-
4*a*b + c^2])]))/(3*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b +
c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^2+c\right)\left(bx^4+cx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+c\right)\left(bx^4+cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+c\right) \left(b\,x^{4}+c\,x^{2}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
[Out] int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^{2}+c\right)\left(bx^{4}+cx^{2}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x^{2}+c\right)\,\left(b\,x^{4}+c\,x^{2}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + e*x^2)*(a + b*x^4 + c*x^2)^p,x)
[Out] int((c + e*x^2)*(a + b*x^4 + c*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)*(b*x**4+c*x**2+a)**p,x)
[Out] Timed out
```

3.403 $\int \left(a+cx^2+bx^4\right)^p \, dx$

Optimal. Leaf size=133

$$x\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\left(a+bx^4+cx^2\right)^pF_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4\cdots}}\right.$$

```
[Out] x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),
-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1
+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)
```

Rubi [A] time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1105, 429}

$$x\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\left(a+bx^4+cx^2\right)^pF_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4
*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c - Sq
rt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (
2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1
+ (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int\left(a+cx^2+bx^4\right)^p\,dx &= \left(\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\right)\int\left(1+\frac{2bx^2}{c-\sqrt{-4a\cdots}}\right.\\ &= x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^pF_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\cdots\right.\end{aligned}$$

Mathematica [A] time = 0.16, size = 161, normalized size = 1.21

$$x\left(\frac{-\sqrt{c^2-4ab}+2bx^2+c}{c-\sqrt{c^2-4ab}}\right)^{-p}\left(\frac{\sqrt{c^2-4ab}+2bx^2+c}{\sqrt{c^2-4ab}+c}\right)^{-p}\left(a+bx^4+cx^2\right)^pF_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\sqrt{c^2\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4
*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/(((c - Sqrt[-4*a*b + c^
2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^
2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4 + cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + cx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + c\,x^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((b*x^4+c*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + cx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^4 + c\,x^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4 + c*x^2)^p,x)
```

```
[Out] int((a + b*x^4 + c*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^4 + cx^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+c*x**2+a)**p,x)
```

```
[Out] Integral((a + b*x**4 + c*x**2)**p, x)
```

3.404 $\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{(a+bx^4+cx^2)^p}{c+ex^2},x\right)$$

[Out] Unintegrable((b*x^4+c*x^2+a)^p/(e*x^2+c),x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]

[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]

Rubi steps

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx = \int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$$

Mathematica [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]

[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(bx^4+cx^2+a)^p}{ex^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="fricas")

[Out] integral((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4+cx^2+a)^p}{ex^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="giac")

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+c\,x^{2}+a\right)^{p}}{e\,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
[Out] int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{4}+cx^{2}+a\right)^{p}}{ex^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b\,x^{4}+c\,x^{2}+a\right)^{p}}{e\,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4 + c*x^2)^p/(c + e*x^2),x)
[Out] int((a + b*x^4 + c*x^2)^p/(c + e*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+c*x**2+a)**p/(e*x**2+c),x)
[Out] Timed out
```

3.405 $\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{(a+bx^4+cx^2)^p}{(c+ex^2)^2},x\right)$$

```
[Out] Unintegrable((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]
[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx$$

Mathematica [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]
[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(bx^4+cx^2+a)^p}{e^2x^4+2\,cex^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="fricas")
[Out] integral((b*x^4 + c*x^2 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4+cx^2+a)^p}{(ex^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+c\,x^{2}+a\right)^{p}}{\left(e\,x^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

```
[Out] int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{4}+cx^{2}+a\right)^{p}}{\left(ex^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b\,x^{4}+c\,x^{2}+a\right)^{p}}{\left(e\,x^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4 + c*x^2)^p/(c + e*x^2)^2,x)
```

```
[Out] int((a + b*x^4 + c*x^2)^p/(c + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+c*x**2+a)**p/(e*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.406 $\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=446

$$\frac{(ef-dg)\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}}+\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}\,eg+\sqrt{c}\,df\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}\,\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\ldots}{\ldots}$$

```
[Out] 1/2*(-d*g+e*f)*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c
*d^4)^(1/2)-1/2*(-d*g+e*f)*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c
*x^4+a)^(1/2))/(a*e^4+c*d^4)^(1/2)-1/4*(-d*g+e*f)*(cos(2*arctan(c^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2)
,1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/e/(e^2*a^(1/2)+d^2*c^(1/2))
/(c*x^4+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(e*g*a^(1/2)+d*f*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*
c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1742, 12, 1248, 725, 206, 1709, 220, 1707}

$$\frac{(ef-dg)\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}}-\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\,\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}+\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\ldots\right)\ldots}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}\,\left(\sqrt{a}\,e^2+\sqrt{c}\ldots\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*S
qrt[-(c*d^4) - a*e^4]) - ((e*f - d*g)*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d
^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[c]*d*f + Sq
rt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt
[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f
 - d*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt
[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1742

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{(-ef+dg)x}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx + \int \frac{df-egx^2}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}\,de(ef-dg)\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx}{\sqrt{c}\,d^2+\sqrt{a}\,e^2} + (-ef+dg)\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx + \frac{\left(\sqrt{c}\right.}{}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} + \frac{\left(\sqrt{c}\,df+\sqrt{a}\,eg\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\right.}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+cx^4}}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} + \frac{\left(\sqrt{c}\,df+\sqrt{a}\,eg\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\right.}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+cx^4}}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\,\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}} + \frac{\left(\sqrt{c}\,df+\sqrt{a}\,eg\right)\left(\right.}{2}$$

Mathematica [C] time = 0.68, size = 258, normalized size = 0.58

$$\frac{\frac{(dg-ef)\left(\sqrt[4]{c}\,de\sqrt{a+cx^4}\,\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\,\sqrt{ae^4+cd^4}}\right)+2\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt{\frac{cx^4}{a}+1}\,\sqrt{ae^4+cd^4}\,\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)\right)}{\sqrt[4]{c}\,d\sqrt{ae^4+cd^4}} - \frac{2ig\sqrt{\frac{cx^4}{a}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{2e\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (((-2*I)*g*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]
]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + ((-(e*f) + d*g)*(c^(1/4)*d*e*Sqrt[a
+ c*x^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
 + 2*(-1)^(1/4)*a^(1/4)*Sqrt[c*d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a]*EllipticPi[
(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1])
)/(c^(1/4)*d*Sqrt[c*d^4 + a*e^4]))/(2*e*Sqrt[a + c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\sqrt{cx^4+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

maple [C] time = 0.02, size = 251, normalized size = 0.56

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,g\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,e}+\frac{\left(-dg+ef\right)\left(\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,e\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\ldots\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,d}\ldots\right)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] g/e/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c
^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I
)+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a
)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(-I/
a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2
)*EllipticPi((I/a^(1/2)*c^(1/2))^(1/2)*x,-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(
1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\sqrt{cx^4+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f+g\,x}{\sqrt{c\,x^4+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((a + c*x^4)^(1/2)*(d + e*x)),x)
```

```
[Out] int((f + g*x)/((a + c*x^4)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/(sqrt(a + c*x**4)*(d + e*x)), x)
```

3.407 $\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=218

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,(ef-dg)\Pi\left(\frac{\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,de\sqrt{cx^4-a}}+\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\,\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}+\frac{\sqrt[4]{a}\,g\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\cdots\right.\right.}{\sqrt[4]{c}\,e\sqrt{cx^4-a}}$$

```
[Out] 1/2*(-d*g+e*f)*arctanh((-c*d^2*x^2+a*e^2)/(-a*e^4+c*d^4)^(1/2)/(c*x^4-a)^(1
/2))/(-a*e^4+c*d^4)^(1/2)+a^(1/4)*g*EllipticF(c^(1/4)*x/a^(1/4),I)*(1-c*x^4
/a)^(1/2)/c^(1/4)/e/(c*x^4-a)^(1/2)+a^(1/4)*(-d*g+e*f)*EllipticPi(c^(1/4)*x
/a^(1/4),e^2*a^(1/2)/d^2/c^(1/2),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/d/e/(c*x^4-a)
^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1742, 12, 1248, 725, 206, 1711, 224, 221, 1219, 1218}

$$\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\,\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,(ef-dg)\Pi\left(\frac{\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,de\sqrt{cx^4-a}}+\frac{\sqrt[4]{a}\,g\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\cdots\right.\right.}{\sqrt[4]{c}\,e\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTanh[(a*e^2 - c*d^2*x^2)/(Sqrt[c*d^4 - a*e^4]*Sqrt[-a + c*x
^4])])/(2*Sqrt[c*d^4 - a*e^4]) + (a^(1/4)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[A
rcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*Sqrt[-a + c*x^4]) + (a^(1/4)*(e
*f - d*g)*Sqrt[1 - (c*x^4)/a]*EllipticPi[(Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e*Sqrt[-a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1711

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + c*x^4], x], x] + Dist[(e*A - d*B)/
e, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e, A, B},
 x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 1742

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \int \frac{(-ef+dg)x}{\left(d^2-e^2x^2\right)\sqrt{-a+cx^4}}\,dx + \int \frac{df-egx^2}{\left(d^2-e^2x^2\right)\sqrt{-a+cx^4}}\,dx$$

$$= \frac{g\int \frac{1}{\sqrt{-a+cx^4}}\,dx}{e} + \frac{(d(ef-dg))\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{-a+cx^4}}\,dx}{e} + (-ef+dg)\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{\ldots}}$$

$$= \frac{1}{2}(-ef+dg)\,\text{Subst}\left(\int \frac{1}{\left(d^2-e^2x\right)\sqrt{-a+cx^2}}\,dx, x, x^2\right) + \frac{\left(g\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\ldots}{e\sqrt{-a+cx^4}}$$

$$= \frac{\sqrt[4]{a}\,g\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,e\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}\,(ef-dg)\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\ldots\right.}{\sqrt[4]{c}\,de\sqrt{-a+cx^4}}$$

$$= \frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cd^4-ae^4}\,\sqrt{-a+cx^4}}\right)}{2\sqrt{cd^4-ae^4}} + \frac{\sqrt[4]{a}\,g\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,e\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}\,(\ldots}{}$$

Mathematica [C] time = 1.26, size = 719, normalized size = 3.30

$$\frac{if\sqrt{-\frac{(1-i)\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[4]{c}x+i\sqrt[4]{a}}}\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+i\sqrt[4]{c}x\right)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2}}\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2\left(\left(\sqrt[4]{a}e-\sqrt[4]{c}d\right)F\left(\sin^{-1}\left(\sqrt{\frac{(1+i)\left(\sqrt[4]{c}x+\sqrt[4]{a}\right)}{2\sqrt[4]{c}x+2i\sqrt[4]{a}}}\right)|2\right)-(1-i)\sqrt[4]{a}e\Pi\left(\frac{(1-i)\left(\sqrt[4]{c}d-i\sqrt[4]{a}e\right)}{\sqrt[4]{c}d-\sqrt[4]{a}e};\sin^{-1}\left(\sqrt{\cdots}\right)\right)\right)\cdots}{\sqrt[4]{a}\left(\sqrt[4]{a}e-\sqrt[4]{c}d\right)\left(\sqrt[4]{a}e+i\sqrt[4]{c}d\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

```
[Out] (((-I)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x
], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*e) + (I*f*(a^(1/4) - I*c^(1/4)*x)^2*Sqrt[
((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/4)*x)]*Sqrt[((1 + I)*(a^
(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1/4)*x)^2]*((-(
c^(1/4)*d) + a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x
))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] - (1 - I)*a^(1/4)*e*EllipticPi[((1 -
 I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin[Sqrt[((1 + I
)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]))/(a^(1/4)*(-(
c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e)) + (d*g*(a^(1/4) - I*c^(1
/4)*x)^2*Sqrt[((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/4)*x)]*Sqr
t[((1 + I)*(a^(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1
/4)*x)^2]*(I*(c^(1/4)*d - a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4
) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] + (1 + I)*a^(1/4)*e*Ell
ipticPi[((1 - I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin
[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]))
/(a^(1/4)*e*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e)))/Sqrt[-a
+ c*x^4]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\sqrt{cx^4-a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

maple [A] time = 0.02, size = 247, normalized size = 1.13

$$\frac{\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,g\,\mathrm{EllipticF}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}\,e}+\frac{(-dg+ef)\left(\frac{\sqrt{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{-\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}\,e\,\mathrm{EllipticPi}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x,-\frac{\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\cdots\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}\,d}\cdots\right)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x)
```

```
[Out] g/e/(-1/a^(1/2)*c^(1/2))^(1/2)*(1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(-1/a^(1/2)*
c^(1/2)*x^2+1)^(1/2)/(c*x^4-a)^(1/2)*EllipticF((-1/a^(1/2)*c^(1/2))^(1/2)*x
,I)+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4-a)^(1/2)*arctanh(1/2*(2*c*d^2/e^2*x^2-2
*a)/(c*d^4/e^4-a)^(1/2)/(c*x^4-a)^(1/2))+1/(-1/a^(1/2)*c^(1/2))^(1/2)/d*e*(
1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(-1/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4-a)^(
1/2)*EllipticPi((-1/a^(1/2)*c^(1/2))^(1/2)*x,-e^2*a^(1/2)/d^2/c^(1/2),(1/a^
(1/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx + f}{\sqrt{cx^4 - a}\,(ex + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f + g\,x}{\sqrt{c\,x^4 - a}\ \ (d + e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((c*x^4 - a)^(1/2)*(d + e*x)),x)
```

```
[Out] int((f + g*x)/((c*x^4 - a)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx}{\sqrt{-a + cx^4}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/(sqrt(-a + c*x**4)*(d + e*x)), x)
```

3.408 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}\,x^2-4}}\right)$$

```
[Out] 1/3*arctanh((1+x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/
2))*(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}\,x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\,dx = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{3\left(1-\sqrt{3}\right)^4+6\left(1-\sqrt{3}\right)^3\left(1+\sqrt{3}\right)+4x^2}\right.\right.$$

$$=\frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\right)$$

Mathematica [C] time = 3.07, size = 685, normalized size = 10.54

$$\frac{\left(x+\sqrt{3}-1\right)^2\sqrt{-x^3+\left(\sqrt{3}-1\right)x^2-2\left(2+\sqrt{3}\right)x+2\left(1+\sqrt{3}\right)}\sqrt{\frac{-\frac{4}{x+\sqrt{3}-1}+\sqrt{3}+1}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\left(2\sqrt{6}\sqrt{\frac{x^2+2\sqrt{3}+4}{\left(x+\sqrt{3}-1\right)^2}}\sqrt{\sqrt{2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] ((-1 + Sqrt[3] + x)^2*Sqrt[2*(1 + Sqrt[3]) - 2*(2 + Sqrt[3])*x + (-1 + Sqrt
[3])*x^2 - x^3]*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + x))/(3 + Sqrt[3] + I*
Sqrt[2*(2 + Sqrt[3])])]*((I*(-1 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]) + (2*(
(2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3])] + Sqrt[6*(2 + Sqrt[3])]))/(-1 + Sqrt[
3] + x))*Sqrt[Sqrt[2*(2 + Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x)
)]*EllipticF[ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + S
qrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])
/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])] + 2*Sqrt[6]*Sqrt[(4 + 2*Sqrt[3] +
 x^2)/(-1 + Sqrt[3] + x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8
/(-1 + Sqrt[3] + x))]*EllipticPi[(2*Sqrt[2*(2 + Sqrt[3])])/(Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[
3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(
2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])]))/((Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3]))*Sqrt[1 + Sqrt[3] - (2 + Sqrt[3])*x + ((-1 + Sqrt[
3])*x^2)/2 - x^3/2]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]*Sqrt[Sqrt[2*(2 + Sqrt[3]
)] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))])
```

fricas [B] time = 1.58, size = 323, normalized size = 4.97

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37x^{12}-204x^{11}+804x^{10}-2408x^9+3708x^8-5472x^7+6432x^6+10944x^5+14832}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3
708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x
^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*
x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7
 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 11
52*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(
7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696
*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11
 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320
*x^3 + 768*x^2 - 384*x + 64))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

maple [C] time = 0.17, size = 327, normalized size = 5.03

$$\frac{\sqrt{-\left(\frac{\sqrt{3}}{2}-1\right)x^2+1}\,\sqrt{-\left(1+\frac{\sqrt{3}}{2}\right)x^2+1}\ \mathrm{EllipticF}\left(\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)x, i\sqrt{1+4\sqrt{3}\,\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{x^4+4\sqrt{3}\,x^2-4}}-2\sqrt{3}\left(-\frac{\sqrt{-\left(\frac{\sqrt{3}}{2}-1\right)}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^
2)^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),I*(
1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*3^(1/
2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*x^2*3^
(1/2)+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/
2)/(-4+x^4+4*x^2*3^(1/2))^(1/2))-1/(1/2*3^(1/2)-1)^(1/2)/(-1-3^(1/2))*(1-(1
/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*x^2*3^(1/2
))^(1/2)*EllipticPi((1/2*3^(1/2)-1)^(1/2)*x,1/(1/2*3^(1/2)-1)/(-1-3^(1/2))^
2,(1+1/2*3^(1/2))^(1/2)/(1/2*3^(1/2)-1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^4+4\sqrt{3}\,x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),x
)
```

```
[Out] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
4)), x)
```

3.409 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}\,x^2-4}}\right)$$

```
[Out] -1/3*arctan((1+x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2
))*(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}\,x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3])]*S
qrt[-4 - 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx &= -\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+3\left(1+\sqrt{3}\right)^4+4x^2\right.\right.\\ &= -\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\right)\end{aligned}$$

Mathematica [C] time = 7.83, size = 876, normalized size = 13.90

$$-\frac{\sqrt{2}\sqrt{\frac{\sqrt{3}-1-\frac{4}{-x+\sqrt{3}+1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\left(-x+\sqrt{3}+1\right)^2\left(\left(\left(\frac{2\left(2i\sqrt{3}\sqrt{i\left(\sqrt{3}+1-\frac{8}{-x+\sqrt{3}+1}\right)+\sqrt{4-2\sqrt{3}}}+\sqrt{6}\sqrt{2\sqrt{4-2\sqrt{3}}-\sqrt{12-6\sqrt{3}}+i\sqrt{3}-i+\frac{8i(-2}{-x+}}\right.}{x-\sqrt{3}-1}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - x))/(-3 + Sqrt[3] - I*Sqrt
[4 - 2*Sqrt[3]])]*(1 + Sqrt[3] - x)^2*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 +
 Sqrt[3] - 8/(1 + Sqrt[3] - x))] + I*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(
1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] + Sqrt[-2*Sqrt[12 - 6*Sqrt[3]] + 4*Sqrt
[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3]
- x)] + (2*((2*I)*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3] - 8/(1
+ Sqrt[3] - x))] + Sqrt[6]*Sqrt[-I + I*Sqrt[3] - Sqrt[12 - 6*Sqrt[3]] + 2*S
qrt[4 - 2*Sqrt[3]] + ((8*I)*(-2 + Sqrt[3]))/(1 + Sqrt[3] - x)] + Sqrt[-2*Sq
rt[12 - 6*Sqrt[3]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 +
 Sqrt[3])*x))/(1 + Sqrt[3] - x)]))/(-1 - Sqrt[3] + x))*EllipticF[ArcSin[Sqr
t[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)*(2
- Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 +
Sqrt[3]))] + 2*Sqrt[6]*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + S
qrt[3] - x))]*Sqrt[(4 - 2*Sqrt[3] + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(2
*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3])), ArcSin[Sqrt
[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)*(2 -
 Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + S
qrt[3]))]))/((Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]))*Sqrt[Sqrt[4 - 2*Sqrt[
3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]
))
```

fricas [B] time = 1.27, size = 112, normalized size = 1.78

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9x^4-30x^3+18x^2-2\sqrt{3}\left(2x^4-10x^3+3x^2-10x+2\right)+24\right)\sqrt{x^4-4\sqrt{3}x^2-4}}{11x^6-42x^5+66x^4-176x^3-132x^2-168x-88}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^4
 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*sq
rt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

maple [C] time = 0.16, size = 311, normalized size = 4.94

$$\frac{\sqrt{-\left(-1-\frac{\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{\sqrt{3}}{2}+1\right)x^2+1}\,\mathrm{EllipticF}\left(\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)x, i\sqrt{1-4\sqrt{3}\,\left(-\frac{\sqrt{3}}{2}+1\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x^4-4\sqrt{3}\,x^2-4}}+2\sqrt{3}\left(-\frac{\sqrt{-\left(-1\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*
x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),I
*(1-4*3^(1/2)*(-1/2*3^(1/2)+1))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*3^(
1/2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*3^(1/
2)*x^2+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)
/(-4+x^4-4*3^(1/2)*x^2)^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-
1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^
2)^(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))/(3^(1/2)-1)
^2,(-1/2*3^(1/2)+1)^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}\,x^2-4}\,\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),x
)
```

```
[Out] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
4)), x)
```

3.410 $\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(2x-\sqrt{3}+1\right)^2}{2\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{4x^4+4\sqrt{3}x^2-1}}\right)$$

```
[Out] 1/3*arctanh(1/2*(1+2*x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-1+4*x^4+4*3^(1/2)*
x^2)^(1/2))*(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(2x-\sqrt{3}+1\right)^2}{2\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{4x^4+4\sqrt{3}x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^
4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3*(-3 + 2*Sqrt[
3])]*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)^4+12\left(1-\sqrt{3}\right)^3\left(1+\sqrt{3}\right)+\right.\right.$$

$$= \frac{1}{3}\sqrt{-3+2\sqrt{3}}\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+2x\right)^2}{2\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-1+4\sqrt{3}x^2+4x^4}}\right)$$

Mathematica [C] time = 1.73, size = 623, normalized size = 8.65

$$\frac{\left(2x+\sqrt{3}-1\right)^2\sqrt{\frac{-\frac{4}{2x+\sqrt{3}-1}+\sqrt{3}+1}{3+\sqrt{3}+i\sqrt{2(2+\sqrt{3})}}}\left(4\sqrt{3}\sqrt{\frac{2x^2+\sqrt{3}+2}{(2x+\sqrt{3}-1)^2}}\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(\frac{8}{2x+\sqrt{3}-1}-\sqrt{3}+1\right)}\Pi\left(\frac{2\sqrt{2(2+}}{\sqrt{2(2+\sqrt{3})}+}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2
+ 4*x^4]),x]
```

```
[Out] ((-1 + Sqrt[3] + 2*x)^2*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + 2*x))/(3 + Sq
rt[3] + I*Sqrt[2*(2 + Sqrt[3])])]*((I*(-1 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3]
)]) + (2*((2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3])] + Sqrt[6*(2 + Sqrt[3])]))/(
-1 + Sqrt[3] + 2*x))*Sqrt[Sqrt[2*(2 + Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 +
Sqrt[3] + 2*x))]*EllipticF[ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[
3] + 8/(-1 + Sqrt[3] + 2*x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2
*(2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])] + 4*Sqrt[3]*Sqrt[
(2 + Sqrt[3] + 2*x^2)/(-1 + Sqrt[3] + 2*x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[3])] -
I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))]*EllipticPi[(2*Sqrt[2*(2 + Sqrt[3]
)])/(Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[2*(2 + Sqrt
[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))]/(2^(3/4)*(2 + Sqrt[3])^(1
/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]
)]))/((Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3]))*Sqrt[-2 + 8*Sqrt[3]*x^2 + 8
*x^4]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))
])
```

fricas [B] time = 1.55, size = 328, normalized size = 4.56

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{2368\,x^{12}-6528\,x^{11}+12864\,x^{10}-19264\,x^{9}+14832\,x^{8}-10944\,x^{7}+6432\,x^{6}+5472\,x^{5}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(2368*x^12 - 6528*x^11 + 12864*x^10 - 19264*x
^9 + 14832*x^8 - 10944*x^7 + 6432*x^6 + 5472*x^5 + 3708*x^4 + 2408*x^3 + 80
4*x^2 + (1728*x^10 - 4800*x^9 + 8208*x^8 - 8928*x^7 + 6048*x^6 - 3024*x^5 -
 504*x^4 - 504*x^3 - 324*x^2 + 2*sqrt(3)*(496*x^10 - 1408*x^9 + 2304*x^8 -
2640*x^7 + 1848*x^6 - 504*x^5 + 336*x^4 + 204*x^3 + 63*x^2 + 26*x + 4) - 72
*x - 15)*sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(4
48*x^12 - 1280*x^11 + 2560*x^10 - 3200*x^9 + 3696*x^8 - 1920*x^7 - 960*x^5
- 924*x^4 - 400*x^3 - 160*x^2 - 40*x - 7) + 204*x + 37)/(64*x^12 + 384*x^11
 + 768*x^10 + 320*x^9 - 720*x^8 - 576*x^7 + 384*x^6 + 288*x^5 - 180*x^4 - 4
0*x^3 + 48*x^2 - 12*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^{4}+4\,\sqrt{3}x^{2}-1}\left(2\,x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(
3) + 1)), x)
```

maple [C] time = 0.16, size = 336, normalized size = 4.67

$$\frac{\sqrt{-\left(2\sqrt{3}-4\right)x^{2}+1}\,\sqrt{-\left(4+2\sqrt{3}\right)x^{2}+1}\,\mathrm{EllipticF}\left(\left(i\sqrt{3}-i\right)x, i\sqrt{1+\sqrt{3}\,\left(4+2\sqrt{3}\right)}\right)}{\left(i\sqrt{3}-i\right)\sqrt{4x^{4}+4\sqrt{3}\,x^{2}-1}}-2\sqrt{3}\left(-\frac{\sqrt{-\left(2\sqrt{3}-\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/(I*3^(1/2)-I)*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(4+2*3^(1/2))*x^2)^(1/2)/(-1
+4*x^4+4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(I*3^(1/2)-I),I*(1+3^(1/2)*(4+2*3^(
1/2)))^(1/2))-2*3^(1/2)*(-1/4/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2*3
^(1/2))^2-1)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1/2-1/2*3^(1/2))^2-2+4*3^(1/2)*
x^2+8*x^2*(-1/2-1/2*3^(1/2))^2)/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2
*3^(1/2))^2-1)^(1/2)/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2))-1/2/(2*3^(1/2)-4)^(1/2
)/(-1/2-1/2*3^(1/2))*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(4+2*3^(1/2))*x^2)^(1/2
)/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2)*EllipticPi((2*3^(1/2)-4)^(1/2)*x,1/(2*3^(1
/2)-4)/(-1/2-1/2*3^(1/2))^2,(4+2*3^(1/2))^(1/2)/(2*3^(1/2)-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^{4}+4\,\sqrt{3}\,x^{2}-1}\left(2\,x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(
3) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^{4}+4\,\sqrt{3}\,x^{2}-1}\,\left(2\,x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3^(1/2) + 1)/((4*3^(1/2)*x^2 + 4*x^4 - 1)^(1/2)*(2*x + 3^(1/2) +
 1)),x)
```

```
[Out] int((2*x - 3^(1/2) + 1)/((4*3^(1/2)*x^2 + 4*x^4 - 1)^(1/2)*(2*x + 3^(1/2) +
 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-\sqrt{3}+1}{\left(2x+1+\sqrt{3}\right)\sqrt{4x^{4}+4\sqrt{3}\,x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3**(1/2))/(1+2*x+3**(1/2))/(-1+4*x**4+4*3**(1/2)*x**2)**(1
/2),x)
```

```
[Out] Integral((2*x - sqrt(3) + 1)/((2*x + 1 + sqrt(3))*sqrt(4*x**4 + 4*sqrt(3)*x
**2 - 1)), x)
```

3.411 $\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(2x+\sqrt{3}+1\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{4x^4-4\sqrt{3}\,x^2-1}}\right)$$

```
[Out] -1/3*arctan(1/2*(1+2*x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-1+4*x^4-4*3^(1/2)*x
^2)^(1/2))*(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(2x+\sqrt{3}+1\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{4x^4-4\sqrt{3}\,x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^
4]),x]
```

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3*(3 + 2*Sqrt[3]
)]*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}\,dx = -\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{12\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+6\left(1+\sqrt{3}\right)^4+\right.\right.$$

$$= -\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}+2x\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}\right)$$

Mathematica [C] time = 6.39, size = 881, normalized size = 12.59

$$-\frac{\sqrt{\frac{\sqrt{3}-1-\frac{4}{-2x+\sqrt{3}+1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\left(-2x+\sqrt{3}+1\right)^2\left(\left(\left(-\frac{2i\left(2\sqrt{3}\sqrt{i\left(\sqrt{3}+1-\frac{8}{-2x+\sqrt{3}+1}\right)+\sqrt{4-2\sqrt{3}}}-i\sqrt{6}\sqrt{-\frac{2i\left(\left(-1+\sqrt{3}\right)x-4\sqrt{3}+7\right)}{-2x+\sqrt{3}+1}+2\sqrt{4-2\sqrt{3}}-\cdots}\right.}{-2x+\sqrt{3}+1}\cdots\right.\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2
+ 4*x^4]),x]
```

```
[Out] -((Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - 2*x))/(-3 + Sqrt[3] - I*Sqrt[4 - 2
*Sqrt[3]])]*(1 + Sqrt[3] - 2*x)^2*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqr
t[3] - 8/(1 + Sqrt[3] - 2*x))] + I*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1
+ Sqrt[3] - 8/(1 + Sqrt[3] - 2*x))] + Sqrt[-2*Sqrt[12 - 6*Sqrt[3]] + 4*Sqrt
[4 - 2*Sqrt[3]] - ((4*I)*(7 - 4*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3] -
 2*x)] - ((2*I)*(2*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3] - 8/(1
 + Sqrt[3] - 2*x))] - I*Sqrt[6]*Sqrt[-Sqrt[12 - 6*Sqrt[3]] + 2*Sqrt[4 - 2*S
qrt[3]] - ((2*I)*(7 - 4*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3] - 2*x)] -
 I*Sqrt[-2*Sqrt[12 - 6*Sqrt[3]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((4*I)*(7 - 4*Sqr
t[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3] - 2*x)]))/(1 + Sqrt[3] - 2*x))*Ellip
ticF[ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*
x))]/(2^(3/4)*(2 - Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sq
rt[3]] + I*(-3 + Sqrt[3]))] + 4*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] -
 8/(1 + Sqrt[3] - 2*x))]*Sqrt[(6 - 3*Sqrt[3] + 6*x^2)/(1 + Sqrt[3] - 2*x)^2
]*EllipticPi[(2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]
)), ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*x
))]/(2^(3/4)*(2 - Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqr
t[3]] + I*(-3 + Sqrt[3]))]))/(Sqrt[2]*(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3
]))*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*x))]*Sqr
t[-1 - 4*Sqrt[3]*x^2 + 4*x^4]))
```

fricas [B] time = 1.41, size = 114, normalized size = 1.63

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(36x^4-60x^3+18x^2-\sqrt{3}\left(16x^4-40x^3+6x^2-10x+1\right)+6\right)\sqrt{4x^4-4\sqrt{3}x^2-1}\cdots}{88x^6-168x^5+132x^4-176x^3-66x^2-42x-11}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(36*x^4 - 60*x^3 + 18*x^2 - sqrt(3)*(16*x^4
 - 40*x^3 + 6*x^2 - 10*x + 1) + 6)*sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*sqrt(2*s
qrt(3) + 3)/(88*x^6 - 168*x^5 + 132*x^4 - 176*x^3 - 66*x^2 - 42*x - 11))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+\sqrt{3}+1}{\sqrt{4x^4-4\sqrt{3}x^2-1}\left(2x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(
3) + 1)), x)
```

maple [C] time = 0.15, size = 337, normalized size = 4.81

$$\frac{\sqrt{-\left(-4-2\sqrt{3}\right)x^2+1}\,\sqrt{-\left(-2\sqrt{3}+4\right)x^2+1}\ \mathrm{EllipticF}\left(\left(i+i\sqrt{3}\right)x, i\sqrt{1-\sqrt{3}\,\left(-2\sqrt{3}+4\right)}\right)}{\left(i+i\sqrt{3}\right)\sqrt{4x^4-4\sqrt{3}\,x^2-1}}+2\sqrt{3}\left(-\frac{\sqrt{-(-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/(I+I*3^(1/2))*(1-(-4-2*3^(1/2))*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(1/2)/(
-1+4*x^4-4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(I+I*3^(1/2)),I*(1-3^(1/2)*(-2*3^
(1/2)+4))^(1/2))+2*3^(1/2)*(-1/4/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1
/2)-1/2)^2-1)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(1/2*3^(1/2)-1/2)^2-2-4*3^(1/2)
*x^2+8*x^2*(1/2*3^(1/2)-1/2)^2)/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1/
2)-1/2)^2-1)^(1/2)/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2))-1/2/(-4-2*3^(1/2))^(1/2)
/(1/2*3^(1/2)-1/2)*(1-(-4-2*3^(1/2))*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(1/2
)/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2)*EllipticPi((-4-2*3^(1/2))^(1/2)*x,1/(-4-2*
3^(1/2))/(1/2*3^(1/2)-1/2)^2,(-2*3^(1/2)+4)^(1/2)/(-4-2*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}\,x^2-1}\left(2\,x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(
3) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}\,x^2-1}\,\left(2\,x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3^(1/2) + 1)/((4*x^4 - 4*3^(1/2)*x^2 - 1)^(1/2)*(2*x - 3^(1/2) +
 1)),x)
```

```
[Out] int((2*x + 3^(1/2) + 1)/((4*x^4 - 4*3^(1/2)*x^2 - 1)^(1/2)*(2*x - 3^(1/2) +
 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1+\sqrt{3}}{\left(2x-\sqrt{3}+1\right)\sqrt{4x^4-4\sqrt{3}\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3**(1/2))/(1+2*x-3**(1/2))/(-1+4*x**4-4*3**(1/2)*x**2)**(1
/2),x)
```

```
[Out] Integral((2*x + 1 + sqrt(3))/((2*x - sqrt(3) + 1)*sqrt(4*x**4 - 4*sqrt(3)*x
**2 - 1)), x)
```

3.412 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=560

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}\,eg+\sqrt{c}\,df\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\cdots\right.}{4\sqrt[4]{a}\cdots}$$

```
[Out] 1/2*(-d*g+e*f)*arctan(x*(-a*e^4-b*d^2*e^2-c*d^4)^(1/2)/d/e/(c*x^4+b*x^2+a)^
(1/2))/(-a*e^4-b*d^2*e^2-c*d^4)^(1/2)-1/2*(-d*g+e*f)*arctanh(1/2*(b*d^2+2*a
*e^2+(b*e^2+2*c*d^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2+a)^(1/
2))/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)-1/4*(-d*g+e*f)*(cos(2*arctan(c^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),
1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^
(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/e/
(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x
/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan
(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(e*g*a^(1/2)+d*f*c^(1
/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
a^(1/4)/c^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1741, 12, 1247, 724, 206, 1708, 1103, 1706}

$$\frac{(ef-dg)\tan^{-1}\left(\frac{x\sqrt{-ae^4-bd^2e^2-cd^4}}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-e^2\left(ae^2+bd^2\right)-cd^4}}-\frac{(ef-dg)\tanh^{-1}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}+\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(\cdots\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - b*d^2*e^2 - a*e^4]*x)/(d*e*Sqrt[a + b*
x^2 + c*x^4])])/(2*Sqrt[-(c*d^4) - e^2*(b*d^2 + a*e^2)]) - ((e*f - d*g)*Arc
Tanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d^4 + b*d^2*e^2 +
a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((S
qrt[c]*d*f + Sqrt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[
a + b*x^2 + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f - d*g)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[
(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c
]*d^2 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1741

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(-ef+dg)x}{\left(d^2-e^2x^2\right)\sqrt{a+bx^2+cx^4}}\,dx + \int \frac{df-egx^2}{\left(d^2-e^2x^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}\,de(ef-dg)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d^2-e^2x^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}\,d^2+\sqrt{a}\,e^2} + (-ef+dg)\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{a+bx^2}}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2\left(bd^2+ae^2\right)}} + \frac{\left(\sqrt{c}\,df+\sqrt{a}\,eg\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right.}}}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2\left(bd^2+ae^2\right)}} + \frac{\left(\sqrt{c}\,df+\sqrt{a}\,eg\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right.}}}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2\left(bd^2+ae^2\right)}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+bd^2e^2+ae^4}\sqrt{a+bx^2+cx^4}}\right.}{2\sqrt{cd^4+bd^2e^2+ae^4}}$$

Mathematica [C] time = 7.87, size = 3652, normalized size = 6.52

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2
- 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(Sqrt[2
]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*e*Sqrt[a + b*x^2 + c*x^4]) + (2*(Sqrt
[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]
/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)^2*Sqrt[(Sqr
t[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]
) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b
^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)
)]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c)
 + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 -
 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b -
 Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - S
qrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]
+ 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/
c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt[b
^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^2
- 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2] - Sqrt[2]*Sqrt[(-b - Sq
rt[b^2 - 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[
2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d + (Sqrt[-(b/c) - Sqrt[b
^2 - 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
 + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2
 - 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] -
```

```
 Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c
] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c]
)/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt
[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^
2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2]))/(Sqrt[(-b - Sqrt[b^
2 - 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) +
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/
Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*Sqrt[a + b*x^
2 + c*x^4]) - (2*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c)
+ Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*d*g*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/S
qrt[2]) + x)^2*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt
[b^2 - 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 -
 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b
/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - S
qrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]
 - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])
/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*
c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b
/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - S
qrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b
- Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(
-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x
))]], (Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])
^2/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2]
 - Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sq
rt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d
 + (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[
b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d -
 (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b -
 Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt
[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2
*x))]], (Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c
])^2/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^
2]))/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/
Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*e*(-d - (Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]
*e)/Sqrt[2])*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx + f}{\sqrt{cx^4 + bx^2 + a}\,(ex + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

maple [A] time = 0.02, size = 437, normalized size = 0.78

$$\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,g\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,e}+\frac{(-dg+ef)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*g/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(
-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+b*d^2/e^
2+a)^(1/2)*arctanh(1/2*(2*c*d^2/e^2*x^2+b*x^2+b*d^2/e^2+2*a)/(c*d^4/e^4+b*d
^2/e^2+a)^(1/2)/(c*x^4+b*x^2+a)^(1/2))+2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)/d*e*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1+1/2*(b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*x,2/(-b+(-4*a*c+b^2)^(1/2))*a/d^2*e^2,(-1/2*(b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\sqrt{cx^4+bx^2+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f+g\,x}{(d+e\,x)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((f + g*x)/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{(d+ex)\,\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.413 $\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=527

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,(ef-dg)\Pi\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\,de\sqrt{-a+bx^2+cx^4}}$$

```
[Out] -1/2*(-d*g+e*f)*arctanh(1/2*(b*d^2-2*a*e^2+(b*e^2+2*c*d^2)*x^2)/(-a*e^4+b*d
^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2-a)^(1/2))/(-a*e^4+b*d^2*e^2+c*d^4)^(1/2)+1
/2*g*(1/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^2)^
(1/2)))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2)/(1+2*
c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/(b-(4*a*c+b^2)^(1/
2)))^(1/2))*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b+(4*a*c+b^2)^(1/2))^(1/2)/e
*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))/(
1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)+1/2*(-d*g+e*f)*EllipticPi(x*2^(1/2)
*c^(1/2)/(-b+(4*a*c+b^2)^(1/2))^(1/2),-1/2*e^2*(b-(4*a*c+b^2)^(1/2))/c/d^2,
((b-(4*a*c+b^2)^(1/2))/(b+(4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(4*a*c+b
^2)^(1/2)))^(1/2)*(-b+(4*a*c+b^2)^(1/2))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^2)^(1
/2)))^(1/2)/d/e*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {1741, 12, 1247, 724, 206, 1710, 1104, 418, 1220, 537}

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,(ef-dg)\Pi\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\,de\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -((e*f - d*g)*ArcTanh[(b*d^2 - 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d
^4 + b*d^2*e^2 - a*e^4]*Sqrt[-a + b*x^2 + c*x^4])])/(2*Sqrt[c*d^4 + b*d^2*e
^2 - a*e^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*g*(1 + (2*c*x^2)/(b - Sqrt[b^2
+ 4*a*c]))*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]
], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*e*Sqrt
[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a
*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*(e*f - d*g
)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*EllipticPi[-((b - Sqrt[b^2 + 4*a*c])*e^2)/(2*c*d^2), ArcSin[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (b - Sqrt[b^2 + 4*a*c])/
(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*e*Sqrt[-a + b*x^2 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1104

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b +
q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (
2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1710

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 1741

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
```

```
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{(-ef+dg)x}{\left(d^2-e^2x^2\right)\sqrt{-a+bx^2+cx^4}}\,dx + \int \frac{df-egx^2}{\left(d^2-e^2x^2\right)\sqrt{-a+bx^2+cx^4}}\,dx$$

$$= \frac{g\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx}{e} + \frac{(d(ef-dg))\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{-a+bx^2+cx^4}}\,dx}{e} + (-ef+dg)\int$$

$$= \frac{1}{2}(-ef+dg)\,\mathrm{Subst}\left(\int \frac{1}{\left(d^2-e^2x\right)\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right) + \frac{\left(g\sqrt{1+\frac{2}{b-\sqrt{}}}\right.}{}$$

$$= \frac{\sqrt{b+\sqrt{b^2+4ac}}\,g\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\middle|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}} +$$

$$= -\frac{(ef-dg)\tanh^{-1}\left(\frac{bd^2-2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+bd^2e^2-ae^4}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2-ae^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\,g\left(1+\right.}{\sqrt{2}\sqrt{c}\,e}$$

Mathematica [C] time = 7.87, size = 3658, normalized size = 6.94

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b
 + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2
+ 4*a*c]))]*x], (-b - Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2
]*Sqrt[-(c/(-b - Sqrt[b^2 + 4*a*c]))]*e*Sqrt[-a + b*x^2 + c*x^4]) + (2*(Sqr
t[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c
]/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x)^2*Sqrt[(Sq
rt[(-b - Sqrt[b^2 + 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2
]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[
b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x
))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]
/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c
) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqr
t[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2
+ 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b
- Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(
-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]
/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] -
Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]
```

```
 + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])
/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt[
b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^2
 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2] - Sqrt[2]*Sqrt[(-b - S
qrt[b^2 + 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt
[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d + (Sqrt[-(b/c) - Sqrt[
b^2 + 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]
) + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^
2 + 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]
 - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/
c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c
])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqr
t[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b
^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2]))/(Sqrt[(-b - Sqrt[b
^2 + 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) +
 Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)
/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*Sqrt[-a + b*
x^2 + c*x^4]) - (2*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c
) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*d*g*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]
/Sqrt[2]) + x)^2*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(-(Sqrt[-(b/c) + Sq
rt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt
[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2
 + 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-
(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c
]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) -
 Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/
c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c
])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*
a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-
(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b -
 Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt
[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2
*x))]], (Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c
])^2/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^
2] - Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) -
Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*
(d + (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqr
t[b^2 + 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d
 - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b
 - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[
(-b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sq
rt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] -
 2*x))]], (Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])
/c])^2/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c]
)^2]))/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c
]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*e*(-d - (Sqrt[-(b/c
) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/
c]*e)/Sqrt[2])*Sqrt[-a + b*x^2 + c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")

[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\sqrt{cx^4+bx^2-a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```

maple [A] time = 0.02, size = 439, normalized size = 0.83

$$\frac{\sqrt{\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,g\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,x}{2},\frac{\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)b}{ac}-4}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+b\,x^2-a}\;e}+\frac{\left(-dg+ef\right)\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x)
[Out] 1/2*g/e/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(2*(-b+(4*a*c+b^2)^(1/2))/a*x^2
+4)^(1/2)*(-2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*El
lipticF(1/2*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1
/2))/a*b/c-4)^(1/2))+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+b*d^2/e^2-a)^(1/2)*arc
tanh(1/2*(2*c*d^2/e^2*x^2+b*x^2+b*d^2/e^2-2*a)/(c*d^4/e^4+b*d^2/e^2-a)^(1/2
)/(c*x^4+b*x^2-a)^(1/2))+1/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)/d*e*(1+1/2
*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1-1/2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*
x,-2/(-b+(4*a*c+b^2)^(1/2))*a/d^2*e^2,1/2*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)
^(1/2)/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\sqrt{cx^4+bx^2-a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f+g\,x}{\left(d+e\,x\right)\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((d + e*x)*(b*x^2 - a + c*x^4)^(1/2)),x)
[Out] int((f + g*x)/((d + e*x)*(b*x^2 - a + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*sqrt(-a + b*x**2 + c*x**4)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*     is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*     antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```